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Further Evidence on the Uncertain (Fractional) Unit Root of Real GNP

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Abstract

In an interesting paper Diebold and Senhadji (1996) showed that U.S. GNP data was not as uniformative as many believed as to whether trend was better described as deterministic (trend-stationarity) or stochastic (unit root). By using long data spans and new econometric techniques, they showed that the unit root hypothesis could be rejected with high power. Using the same data set we first show that, if the hypotheses are reversed, also the trend stationary model can be easily rejected. This suggests that neither model provides a good characterization of this data. Long memory (ARFIMA) as well as non-linear models are considered as alternatives. Economic as well as statistical justification for the presence of these features in the data is provided. It turns out that the latter models and in general prefered to the former. Finally, a new technique is also applied to discriminate between these the long memory and the structural break models. It is shown that for both real GNP and real GNP per capita the prefered model turns out to be a fractionally integrated model with a memory parameter, d, around 0.7. This implies that these series are non-stationary, highly persistent but with no permanent shocks. Some macroeconomic implications of these findings are also discussed.

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KEY WORDS: GNP, long memory, unit roots, fractional integration, structural change.

INTRODUCTION

Questions about the persistence of shocks in macroeconomic series and, in particular, in GNP have occupied a very important place in economics and have given rise to a vast literature on the subject. In spite of this fact, important issues remain unclear. Until the early 80's, economists were in broad agreement that business fluctuations could be studied separately from the secular growth of the economy. This was justified on the basis that factors underlying trend growth were assumed to be stable at business-cycle frequencies. After the influential work of Nelson and Plosser (1982) autoregressive unit roots became a popular feature of macroeconometric models, partly because of stationarity could be induced by either differencing or forming cointegrating linear combinations of economic variables. Under this characterization, any stochastic shock has a permanent shift in the level of the series. Then, it is not possible to separate between trend and business cycles as it had been done under the trend-stationary parametrization.

However, many important series do not seem to fall, logically or empirically, into either of the $I(0)$ or $I(1)$ categories. Unlike in finance, where the hypothesis of efficient markets is assumed to hold, there are not theoretical underpinnings for an exact unit root in macroeconomic series. In an important paper, Rudebush (1993) argued that the widespread acceptance of the difference-stationary (D-ST henceforth) model for aggregate output was due to the fact that unit root tests could not reject a unit root in real GNP. But he argued that the available tests lack power to distinguish between trend and difference stationary models not only when both hypothesis are close but also when they are distant[1]. Using quarterly real GNP per capita, he showed that the best-fitting trend stationary and difference-stationary models implied a very different medium and long term dynamics. Then, using bootstrap, he computed the small

[1] See Phillips and Xiao (1998) for a discussion on the finite sample properties of unit root tests.

sample distributions of the Dickey-Fuller (henceforth DF) test corresponding to both best-fitting models and he showed that were very similar. This finding contributed to the "we don't know" literature initiated by Christiano and Eichenbaum (1990) since it implied that the DF test had low power to distinguish between both hypotheses.

Diebold and Senhadji (1996) challenged Rudebush (1993) conclusions by considering longer spans of annual US data. They replicated the latter analysis using annual data ranging from 1869 to 1993 and they showed that the unit root hypothesis was clearly rejected in favor of the trend stationary (T-ST) one and that the DF test had power in this case to distinguish between these two models.

As it is well-known, rejecting a hypothesis in favor of an alternative one does not imply that the latter is a good description of the data. This paper starts by noticing that, if the T-ST model is tested against the unit root model and using the same procedure as in the papers above, evidence against the former hypothesis also arises (see Section 3). This implies that neither the $I(1)$ paradigm nor the $I(0)$ one are good models for these data. This is not the only motivation to go beyond the $I(1)-I(0)$ paradigm. In this framework, whenever a series displays a high degree of persistence is classified as $I(1)$. But from an economic point of view, several inconsistencies resulting from the unit root (or the fully persistent) model arise. Just to mention some examples, it has been found conflicting evidence on persistence measures from unrestricted ARIMA and Unobserved Component models that are difficult to reconcile under the I(1) formulation (see Campbell and Mankiw (1987) and Watson (1986)). As a second example, Michelacci and Zaffaroni (2000) pointed out the theoretical inconsistency between the existence of a unit root in output and its convergence to a steady state (beta convergence). A further example is the so-called Deaton´s paradox. With a fully persistent specification of income, the Permanent Income Hypothesis (PIH) under rational expectations implies that innovations to consumption should have larger variance than innovations to income (see Deaton (1987), Campbell and Deaton, (1989) and Diebold and Rudebush, (1991a)). Nevertheless, an excessive smoothness in consumption, compared to the implications of the PHI, has been observed. These and other inconsistencies have (partially or totally) been solved by introducing models that depart from the unit root hypothesis and these new alternatives will the focus of this study.

From a purely statistical point of view, it is also crucial to have an accurately knowledge of the persistence of the series. For instance, Elliot (1995) emphasizes the non-robustness of cointegration methods to deviations of variables from difference-stationary (D-ST). He shows how even very small deviations from DS can invalidate the inferential procedures associated with conventional tests.

The objective this paper is to shed some further light on the controversy about the existence of a unit root in real GNP by using more recent and powerful techniques and, more generally, to identify the approach that captures better the properties of these series. To this end, long memory processes and models displaying breaking trends will also be considered in the analysis. In contrast to unit root models, long memory models allow for a more flexible characterization of the persistence of shocks permitting a wider range of behaviors (such as hyperbolic decay of correlations, mean-reverting non-stationary series, etc.). In turn, models showing breaking trends assume that just a few shocks have a permanent effect while the others have a short and transitory one.

To accomplish this objective, both the DS and the T-ST hypotheses have been tested against the above-mentioned models. It turns out that the latter are generally prefered to the former and the results are robust across different econometric procedures. Since it is well known that some forms of breaking trend models can produce spurious long memory and vice versa, a crucial step would be to disentagle between these two approches. This has been done by applying a recent technique introduced by Bai and Mayoral (2003). It turns out that the preferred model is a FI process with a memory parameter around 0.7, which implies a highly persistent, non-stationary but mean reverting behavior. This result is also very robust across all the series employed in the analyses.

The outline of the paper is as follows. Section 2 introduces the data and considers some classical unit roots tests of $I(1)$ vs. $I(0)$ and vice versa. Section 3 describes the main characteristics of fractionally integrated models and consider several procedures for testing the fractional integration hypothesis against both the $I(1)$ and the $I(0)$ ones. Section 4 analyzes the existence of breaks in the data. Section 5, in turn, presents a procedure for testing whether the series is better characterized by a FI or by a breaking trend model. Finally, Section 6 put forward some concluding remarks.

