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Welfare Implications of the Interaction between Habits and Consumption Externalities

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Abstract

We analyze the welfare properties of the equilibrium path of a growth model where both habits and consumption externalities affect the utility of consumers. Our analysis highlights the crucial role played by complementarities between externalities and habits in order to generate an inefficient dynamic equilibrium. In particular, we show that the competitive equilibrium is inefficient when consumption externalities and habit adjusted consumption are not perfect substitutes.

JEL classification codes: D91, O40.

Keywords: Habit Formation, Consumption Externalities.

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1. Introduction

In this paper we analyze the welfare properties of a deterministic endogenous growth model where individual preferences are subjected to a process of habit formation and the average level of consumption of the economy affects individuals' felicity. These two departures from standard specifications of preferences have been introduced in several models in order to account for some empirical phenomena that cannot be explained under more traditional forms of the utility function.

On the one hand, our consumers will form habits so that they will not derive utility from the absolute level of their consumption but from the comparison of the level of current consumption with that in the previous period. The presence of this process of habit formation has qualitative consequences for the dynamic optimization problem faced by consumers since, when they choose their current consumption, they are also selecting a standard of living that will be compared with the level of future consumption. Moreover, since past consumption becomes now a state variable, the dynamic properties of the economy will also be affected by the introduction of habits.

On the other hand, the consumers' utility will depend on the average level of consumption in the economy. These spillovers from the others' consumption could either increase or decrease the marginal utility of own (habit adjusted) consumption. In the first case, preferences display the typical "keeping up with the Joneses" feature since consumption of other individuals makes more valuable a marginal increase of own consumption (see Galí, 1994).

The growth model we will use in this paper is a very stylized one. Growth of income per capita will arise from an Ak-type production function as in Rebelo (1991). Under standard preferences, the growth rate of this model displays no transition. This is so because the interest rate is constant and, thus, the rate of consumption growth immediately jumps to its stationary value. However, when habit formation is present, the stock of past consumption at a given period is fixed and, thus, the process of capital accumulation leads to a non-instantaneous adjustment of this consumption reference. Therefore, in our model transitional dynamics will be exclusively driven by preferences, and this will allow us to analyze more clearly the effects of consumption spillovers off the balanced growth path.

Consumption externalities constitute an obvious potential source of inefficiency since individuals do not take them into account when they choose their individual consumption paths. In a centralized economy a social planner internalizes those consumption spillovers and, hence, the resulting consumption path could not coincide with the competitive one. However, in absence of habit formation, we will show that if both the competitive economy and the socially planned economy have balanced growth paths (which requires homogeneous partial derivatives of the instantaneous utility function), then the competitive and the socially planned paths of consumption coincide. Thus, consumption externalities turn out to be irrelevant in terms of the welfare properties of the competitive equilibrium. The reason for this irrelevance is that, if there exist competitive and efficient paths for which consumption is growing at a constant rate, then the functional form of the marginal rate of substitution between consumption at different dates of an individual behaving competitively must be identical to that of the social planner. However, we will see that when we add

a process of habit formation to individual preferences, the competitive equilibrium might fail to be efficient. Even if we preserve the existence of competitive and efficient balanced growth paths, inefficiencies arise whenever habit adjusted consumption and average consumption enter as not perfect substitutes in the utility function of individuals (like, for instance, in the multiplicative specifications of Galí, 1994; and Carroll et al., 1997, 2000). Inefficiency will appear in our model because the interaction between externalities and habits modifies the optimal elasticity of intertemporal substitution and, thus, the optimal speed of convergence. Note that the nature of this distortion is essentially intertemporal and this contrasts with the intratemporal inefficiency appearing in the models of Ljungqvist and Uhlig (2000) and Dupor and Liu (2003) where consumption externalities distort the consumption-leisure choice. Moreover, the use of an Ak production function allows us to highlight the dynamic inefficiency brought about by the interaction between habits and consumption externalities since the dynamic adjustment to the balanced growth path is entirely driven by preferences and not by technological decreasing returns to scale. In fact, we must use an Ak production function in order to obtain a transitional dynamics governed just by the consumers' preferences.

The plan of the paper is the following. Section 2 presents the endogenous growth model with only consumption spillovers. Section 3 adds to the previous model a simple process of habit formation in consumption. Section 4 concludes the paper. All the proofs and lengthy computations are in the Appendix.

