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A test between matching theories

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Abstract

This paper tests whether aggregate matching is consistent with random matching or stock-flow matching. Using U.K. matching data and correcting for temporal aggregation bias, estimates of the random matching function are consistent with previous work in this field. The data however support the 'stock-flow' matching hypothesis. Estimates find that around 50% of newly unemployed workers match quickly. The remaining workers match slowly, their re-employment rates depending statistically on the inflow of new vacancies and not on the vacancy stock. The interpretation is that these latter workers, having failed to match with existing vacancies, wait for new, more suitable vacancies to come onto the market. The results have important policy implications, particularly for long term unemployment and the design of optimal unemployment insurance programs.

Keywords: Stock-flow matching, Random matching, Job queues, Temporal aggregation.

JEL Classification: J3, J6.

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1   Introduction

The random matching approach has provided an important framework for analyzing labor market policy (Pissarides, 2000). But the empirical literature, when estimating the random matching function, rarely tests the matching function against a meaningful alternative (see for example Blanchard and Diamond, 1989, and Petrongolo and Pissarides, 2001, for a recent survey). This paper uses matching data to test between the random matching hypothesis and stock-flow matching.

Stock-flow matching assumes that when laid-off, a worker contacts friends, consults situations vacant columns in newspapers, perhaps registers with job agencies, and so observes the stock of vacancies currently on the market. If the worker is lucky and a suitable vacancy already exists, the worker can quickly exit unemployment. If a suitable vacancy does not exist, the worker then has to wait for something suitable to come onto the market. This worker does not then match with the stock of vacancies - the stock has already been sampled and no match exists. The worker instead matches with the inflow of new vacancies coming onto the market. The problem is symmetric for vacancies. If a firm has a vacancy, it might first ask employees if they know someone suitable, advertise the post in a situations vacant column etc.. If the firm is lucky, a suitable worker already exists in the stock of unemployed workers and the post is quickly filled. If not, the firm has to wait for someone suitable to come onto the market. This implies "stock-flow" matching as the stock of unmatched agents on one side of the market matches with the inflow of new agents on the other side. Papers in this literature include Taylor (1995), Coles and Smith (1998), Coles and Muthoo (1998), Coles (1999), Lagos (2000) and Gregg and Petrongolo (2005), but also see Jones and Riddell (1999).

Lagos (2000) perhaps provides the most useful perspective for understanding the results obtained here. Using a taxi market analogy, he supposes that cabs meet potential customers at taxi ranks. At the micro-level there is stock-flow matching, i.e. at any given taxi rank there is either a stock of customers waiting for cabs, or a stock of cabs waiting for customers. Aggregation over all taxi ranks and the restriction to *steady state* imply that at the macro-level, the flow number of cab rides depends only on the total stock of taxi cabs and on the total stock of potential customers in the market. Casual introspection also suggests there will be constant returns to matching: doubling the total number of participants should double the (steady state flow) number of taxi rides. Aggregate matching seemingly has the properties of a standard random matching function, even with stock-flow matching at the micro-level.

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This view of matching is consistent with the fact that around 25-30% of new vacancies posted in U.K. Job Centers are filled on the first day (Coles and Smith, 1998). Burdett and Cunningham (1997) also report for the U.S. that most vacancies (55%) are filled within a week. This suggests that for many vacancies matching frictions may not be a significant factor. Instead such vacancies are snapped up by workers who have been waiting for something suitable to come onto the market. This view of matching also reconciles the McDonald's problem: that McDonald's invariably have vacancies and everyone knows where McDonald's is, so how can unemployment be frictional? It cannot be argued that McDonald's jobs are 'bad' jobs, otherwise no-one would work there.[1] Stock-flow matching instead implies that most unemployed workers are better qualified to do different work and wait for more suitable vacancies to come onto the market. McDonald's then hires from the inflow of workers into unemployment, hiring those who have a comparative advantage in working for them. Coles (1999) establishes that trade in such markets is characterised by a turnover externality: higher entry rates of new participants reduces the time lost waiting for suitable matches to enter the market.

The equivalence between random matching and stock-flow matching, as suggested by Lagos (2000), only holds in steady state. Outside of a steady state, stock-flow matching at the micro-level implies that a higher inflow of new vacancies, say into an unemployment black spot, will yield an immediate increase in matches. Consider for example Figure 1 which describes aggregate matching in the U.S. manufacturing sector (a monthly time series, taken from Blanchard and Diamond, 1989). A critical feature of this time series is that *the number of matches in each month is much more volatile than the stock of vacancies.* This implies the inflow of new vacancies cannot be smooth over time - if it were, a large increase in the number of matches would necessarily result in a large fall in the stock of vacancies. These data therefore imply that a spike in the number of matches coincides with a spike in the inflow of new vacancies and *not* with a spike in the stock of vacancies. The standard random matching approach is inconsistent with this interpetation of the data.

The underlying point is that outside of a steady state, random matching and stock-flow matching imply quite different equilibrium hazard rates of re-employment. In Shapiro and Stiglitz (1984) for example, the re-employment hazard rate of an unemployed worker, denoted λ, is simply $\lambda = v/U$ where v is the flow of new vacancies into the market, which match immediately (and randomly) with one unemployed worker, and U describes the current stock of unemployed workers. We shall refer to this case as job queueing. In contrast,

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[1]Although a popular term, the notion of a 'good' or 'bad' job is not particularly helpful as jobs are either well or badly compensated. Stock-flow matching implies the monopsonist chooses wages to maximise expected profit given the inflow of new entrants into the market.
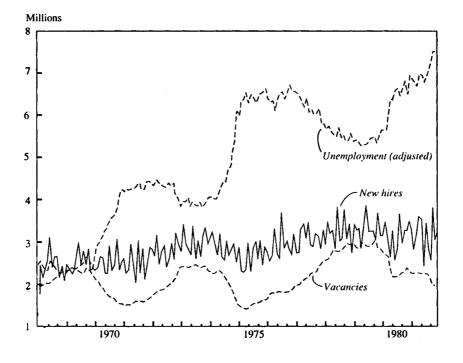
Figure 1: New hires, unemployment and vacancies in the U.S., 1968-1981. Source: Blanchard and Diamond (1989).

the random matching approach assumes the re-employment hazard rate depends on the vacancy/unemployment ratio, where frictions imply it is the stock of vacancies V which is the relevant state variable (throughout upper case refers to stock variables, lower case to flow variables). The stock-flow explanation instead implies there are two parameters of interest. Proportion p of laid-off workers are on the short side of their market and are re-employed immediately (or at least within a very short period of time). Proportion $(1-p)$ are on the long-side of their market and so chase new vacancies. Their subsequent hazard rate λ depends on the flow of vacancies into their particular specialization. Clearly this latter hazard rate λ has a similar structure to the job queueing hazard, depending on v rather than V.

Consistent with Lagos (2000), we find that the random matching function does indeed provide a good fit of the UK aggregate matching data and the hypothesis of constant returns is accepted (see column 2, Table 1 in the text). The critical over-identifying test for the random matching approach is that the inflow of new vacancies should not have any additional explanatory power for the matching rates of workers (given we control for temporal aggregation of the data). We find instead that the vacancy inflow coefficient is highly significant. The principal difficulty for the random matching approach, however, is that once vacancy inflow is included as a conditioning variable, the estimated vacancy stock term becomes wrong–signed. The results instead find that a stock-flow matching specification provides a much better fit of the data. The central finding is that the longer-term unemployed match with the inflow of new vacancies which implies these workers are waiting for suitable vacancies rather than facing high matching frictions. As pointed out in the conclusion, this has important implications for the design of optimal unemployment insurance schemes.

The paper is structured as follows. Section 2 discusses identification. It argues that with unobserved search effort, a distinguishing test between the two matching frameworks is not possible using microdata, but that identification is possible on macrodata. Section 3 shows how to distinguish between competing matching frameworks, while controlling econometrically for temporal aggregation bias. Section 4 describes the data used. Section 5 presents our results. Section 6 concludes.

2 Identification on aggregate data

The hazard function literature establishes there is substantial variation in re-employment rates across unemployed workers (see Machin and Manning (1999) for a recent survey). To understand how worker heterogeneity may affect identification, suppose unemployed worker

i at time t makes search effort k_{it} and let

$$K_t = \sum_{i=1}^{U_t} k_{it}$$

denote aggregate search effort across the U_t unemployed. The standard random matching approach (e.g. Pissarides (2000)) assumes the re-employment rate of this worker, denoted λ_{it}, is

$$\lambda_{it} = \frac{k_{it}}{K_t} M(K_t, V_t)$$

where V_t is the stock of vacancies at time t and $M(.)$ is a standard matching function.