THE DATA AND PRELIMINARY TESTS

We consider the same data set employed in Diebold and Senhaji (1996) (DS henceforth): the annual real GNP series reported in table 1.10 of the National Income and Product Accounts of the United States, measured in billions of 1987 dollars, ranging from 1929 to 2001 (8 new observations have been added with respect to DS analysis). As in DS, these series has been spliced to the 1869-1929 real GNP series of Balke and Gordon (1989) or Romer (1989) given rise to two different series of Real GNP, each containing 133 annual observations. Per capita GNP has also been considered and in order to construct the series, total population residing in the United States (in thousands of people) has been taken from table A-7 of *Historical Statistics of the United States* for years ranging from 1869 to 1970 and for the years 1971-2001, data has been taken from the *Census Bureau's Current Population Reports*, Series P-25. All series are in natural logs.

We keep the same notation as in DS and we define:

GNP-BG ("GNP-Balke-Gordon"). Gross national product. Pre-1929 values from Balke-Gordon.

GNP-R ("GNP-Romer"). Gross national product. Pre-1929 values from Romer.

GNP-BGPC ("GNP-Balke-Gordon, per capita"). Gross national product per capita. Pre-1929 values from Balke-Gordon.

GNP-RPC ("GNP-Romer, per capita"). Gross national product per capita. Pre-1929 values from Romer.

The post-1929 series values of both the GNP-R and GNP-BG are identical but pre-1929 values differ slightly due to the differing assumptions underlying their construction. The same is true for the per capita series GNP-RPC and GNP-BGPC.

Some preliminary unit root tests

DS (1996) provided robust and conclusive evidence against the hypothesis of a unit root versus trend stationarity in the series defined above. They followed Rudebush (1993) approach and computed the best fitting T-ST and D-ST models for each of the four series. They first showed that the former models implied very different dynamics (therefore they were quite distant alternatives). Then, for each series they computed the exact finite sample distribution of the corresponding t-statistics from an augmented Dickey-Fuller (ADF) regression under both best-fitting models. They showed that the p-value associated to the former statistic was very small under the D-ST model but quite large under the trend stationary one. In other words, since the sample value of the ADF test was very unlikely under the difference-stationary formulation, it was possible to reject the null hypothesis of a unit root with reasonable power. But, as noticed by the authors, rejecting the null hypothesis does not mean that the alternative is a good characterization for this data set. Table I and Figure I show that, when the hypotheses are reverse, the T-ST vs. the D-ST hypothesis is also rejected. Table I also presents the results of other classical unit root tests. The first three columns have the unit root as null hypothesis. In addition to the Augmented Dickey-Fuller test employed by DS, also the Phillips-Perron (P-P) (1988) and the efficient DF-GLS method proposed by Elliot, Rothemberg and Stock (1996) are considered. Not surprisingly, the former hypothesis is rejected for the four extended series employed in this article, confirming DS findings. The last column reports the result of the KPSS test for the null hypothesis of trend stationarity versus a unit root.

Interestingly, the I(0) hypothesis is also rejected in all the four series.[2]

TABLE I

UNIT ROOT TESTS

	H_0: Unit Root			H_0: T-ST
Data | Test	*ADF*	P-P	DF-GLS	KPSS
GNP-R	-4.70**	-3.53*	-3.34*	0.190*
GNP-RPC	-4.63**	-4.27*	-4.61**	0.168*
GNP-BG	-4.16**	-3.65*	-3.34*	0.181*
GNP-BGPC	-4.79**	-3.65*	-4.76**	0.182*

*Rejection at the 5% level;**Rejection at the 1% level;

Figure 1 replicates DS' Figure 2 and contains the exact finite-sample distribution of the KPSS statistic under the best-fitting trend-stationary and difference-stationary models for real GNP per capita (Pre-1929 values are from Romer, (1989))[3]. The p-values corresponding to the value of the statistic in the sample (0.168) are 19.08% and 96% for the trend stationary and the difference stationary hypothesis, respectively. This plot confirms DS conclusions in the sense that the value of the KPSS statistic obtained with this data is very unlikely under the best-fitting unit root models but is also shows that this value is also unlikely under the trend stationary hypothesis.

A plausible explanation of the rejection of both hypotheses stems from noticing that the tests above still have power against other alternative processes. For instance, DF tests have some (although little) power against fractionally integrated processes (see Diebold and Rudebush, 1991b) and also against some types of breaks in the data (cf. Perron, 1989). Similar properties have been found to hold for the KPSS test (see Lee and Schmidt (1996)). In other words, these results may be suggesting that the data display some kind of 'intermediate' behavior. As we have seen, a unit root implies

[2] For the DF and the DF- GLS tests, the number of lags was chosen according to the BIC criterion and in all cases a number equal to 2 was set. For the remaining tests, also two lags were considered to compute the Newey-West variance-covariance estimator. Different values were tried and the results remained qualitatively identical.

[3] The best-fitting T-ST and D-ST models are reproduced in Table VIII in the Appendix.

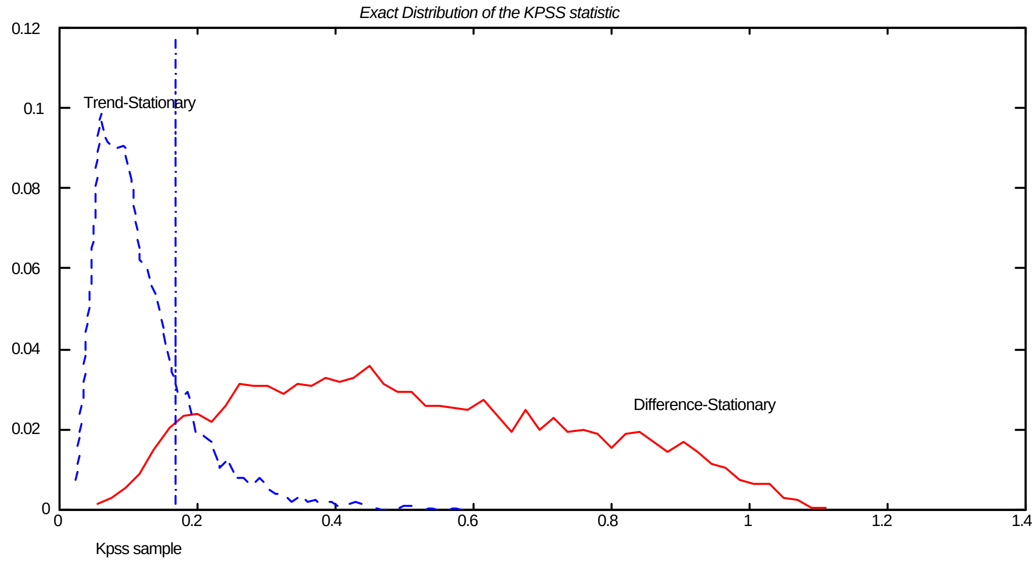

FIG. 1.