2. Consumption Externalities and Balanced Growth

Let us consider an infinite horizon economy in discrete time. The economy is populated by a continuum of identical individuals facing also an infinite horizon. Each individual maximizes the discounted sum of instantaneous utilities and the discount factor is $\beta \in (0,1)$. Individual preferences exhibit consumption externalities so that the average consumption in the economy affects the utility of agents as in Galí (1994), Harbaugh (1996), Abel (1999), Ljungqvist and Uhlig (2000) and Dupor and Liu (2003), among many others. Therefore, each individual chooses the sequence of per capita consumption $\{c_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, \bar{c}_t\right), \tag{2.1}$$

where $\bar{c}_t$ is the consumption per capita of the economy at period t. The utility function u is twice continuously differentiable and satisfies: (i) $u_c\left(c,\bar{c}\right) > 0$ and $u_{cc}\left(c,\bar{c}\right) < 0$ for all $c > 0$ and $\bar{c} > 0$; (ii) $u_c(c,\bar{c}) + u_{\bar{c}}(c,\bar{c}) > 0$ when $c = \bar{c} > 0$; and (iii) $u_{cc}(c,\bar{c}) \cdot u_{\bar{c}\bar{c}}(c,\bar{c}) - [u_{c\bar{c}}(c,\bar{c})]^2 > 0$ when $c = \bar{c} > 0$, where the subindexes denote the variables with respect to which the partial derivatives are taken. The second assumption implies that utility rises if everyone's consumption is identical and increases, whereas the third assumption implies that u is strictly concave. Moreover, the following Inada conditions hold: (i) $\lim_{c\to 0} u_c\left(c,\bar{c}\right) = \infty$ and $\lim_{c\to\infty} u_c\left(c,\bar{c}\right) = 0$ for all $\bar{c} > 0$; and (ii) $\lim_{c\to 0} [u_c(c,\bar{c}) + u_{\bar{c}}(c,\bar{c})] = \infty$ and $\lim_{c\to\infty} [u_c(c,\bar{c}) + u_{\bar{c}}(c,\bar{c})] = 0$ when $c = \bar{c} > 0$.

Following Rebelo (1991), we will assume that the gross production function per capita is

$$y_t = Ak_t \quad \text{with } A > 0,$$

where k_t is the capital per capita and y_t is the corresponding output. The depreciation rate of capital is $\delta \in [0, A]$, which implies that the net productivity is positive. To keep the analysis simple, we assume that there is no population growth.

The budget constraint faced by an individual is thus

$$c_t = Ak_t - k_{t+1} + (1-\delta)k_t, \tag{2.2}$$

which can be written as

$$\frac{k_{t+1}}{k_t} = (1 + A - \delta) - \frac{c_t}{k_t}. \tag{2.3}$$

Taking as given the initial capital per capita k_0 and the sequence $\{\bar{c}_t\}_{t=0}^{\infty}$ of average consumption, each individual maximizes (2.1) subject to the budget constraint (2.2). The solution to the individual problem along a symmetric equilibrium (with $c_t = \bar{c}_t$) involves the Euler equation

$$\frac{u_c(c_{t+1}, c_{t+1})}{u_c(c_t, c_t)} = \frac{1}{\beta(1 + A - \delta)}, \tag{2.4}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t u_c(c_t, c_t) k_{t+1} = 0. \tag{2.5}$$

The competitive equilibrium is thus given by the sequence $\{c_t, k_t\}_{t=0}^{\infty}$ satisfying (2.4), (2.2), and the transversality condition (2.5) with the initial capital per capita k_0 exogenously given.

Let us characterize now the solution that a benevolent social planer would implement in this economy. This social planer internalizes the spillovers from average consumption so that he is facing the instantaneous utility function $\hat{u}(c) \equiv u(c, c)$, which is strictly increasing and strictly concave as follows from the assumptions imposed on u. The resource constraint of the planner's problem is also (2.2). Following the same steps as before, it is straightforward to see that optimality requires

$$\frac{\hat{u}'(c_{t+1})}{\hat{u}'(c_t)} = \frac{1}{\beta(1 + A - \delta)}, \tag{2.6}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t \hat{u}'(c_t) k_{t+1} = 0. \tag{2.7}$$

The social planner solution is thus given by the sequence $\{c_t, k_t\}_{t=0}^{\infty}$ satisfying (2.6), (2.2) and the transversality condition (2.7) with k_0 exogenously given. The path chosen by the social planner is also called the efficient path.

At a balanced growth path (BGP) the output per capita grows at a constant rate, which implies that the gross rate of growth of capital k_{t+1}/k_t is constant. Hence, we see from (2.3) that the ratio c_t/k_t is also constant and that both consumption and capital grow at the same rate along a BGP. As is customary in the economic growth literature, we will assume that both the competitive economy and the socially planned economy have a BGP. Regarding the competitive economy, this assumption means that there exists a sequence $\{c_t, k_t\}_{t=0}^{\infty}$ satisfying (2.4), (2.2) and (2.5) along which the variables c_t and k_t grow at constant rates. The existence of a BGP for the socially planned economy means that there exists a sequence $\{c_t, k_t\}_{t=0}^{\infty}$ satisfying (2.6), (2.2) and (2.7) along which the variables c_t and k_t grow also at constant rates. Obviously, the BGP's of these two economies are not necessarily equal.