Consider instead a job queueing framework. In this world the stock of unemployed workers match with new vacancies as they are created. Using the taxi rank analogy, suppose unemployed worker i makes effort k_{it} to catch the next job. Then this worker's re-employment rate is

$$\lambda_{it} = \frac{k_{it}}{K_t} v_t$$

where v_t is the inflow of new vacancies and k_{it}/K_t is the probability that worker i gets the next job to come onto the market.

The aim of the paper is to identify which better explains the re-employment rates of unemployed workers: is it the stock of vacancies (as implied by random matching) or the inflow of new vacancies (as implied by job waiting)? Unfortunately it is not possible to identify either of these processes on microdata. For most individuals we have only one data point - that individual's observed unemployment spell (a starting and end-date) - and we cannot control for the fixed effect k_{it}. We might potentially control for the fixed effect by focussing on individuals with repeat unemployment spells, but such individuals are a biased sample of the market.

Note also that this matching structure generates congestion effects, where greater job search effort by one worker reduces the matching probability of other workers (via an increase in aggregate K). Unobserved search heterogeneity implies a negative correlation in individual worker re-employment rates - if one worker gets the job, the others do not. Aggregating across workers, however, nets out these congestion effects. For example job queueing implies individual hazard rate $\lambda_{it} = k_{it}v_t/K_t$ but aggregating over i implies average re-employment rate

$$\overline{\lambda}_t = \frac{1}{U_t} \sum_{i=1}^{U_t} \lambda_{it} = \frac{1}{U_t} \sum_{i=1}^{U_t} \frac{k_{it}}{K_t} v_t = \frac{v_t}{U_t}.$$

Note the average re-employment rate is driven by the inflow of new vacancies, there is

crowding out by the unemployment stock, but pure crowding out implies the average re-employment rate is independent of the distribution of search efforts $\{k_{it}\}$.[2]

The random matching approach instead implies $\lambda_{it} = \frac{k_{it}}{K_t} M(K_t, V_t)$ and aggregating over i implies average re-employment rate

$$\overline{\lambda}_t \equiv \frac{1}{U_t} \sum_{i=1}^{U_t} \lambda_{it} = \frac{1}{U_t} \sum_{i=1}^{U_t} \frac{k_{it}}{K_t} M(K_t, V_t) = \frac{1}{U_t} M(K_t, V_t).$$

If there are constant returns to matching then the average re-employment rate simplifies to

$$\overline{\lambda}_t = M(\overline{k}_t, \frac{V_t}{U_t})$$

where $\overline{k}_t = K_t/U_t$ is average search effort. As there is only partial crowding out, the average re-employment rate depends on average search effort but is otherwise independent of the distribution of search efforts $\{k_{it}\}$.

As $\overline{k}_t$ is unobserved, a critical identifying assumption is that $\overline{k}_t$ changes slowly and systematically over time. This seems reasonable as the unemployment and vacancy stocks change slowly over time (see Figure 1 for the U.S.).[3] In that case we can control for unobserved changes in aggregate job search effort by using time dummies. In particular using month dummies to capture seasonal variations in job search effort (for example the job market is quiet in August) and year dummies to capture business cycle variations, random matching implies observed short-run variations in the average re-employment rates of unemployed workers are driven by variations in the vacancy-unemployment ratio. Job queueing instead implies those variations are driven by variations in vacancy inflow. A second approach, discussed further in the data section, is to use an HP filter which takes out all trends.

3 The Empirical Framework

The data do not record unemployment and vacancy stocks over the month. Instead we have information on the stocks available at the start of each month and the gross inflows during that month. From now on we adopt the following time notation. U_n denotes the stock of unemployed workers at the beginning of month $n \in \mathcal{N}$ and V_n denotes the stock of vacancies. u_n denotes the total inflow of newly unemployed workers within the month and v_n denotes the inflow of new vacancies.

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[2] This is not true for the distribution of expected unemployment spells. As $1/\lambda_{it}$ is a convex function of k_{it}, a mean preserving spread in k_{it} yields an increase in the average expected unemployment spell. Indeed if $k_{it} = 0$ for one individual, the average spell is infinity.

[3] In a job search framework this implies job offer arrival rates change slowly over time and so aggregate job search effort will also change slowly over time.

We suppose that at time $t \in [n, n+1]$ in month n, the average re-employment probabilities of unemployed workers are denoted by a pair $(p(t), \lambda(t))$. $p(t)$ is the proportion of workers laid off at date t who find immediate re-employment, while $\lambda(t)$ is the average re-employment rate of workers who have been unemployed for some (strictly positive) period of time. The competing theories suggest alternative functional forms for p, λ.

As previously described, the random matching approach implies $p = 0$ - it takes time to find work - and the **average** re-employment rate, now simply denoted λ, is $\lambda = M(kU, V)/U$, where k is average search effort. Constant returns to matching implies a functional form $\lambda = \lambda^M(V/U, k)$.

Pure job queueing as described by Shapiro and Stiglitz (1984) also implies $p = 0$ - it takes time to find work - but in this case the average re-employment rate $\lambda = v/U$ where the stock of unemployed workers U match with the inflow of new vacancies v. For econometric purposes we consider a more general specification of the form $\lambda = \lambda^Q(v, U)$.

Stock-flow matching is a generalization of the job queueing approach. Shapiro and Stiglitz (1984) implies all unemployed workers are on the long side of the market. In reality, some suitably skilled workers may find themselves on the short side of the market and can quickly find re-employment. We suppose with probability $p = p^{SF}$, a newly unemployed worker finds there is a suitable vacancy already on the market and so becomes (very quickly) re-employed. With probability $1 - p^{SF}$ there is no suitable vacancy and the worker must then wait for a suitable vacancy to come onto the market. Job queueing implies average re-employment rate $\lambda = \lambda^{SF}(v, U)$. The same argument implies that each new vacancy may be either on the short or long side of the market. Vacancy queueing - waiting for a suitably skilled worker to come onto the market - implies average matching rate $\mu = \mu^{SF}(u, V)$ where u is the inflow of newly unemployed workers onto the market. Symmetry suggests functional form $p^{SF} = p^{SF}(V, u)$ where laid off workers on the short side of the market match with vacancies on the long side. Pure job queueing (e.g. Shapiro and Stiglitz (1984)) implies one-sided stock flow matching and the testable restriction $p^{SF} = 0$.

We now show how to identify (p, λ) using data which is reported as a monthly time series. As the identifying equations depend on the assumed theory, we consider each case separately, starting with random matching.

3.1   Temporal Aggregation with Random Matching.

To obtain a discrete time representation of the underlying continuous time matching process, we assume the inflows of new agents, u_n and v_n, are constant within a month. We construct at risk measures for the stock of vacancies and unemployed workers by considering a rep-

resentative worker who chooses average search effort $\overline{k}_n$ over the month and so matches at average rate λ_n. Temporal aggregation then implies the expected total number of matches in month n is

$$M_n = U_n[1 - e^{-\lambda_n}] + \int_{t=0}^{1} u_n \left[1 - e^{-\lambda_n(1-t)}\right] dt$$

where the first term describes the number of workers in the initial unemployment stock who match within the month, and the second describes those who become unemployed at time $t \in [0,1]$ within the month and who match by the end of it. Integration implies

$$M_n = \overline{U}_n[1 - e^{-\lambda_n}] \tag{1}$$

where

$$\overline{U}_n = U_n + \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n[1 - e^{-\lambda_n}]} u_n. \tag{2}$$

(1) implies the expected number of matches in month n is $\overline{U}_n$, the number of unemployed workers 'at-risk' in month n, times $[1 - e^{-\lambda_n}]$, which is the matching probability of an unemployed worker over the entire month. To see that $\overline{U}_n$ is an 'at-risk' measure, suppose $\lambda_n \to 0$ (i.e. each unemployed worker in month n matches very slowly). The definition of $\overline{U}_n$ (equation (2)) implies $\overline{U}_n \to U_n + 0.5u_n$. Given nobody finds work ($\lambda_n = 0$) then each newly unemployed worker in month n is, on average, unemployed in that month for exactly half of it (given the identifying assumption that newly unemployed workers enter the market at a uniform rate). Hence $U_n + 0.5u_n$ measures the average number of unemployed workers at risk over the whole month.

Suppose instead $\lambda_n \to \infty$; all workers immediately find work. (2) implies $\overline{U}_n \to U_n + u_n$. $U_n + u_n$ is the relevant at-risk measure of unemployment for this case as each unemployed worker who enters the market matches immediately. (2) therefore computes the 'at risk' measure of unemployment for any arbitrary matching rate $\lambda_n \geq 0$ (given data U_n, u_n), and $M_n = \overline{U}_n[1 - e^{-\lambda_n}]$ is then the predicted number of matches.