that all shocks are permanent whereas I(0) means that all shocks vanish exponentially fast. In the following sections we will explore models in which shocks may behave quite differently. Firstly, the possibility of very persistent although non-permanent shocks will be explored. To this end, long memory models (in particular, Fractionally Integrated models, henceforth FI) will be considered. As it will be seen in Section 3, they can represent a wider variety of behaviors (mean-reverting non-stationary series, long memory stationarity, etc.). The second approach deals with the possibility that just a few shocks have a permanent effect on the series (shocks such as wars, oil shocks, etc.) whereas all the others die out very fast. This behavior would be captured by using a non-linear model whose non-linearity behavior comes from the existance of a breaking trend. Both alternatives seem plausible and are well motivated both from a statistical and from an economic point of view.

LONG MEMORY MODELS

In standard Business Cycle models, memory is infinite (unit roots) or decays exponentially ('stationary' AR), but nothing in between. In many cases macroeconomic processes require a more elaborate and flexible characterization. A class that embeds both behaviors but is also able to bridge the gap between these two extreme situations is given by strongly dependent processes, also known as long memory or long range dependent processes (see Baillie, 1996 and Henry and Zaffaroni, (2002) for recent surveys on this topic). For stationary processes, long memory means that observations far away from each other are still strongly correlated. More specifically, a (stationary) series y_t is said to display long-memory if

$$\rho(k) \sim ck^{2d-1} \text{ for large } k, d \in (0, 0.5), \tag{1}$$

where $\rho(.)$ denotes the autocorrelation function and c is a constant. In other words, long memory is implied by a hyperbolic decay of correlations. The parameter d governs the speed of the decay and for that reason is called the *memory parameter*. This concept has also been employed in the literature for non-stationary series that, although very persistent, are eventually mean reverting..

The most popular parametric model that can represent this slow decay of the correlations (long memory) is the ARFIMA processes. They were independently introduced by Granger and Joyeux (1980) and Hosking (1981) and can be interpreted as a generalization of the ARIMA models. A series y_t is said to be an ARFIMA(p,d,q) process if

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t, \ \varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$$

where $\Phi(L) = 1-\phi_1 L - ... - \phi_p L^p$ and $\Theta(L) = 1-\theta_1 L - ... - \theta_q L^q$ are the autoregressive and moving average polynomials respectively with all their roots lying outside the unit circle. The parameter d determines the integration order of the series and is allowed to take values in the real, as opposed to the integer, set of numbers. Then, fractional integration generalizes the rigid framework of $I(1)$ vs. $I(0)$, since it allows to consider the whole range of intermediate integration orders. This provides for parsimonious yet flexible modeling of low frequency variation. Stationarity and invertibility require $|d| > 1/2$, which can always be achieved by taking a suitable number of (integer) differences. Long memory occurs whenever d belongs to the (0,0.5) interval, since correlations decay at a hyperbolic rate. The case $d = 0$ corresponds to the standard $I(0)$ model where correlations decay exponentially fast. For values of $d \in [0.5, 1)$ the process is not stationary but shocks doesn't have a permanent effect and then the series is mean reverting, whereas shocks have a permanent effect if $d \geq 1$.

Operationally, a binomial expansion of the operator $(1-L)^d$ is used in order to differentiate fractionally a time series:

$$(1-L)^d = \sum_{i=0}^{\infty} \pi_i(d) L^i$$

where,

$$\pi_i = \Gamma(i-d)/\Gamma(-d)\Gamma(j+1) \tag{2}$$

and $\Gamma(.)$ denotes the gamma function. When $d = 1$, this is just the usual first-differencing filter. For non-integer d, the operator $(1-L)^d$ provides an infinite-order lag-operator polynomial with coefficients that decay very slowly. Since the expansion is infinite, a truncation is needed in order to differentiate fractionally a series in practice (see Dolado, Gonzalo and Mayoral (2002) for details on the consequences of the truncation).

The long memory paradigm appears to be a suitable description for many economic series since it naturally characterizes time series displaying a high degree of persistence, in the form of a long lasting effect of unanticipated shocks, yet exhibiting mean reversion. Although it had been successfully employed in other physical sciences such as hydrology and geology (see Hurst, Mandelbrot and Wallis (1969) among others), it only became popular in economics after the work of Granger (1980) and Robinson (1978), who showed how this behavior could arise as a consequence of aggregation over heterogeneous entitities (see also Lippi and Zaffaroni (2000), Forni and Reichlin (1998) and Lewbel (1994)).

One of the major debates in macroeconomic research concerns the persistence of economic shocks on income and the nature on the long cycles observed in output. As Granger (1966) pointed out, one of the most pervasive characteristics of macroeconomic time series is a concentration of power in low frequencies (the so-called "typical spectral shape of economic time variables"). This behaviour would be consistent with the presence of a unit root (which would imply the total persistence of shocks) if it were not often associated with first differenced series with very low power around zero frequency, which is a sign of overdifferentiation. This would suggest that the series in levels could be integrated of order less than one. Besides, unlike the case of financial series with efficient markets hypothesis, the theoretical underpinnings for an exact unit root in macroeconomic series are more controversial.

Why output could display long-run dependence? One plausible explanation is that production shocks themselves also display this behavior. Long-range dependence (LRD) shocks (possibly inherited from underlying geophysical processes) in a real business cycle model of the economy (see Kydland and Prescot (1982)) can account for the presence of such behaviour in aggregate income series. On the other hand, the presence of LRD in geophysical series, such as rainfall, riverflow and climatic series is well documented (see Mandelbrot and Wallis (1969), Lawrence and Kottegoda (1977) and Hipel and McLeod (1978) among others).