On the one hand, the assumption of existence of a BGP for the competitive economy stems from the fact that the equilibrium path should be consistent with Kaldor's stylized facts. In particular, the economy should exhibit a constant rate of growth in the long run. On the other hand, the requirement of existence of a BGP for the socially planned economy is usually justified by an argument running in the opposite direction, namely, that tax rates aimed at implementing the efficient path should become stationary in the long run.

Note that, if the competitive economy has a BGP, we must impose that $v_1(c) \equiv u_c(c,c)$ be an homogeneous function in order to satisfy the Euler equation (2.4) when c_t is growing at a constant rate. Similarly, the existence of a BGP for the socially planned economy implies that the function $\hat{u}'(c)$ must be also homogeneous so as to allow the Euler equation (2.6) to hold when c_t is growing at a constant rate.

Proposition 2.1. *Let $v_2(c) \equiv u_{\bar{c}}(c,c)$. Assume that the functions v_1 and $\hat{u}'$ are both homogeneous and $v_2(c) \neq 0$ for all c, and that the initial capital k_0 is the same for both the competitive economy and the socially planned economy. Then, the equilibrium paths of consumption and capital $\{c_t, k_t\}_{t=0}^{\infty}$ for the socially planned economy and for the competitive economy coincide.*

We have thus shown that the existence of BGP's for the competitive economy and for the socially planned one leads to the efficiency of the competitive accumulation path even if consumption externalities are present. This means that public intervention is not needed in order to implement an efficient path. Note that in the proof of the previous proposition we show that the function v_2 must be homogenous, which together with the assumed homogeneity of v_1, implies that the function $u(c,\bar{c})$ is homothetic with respect to its two arguments along the 45°-degree line, i.e., when $c = \bar{c}$ (see (A.3)). This kind of "restricted homotheticity" constitutes in fact the necessary and sufficient condition discussed in Fisher and Hof (2000) for having a competitive solution identical to its socially planned counterpart when consumption spillovers affect the utility of individuals

In our basic model contemporaneous consumption spillovers have symmetric intertemporal effects and thus they do not generate inefficient competitive paths. However, inefficiency could arise when a distortion is introduced on either intratemporal or intertemporal decisions. For instance, Ljungqvist and Uhlig (2000) and Dupor and Liu (2003) consider a departure of our model where consumption spillovers affect the intratemporal consumption-leisure choice. Concerning the

introduction of asymmetries on the intertemporal decisions, they can be achieved by assuming time dependent preferences. We will thus modify our basic setup in the next section by assuming that private consumption is subjected to a process of habit formation. With this modification the existence of BGP's is not longer incompatible with inefficiencies in the capital accumulation process when consumption externalities are present.

3. The Model with Consumption Externalities and Habit Formation

We will now introduce the assumption that individuals will not derive utility from their absolute level of consumption at a given period but also from the change of consumption with respect to their past experience. Therefore, individuals care about the lagged values of their own consumption, as in the seminal paper of Ryder and Heal (1973) and the models with rational addiction of Becker and Murphy (1988) and Orphanides and Zervos (1995). In particular, we will assume that the instantaneous utility function of individuals is $u(h_t, \bar{c}_t)$, where $h_t = c_t - \gamma c_{t-1}$ with $\gamma \in (0,1)$. This means that consumption in the previous period becomes a standard of living that is used to evaluate the utility accruing from current consumption. The parameter γ measures thus how important is the reference set by past consumption. We will assume that the utility funcition u is twice continuously differentiable and satisfies: (i) $u_h(h,\bar{c}) > 0$ and $u_{hh}(h,\bar{c}) < 0$ for all $h > 0$ and $\bar{c} > 0$; (ii) $u_h(h,\bar{c}) + u_{\bar{c}}(h,\bar{c}) > 0$ when $h = \bar{c} - \gamma\underline{c} > 0$, for all $\underline{c} > 0$; and (iii) $u_{hh}(h,\bar{c}) \cdot u_{\bar{c}\bar{c}}(h,\bar{c}) - [u_{h\bar{c}}(h,\bar{c})]^2 > 0$ when $h = \bar{c} - \gamma\underline{c} > 0$, for all $\underline{c} > 0$. The second assumption implies that utility rises if everyone's present consumption is identical and increases, whereas the third assumption implies that u is strictly concave. Moreover, the following Inada conditions hold: (i) $\lim_{h\to 0} u_h(h,\bar{c}) = \infty$ and $\lim_{h\to\infty} u_h(h,\bar{c}) = 0$, for all $\bar{c} > 0$; and (ii) $\lim_{h\to 0} [u_h(h,\bar{c}) + u_{\bar{c}}(h,\bar{c})] = \infty$ and $\lim_{h\to\infty} [u_h(h,\bar{c}) + u_{\bar{c}}(h,\bar{c})] = 0$ when $h = \bar{c} - \gamma\underline{c} > 0$, for all $\underline{c} > 0$. Finally, as follows from our discussion in the previous section, we will assume that the partial derivatives of u with respect to its two arguments are homogeneous in order to guarantee the existence of BGP's for the competitive economy and for the socially planned one.