The above temporal aggregation argument also applies to vacancies. If vacancies match at average rate μ_n in month n and v_n describes the total inflow of new vacancies within the month, then the relevant at-risk measure for vacancies is

$$\overline{V}_n = V_n + \frac{e^{-\mu_n} - 1 + \mu_n}{\mu_n[1 - e^{-\mu_n}]} v_n, \tag{3}$$

and expected matches are given by

$$M_n = \overline{V}_n[1 - e^{-\mu_n}].$$

Note, this implies the identifying restriction

$$\overline{U}_n[1 - e^{-\lambda_n}] = \overline{V}_n[1 - e^{-\mu_n}], \tag{4}$$

as the expected number of workers who match must equal the expected number of vacancies that match.

Given data $\{U_n, V_n, u_n, v_n\}$, we estimate λ_n by adopting the functional form

$$\lambda_n = \lambda^M(\overline{U}_n, \overline{V}_n; \theta). \tag{5}$$

This specification says that the average re-employment rate λ_n in month n does not depend on the initial stocks U_n, V_n, but on the number of unemployed workers and vacancies who try to match over the entire month. The parameter set θ potentially contains time dummies to proxy for systematic variations in average search effort.

Given parameters θ and the data, equations (2)-(5) can be solved numerically for the 4 unknowns $\overline{U}_n, \overline{V}_n, \lambda_n$ and μ_n. Conditional on θ, the predicted number of matches is

$$M_n(\theta) = \overline{U}_n[1 - e^{-\lambda_n}], \tag{6}$$

where the identifying restriction (4) implies the matching rates of workers are consistent with the matching rates of vacancies.[4] Given data on actual matches, Section 5 provides maximum likelihood estimates of θ. The next section, however, derives the identifying equations for stock-flow matching.

3.2   Temporal Aggregation with Stock-Flow Matching.

Suppose now that during month n, proportion p_n of newly unemployed workers are on the short side and so match immediately (within the month), and all other unemployed workers are on the long side and match at average rate λ_n. Given (p_n, λ_n), the expected number of matches is

$$M_n = U_n[1 - e^{-\lambda_n}] + p_n u_n + \int_{t=0}^{1} (1 - p_n)u_n \left[1 - e^{-\lambda_n(1-t)}\right] dt.$$

The first term describes the number of workers in the original stock who match within the month, the second describes those who become unemployed but being on the short side of the market quickly re-match, the third describes those who do not find employment immediately but manage to find employment before the end of the month. Integration implies the temporally aggregated matching function

$$M_n = U_n[1 - e^{-\lambda_n}] + u_n(1 - p_n)\frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n} + p_n u_n.$$

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[4]In contrast, Gregg and Petrongolo (2005) do not compute these at risk measures and instead estimate (3) assuming $\lambda = \lambda^M(U_n, V_n; \theta)$ which ignores the matching effects due to the inflow of new vacancies.

In this case, the appropriate at-risk measure for unemployment is

$$\overline{U}_n = U_n + u_n(1 - p_n)\frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n[1 - e^{-\lambda_n}]}, \tag{7}$$

as the temporally aggregated matching function then reduces to

$$M_n = \overline{U}_n[1 - e^{-\lambda_n}] + p_n u_n.$$

Note, stock-flow matching implies matches can be decomposed into those workers who are on the long side of the market and who match slowly (at average rate λ_n) and those newly unemployed who are on the short side and match quickly (proportion p_n).

The argument is symmetric for vacancies. Suppose proportion q_n of new vacancies match immediately (with the stock of longer term unemployed workers), and all other vacancies match at average rate μ_n. Define the at-risk measure for the stock of vacancies

$$\overline{V}_n = V_n + v_n\frac{e^{-\mu_n} - 1 + \mu_n}{\mu_n[1 - e^{-\mu_n}]}(1 - q_n). \tag{8}$$

The argument above implies expected matches satisfy

$$M_n = \overline{V}_n[1 - e^{-\mu_n}] + q_n v_n.$$

As in the random matching case, the expected number of unemployed workers who match must equal the expected number of vacancies that match. Stock-flow matching, however, implies two identifying restrictions. First as the stock of (longer term) unemployed workers matches with the inflow of new vacancies, we have

$$q_n v_n = (1 - e^{-\lambda_n})\overline{U}_n. \tag{9}$$

Second, newly unemployed workers (who match immediately) match with the stock of vacancies and so

$$p_n u_n = (1 - e^{-\mu_n})\overline{V}_n. \tag{10}$$

A closed form econometric structure is then obtained by specifying

$$\lambda_n = \lambda^{SF}(v_n, \overline{U}_n; \theta); \tag{11}$$
$$p_n = p^{SF}(u_n, \overline{V}_n; \theta) \tag{12}$$

which says that the stock of (longer-term) unemployed workers $\overline{U}_n$ matches with the inflow of new vacancies v_n, while proportion p_n of newly laid-off workers u_n successfully match with current vacancies $\overline{V}_n$.

Given these specifications for λ^{SF}, p^{SF}, the parameters θ and period n data, (7)-(12) jointly determine $(\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n, p_n, q_n)$. Expected matches are then

$$M_n(\theta) = \overline{U}_n(1 - e^{-\lambda_n}) + p_n u_n \qquad (13)$$

and we use MLE techniques to estimate θ.

4   The data

Construction of the 'at risk' measures $\overline{U}_n, \overline{V}_n$ requires data which distinguish between flows (u, v) and stocks (U, V). Using inches of help-wanted advertisements to measure vacancies, as is the general procedure for the United States,[5] is not sufficient as there is no information on whether a particular job advertisement is new or is a re-advertisement. Job Center data, however, provides this information for the U.K. labor market.

The U.K. Job Center system is a network of government funded employment agencies, where each town/city typically has at least one Job Center. A Job Center's services are free of charge to all users, both to job seekers and to firms advertising vacancies. To be entitled to receive welfare payments, an unemployed benefit claimant in the United Kingdom is required to register at a Job Center.[6]

The vast majority of Job Center vacancy advertisements are for unskilled and semi-skilled workers. Certainly the professionally trained are unlikely to find suitable jobs there. Nevertheless, as the bulk of unemployment is experienced by unskilled and semi-skilled workers [rather than by professionals], it seems reasonable that understanding the determinants of re-employment hazard rates at this level of matching provides useful differentiating information between competing theories of equilibrium unemployment.

The data is a monthly time series running from September 1985 to December 1999, [172 observations]. The data record not only the number unemployed (U_n) and number of unfilled vacancies (V_n) carried over from the previous month in the United Kingdom, but also the number of new registered job seekers (u_n) and new vacancies (v_n) which register within each month n. The data also record the number of workers who leave unemployment, and the number of vacancies which are filled, and either series might be used as the measure of matches (M_n).

All data used are extracted from the Nomis databank and not seasonally adjusted. The series are plotted in Figures 2 and 3. To improve visual inspection of our data, the data

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[5]See Abraham (1987) for a description of U.S. vacancy data.

[6]Gregg and Wadsworth (1996) report that Job Centres are used by roughly 80-90 percent of the claimant unemployed, 25-30 percent of employed job seekers and 50 percent of employers.

series in these Figures are seasonally adjusted, while raw data are used in estimation. As also suggested by the Blanchard and Diamond (1989) data, Figure 3 establishes that the monthly vacancy outflow is very highly correlated with the inflow of new vacancies, and more weakly correlated with the vacancy stock. Correlation coefficients on raw data are 0.93 and 0.55 respectively. When only including vacancies which are filled, the correlation between filled and new vacancies becomes 10 times higher than that between vacancies filled and the vacancy stock (0.78 and 0.08 respectively). For the unemployed, the correlation coefficient between the inflow and the outflow is 0.63, and the one between the outflow and the stock is 0.54. Data also show a much higher turnover rate for vacancies than for the unemployed: the relevant monthly inflow/stock ratio being 0.15 for the unemployed and 1.12 for vacancies.