However, a more satisfactory explanation is provided by models that produce long memory despite white noise residuals. All these models make use of the above-mentioned aggregation results over heterogeneous entities. Michelacci and Zaffaroni (2000) showed that long memory could arise in GDP per capita in a Solow-Swan growth model just by allowing for cross sectional heterogeneity in the speed with which different units in the same countries adjust. Abadir and Talmain (2002) consider a monopolistically-competitive Real Business Cycle model and, by allowing for firm heterogeneity, they show that GDP turns out to be very persistent although mean-reverting. Haubrich and Lo (2001) discuss a multiple-sector real business cycle model along the lines of Long and Plosser (1983) and find a similar behavior.[4]

From an empirical point of view, several papers have also tested the existence of long memory in output with somehow mixed results. Diebold and Rudebush (1989) analyzed quarterly post-war real GNP and quarterly post-war real GNP per capita. By applying the semiparametric method of Geweke and Porter-Hudak, they obtained estimates of the memory parameter around 0.9 and 0.7 for the former and the latter respectively. In both cases, the null hypothesis of $I(1)$ could not be rejected whereas the hypothesis of trend stationarity could. Sowell (1992a) criticized these results on

[4] Other interesting papers that, through aggregation, get long memory series in different contexts such as inflation series, opinion poll data, etc., are Backus and Zin (1993) , Michelacci (1999), Byers, Davidson and Peel. (1997), Dolado, Gonzalo and Mayoral (2003b) among others.

the basis that the estimation method employed is biased due to the influence of the short-run dynamics of the series that gave misleading measures of the long run. He computed new estimates using parametric exact maximum likelihood on the quaterly real US GNP from 1947:1 to 1989:IV and obtained a point estimate of the memory parameter $d = 0.41$. His results were in line with Rudebush's (1993) and Christiano and Eichembaum (1990) conclusions in the sense that the confidence interval of the estimate included both the unit root and the trend stationary models.

In the next subsections, the plausibility of this alternative will be explored in this data set. To try to clarify this controversy, a longer annual data set along with a wider set of econometric techniques will be employed. Firstly, $FI(d)$ models will be fited to the data and then, several tests will be conducted to shed further ligth on the issue.

Estimation of $FI(d)$ models

There is a broad literature on the parametric and semiparametric estimation of ARFIMA models that has been developed both in the time as well as in the frequency domain. The four series of this analysis have been estimated according to several fully parametric and also semiparametric techniques. In particular, the parametric time domain Minimum Distance (MD) (cf. Mayoral, 2003) and exact maximum likelihood (ML) (Sowell, (1992b)) techniques together with the frequency domain Whittle with tapered data procedure (WT, cf. Velasco and Robinson, 2000) are employed. Finally, the semiparametric method proposed by Geweke and Porter Hudak (1983) has been also examined (see also Robinson (1994) and Velasco (1999)). Table II gathers the main results.[5]

TABLE II

Estimation of $FI(d)$ models

Method-Data	GNP-R	GNP-BG	GNP-RPC	GNP-BGPC
MD	$d = \underset{(0.21)}{0.59}$	$d = \underset{(0.15)}{0.711}$	$d = \underset{(0.20)}{0.510}$	$d = \underset{(0.21)}{0.521}$
	$\phi_1 = \underset{(0.13)}{0.81}$	$\phi_1 = \underset{(0.12)}{0.554}$	$\phi_1 = \underset{(0.14)}{0.797}$	$\phi_1 = \underset{(0.13)}{0.857}$
	$\phi_2 = \underset{(0.12)}{-0.18}$	–	$\phi_2 = \underset{(0.11)}{-0.198}$	$\phi_2 = \underset{(0.11)}{-0.218}$
ML	$d = \underset{(0.16)}{0.68}$	$d = \underset{(0.16)}{0.65}$	$d = \underset{(0.15)}{0.65}$	$d = \underset{(0.15)}{0.63}$
	$\phi_1 = \underset{(0.11)}{0.60}$	$\phi_1 = \underset{(0.12)}{0.63}$	$\phi_1 = \underset{(0.12)}{0.61}$	$\phi_1 = \underset{(0.12)}{0.61}$
Whittle	$d = \underset{(0.22)}{0.731}$	$d = \underset{(0.24)}{0.627}$	$d = \underset{(0.24)}{0.731}$	$d = \underset{(0.26)}{0.731}$
	$\phi_1 = \underset{(0.21)}{0.625}$	$\phi_1 = \underset{(0.21)}{0.63}$	$\phi_1 = \underset{(0.21)}{0.62}$	$\phi_1 = \underset{(0.21)}{0.621}$
GPH	$d = \underset{(0.27)}{0.92}$	$d = \underset{(0.27)}{0.91}$	$d = \underset{(0.263)}{0.89}$	$d = \underset{(0.271)}{0.88}$

Two main conclusions might be drawn from the inspection of the table above. Firstly, fractional values below unity have been found for all series across all techniques employed. Secondly, although the values differ slightly across techniques, values of d in the interval (0.5,1) are always found. More specifically, all parametric methods show values of d around 0.6-0.7 whereas the semiparametric GPH estimator delivers higher values around 0.8-0.9. This fact is not surprising since it is well-known that short-run autocorrelation may bias this estimator (see Sowell (1992b) and Agiakloglou et Al. (1993)). On the other hand, if the GPH is computed on the first differences of the series, the estimates change radically (they are, after adding unity, around 0.2-0.3). This difference can be accounted for by noticing that deterministic trends can substantially bias the estimator (cf. Sibbertsen, 2003). Summarizing, the finding of fractional integration seems to be quite robust in these series, with an integration

[5] All parametric models have chosen according to the BIC criterion. The exact ML has been computed in first differences and unity has been added to the estimated value of d. Following Velasco and Robinson (2000), tapering has been employed to compute the Whittle estimator since non-stationary was suspected. Finally, the number of frequencies used in the calculus of the GPH estimator was set equal to $T^{0.5}$.