Taking as given k_0, c_{-1}, and the sequence $\{\bar{c}_t\}_{t=0}^{\infty}$ of average consumption, each dynasty chooses the sequence of per capita consumption $\{c_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t - \gamma c_{t-1}, \bar{c}_t\right),$$

subject to the budget constraint (2.2). To ease the notation we define $u(t) = u(h_t, \bar{c}_t)$ and $u_h(t) = u_h(h_t, \bar{c}_t)$. In order to have an objective function increasing in current consumption, we need to impose that

$$u_h(t) - \beta\gamma u_h(t+1) > 0 \tag{3.1}$$

for all t. The Euler condition of the individual's problem is

$$\frac{u_h(t+1) - \beta\gamma u_h(t+2)}{u_h(t) - \beta\gamma u_h(t+1)} = \frac{1}{\beta(1 + A - \delta)}. \tag{3.2}$$

Note that the previous equation differs from the Euler equation appearing in standard models of capital accumulation in the fact that individuals take into account the effect that present consumption has in setting the reference for next period consumption. The Euler equation (3.2) characterizes the equilibrium paths of c_t and k_t when they are combined with the initial conditions on k_0 and c_{-1}, the budget constraint (2.2), the equilibrium condition $c_t = \bar{c}_t$, and the following transversality conditions:

$$\lim_{t\to\infty} \left[\beta^t u_h(t) - \beta^{t+1}\gamma u_h(t+1)\right] k_{t+1} = 0 \tag{3.3}$$

and

$$\lim_{t\to\infty} \beta^t u_h(t) c_t = 0. \tag{3.4}$$

The instantaneous utility function perceived by the social planner is

$$\hat{u}(t) \equiv \hat{u}(c_t, c_{t-1}) = u(c_t - \gamma c_{t-1}, c_t), \tag{3.5}$$

where $\hat{u}$ is strictly increasing in its first argument and strictly concave, as follows from the properties of the function $u(h_t, \bar{c}_t)$ discussed above. Let us define $\hat{u}_1(t) = \dfrac{\partial \hat{u}(c_t, c_{t-1})}{\partial c_t}$ and $\hat{u}_2(t) = \dfrac{\partial \hat{u}(c_t, c_{t-1})}{\partial c_{t-1}}$. In order to have an objective function increasing in current consumption from the social planner viewpoint, we need to impose that

$$\hat{u}_1(t) + \beta \hat{u}_2(t+1) > 0$$

for all t. The paths $\{c_t, k_t\}_{t=0}^{\infty}$ chosen by the social planner would be thus characterized by the Euler condition

$$\frac{\hat{u}_1(t+1) + \beta \hat{u}_2(t+2)}{\hat{u}_1(t) + \beta \hat{u}_2(t+1)} = \frac{1}{\beta(1 + A - \delta)}, \tag{3.6}$$

the constraint (2.2), the transversality conditions

$$\lim_{t\to\infty} \left[\beta^t \hat{u}_1(t) + \beta^{t+1} \hat{u}_2(t+1)\right] k_{t+1} = 0$$

and

$$\lim_{t\to\infty} \beta^t \hat{u}_1(t) c_t = 0,$$

and the initial conditions on k_0 and c_{-1}.

The following proposition provides, under the previously discussed homogeneity condition aimed at allowing the existence of BGP's, necessary and sufficient conditions for efficiency:

Proposition 3.1. *Assume that the instantaneous utility function $u(h, \bar{c})$ has partial derivatives that are homogeneous of the same degree. Then, the dynamic competitive equilibrium is efficient if and only if the ratio $\dfrac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)}$ is constant along the competitive equilibrium path.*

The previous proposition extends the "restricted homotheticity" condition (A.3) to a situation where habits are present. An immediate implication of the previous proposition is the following corollary:

Corollary 3.2. *Assume that the instantaneous utility function $u(h, \bar{c})$ has partial derivatives that are homogeneous of the same degree.*

(a) If the economy starts at a BGP then the competitive equilibrium is efficient.

(b) If the two arguments of $u(h, \bar{c})$ are perfect substitutes then the competitive equilibrium is efficient.

The previous Corollary tells us that some kind of complementarity between habit adjusted consumption and consumption externalities is necessary to generate inefficiency during the transition towards the steady state.