There are several data issues. First, we prefer to use unemployment outflow, rather than vacancy outflow, as our measure of matches (M_n). Given the high rate of vacancy turnover, where the average duration of a vacancy coincides with the length of the data period (one month), then having monthly vacancy inflows and outflows on the two sides of the regression equation potentially produces spurious regression results.[7]

Simple OLS estimates of the random matching function, using unemployment outflow as the left hand side variable, generates results which are reasonably consistent with the literature. In particular, estimating a log-linear matching function à la Blanchard and Diamond (1989) gives

$$\ln M_n \;=\; \underset{(3.819)}{-2.207} \;+\; \underset{(0.149)}{0.235 \ln V_n} \;+\; \underset{(0.197)}{0.809 \ln U_n}, \tag{14}$$

where constant returns are not rejected ($F = 0.88$) and $R^2 = 0.74$ (the regression includes both monthly and yearly dummies, with standard errors reported in brackets). These results are fairly close to those obtained by Pissarides (1986) on a similar log linear specification for the U.K.[8].

A second issue is that vacancies advertised at Job Centres are only a fraction of existing job openings. Gregg and Wadsworth (1996) report that Job Centres are used by roughly 50 percent of employers. As we use log-linear functional forms, this mismeasurement of total vacancies does not bias the results (apart from the constant term) as long as we assume the

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[7]Results (not reported here) which use vacancy outflow as the measure of matches provide even stronger support for stock flow matching.

[8]Note that in both equation (14) and Pissarides (1986) the vacancy elasticity is lower and the unemployment elasticity is higher than in the findings of Blanchard and Diamond (1989), who obtain estimates around 0.6 and 0.4, respectively. These differences are due to the different choice of dependent variable, see Petrongolo and Pissarides (2001, Section 4.2) for a discussion.
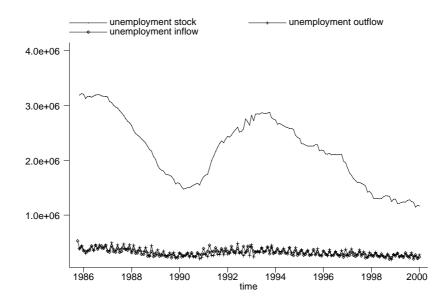
Figure 2: Monthly unemployment stock, inflow and outflow in Britain, September 1985-December 1999. Source: Nomis. Data seasonally adjusted.
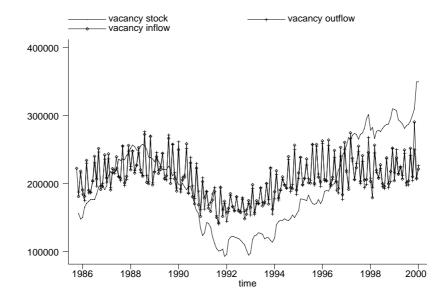

Figure 3: Vacancy stock, inflow and outflow in Britain, September 1985-December 1999. Source: Nomis. Data seasonally adjusted.

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fraction of vacancies advertised in U.K. Job Centers remains constant over time. Nevertheless, we rescale the vacancy measures so that the identifying restriction - that the measured number of vacancies which match is equal to the number of unemployed job seekers that find work - is not unreasonable. By constructing a series for total hires in the economy, we find that filled Job Center vacancies account, on average, for 44% of total new hires in the United Kingdom.[9] We therefore rescale both Job Center vacancy measures V_n, and v_n, by dividing through by 0.44. This rescaling, however, is largely cosmetic - the results are qualitatively identical without it.

The time series for the stocks of unemployment and vacancies are not stationary.[10] Indeed there is quite a literature on so-called shifting 'Beveridge curves', which hints at structural breaks in the long-run unemployment-vacancy relationship in Britain and elsewhere (see Jackman et al. 1990 for a multi-country study). The matching structure defined above describes short-run variations in matching rates due to short-run variations in labour market conditions. It cannot be used to explain long-run matching trends due to, say, changes in the composition of the workforce [more workers now attend higher education] or regional migration.

To focus on explaining the short-run variations on observed matching rates, an obvious approach is to include month and year dummies. Tables 1 and 2 report the regression results when year dummies are included. This approach has the disadvantage of generating discontinuous "jumps" at arbitrary discontinuity points, instead of a smooth long-run trend. In our second set of results, presented in Tables 3 and 4 in the Appendix, we do not include year dummies but detrend the data series instead, as already done in the matching literature by Yashiv (2000), by filtering all time series with a Hodrick-Prescott (1997) filter, with smoothing parameter equal 14400. To preserve series means we have added to the detrended series their sample averages. While data filtering fits a smooth long run trend through the data series instead of discontinuous jumps, including year dummies on raw data has the advantage of estimating structural breaks and matching function parameters simul-

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[9]Total hires can be proxied by $H_n = u_n + \Delta N_n$, where ΔN_n is the net change in aggregate employment and u_n is the inflow into unemployment in month n. If M_n^v denotes total vacancies filled in U.K. Job Centers, then M_n^v/H_n is the fraction of hires accounted for by Job Center matching. This statistic has average value 0.44 and has no discernible trend over the sample period.

A different approach is to note that if vacancy outflow described total U.K. matches, then $\lambda_n = M_n^v/U_n$ would be the average exit rate out of unemployment in month n, and hence $1/\lambda_n = U_n/M_n^v$ would be the average expected duration of unemployment [measured in months]. Computing this statistic implies an average duration of unemployment of around 14 months. In contrast, the actual average duration of unemployment for this period is around 6.5 months. This ratio, [6.5]:[14] equals 0.46, and so suggests that Job Center vacancies account for 46% of the total in the U.K..

[10]ADF statistics (with 4 lags) are -1.181 and -0.806, respectively, against a 5% critical value of -3.12.

taneously, allowing for possible correlation between shift variables and other right-hand side variables.

As one would expect, the estimates using the filtered data imply predictions which at times drift away from actual matches, but do a good job at reproducing the short-run fluctuations. In contrast, the estimates using non-filtered data and year dummies do not explain the short-run fluctuations so well, but do not drift so much from the actual series. Most of the discussion that follows focusses on the non-filtered data with year dummies. At the end we discuss the results using filtered data instead and shall establish that the estimates and insights are qualitatively identical.

5 Results

5.1 Random Matching

Given some initial parameters θ_0, then for each observation $n = 1, ..., 172$, we solve numerically (2), (3), (4) and (5) for $\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n$. Predicted matches for each n are then $M_n(\theta_0) = \overline{U}_n[1 - \exp(-\lambda_n)]$. Assuming residual errors are Normally distributed, a maximum likelihood estimator for θ is obtained using a standard hill-climbing algorithm.

Given the identifying restrictions for random matching, Table 1 describes the MLE results using various functional forms for $\lambda_n = \lambda^M(.)$. As the data is not seasonally adjusted, all estimated equations include monthly dummies, which turn out to be jointly significant in all specifications.[11]

Column 1 assumes the standard Cobb-Douglas specification

$$\lambda_n = \exp\left[\alpha_0 + \alpha_1 \ln \overline{V}_n + \alpha_2 \ln \overline{U}_n\right]$$

plus time dummies. The coefficients on (time-aggregated) vacancies and unemployment have the expected sign and are significantly different from zero. Estimated matching elasticities around 0.5 are very much in line with the previous matching function estimates (see Petrongolo and Pissarides 2001), and constant returns to scale in the matching function are not rejected, given a virtually zero Wald test statistics on the restriction $\alpha_1 = -\alpha_2$. The extremely low value of this test statistic, however, together with a non-significant constant term in λ_n makes one doubt that the elasticities on $\overline{V}_n$ and $\overline{U}_n$ are separately identified. In Column 2 we impose constant returns to scale: the constant term is now precisely determined, and the goodness of fit remains unchanged. In both Columns 1 and 2 the predicted value of λ_n is consistent with an expected unemployment duration just below 6 months (computed as

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[11]The exact specification used for predicted matches is $M_n(\theta) = \overline{U}_n[1 - \exp(-\lambda_n)] +$ dummies.

Table 1: Estimation results under random matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.152 (3.055)	-1.120 (0.108)	-1.276 (2.366)	-1.217 (1.641)	-0.710 (0.058)
	$\ln \overline{V}_n$	0.539 (0.129)	0.524 (0.064)	-0.347 (0.142)	-	-
	$\ln v_n$	-	-	0.980 (0.090)	0.694 (0.058)	0.673 (0.035)
	$\ln \overline{U}_n$	-0.534 (0.120)	-0.524^a	-0.597 (0.086)	-0.657 (0.074)	-0.673^a
Log-likelihood		-0.03582	-0.03593	-0.01424	-0.01567	-0.01610
R^2		0.865	0.864	0.946	0.941	0.939
CRS^b		0.001	-	0.047	0.102	-
monthly dummies $= 0^c$		150.2	181.0	176.3	184.4	204.4
yearly dummies $= 0^d$		32.2	37.0	120.0	100.1	97.3
ADF^e		-7.520	-4.523	-4.047	-4.065	-4.043
Sample averages:						
λ_n		0.189	0.184	0.156	0.176	0.177
$1/\lambda_n$		5.6	5.8	6.8	6.1	6.1
μ_n		0.828	0.800	0.682	0.758	0.761
$1/\mu_n$		1.4	1.4	1.8	1.5	1.5

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $1/\lambda_n$ and $1/\mu_n$, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$, $\ln v_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% singificance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

the sample average of $1/\lambda_n$), which is roughly in line with the actual unemployment duration during the sample period (6.5 months).