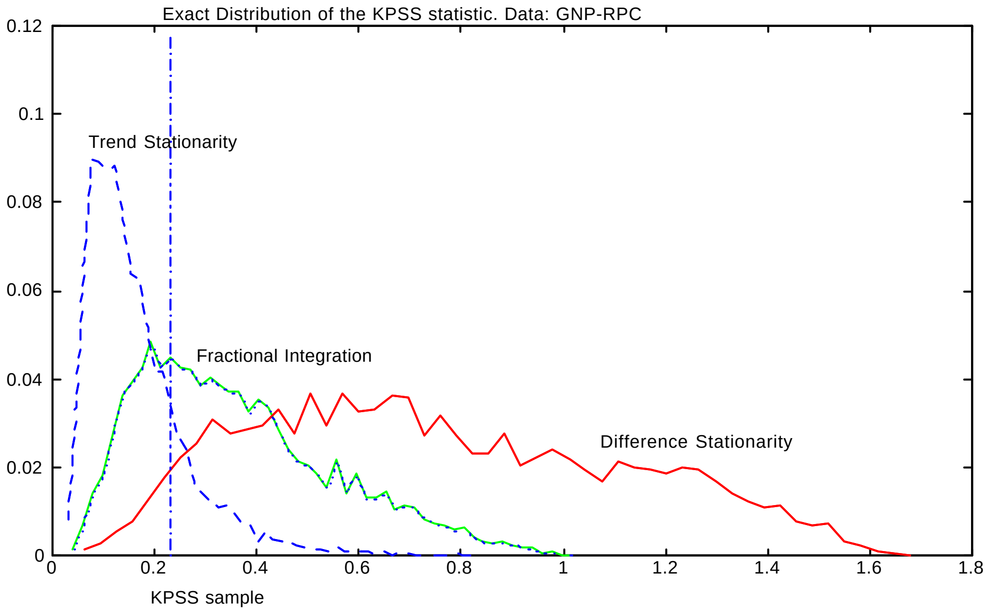

FIG. 2.

order close to 0.7. This implies that the series are non-statioanary, highly persistent although mean-reverting.

Figure 2 reproduces Figure I above but the exact distribution of the best-fitting FI(d) model (according to the exact ML procedure) has been included. The p-value associated to the KPSS test statistic is in this case equal to 0.66, which implies that this value is very reasonable under the $FI\,(d)$ hypothesis.

Although Table II provides some evidence in favor of the hypothesis of mean-reverting series, tests based on the confidence intervals of these estimated values are not very powerful (see Dolado et Al. (2002) for a discussion on this subject). Parametric methods rely very heavily on the correct specification of the model and the estimated value of d is usually very sensitive to this specification. On the other hand, semiparametric methods do not present in principle problems related to the specification of the short-run component (although it is well-known that they can present severe biases due to these components) but at the price of having very large standard deviations and therefore, low power. In the following section, we will present the results of some tests that have been developed in the literature to cope with these problems. The hypothesis of FI(d) would be tested both against the unit root and the trend stationarity ones.

Testing $I\left(1\right)$ versus $FI\left(d\right)$

Two groups of outstanding contributions have been proposed in the literature for testing these hypotheses with sufficient power. Within the first one, Lagrange Multiplier (LM) tests have been introduced by Robinson (1994*b*) and Tanaka (1999) in the frequency and the time domain respectively. A distinctive feature of this approach is that, in contrast to the classical unit root tests, the statistics have standard asymptotic distributions. Moreover, under gaussianity, these tests are locally optimal. A drawback of this technique, however, is that it is fully parametric and consequently the results may rely heavily in the parametric specification of the model. A different approach was introduced by Dolado, Gonzalo and Mayoral $(2002, 2003)$ who generalized the traditional Dickey-Fuller test of $I\left(1\right)$ against $I\left(0\right)$ to the more general framework of $I\left(1\right)$ versus FI(d). The so-called Fractional Dickey-Fuller test is based upon the t-ratio associated to the coefficient of $\left(1-L\right)^{d} y_{t-1}$ in a regression of $\left(1-L\right) y_{t}$ on $\left(1-L\right)^{d} y_{t-1}$, and possibly, some lags of $\left(1-L\right) y_{t}$ to account for the short run autocorrelation of the process and/or some deterministic components if the series displays a trending behaviour or initial conditions different from zero. Besides its simplicity, two additional features stand out in this approach: as in the Dickey-Fuller approach (see Said and Dickey, 1984), the test allows for a semiparametric specification of the short term structure and secondly, although not locally optimal under gaussianity, it presents a high power in finite samples that in general outperforms the above-mentioned methods.

Table III presents the outcome of the tests. With respect to the FDF test, the invariant regression described in Dolado et Al. (2003)[6] has been performed against

[6]The FDF invariant regression that has been run is equal to $\Delta y_{t} = \alpha_{1} + \alpha_{2}\tau_{t-1}\left(d\right) + \alpha_{3}\tau_{t-1}\left(d-1\right) + \phi\Delta^{d}y_{t-1} + \sum_{j=1}^{k}\Delta y_{t-j} + a_{t}$ and a number of lags of Δy_{t} equal to two was chosen according to the BIC criterion. Coefficients α_{1}, α_{2} and α_{3} are associated to different deterministic components that result from introducing a constant and a time trend in the DGP (see

several non-stationary fractional hypotheses. For all the values of d considered under the alternative (from 0.6 to 0.9), the null hypothesis of a unit root was rejected. Similar results were obtained by applying the time domain LM[7] test. Then, both tests supports the hypothesis that these series do not contain a unit root.

TABLE III

TEST OF I(1) VERSUS FI(d).

	FD-F Test				LM Test (Tanaka,1999)
H_1 :	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$	$d < 1$
GNP-R	-4.06*	-3.92*	-3.75*	-3.58*	-1.71*
GNP-RPC	-3.85*	-3.67*	-3.49*	-3.32*	-1.91*
GNP-BG	-2.89*	-2.78*	-2.66*	-3.51*	-1.91*
GNP-BGPC	-3.99*	-3.81*	-3.63*	-3.46*	-1.89*
Crit. Values (5% $S.L.$)	-2.45	-2.23	-2.11	-1.97	-1.64

Testing FI versus Trend stationarity

Once the unit root has been rejected, we consider the problem of directly testing for FI versus T-ST. As mentioned above, the KPSS is consistent against some types of long memory processes (see Lee and Schmidt, 1996), therefore the rejection of the null of $I(0)$ reported in Table I is consistent with the finding of fractional integration in the data reported in Tables II and III. To formally test these hypotheses, LM tests have again been considered, setting in this case the T-ST as null hypothesis versus the alternative of a bigger integration order. Table IV presents the outcome of the LM test for testing the hypothesis of $I(0)$ vs. a bigger integration order. Not surprisingly,

Dolado, Gonzalo and Mayoral, 2003). The definition of the trends is $\tau_t(d) = \sum_{i=0}^{t-1} \pi_i(d)$ and $\tau_t(d-1) = \sum_{i=0}^{t-1} \pi_i(d-1)$, where the coefficients $\pi_i(\delta)$ come from the expansion of $(1-L)^{\delta}$ as defined in equation (2).