Consider the following parametrization of the utility function $u\left(h_t, \bar{c}_t\right)$:

$$u\left(h_t, \bar{c}_t\right) = \frac{\left(h_t - \theta \bar{c}_t\right)^{1-\sigma}}{1-\sigma}, \qquad \sigma > 0. \tag{3.7}$$

Note that no restriction is imposed on the sign of the parameter θ so that, if $\theta > 0$ ($\theta < 0$) average consumption decreases (increases) the utility level and increases (decreases) the marginal utility of an additional unit of the individual's habit adjusted consumption. It should also be pointed out that the functional form (3.7) collapses in a single function both the additive specification of consumption externalities found in Ljungqvist and Uhlig (2000) and the traditional specification of additive habit formation. Finally, note that the utility function (3.7) satisfies

$$\frac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)} = -\theta,$$

and, hence, the competitive equilibrium is efficient. Clearly, the two arguments of the utility function (3.7) are perfect substitutes.

Let us now consider a specification of preferences involving complementarities between the two arguments of the utility function so that the marginal rate of substitution between average consumption $\bar{c}_t$ and the habit adjusted private consumption h_t will not be constant. We generalize thus the parametrization in Galí (1994), who only considered externalities in consumption, by positing the instantaneous utility function

$$u\left(h_t, \bar{c}_t\right) = \frac{\left(h_t\right)^{1-\sigma}\left(\bar{c}_t\right)^{\theta\sigma}}{1-\sigma}, \qquad \sigma > 0. \tag{3.8}$$

The concavity of u with respect to its first argument and the linearity of h_t imply the joint concavity with respect to c_t and c_{t-1} of the function $u\left(c_t - \gamma c_{t-1}, \cdot\right)$, which is the relevant concavity needed to solve the consumer's problem in a competitive economy. Moreover, we should also impose the conditions $\theta < 1$ and $\frac{\theta}{1-\sigma} \geq 0$, which guarantee the concavity of the utility function $\hat{u}$ perceived by the social planner,

$$\hat{u}\left(c_t, c_{t-1}\right) \equiv u(c_t - \gamma c_{t-1}, c_t) = \frac{\left(c_t - \gamma c_{t-1}\right)^{1-\sigma}\left(c_t\right)^{\theta\sigma}}{1-\sigma}. \tag{3.9}$$

Note that the case $\theta > 0$ corresponds to the typical "keeping up with the Joneses" formulation since the average consumption of the other individuals makes more valuable an additional unit of own (habit adjusted) consumption. In the case $\theta < 0$

average consumption lowers the marginal utility of own consumption. We see thus that the consumption externality introduces a scale factor to the marginal utility derived from present consumption (once it has been adjusted by the corresponding past reference).[1] Under this formulation, we have,

$$\frac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)} = \left(\frac{\theta\sigma}{1-\sigma}\right)\left(\frac{h_t}{c_t}\right) = \frac{\theta\sigma}{1-\sigma}\left(1 - \frac{\gamma}{x_t}\right),$$

where $x_t \equiv c_t / c_{t-1}$ is the gross rate of consumption growth. We show in the Appendix B that the model with the utility function (3.8) exhibits saddle path stability and a non-constant growth rate x_t during the transition. Therefore, since the gross rate x_t of consumption growth is not constant off the BGP, we can conclude that in this case the competitive path is not efficient during the transition. However such an inefficiency vanishes in the long run as x_t approaches its stationary value x. In fact, it can be shown that the rate of convergence of the competitive economy is lower than that of the corresponding socially planned economy under this specification of the instantaneous utility function (see Alonso et al., 2001). The intuition behind this suboptimality low speed of convergence lies in the fact that consumption externalities generate inefficiency because they affect the interaction between habits and current consumption. To see this, we just have to observe that present consumption has two countervailing effects on the objective function. Present consumption increases current utility whereas it reduces future utility. The latter effect is due to the increase in the standard of living respect to which future consumption will be compared with. Decision makers try to minimize the effect of habits on current utility while maximizing simultaneously the utility from current consumption. In other words, they also use current consumption to outweigh the negative effect of habits. Note that habits become less important when the marginal utility of consumption increases. In the presence of consumption externalities, the marginal utility of consumption in the socially planned economy differs from the one in the competitive economy. More precisely, since the concavity of the utility perceived by the social planner requires that $\frac{\theta}{1-\sigma} \geq 0$, the marginal utility of consumption in the socially planned economy is always larger than in the competitive one, i.e., $\hat{u}_1(t) = u_h(t) + u_{\bar{c}}(t) > u_h(t)$. Therefore, in the socially planned solution habits turn out to be less important than in the competitive equilibrium and, as the transition is driven by the habits, the rate of convergence increases when habits become less important.

One implication of the previous discussion is that optimal taxation in this scenario should consist on accelerating the rate of convergence when the economy is adjusting towards its BGP. This could be achieved for instance by taxing (subsidizing) capital income if the economy is growing faster (slower) than in its BGP.