Comparing these results with those obtained using OLS (see the data description section above) finds that the estimated vacancy coefficient is much larger (0.52 rather than 0.24), is highly significant and the fit is much improved ($R^2 = 0.86$ rather than 0.74). Ignoring temporal aggregation implies a significant downward bias in the vacancy coefficient. The reason is that the initial vacancy stock V_n is a poor proxy for the total number of vacancies at risk over the month. For example, equation (3) with $\mu = 0.8$ (the mean value of μ_n estimated in Column 2) implies at risk measure $\overline{V}_n = V_n + 0.57 v_n$. This 'at risk' weighting (around one half) reflects that the entire stock of vacancies is 'at risk' from the very start of the month, while new vacancies only enter the market gradually during the month. The average monthly inflow to stock ratio, v_n/V_n, is large (equal to 1.12) which implies V_n is a poor proxy for $\overline{V}_n$. Further, as the unemployment outflow is highly correlated with the vacancy inflow during the month, correcting for temporal aggregation bias results in a much better fit and a higher estimated vacancy coefficient.

Column 3 is a test of an overidentifying restriction - that random matching implies the matching rate of individual workers does not depend directly on the inflow of new vacancies. Column 3 asks whether including the flow of new vacancies as an added explanatory variable for λ_n improves the fit. In fact the fit is not only much improved, it is important to note that the vacancy stock coefficient becomes wrong signed. Column 4 drops the vacancy stock term and the fit is essentially unchanged. In both Columns 3 and 4 constant returns in the matching function are not rejected, and this restriction is again imposed in Column 5.

Lagos (2000) provides a useful perspective for this result. He shows that even with stock flow matching at the micro-level, aggregation over locations may yield, in a *steady state,* aggregate matching behavior which appears consistent with a standard random matching function. Column 2 establishes that a standard random matching function fits the aggregate data well ($R^2 = 0.86$) and implies constant returns. But the identifying equations are not consistent with the 'out-of-steady-state' matching dynamics. The next set of results establish that the stock-flow identifying equations with spatial mismatch explain better the observed time series variation in unemployment outflow.

5.2   Stock-Flow Matching

Given some initial parameters θ_0, then for each observation n we solve numerically (7)-(12) for $\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n, p_n, q_n$. Predicted matches, $M_n(\theta_0)$ are given by (13). A standard hill climbing algorithm then identifies the MLE for θ.

Assuming errors are Normally distributed, the results for stock-flow matching are reported in Table 2 under alternative specifications for $\lambda_n = \lambda^{SF}(.)$ and $p_n = p^{SF}(.)$. Recall that in contrast to random matching, stock flow matching implies λ_n depends on the vacancy inflow and not on the stock of vacancies. The pure job queueing hypothesis in addition predicts $p_n = 0$.

Column 1 adopts the functional form

$$\lambda_n = \exp\left(\alpha_0 + \alpha_1 \ln \overline{V}_n + \alpha_2 \ln v_n + \alpha_3 \ln \overline{U}_n\right)$$

while p_n is estimated as a constant parameter, and constrained to be non-negative, i.e. $p_n = \exp(\beta_0)$. Consistent with stock flow matching, Column 1 in Table 2 finds that λ is driven by the inflow of new vacancies, and that the vacancy stock effect is insignificant [and wrong-signed]. Column 2 drops the vacancy stock from the specification of λ_n and re-estimates. The results establish that the exit rates of the longer term unemployed, λ_n, are driven by the inflow of new vacancies with an estimated elasticity around 0.7, with crowding out by other unemployed job seekers. Further, the pure job queueing hypothesis is rejected - the matching probability of the newly unemployed, p_n, is around 0.4, and is significantly different from zero, with a standard error of 0.059.[12]

Columns 3-5 consider a more general specification for

$$p_n = \exp\left(\beta_0 + \beta_1 \ln \overline{V}_n + \beta_2 \ln v_n + \beta_3 \ln u_n\right)$$

while leaving the specification of λ_n as in column 2, which is consistent with the identifying assumptions. Unfortunately the parameter estimates only converge when we impose constant returns on the estimation routine; i.e. set $\alpha_2 + \alpha_3 = 0$ and $\beta_1 + \beta_2 + \beta_3 = 0$.[13] We are therefore unable to test for constant returns to matching. When imposing constant returns in columns 3-5, we still get a positive p_n, but no variables seem to explain it well. Column 4 is the 'stock-flow' specification, that p depends on the vacancy stock but not the inflow, but the vacancy variable is wrong signed and insignificant. The results for p are therefore a little disappointing, but we note throughout that the estimates for λ are robust to these variations.

As all specifications provide an identical fit, Column 2, being the most parsimonious, is the most preferred. It implies that around 40% of newly unemployed workers quickly find work. The exit rates of the longer-term unemployed, λ, are driven by the inflow of new vacancies with crowding out by other competing job seekers. The overall fit ($R^2 = 0.96$) is

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[12]Using the delta method: $var(p_n) = \exp(2 * \beta_0) var(\beta_0) = 0.003$.

[13]This perhaps reflects a multi-collinearity problem between $\overline{V}_n$ and v_n.

Table 2: Estimation results under stock-flow matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.286 (3.411)	-1.311 (2.330)	-1.039 (0.143)	-1.011 (0.112)	-0.994 (0.191)
	$\ln \overline{V}_n$	-0.159 (0.152)	—	—	—	—
	$\ln v_n$	0.840 (0.052)	0.724 (0.094)	0.746 (0.048)	0.731 (0.040)	0.734 (0.056)
	$\ln \overline{U}_n$	-0.677 (0.124)	-0.717 (0.095)	-0.746^a	-0.731^a	-0.734^a
$\ln p_n$	constant	-0.932 (0.150)	-0.848 (0.125)	-0.996 (0.153)	-1.107 (0.174)	-1.124 (0.203)
	$\ln \overline{V}_n$	—	—	-0.193 (0.176)	-0.236 (0.182)	
	$\ln v_n$	—	—	-0.111 (0.249)	—	-0.161 (0.354)
	$\ln u_n$			0.304^a	0.236^a	0.161^a
log-likelihood		-0.01039	-0.01067	-0.01053	-0.01061	-0.01091
R^2		0.961	0.960	0.960	0.960	0.959
CRS^b		0.0002	0.002	—	—	—
monthly dummies $= 0^c$		276.6	300.7	304.5	294.3	334.1
year dummies $= 0^d$		90.8	100.2	132.9	128.7	121.8
ADF^e		-5.382	-5.435	-5.412	-5.474	-8.668
Sample averages:						
λ_n		0.102	0.102	0.117	0.122	0.124
p_n		0.393	0.428	0.316	0.289	0.297
$(1 - p_n)/\lambda_n$		6.4	6.1	6.3	6.3	6.1
μ_n		0.273	0.300	0.235	0.214	0.219
q_n		0.442	0.438	0.498	0.522	0.529
$(1 - q_n)/\mu_n$		2.7	2.5	3.3	3.4	3.0

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $(1-p_n)/\lambda_n$ and $(1-q_n)/\mu_n$, respectively. No. observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

also better than all specifications in Table 1, even those that (inconsistently) include vacancy inflow.

5.3 The Results Using HP Filtered Data.

We quickly discuss Tables 3 and 4 which describe the results when the data is first passed through an HP filter and the identifying equations estimated without year dummies. The results are qualitatively identical. For the random matching case (Table 3), Column 2 accepts constant returns to matching and estimates a slightly higher vacancy coefficient (0.64). Column 3 is the over-identifying test which says that the vacancy inflow term should have no significant impact on λ. As before including vacancy inflow results in a much better fit and the vacancy stock term becomes wrong signed. Omitting the vacancy stock variable does not affect the goodness of fit.

Table 4 estimates stock flow matching. As in Table 2, the estimates of λ are robust across all specifications and implies that the stock of longer term unemployed workers are waiting for new suitable vacancies to come onto the market. As in Table 2, no variables seem to explain p well, and Column 2 is again the preferred specification.