[7] Tanaka's (1999) time domain version has been computed instead of Robinson's (1994) original frequency domain test since Monte Carlo simulations show that the former sligthly outperforms its frequency domain counterpart in finite samples (cf. Tanaka, (1999)). The test statistic is given by $\sqrt{T}\sum_{k=1}^{T-1}\frac{1}{k}\hat{\rho}_k$, where $\hat{\rho}_k$ is the autocorrelation function of the residuals of a FI(d) parametric model, and it is asymptotically normally distributed.

the null hypothesis of T-ST is rejected.

TABLE IV

TEST OF I(0) VERSUS FI(d): LM TEST, (TANAKA (1999))

H_1 :	$d > 0$
GNP-R	-1.95*
GNP-RPC	-1.82*
GNP-BG	-1.99*
GNP-BGPC	-1.88*
Crit. Values (5% *S.L.*)	-1.64

From Kwiatkowsky et al. (1992) it is known the importance of reversing the null and the alternative hypothesis. The above-described tests do not allow to do this. The LM tests (cf. Robinson (1994*b*), Tanaka (1999)) are designed to test if y_t is $I(d_0)$ versus the alternative of $d_0 \neq d$. Then, if the null is rejected it does not provide any information about d being equal or different from zero. The FDF test is been developed for the case I(1) vs. $I(d)$ and, although the results can be extended to cover the case $FI(d)$ vs. FI(0), the asymptotic properties of the statistics in this case are still under investigation. Recently, Mayoral (2004) has introduced a point-optimal test that is able to deal with this issue. The problem of testing for $FI(d)$ vs. $I(0)$ can be seen as a simple hypotheses test. Under this perspective, the natural way of testing it would be to carry out a Neyman-Pearson test. Consider the simplest case where the series is a pure fractional white noise process,

$$\Delta^{d_0} y_t = u_t, \tag{3}$$

where $u_t = \varepsilon_t$ is i.i.d. Under gaussianity, minus two times the log-likelihood function is (except for an additive constant) given by

$$L(d, \sigma)|_{H_0} = \sigma^{-2} \sum_{t=2}^{T} \left(\Delta^{d_0} y_t\right)^2.$$

Analogously, under the alternative hypothesis $d = 0$ the likelihood becomes,

$$L(d, \sigma)|_{H_1} = \sigma^{-2} \sum_{t=1}^{T} y_t^2.$$

By the Neyman-Pearson Lemma, the most powerful (MP) test of the null hypothesis of $d = d_0$ rejects the null hypothesis for small values of the likelihood ratio statistic $L(d,\sigma)|_{H_1} - L(d,\sigma)|_{H_0}$. Replacing σ^2 by a consistent estimator under the null hypothesis and rearranging terms in follows that the MP test rejects the null hypothesis for each finite T whenever

$$\frac{\sum\left((1-L)^{d_0} y_t\right)^2}{\sum y_t^2} > k'_T. \tag{4}$$

This approach is similar to the one presented by Elliot et al. (1996) and the resulting test statistics coincide with the Von-Neumann ratio proposed in the framework of efficient unit root tests (cf. Sargan and Bhargava (1983)), Bhargava (1986)). This very simple framework can easily accommodate the general case where deterministic components and short term autocorrelation are allowed for. In this case more general case, the most powerful test has as critical region

$$\frac{\sum(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t))^2}{\sum(y_t - \tilde{\alpha}_1 - \tilde{\beta}_1 t)^2} > k_T \tag{5}$$

The asymptotic distribution of this statistic (scaled by T^{1-2d}) is not standard and critical values can be found in Mayoral (2004) for the case where $u_t = \varepsilon_t$. Where u_t is a linear short memory process with a Wold representation $u_t = \Psi(L)\varepsilon_t$, where the coefficients ψ_j are such that $\sum_{j=0}^{\infty} j\left|\psi_j\right| < \infty$, a nuisance term (equal to γ_0/λ^2, with $\gamma_0 = \sigma^2 \sum_{i=0}^{\infty} \psi_j^2$ and $\lambda = \sigma\Psi(1)$) appears multiplying the asymptotic distribution function above. To get rid of this term, the statistic in (5) should be multiplied by a consistent estimator of $(\gamma_0/\lambda^2)^{-1}$. The numerator γ_0 can be estimated under the null hypothesis simply by $\sum\left((1-L)^{d_0} y_t\right)^2/T$ whereas the denominator can be rewritten as:

$$\lambda^2 = \gamma_0 + 2\sum_{i=1}^{\infty}\gamma_i = 2\pi s_u(0).$$

where $s_u(0)$ is the spectral density of u_t evaluated at zero frequency. The Newey-West estimator can be employed to estimate this quantity and then,

$$\hat{\lambda}^2 = \hat{\gamma}_0 + 2\sum_{i=1}^{q}(1 - j/(q+1))\hat{\gamma}_i$$

where $\hat{\gamma}_i = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}$.

The following table gathers the results of applying the above-described test to the series used in this paper. To interpret the results notice that the test is consistent (rejects the null hypothesis of $FI(d_0)$) if the true integration order, d^*, is smaller than the integration order used as null hypothesis. Consequently, whenever $d_0 > d^*$, the test will reject the $FI(d_0)$ hypothesis. The opposite is also true, that is, whenever the value employed to run the test is smaller than the true value, $(d_0 < d^*)$, the test will not reject the hypothesis of $FI(d_0)$.

TABLE V

LR TESTS FI(d) VS I(0)[8]. SL:5%

H_0 :	d_0 =0.6	d_0 =0.7	d_0 =0.8	d_0 =0.9	d_0 =1.0
GNP-R	1.73	3.83	8.41	18.79	43.67*
GNP-RPC	1.96	4.33	9.70	21.96*	50.05*
GNP-BG	1.73	3.89	8.63	19.47	45.32*
GNP-BGPC	1.96	4.34	9.71	22.02*	50.24*
Critical Values (5%-10%)	(2.13-1.74)	(4.78-4.10)	(10.32-9.08)	(20.85-18.63)	(41.25-34.05)

The results clearly confirm the output from the estimation methods. The null hypothesis of FI cannot be rejected for values of d_0 around 0.6, 0.7. For bigger values, the null hypothesis is rejected. This is the result that we expected since whenever the value used in the test, d_0, is bigger than the true integration order, d, the test diverges towards ∞ at a rate $T^{2(d-d_0)}$ and therefore the null hypothesis is rejected. This explains the high values obtained for the cases where d is bigger than 0.7.