[1] The functional form of u given in (3.8) could be written as

$$u(h_t, \bar{c}_t) = \frac{(h_t)^{\sigma_1}(\bar{c}_t)^{\sigma_2}}{\sigma_1}, \quad \sigma_1 < 1.$$

As in Galí (1994), we make $\sigma_1 = 1-\sigma$ and $\sigma_2 = \theta\sigma$ so that σ can be interpreted as the intertemporal elasticity of substitution of consumption if both habits and consumption spillovers were absent, and θ is the ratio of the elasticities of marginal utility of habit adjusted consumption with respect to average consumption and with respect to habit adjusted consumption.

4. Conclusion

In this paper we have shown that consumption externalities are not necessarily a source of inefficiency. In particular, when habits are not present and both the competitive and the socially planned economy exhibit a BGP, consumption spillovers do not generate any kind of sub-optimality. This is so because the existence of a BGP's makes the functional form of the competitive marginal rate of substitution of consumption between two periods identical to the efficient marginal rate of substitution. When habits are introduced in the individuals' utility function in such a way that habit adjusted consumption is a perfect substitute for the average consumption in the economy, the previous identity between the two marginal rates of substitution is preserved and, again, no public intervention is needed to restore efficiency. However, such an identity between marginal rates of substitution is not longer obtained when habit adjusted consumption and average consumption are not perfect substitutes.

A possible extension of our analysis will be the introduction of "external habits". Under this kind of habits the average past consumption of the economy becomes the relevant standard of living that is used to evaluate the utility accruing from present consumption.[2]

[2]External habits are used in the stochastic models of Constantinides (1990), Abel (1999), Campbell and Cochrane (1999), and Ljungqvist and Uhlig (2000). Moreover, the social norms appearing in the capital accumulation model of de la Coix (1998) play also the role of external habits.

Appendix

A. Proofs

Proof of Proposition 2.1. The first step of the proof is to show that the functions v_1, v_2 and $\hat{u}'$ are all homogeneous of the same degree. Note that

$$\hat{u}'(c) = u_c(c,c) + u_{\bar{c}}(c,c) = v_1(c) + v_2(c), \tag{A.1}$$

so that, if the functions v_1 and $\hat{u}'$ are homogeneous of degree κ_1 and κ_2, respectively, we have that

$$\mu^{\kappa_2}\hat{u}'(c) = \hat{u}'(\mu c) = v_1(\mu c) + v_2(\mu c) =$$
$$\mu^{\kappa_1} v_1(c) + v_2(\mu c), \quad \text{for all } \mu \in \mathbb{R}_{++} \text{ and } \ c \in \mathbb{R}_{++}.$$

Let us proceed by contradiction and assume that $\kappa_1 \neq \kappa_2$. Then, after dividing by μ^{κ_2} and rearranging, the previous expression becomes

$$\hat{u}'(c) - \mu^{\kappa_1-\kappa_2} v_1(c) = \frac{1}{\mu^{\kappa_2}} v_2(\mu c), \qquad \text{for all } \mu \in \mathbb{R}_{++} \text{ and } \ c \in \mathbb{R}_{++}. \tag{A.2}$$

Hence, for any arbitrarily given value $c \in \mathbb{R}_{++}$, there exists a value $\mu^* \in \mathbb{R}_{++}$ such that $\hat{u}'(c) - (\mu^*)^{\kappa_1-\kappa_2} v_1(c) = 0$, which in turn implies that $\dfrac{1}{\mu^{\kappa_2}} v_2(\mu^* c) = 0$, and this is impossible by assumption. Thus, $\kappa_1 = \kappa_2$, so that (A.2) becomes

$$\hat{u}'(c) - v_1(c) = \frac{1}{\mu^{\kappa_1}} v_2(\mu c), \qquad \text{for all } \mu \in \mathbb{R}_{++} \text{ and } \ c \in \mathbb{R}_{++},$$

which combined with (A.1) implies that

$$v_2(\mu c) = \mu^{\kappa_1} v_2(c),$$

and this is the desired conclusion.

As the functions v_1 and v_2 are homogeneous of the same degree, the ratio $\frac{v_2(c)}{v_1(c)}$ is constant. Clearly, for all pairs $(c, c') \in \mathbb{R}^2_{++}$ we have that

$$\frac{v_2(c')}{v_1(c')} = \frac{\left(\frac{c'}{c}\right)^{\kappa} v_2(c)}{\left(\frac{c'}{c}\right)^{\kappa} v_1(c)} = \frac{v_2(c)}{v_1(c)}. \tag{A.3}$$

Let us define the constant $\varsigma = \dfrac{v_2(c)}{v_1(c)}$. Note that $\varsigma > -1$ since $\hat{u}'(c) > 0$, $v_1(c) > 0$ and

$$\hat{u}'(c) = v_1(c) + v_2(c) = (1+\varsigma) v_1(c). \tag{A.4}$$

We see that the right hand sides of the Euler equations (2.4) and (2.6) are identical. Moreover, their left hand sides have also the same functional form since

$$\frac{\hat{u}'(c_{t+1})}{\hat{u}'(c_t)} = \frac{v_1(c_{t+1}) + v_2(c_{t+1})}{v_1(c_t) + v_2(c_t)} = \frac{(1+\varsigma) v_1(c_{t+1})}{(1+\varsigma) v_1(c_t)} =$$

$$\frac{v_1(c_{t+1})}{v_1(c_t)} = \frac{u_c\left(c_{t+1}, c_{t+1}\right)}{u_c\left(c_t, c_t\right)}.$$