In fact the results described in Column 2, Table 4, are interesting for several reasons. Over this data period, the average completed spell of unemployment was 6.5 months. Further Coles at al (2003) report that the average uncompleted spell of unemployment across the stock of unemployed workers had a mean value of around 14 months. Note, Column 2 estimates mean $\lambda = 0.07$. Hence conditional on remaining unemployed, it predicts an average spell of unemployment $1/\lambda = 14$ months. Column 2 also implies that conditional on becoming unemployed, the mean expected duration of unemployment is $(1 - p)/\lambda = 6.8$ months. A third surprising feature is that it predicts mean $q = 0.31$ which is also on the button; Coles and Smith (1998) report that approximately 30% of new vacancies are filled on the first day of being posted. Finally, note the estimated intercept has value exp(-1.2)=0.30, which suggests functional form $\lambda \simeq 0.3v/U$; i.e. the stock of longer term unemployed workers match with around 30% of all new vacancies, which is consistent with the prediction that 30% of new vacancies match immediately.

The only sample mean Column 2 fails to predict adequately is the average duration of a vacancy. The sample average is between 3 and 4 weeks, but the final row of Column 2 predicts average duration $(1-q)/\mu = 2.2$ months. At first sight, this seems to be an important failure of the model. Note, however, that all specifications in Tables 1-4 overestimate this statistic (see the bottom rows in each Table). A potential explanation for this is that approximately 1/3 of all vacancies are withdrawn unfilled from Job Centres [measured as the difference

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Table 3: Estimation results under random matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.130 (3.045)	-1.122 (0.090)	-1.290 (2.352)	-1.222 (1.896)	-0.826 (0.083)
	$\ln \overline{V}_n$	0.610 (0.135)	0.637 (0.089)	-0.269 (0.142)	-	-
	$\ln v_n$	-	-	0.856 (0.101)	0.700 (0.072)	0.680 (0.057)
	$\ln \overline{U}_n$	-0.612 (0.134)	-0.637^a	-0.559 (0.103)	-0.671 (0.105)	-0.680^a
Log-likelihood		-0.03616	-0.03618	-0.02225	-0.02297	-0.02327
R^2		0.663	0.663	0.793	0.786	0.783
CRS^b		0.00005	-	0.030	0.047	-
monthly dummies $= 0^c$		153.9	169.8	117.9	126.0	132.4
ADF^d		-6.160	-3.703	-3.358	-3.364	-3.362
Sample averages:						
λ_n		0.155	0.155	0.148	0.149	0.149
$1/\lambda_n$		6.5	6.5	6.9	6.8	6.8
μ_n		0.623	0.623	0.588	0.592	0.592
$1/\mu_n$		1.6	1.6	1.7	1.7	1.7

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $1/\lambda_n$ and $1/\mu_n$, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$, $\ln v_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% singificance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

Table 4: Estimation results under stock-flow matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.252 (4.198)	-1.201 (3.301)	-1.063 (0.322)	-1.323 (0.236)	-1.342 (0.577)
	$\ln \overline{V}_n$	-0.199 (0.198)	$-$	$-$	$-$	$-$
	$\ln v_n$	0.852 (0.142)	0.792 (0.125)	0.815 (0.072)	0.889 (0.094)	0.854 (0.124)
	$\ln \overline{U}_n$	-0.676 (0.170)	-0.810 (0.173)	-0.815^a	-0.889^a	-0.854^a
$\ln p_n$	constant	-0.636 (0.116)	-0.600 (0.108)	-0.829 (0.208)	-0.603 (0.134)	-0.597 (0.235)
	$\ln \overline{V}_n$	$-$	$-$	-0.151 (0.284)	-0.012 (0.139)	
	$\ln v_n$	$-$	$-$	-0.318 (0.404)	$-$	-0.090 (0.395)
	$\ln u_n$			0.469^a	0.012^a	0.090^a
log-likelihood		-0.01290	-0.01277	-0.01275	-0.01287	-0.01274
R^2		0.880	0.881	0.881	0.880	0.881
CRS^b		0.006	0.006	$-$	$-$	$-$
monthly dummies $= 0^c$		251.3	283.6	248.6	260.4	324.0
ADF^d		-3.870	-3.867	-3.882	-3.772	-7.289
Sample averages:						
λ_n		0.071	0.068	0.097	0.068	0.070
p_n		0.529	0.549	0.348	0.542	0.517
$(1-p_n)/\lambda_n$		6.8	6.8	6.9	7.0	7.1
μ_n		0.310	0.322	0.208	0.317	0.312
q_n		0.321	0.308	0.441	0.307	0.317
$(1-q_n)/\mu_n$		2.3	2.2	2.9	2.3	2.3

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $(1-p_n)/\lambda_n$ and $(1-q_n)/\mu_n$, respectively. No. observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% singificance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

between vacancy outflow and vacancies filled]. To correct our estimates for the average duration of a vacancy, suppose that vacancies are withdrawn exogenously according to a Poisson rate s. Further suppose that $sV = (1/3)v$, so that on average one third of all vacancies are withdrawn unfilled. As the sample average $v/V = 1.12$, this suggests $s \approx 0.37$. We now use $s = 0.37$ to correct the estimates of the average duration of a vacancy by replacing the estimated exit rate μ with the gross exit rate $\mu + s$.

For example, Column 2 in Table 1, representing random matching, finds $\mu = 0.8$ which, uncorrected, predicts an average vacancy duration $1/\mu = 1.4$ months, but corrected implies $1/(\mu + s) = 0.85$ which is then consistent with the sample average. Similarly, column 2 in Table 2, testing for stock-flow matching, predicts an average vacancy duration $(1-q)/\mu = 2.5$ months. The correction implies expected duration $(1-q)/(\mu+s) = 0.84$ months. Column 4 in Table 2 seems to do the most badly, predicting an average vacancy duration of 3.4 months, but when corrected yields $(1-q)/(\mu+s) = 0.82$ months. Finally, this correction for Column 2 in Table 4 implies an average vacancy duration of one month, which is slightly high but is clearly consistent with the sample mean.

6   Conclusion

As described in the Introduction, Lagos (2000) considers an equilibrium trading framework where matching at the micro-level implies stock-flow matching, but aggregation over locations and a restriction to *steady state* implies aggregate matching appears consistent with a standard random matching function. Our results are consistent with that view. The random matching function fits the aggregate data reasonably well and constant returns to matching are accepted. The random matching equations, however, are not consistent with observed 'out-of-steady-state' matching dynamics. Instead stock-flow matching with spatial mismatch provides a better explanation for the observed time series variations in unemployment outflow.

The stock-flow matching specification in Table 4, column 2 predicts (i) around 30% of new vacancies are filled immediately, (ii) the longer-term unemployed experience average unemployment spells of around 14 months and (iii) the average completed spell is around 7 months. All of these results are consistent with the empirical means. The estimates of λ are particularly interesting. Table 4, column 2 suggests $\lambda \simeq 0.3v/U$, which implies that the longer-term unemployed chase new vacancies as they come onto the market (job queues), and that around 30% of those new vacancies provide suitable work opportunities for the long term unemployed. This is clearly consistent with the mean prediction that 30% of new

vacancies are immediately snapped up by unemployed job seekers. Estimated $p \approx 0.5$ and is also highly significant. Although unemployment rates for this period were rather high (and average unemployment spells were long), this estimate implies that for the period 1986-99 in the U.K., on average around half of newly unemployed workers were able to find work quickly, the other half became long term unemployed.

Perhaps the major contribution of this paper is that the matching behavior of the longer term unemployed is robustly identified. As in Shapiro and Stiglitz (1984), estimates imply the long term unemployed have to wait for suitable new vacancies to come onto the market. This insight is important for policy purposes. There is a large optimal unemployment insurance (UI) literature which argues that unemployment benefit payments should be reduced with unemployment duration to encourage greater search effort (e.g. Shavell and Weiss (1979), Layard et al (1991), Frederiksson and Holmlund (2001) but also see Hopenhayn and Nicolini (1997), Cahuc and Lehmann (2000). But with pure crowding out in a job queueing framework, creating incentives to increase job search effort is undesirable. As in the taxi market analogy, encouraging customers to fight harder for each taxi is counterproductive.