MODELS CONTAINING BREAKS

The consensus about the existence of a unit root in many macroeconomic series that followed the work of Nelson and Plosser (1982) prompted several counterchallenges.

[8] The values reported are computed by including three lags in the computation of the Newey-West variance-covariance function. Other numbers of lags were also tried and the results remain qualitatively identical.

In addition to the fractional models analyzed above, one of the most constructive alternatives was first presented by Perron (1989). He claimed that the trend in most of these series could be explained by a few (one or two) structural breaks in an otherwise constant linear trend. His explanation seemed plausible since a trend break produces serial correlation properties that are similar to those of a random walk. Perron (1989) developed a procedure for testing the random walk hypothesis against the trend-break model (under the assumption of known break date) and by applying it to the Nelson-Plosser data set he was able to reject the unit root hypothesis for many of the series. However, this approach was disputed by a collection of papers (cf. Zivot and Andrews (1992), Christiano, (1992), Banerjee et al. (1992) among others). These authors argued that it is inappropriate to specify the breakdate as known and they collectively suggested to select the breakdate that produces the largest value of the test. The literature on this area is very large, see Hansen (2001) for a recent survey.

As argued by DS (1996), in the context of the $I(1)$ vs. $I(0)$ + trend hypotheses allowing for a breaking trend in the spirit of the papers above only strengthens the rejection of the null hypothesis of unit root. For the sake of brevity the results of these tests are not included but, not surprisingly all lead to the rejection of the unit root hypothesis.

Taking these results as starting point, we now explore in more detail the breaking-trend model. This parametrization amounts to consider that just a few shocks have a permanent effect on the series while the others die out very fast. Since the data span covers a very long period of time, it is reasonable to allow for the existence of several permanent shocks in the data. For this reason, we will adopt the approach proposed by Bai and Perron (1998) that allows for multiple structural changes. Some of the procedures developed in this paper are not valid when trending regressors are allowed for as it is the case of these data. Nevertheless, the consistency, the rate of convergence and the confidence intervals of the estimated breaks points still hold. The case with trending data is discussed in Bai (1999) and yields different asymptotic distributions for the tests of no break versus a fixed number of tests. But, as discussed in Bai and Perron (2001), the asymptotic distributions in the two cases are fairly similar, especially in the tail (where critical values are obtained) and simulations confirm that the size distortions are minor.

To compute the tests, a Gauss code programmed by one of the authors has been employed ((available from his webpage at http://econ.bu.edu/perron/code.html). A number of breaks equal or less than 5 is allowed for and both the mean and the trend are allowed to change while the ARMA parameters were assumed to remain fixed. The following table gathers the main results[9]. The main conclusion that can be drawn from the table below is that the test does not identify any break in the data for real GNP and just one break in both per-capita series, located around 1939, coinciding with the beginning of World War 2. Tests of no-break versus an unknown number of breaks and sequential tests were performed and their conclusions were similar as the above-described ones.

TABLE VI

SUPF F TESTS FOR A FIXED NUMBER OF BREAKS. S.L.:5%

H_0 : no break				
	GNP-R	GNP-RPC	GNP-BG	GNP-BGPC
H_1 : 1 break	6.53	15.84**	5.53	16.26**
H_1 : 2 breaks	9.44	8.06	9.43	7.39
H_1 : 3 breaks	7.35	6.13	7.07	5.49
Break date (BD)	_	1941	-	1939
C. I. of B. D	_	(1938-1956)	-	(1936-1958)

*Rejection at the 5% level; **Rejection at the 1% level.

LONG MEMORY OR STRUCTURAL BREAKS

So far, the main results that have obtained can be summarized as follows: First, the rejection of the unit root hypothesis in all the four series is very robust across

[9] For the sake of brevity, we only report the results of the tests with three breaks or less, but the hypotheses of 4 and 5 breaks were also considered and rejected in favor of the no-break hypothesis in all cases.

all the different alternatives considered (T-ST, FI and structural breaks) and across the different techniques employed; Second, the T-ST hypothesis is also rejected for all series against all alternatives with the exception of real GNP, for which the former hypothesis is not rejected when tested against the breaking-trend model. Third, the best-fitting FI models show an integration order around 0.7, which imply that the series are very persistent, non-stationary although mean reverting. Finally, with respect to the model with breaks, there is no clear evidence of breaks in real GNP although evidence of a break has been found in per capita GNP corresponding to the beginning of World War II.

These findings suggest that the series seem to be non-stationary and that both the breaking trend and the FI model are in general preferred to the $I(1)$ or the $I(0)$ parametrizations. Clearly, it remains to shed some further light on the reasons that drive the non-stationarity of these series: whether it is due to strong persistence of the shocks or whether to the existence of very few permanent shocks in the series.

It is been argued by several authors that in many situations it is not clear whether the observed dependence structure is real long memory or some other phenomena such as structural breaks or deterministic trends, since the latter can cause spurious long memory and viceversa. Granger and Hyung (1999) argue that breaks in the data cause the long memory structure that has been commonly found in the Standard and Poor's 500 composite index. Diebold and Inoue (2001) introduced a model where processes are stationary and short memory but exhibit periodic regime shifts (random changes in their mean) and they argued that if switches occur with a low probability related to the sample size, then the variance of the partial sums will be related to the sample size in the same way as a fractionally integrated process. A similar approach is adopted in Gourieroux and Jasiak (2001). Davidson and Sibbertsen (2001) show that a fairly general class of nonlinear processes can exhibit the covariance structure associated with long memory but they also point out that they are not observationally equivalent to fractionally integrated processes (as claimed by the above-mentioned authors). This is due to the fact that, although having the same covariance structure, they do not admit a linear representation and hence their partial sums do not converge to fBM, which is characteristic of FI models. They underline the necessity of cross-sectional aggregation to achieve observational equivalence since in this case the process may be linearized by virtue of their Gaussianity. See also Sibbertsen (2003) for a review of the literature on this problem.

The problem for disguishing for structural breaks and fractional integration has been largely overlooked in the literature, where most of the literature has focused on different problem of testing for structural breaks in the presence of long-memory innovations (see for instance, Hidalgo and Robinson, (1996) and Teverovsky and Taqqu (1994)). Recently, Mayoral (2004) has introduced a test that is able to discriminate between both hypothesis. In the following, this new test will be implemented on this data set.