Furthermore, the transversality conditions (2.5) and (2.7) are also equivalent as can be seen from (A.4). Recall also that both economies face the same constraint (2.2). Therefore, given the same initial condition on k_0, the path $\{c_t, k_t\}_{t=0}^{\infty}$ that solves the social planner's problem constitutes a competitive equilibrium. ■

Proof of Proposition 3.1. Recall that the Euler equation for the individual problem in a competitive economy is (3.2). The Euler equation (3.6) for the socially planned economy becomes

$$\frac{u_h(t+1) + u_{\bar{c}}(t+1) - \beta\gamma u_h(t+2)}{u_h(t) + u_{\bar{c}}(t) - \beta\gamma u_h(t+1)} = \frac{1}{\beta(1+A-\delta)}, \tag{A.5}$$

since $\hat{u}_1(t) = u_h(t) + u_{\bar{c}}(t)$, $\hat{u}_2(t+1) = -\gamma u_h(t+1)$ when $c_t = \bar{c}_t$. As the right hand sides of the two Euler equations (3.2) and (A.5) are identical, the competitive allocation will coincide with the one selected by the social planner if and only if the left hand sides of (3.2) and (A.5) have the same functional form along the competitive consumption path. Therefore, taking into account that in equilibrium $\bar{c}_t = c_t$, the competitive path of consumption $\{c_t\}_{t=0}^{\infty}$ is efficient if and only if

$$\frac{u_h(h_{t+1}, c_{t+1}) - \beta\gamma u_h(h_{t+2}, c_{t+2})}{u_h(h_t, c_t) - \beta\gamma u_h(h_{t+1}, c_{t+1})} = \frac{u_h(h_{t+1}, c_{t+1}) + u_{\bar{c}}(h_{t+1}, c_{t+1}) - \beta\gamma u_h(h_{t+2}, c_{t+2})}{u_h(h_t, c_t) + u_{\bar{c}}(h_t, c_t) - \beta\gamma u_h(h_{t+1}, c_{t+1})}.$$

for all t. The previous expression simplifies to

$$\frac{u_{\bar{c}}(h_{t+1}, c_{t+1})}{u_{\bar{c}}(h_t, c_t)} = \frac{u_h(h_{t+1}, c_{t+1}) - \beta\gamma u_h(h_{t+2}, c_{t+2})}{u_h(h_t, c_t) - \beta\gamma u_h(h_{t+1}, c_{t+1})}.$$

That is, the competitive solution will be efficient if and only if

$$u_{\bar{c}}(h_t, c_t) = \varsigma\left[u_h(h_t, c_t) - \gamma\beta u_h(h_{t+1}, c_{t+1})\right], \tag{A.6}$$

for all t and for some constant ς along the competitive equilibrium path of consumption. Define the gross rate of growth of the marginal utility of habit adjusted consumption,

$$f_t = \frac{u_h\left(t+1\right)}{u_h\left(t\right)},$$

and divide (A.6) by $u_h(h_t, c_t)$ to obtain

$$\frac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)} = \varsigma\left[1 - \gamma\beta f_t\right]. \tag{A.7}$$

Using the functional form of h_t, we can write the Euler equation (3.2) for the competitive economy as

$$f_{t+1} = \frac{1}{\beta(1+A-\delta)}\left(1 - \frac{1}{\beta\gamma f_t}\right) + \frac{1}{\beta\gamma}. \tag{A.8}$$

The difference equation (A.8) has two stationary equilibria: $f = \frac{1}{\beta(1+A-\delta)}$ and $\breve{f} = \frac{1}{\beta\gamma}$, with $\breve{f} > f$ since

$$1 + A - \delta > 1 > \gamma.$$

On the one hand, the stationary equilibrium $\breve{f}$ is locally stable but violates the monotonicity condition (3.1) . To see this, we only have to notice that (3.1) becomes

$$\beta\gamma f < 1.$$

at a BGP. On the other hand, the stationary equilibrium f is unstable and, thus, the equilibrium path of the variable f_t exhibits no transition. Since $f_t = f$ for all t, condition (A.7) becomes

$$\frac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)} = \varsigma\left[1 - \beta\gamma f\right] \equiv \vartheta, \tag{A.9}$$

for some constant ϑ. Obviously both the competitive economy and the planned one face the same budget constraint (2.2). Furthermore, it is immediate to see that the transversality conditions of the two economies are equivalent under our assumptions. ■

Proof of Corollary 3.2.