We do not claim that our results are conclusive - the evidence on the matching behavior of workers on the short-side is weak. This is perhaps not surprising - rather than match immediately, one might expect that workers on the short-side take a little time to locate their preferred vacancy and start work. A preferable econometric structure might be that proportion p of entrants match at rate λ_s and $1 - p$ match at rate λ_L where $\lambda_s > \lambda_L$ (e.g. Lancaster and Nickell (1980), Heckman and Singer (1984)). Unfortunately such a structure cannot be identified on the data used here.[14] Nevertheless, suppose instead that workers on the short side match quickly but not immediately, say around one month $\lambda_s = 1$. Fitting the average duration of a completed spell of unemployment (6.5 months) and the average uncompleted spell of unemployment (14 months) in a steady state requires values

$$1 - p \simeq 0.4 \text{ and } \lambda_L \simeq 1/15.$$

For these parameter values, 40% of entrants become long term unemployed (i.e. match at rate λ_L) and steady state implies that 91% of workers in the unemployment stock are long term unemployed. The average completed spell is then 0.4[1]+0.6[15]=6.6 months while the average uncompleted spell is [0.09][1]+[0.91][15]=13.7 months. A possible interpretation of the overidentifying test for the random matching function (Tables 1 and 3, column 3) is that

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[14]The complication is that workers on the short side may then match with the inflow of new vacancies. As types (short or long) are not observed, this generates non-linear compositional dynamics which are not identified on the data.

it shows the majority of workers in the unemployment stock match with the inflow of new vacancies.

The stock-flow matching equations instead identify the data by setting $\lambda_s = \infty$ and Column 2, Table 4 then estimates

$$1 - p = 0.45 \text{ and } \lambda_L = 1/14.7$$

Although this approach provides little information on the matching behavior of the short-term unemployed, the estimates for λ_L are seemingly robust.

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A test between matching theories.[*]

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Abstract

This paper tests whether aggregate matching is consistent with random matching or stock-flow matching. Using U.K. matching data and correcting for temporal aggregation bias, estimates of the random matching function are consistent with previous work in this field. The data however support the 'stock-flow' matching hypothesis. Estimates find that around 50% of newly unemployed workers match quickly. The remaining workers match slowly, their re-employment rates depending statistically on the inflow of new vacancies and not on the vacancy stock. The interpretation is that these latter workers, having failed to match with existing vacancies, wait for new, more suitable vacancies to come onto the market. The results have important policy implications, particularly for long term unemployment and the design of optimal unemployment insurance programs.

Keywords: Stock-flow matching, Random matching, Job queues, Temporal aggregation.

JEL Classification: J3, J6.

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1 Introduction

The random matching approach has provided an important framework for analyzing labor market policy (Pissarides, 2000). But the empirical literature, when estimating the random matching function, rarely tests the matching function against a meaningful alternative (see for example Blanchard and Diamond, 1989, and Petrongolo and Pissarides, 2001, for a recent survey). This paper uses matching data to test between the random matching hypothesis and stock-flow matching.

Stock-flow matching assumes that when laid-off, a worker contacts friends, consults situations vacant columns in newspapers, perhaps registers with job agencies, and so observes the stock of vacancies currently on the market. If the worker is lucky and a suitable vacancy already exists, the worker can quickly exit unemployment. If a suitable vacancy does not exist, the worker then has to wait for something suitable to come onto the market. This worker does not then match with the stock of vacancies - the stock has already been sampled and no match exists. The worker instead matches with the inflow of new vacancies coming onto the market. The problem is symmetric for vacancies. If a firm has a vacancy, it might first ask employees if they know someone suitable, advertise the post in a situations vacant column etc.. If the firm is lucky, a suitable worker already exists in the stock of unemployed workers and the post is quickly filled. If not, the firm has to wait for someone suitable to come onto the market. This implies "stock-flow" matching as the stock of unmatched agents on one side of the market matches with the inflow of new agents on the other side. Papers in this literature include Taylor (1995), Coles and Smith (1998), Coles and Muthoo (1998), Coles (1999), Lagos (2000) and Gregg and Petrongolo (2005), but also see Jones and Riddell (1999).

Lagos (2000) perhaps provides the most useful perspective for understanding the results obtained here. Using a taxi market analogy, he supposes that cabs meet potential customers at taxi ranks. At the micro-level there is stock-flow matching, i.e. at any given taxi rank there is either a stock of customers waiting for cabs, or a stock of cabs waiting for customers. Aggregation over all taxi ranks and the restriction to *steady state* imply that at the macro-level, the flow number of cab rides depends only on the total stock of taxi cabs and on the total stock of potential customers in the market. Casual introspection also suggests there will be constant returns to matching: doubling the total number of participants should double the (steady state flow) number of taxi rides. Aggregate matching seemingly has the properties of a standard random matching function, even with stock-flow matching at the micro-level.

This view of matching is consistent with the fact that around 25-30% of new vacancies posted in U.K. Job Centers are filled on the first day (Coles and Smith, 1998). Burdett and Cunningham (1997) also report for the U.S. that most vacancies (55%) are filled within a week. This suggests that for many vacancies matching frictions may not be a significant factor. Instead such vacancies are snapped up by workers who have been waiting for something suitable to come onto the market. This view of matching also reconciles the McDonald's problem: that McDonald's invariably have vacancies and everyone knows where McDonald's is, so how can unemployment be frictional? It cannot be argued that McDonald's jobs are 'bad' jobs, otherwise no-one would work there.[1] Stock-flow matching instead implies that most unemployed workers are better qualified to do different work and wait for more suitable vacancies to come onto the market. McDonald's then hires from the inflow of workers into unemployment, hiring those who have a comparative advantage in working for them. Coles (1999) establishes that trade in such markets is characterised by a turnover externality: higher entry rates of new participants reduces the time lost waiting for suitable matches to enter the market.

The equivalence between random matching and stock-flow matching, as suggested by Lagos (2000), only holds in steady state. Outside of a steady state, stock-flow matching at the micro-level implies that a higher inflow of new vacancies, say into an unemployment black spot, will yield an immediate increase in matches. Consider for example Figure 1 which describes aggregate matching in the U.S. manufacturing sector (a monthly time series, taken from Blanchard and Diamond, 1989). A critical feature of this time series is that *the number of matches in each month is much more volatile than the stock of vacancies*. This implies the inflow of new vacancies cannot be smooth over time - if it were, a large increase in the number of matches would necessarily result in a large fall in the stock of vacancies. These data therefore imply that a spike in the number of matches coincides with a spike in the inflow of new vacancies and *not* with a spike in the stock of vacancies. The standard random matching approach is inconsistent with this interpetation of the data.

The underlying point is that outside of a steady state, random matching and stock-flow matching imply quite different equilibrium hazard rates of re-employment. In Shapiro and Stiglitz (1984) for example, the re-employment hazard rate of an unemployed worker, denoted λ, is simply $\lambda = v/U$ where v is the flow of new vacancies into the market, which match immediately (and randomly) with one unemployed worker, and U describes the current stock of unemployed workers. We shall refer to this case as job queueing. In contrast,

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[1] Although a popular term, the notion of a 'good' or 'bad' job is not particularly helpful as jobs are either well or badly compensated. Stock-flow matching implies the monopsonist chooses wages to maximise expected profit given the inflow of new entrants into the market.
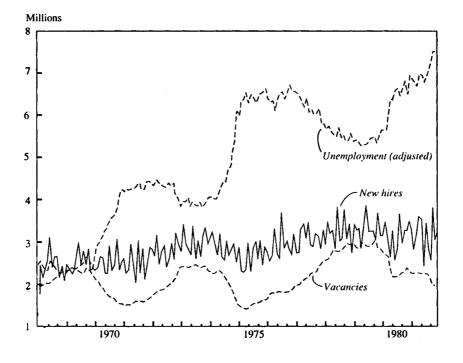
Figure 1: New hires, unemployment and vacancies in the U.S., 1968-1981. Source: Blanchard and Diamond (1989).

the random matching approach assumes the re-employment hazard rate depends on the vacancy/unemployment ratio, where frictions imply it is the stock of vacancies V which is the relevant state variable (throughout upper case refers to stock variables, lower case to flow variables). The stock-flow explanation instead implies there are two parameters of interest. Proportion p of laid-off workers are on the short side of their market and are re-employed immediately (or at least within a very short period of time). Proportion $(1-p)$ are on the long-side of their market and so chase new vacancies. Their subsequent hazard rate λ depends on the flow of vacancies into their particular specialization. Clearly this latter hazard rate λ has a similar structure to the job queueing hazard, depending on v rather than V.

Consistent with Lagos (2000), we find that the random matching function does indeed provide a good fit of the UK aggregate matching data and the hypothesis of constant returns is accepted (see column 2, Table 1 in the text). The critical over-identifying test for the random matching approach is that the inflow of new vacancies should not have any additional explanatory power for the matching rates of workers (given we control for temporal aggregation of the data). We find instead that the vacancy inflow coefficient is highly significant. The principal difficulty for the random matching approach, however, is that once vacancy inflow is included as a conditioning variable, the estimated vacancy stock term becomes wrong–signed. The results instead find that a stock-flow matching specification provides a much better fit of the data. The central finding is that the longer-term unemployed match with the inflow of new vacancies which implies these workers are waiting for suitable vacancies rather than facing high matching frictions. As pointed out in the conclusion, this has important implications for the design of optimal unemployment insurance schemes.