A test for fractional integration versus structural break

The test that will be applied is an extension of the approach considered in Section 3 to test for $FI(d)$ against $I(0)$. As in the unit root case, the aim of the test is to disentangle whether the source of the persistence is due to (fractional) integration in the data or to changes of regime that provoke a similar correlation structure. For that reason we will consider the same null hypothesis as before, namely $FI(d)$ processes with $d > 0.5$ versus I(0) processes with possibly one trend break.

Let us call T_B the time when the break occurs and let $\omega = T_B/T$ be the location of the break point in the sample. Let us also define the dummy variable $D_t(\omega)$ =1 if $t < T_B$ and viceversa. Therefore, under the alternative hypothesis of one structural break in the sample, minus two times the likelihood function could be written as,

$$\sigma^{-2} \sum (y_t - \alpha_1 - \alpha_2 D_t(\omega) - \beta_1 t - \beta_2 t D_t(\omega))^2.$$

Since the date break is considered to be unknown, we consider as candidate for break point the one that maximizes the likelihood (or alternatively, that minimizes the variance). The minimization is carried out in $\omega \in$, where $= [\omega_L, \omega_H]$ for $0 < \omega_L < \omega_H < 1$. The test as a critical region,

$$\frac{\min_{\alpha,\beta} \sum \left(\Delta^d (y_t - \alpha - \beta t)\right)^2}{\inf_\omega (\min_{\alpha,\beta} \sum (y_t - \alpha_1 - \alpha_2 D_t(\omega) - \beta_1 t - \beta_2 t D_t(\omega))^2)} \geq k_T \qquad (6)$$

The distribution of the statistic in (6) scaled by T^{2d-1} is non-standard and critical values can be found in Bai and Mayoral (2003). Again, when short term structure is allowed for, the statistic should be multiplied by an estimator of the quantity $(\gamma_0/\lambda^2)^{-1}$ (see Section 3 for details). The following table gathers the results of the above-described test on the data analyzed in this article. The main conclusion that can be drawn from Table VII is that the null hypothesis of fractional integration cannot be rejected in the range of values of d around 0.7 or smaller for all the series considered. This is very interesting since these are the suspected values for d, according to the results of Table II in Section 3. The null hypothesis is of $FI\,(0.8)$ is rejected for per-capita series, although it is not for real GNP. Finally, for a null hypothesis closer to one, $d = 0.9$, the test rejects fractional integration in favor of a structural break in all the four series. These results are in good agreement with the above-obtained results. They suggest that fractional integration of an order around 0.7 is present in the four series. Nevertheless, for values of d closer to the unit root, the test non-surprisingly can reject the null hypothesis.

TABLE VII

LR TESTS FI(d) VS I(0) WITH ONE BREAK. S.L.:5%

$Data/\mathrm{H}_0$:	0.6	0.7	0.8	0.9
GNP-R	0.67	1.98	5.96	18.14*
GNP-RPC	0.89	2.64	7.92*	23.99*
GNP-BG	0.71	2.09	6.22	18.72*
GNP-BGPC	0.94	2.77	8.27*	24.94*
Critical Values (5%)	1.63	3.34	6.60	12.18

From a macroeconomic point of view, the non-rejection of the $FI\,(d)$ model against the structural change one means that the growth rate of GDP per capita is well characterized as a process with little persistence and a constant mean. Following Jones (1995), this evidence is inconsistent with the two types of endogenous growth theories, the AK and the R&D-based models. According to both approaches, permanent changes in certain policy variables have permanent effects on the rate of economic growth. But, as shown above, no permanent changes seem to have occurred in per capita GNP which in turn implies that no such changes are present in the growth rates.

CONCLUSIONS

This paper has tried to shed some light on the issue of the statistical properties of real GNP. Taking as starting point the conclusions in Diebold and Senhaji (1996) that underline the fact that long spans of GNP's annual data are not as uninformative as many believe, we have completed their analysis by first, considering other type of alternatives to the unit root model, namely, models showing strong persistence and non-linear models. In agreement to their results, for all alternatives and across different techniques, the unit root hypothesis was rejected for all the series. But, interestingly, also the trend-stationary hypothesis can be easily rejected in these data. This led us to analyze in depth the proposed alternatives: fractionally integrated processes and processes containing breaks. Although these models may present a similar correlation structure under certain conditions, they present very different properties and also very different long-run implications. It has been also argued that one can spuriously generate the other, which implies the difficulty of distinguishing between them. Applying very recent techniques, a test of FI vs. structural break has performed. The final conclusion is that the finding of fractional integration in the series is robust and it is preferred to the structural break model. This result also suggest that the evidence that has been found in other studies supporting the existence of breaks could be the result of the fractional integration of the data, and therefore, spurious.

From a economic point of view, the implications of these findings are important. As we have seen, long memory can appear in macroeconomic series after aggregating heterogeneous individual entities. This suggests that moving from the representative agent assumption to a multiple-sector real business cycle model introduces not unmanageable complexity, but qualitatively new behaviour that should be taken into account. On the other hand, calibrations aimed at matching only a few first and second order moments can similarly hide major differences between models and the data, missing long range dependence properties (which is basically characterized by the slow rate of decay of covariances). Finally, the lack of structural breaks in the data together with the finding of integration orders of around 0.7 for per capita series imply that the growth rate of GDP per capita is well characterized as a process with little persistence and a constant mean. As Jones (1995) first suggested, this evidence is inconsistent with endogenous growth theories for which permanent changes in certain policy variables have permanent effects on the rate of economic growth.

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APPENDIX

TABLE VIII

ESTIMATED BEST FITTING TREND AND DIFF. STAT. MODELS

	Regressor				
Data series	c	t	y_{t-1}	y_{t-2}	Δy_{t-1}
Trend stationary (dependent variable y_t)					
GNP-R	0.797 (0.187)	0.0052 (0.0013)	1.306 (0.082)	−0.489 (0.07)	–
GNP-RPC	−1.21 (0.25)	0.0032 (0.007)	1.306 (0.07)	−0.492 (0.077)	–
GNP-BG	0.805 (0.187)	0.0052 (0.0013)	1.207 (0.042)	−0.377 (0.187)	–
GNP-BGPC	−1.12 (0.023)	0.0032 (0.007)	1.306 (0.0.077)	−0.481 (0.077)	–
Difference-stationary (dependent variable Δy_t)					
GNP-R	0.019 (0.0053)	–	–	–	0.401 (0.083)
GNP-RPC	0.001 (0.080)	–	–	–	0.315 (0.080)
GNP-BG	0.022 (0.0053)	–	–	–	0.30 (0.083)
GNP-BGPC	0.011 (0.080)	–	–	–	0.39 (0.080)