(a) If all the partial derivatives of u are homogeneous of degree κ then, along a BGP with a gross rate of consumption growth x (and, thus, with $f = x^{\kappa}$), it holds that

$$\frac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)} = \frac{(c_t)^{-\kappa} u_{\bar{c}}\left(\left(\frac{h_t}{c_t}\right), 1\right)}{(c_t)^{-\kappa} u_h\left(\left(\frac{h_t}{c_t}\right), 1\right)} = \frac{u_{\bar{c}}\left(\left(\frac{h_t}{c_t}\right), 1\right)}{u_h\left(\left(\frac{h_t}{c_t}\right), 1\right)} = \frac{u_{\bar{c}}\left(1 - \frac{\gamma}{x}, 1\right)}{u_h\left(1 - \frac{\gamma}{x}, 1\right)} = \vartheta,$$

for all t and for some constant ϑ. Therefore, condition (A.9) holds at a BGP.

(b) If the two arguments of the function u are perfect substitutes, the marginal rate of substitution $\dfrac{u_{\bar{c}}(h_t, c_t)}{u_h(h_t, c_t)}$ is constant and, thus, condition (A.9) holds. Therefore, when consumption externalities interact additively with the habit adjusted consumption, the competitive equilibrium is always efficient. ■

B. Dynamic Analysis

Using the functional form (3.8) for the instantaneous utility function and the condition $c_t = \bar{c}_t$ for a symmetric equilibrium, we obtain that the marginal utility appearing in the Euler equation (3.2) becomes in equilibrium

$$u_h(t) = (c_t - \gamma c_{t-1})^{-\sigma} c_t^{\theta\sigma}.$$

Therefore, using the previously defined variables x_t and f_t, we obtain the following difference equation for this specification of u:

$$g(x_{t+1}, x_t, f_t) \equiv \left(\frac{x_t - \gamma}{x_{t+1} - \gamma}\right)(x_{t+1})^{\theta} - x_t (f_t)^{\frac{1}{\sigma}} = 0. \tag{B.1}$$

Moreover, defining the variable $z_t \equiv k_t / c_{t-1}$, the budget constraint (2.2) becomes

$$z_{t+1} = \left(\frac{z_t}{x_t}\right)(1 + A - \delta) - 1. \tag{B.2}$$

The system of first order difference equations (A.8), (B.1) and (B.2), together with the initial condition $z_0 = k_0 / c_{-1}$ and the transversality conditions (3.3) and (3.4), fully describes the equilibrium path of the variables f_t, x_t, and z_t. The system has two control variables, f_t and x_t, and one state variable, z_t.

Since along a BGP consumption and capital grow at constant rates, it follows from (2.3) that the ratio c_t / k_t should be constant. Hence, capital, consumption and income per capita must all grow at the same rate along a BGP. Let x be this common stationary rate of growth. From the definition of z_t, if follows that z_t is constant along a BGP. Finally, it is also clear from (B.1) that f_t is also constant along a BGP. In fact, we have also shown in the proof of Proposition 3.1 that f_t is constant for all t. Let f and z be the steady state values of f_t and z_t. Making $x_t = x$, $f_t = f$, and $z_t = z$ for all t in the system of equations (A.8), (B.1) and (B.2), and solving for f, x and z, we get the following steady state values of the transformed variables of the model:

$$f = \frac{1}{\beta(1 + A - \delta)},$$

$$x = f^{\frac{-1}{\sigma(1-\theta)}},$$

and

$$z = \frac{x}{(1 + A - \delta) - x}.$$

It can also be checked easily that the transversality conditions (3.3) and (3.4) are satisfied by an equilibrium path converging to the BGP.

Since the variable f_t displays no transition, we can linearize around its steady state the system formed by the difference equations (B.1) and (B.2) with $f_t = f$. The eigenvalues of the corresponding matrix of partial derivatives are the following:

$$\lambda_1 = \frac{\partial x_{t+1}}{\partial x_t} = -\frac{\dfrac{\partial g}{\partial x_t}}{\dfrac{\partial g}{\partial x_{t+1}}} = \frac{\gamma}{x - \theta(x - \gamma)},$$

and

$$\lambda_2 = \frac{\partial z_{t+1}}{\partial z_t} = \frac{1 + A - \delta}{x}.$$

Note that the inequality $x > \gamma$ must hold in equilibrium in order to have a well defined function u along the BGP, that is, with $h_t > 0$. Thus, since $\theta < 1$, $\gamma \in (0, 1)$ and $x > \gamma$, we have that $\lambda_1 \in (0, 1)$. Finally, we obtain that $\lambda_2 > 1$, since $z > 0$. Therefore, we can immediately conclude that the steady state of the previous system of difference equations is locally saddle path stable. Therefore, the variables x_t and z_t exhibit transition off the BGP.

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