The paper is structured as follows. Section 2 discusses identification. It argues that with unobserved search effort, a distinguishing test between the two matching frameworks is not possible using microdata, but that identification is possible on macrodata. Section 3 shows how to distinguish between competing matching frameworks, while controlling econometrically for temporal aggregation bias. Section 4 describes the data used. Section 5 presents our results. Section 6 concludes.

2   Identification on aggregate data

The hazard function literature establishes there is substantial variation in re-employment rates across unemployed workers (see Machin and Manning (1999) for a recent survey). To understand how worker heterogeneity may affect identification, suppose unemployed worker

i at time t makes search effort k_{it} and let

$$K_t = \sum_{i=1}^{U_t} k_{it}$$

denote aggregate search effort across the U_t unemployed. The standard random matching approach (e.g. Pissarides (2000)) assumes the re-employment rate of this worker, denoted λ_{it}, is

$$\lambda_{it} = \frac{k_{it}}{K_t} M(K_t, V_t)$$

where V_t is the stock of vacancies at time t and $M(.)$ is a standard matching function.

Consider instead a job queueing framework. In this world the stock of unemployed workers match with new vacancies as they are created. Using the taxi rank analogy, suppose unemployed worker i makes effort k_{it} to catch the next job. Then this worker's re-employment rate is

$$\lambda_{it} = \frac{k_{it}}{K_t} v_t$$

where v_t is the inflow of new vacancies and k_{it}/K_t is the probability that worker i gets the next job to come onto the market.

The aim of the paper is to identify which better explains the re-employment rates of unemployed workers: is it the stock of vacancies (as implied by random matching) or the inflow of new vacancies (as implied by job waiting)? Unfortunately it is not possible to identify either of these processes on microdata. For most individuals we have only one data point - that individual's observed unemployment spell (a starting and end-date) - and we cannot control for the fixed effect k_{it}. We might potentially control for the fixed effect by focussing on individuals with repeat unemployment spells, but such individuals are a biased sample of the market.

Note also that this matching structure generates congestion effects, where greater job search effort by one worker reduces the matching probability of other workers (via an increase in aggregate K). Unobserved search heterogeneity implies a negative correlation in individual worker re-employment rates - if one worker gets the job, the others do not. Aggregating across workers, however, nets out these congestion effects. For example job queueing implies individual hazard rate $\lambda_{it} = k_{it} v_t / K_t$ but aggregating over i implies average re-employment rate

$$\overline{\lambda}_t = \frac{1}{U_t} \sum_{i=1}^{U_t} \lambda_{it} = \frac{1}{U_t} \sum_{i=1}^{U_t} \frac{k_{it}}{K_t} v_t = \frac{v_t}{U_t}.$$

Note the average re-employment rate is driven by the inflow of new vacancies, there is

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crowding out by the unemployment stock, but pure crowding out implies the average re-employment rate is independent of the distribution of search efforts $\{k_{it}\}$.[2]

The random matching approach instead implies $\lambda_{it} = \frac{k_{it}}{K_t} M(K_t, V_t)$ and aggregating over i implies average re-employment rate

$$\overline{\lambda}_t \equiv \frac{1}{U_t} \sum_{i=1}^{U_t} \lambda_{it} = \frac{1}{U_t} \sum_{i=1}^{U_t} \frac{k_{it}}{K_t} M(K_t, V_t) = \frac{1}{U_t} M(K_t, V_t).$$

If there are constant returns to matching then the average re-employment rate simplifies to

$$\overline{\lambda}_t = M(\overline{k}_t, \frac{V_t}{U_t})$$

where $\overline{k}_t = K_t/U_t$ is average search effort. As there is only partial crowding out, the average re-employment rate depends on average search effort but is otherwise independent of the distribution of search efforts $\{k_{it}\}$.

As $\overline{k}_t$ is unobserved, a critical identifying assumption is that $\overline{k}_t$ changes slowly and systematically over time. This seems reasonable as the unemployment and vacancy stocks change slowly over time (see Figure 1 for the U.S.).[3] In that case we can control for unobserved changes in aggregate job search effort by using time dummies. In particular using month dummies to capture seasonal variations in job search effort (for example the job market is quiet in August) and year dummies to capture business cycle variations, random matching implies observed short-run variations in the average re-employment rates of unemployed workers are driven by variations in the vacancy-unemployment ratio. Job queueing instead implies those variations are driven by variations in vacancy inflow. A second approach, discussed further in the data section, is to use an HP filter which takes out all trends.

3   The Empirical Framework

The data do not record unemployment and vacancy stocks over the month. Instead we have information on the stocks available at the start of each month and the gross inflows during that month. From now on we adopt the following time notation. U_n denotes the stock of unemployed workers at the beginning of month $n \in \mathcal{N}$ and V_n denotes the stock of vacancies. u_n denotes the total inflow of newly unemployed workers within the month and v_n denotes the inflow of new vacancies.

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[2]This is not true for the distribution of expected unemployment spells. As $1/\lambda_{it}$ is a convex function of k_{it}, a mean preserving spread in k_{it} yields an increase in the average expected unemployment spell. Indeed if $k_{it} = 0$ for one individual, the average spell is infinity.

[3]In a job search framework this implies job offer arrival rates change slowly over time and so aggregate job search effort will also change slowly over time.

We suppose that at time $t \in [n, n+1]$ in month n, the average re-employment probabilities of unemployed workers are denoted by a pair $(p(t), \lambda(t))$. $p(t)$ is the proportion of workers laid off at date t who find immediate re-employment, while $\lambda(t)$ is the average re-employment rate of workers who have been unemployed for some (strictly positive) period of time. The competing theories suggest alternative functional forms for p, λ.

As previously described, the random matching approach implies $p = 0$ - it takes time to find work - and the **average** re-employment rate, now simply denoted λ, is $\lambda = M(kU, V)/U$, where k is average search effort. Constant returns to matching implies a functional form $\lambda = \lambda^M(V/U, k)$.

Pure job queueing as described by Shapiro and Stiglitz (1984) also implies $p = 0$ - it takes time to find work - but in this case the average re-employment rate $\lambda = v/U$ where the stock of unemployed workers U match with the inflow of new vacancies v. For econometric purposes we consider a more general specification of the form $\lambda = \lambda^Q(v, U)$.

Stock-flow matching is a generalization of the job queueing approach. Shapiro and Stiglitz (1984) implies all unemployed workers are on the long side of the market. In reality, some suitably skilled workers may find themselves on the short side of the market and can quickly find re-employment. We suppose with probability $p = p^{SF}$, a newly unemployed worker finds there is a suitable vacancy already on the market and so becomes (very quickly) re-employed. With probability $1 - p^{SF}$ there is no suitable vacancy and the worker must then wait for a suitable vacancy to come onto the market. Job queueing implies average re-employment rate $\lambda = \lambda^{SF}(v, U)$. The same argument implies that each new vacancy may be either on the short or long side of the market. Vacancy queueing - waiting for a suitably skilled worker to come onto the market - implies average matching rate $\mu = \mu^{SF}(u, V)$ where u is the inflow of newly unemployed workers onto the market. Symmetry suggests functional form $p^{SF} = p^{SF}(V, u)$ where laid off workers on the short side of the market match with vacancies on the long side. Pure job queueing (e.g. Shapiro and Stiglitz (1984)) implies one-sided stock flow matching and the testable restriction $p^{SF} = 0$.

We now show how to identify (p, λ) using data which is reported as a monthly time series. As the identifying equations depend on the assumed theory, we consider each case separately, starting with random matching.

3.1   Temporal Aggregation with Random Matching.

To obtain a discrete time representation of the underlying continuous time matching process, we assume the inflows of new agents, u_n and v_n, are constant within a month. We construct at risk measures for the stock of vacancies and unemployed workers by considering a rep-

resentative worker who chooses average search effort $\overline{k}_n$ over the month and so matches at average rate λ_n. Temporal aggregation then implies the expected total number of matches in month n is

$$M_n = U_n[1 - e^{-\lambda_n}] + \int_{t=0}^1 u_n \left[1 - e^{-\lambda_n(1-t)}\right] dt$$

where the first term describes the number of workers in the initial unemployment stock who match within the month, and the second describes those who become unemployed at time $t \in [0, 1]$ within the month and who match by the end of it. Integration implies

$$M_n = \overline{U}_n[1 - e^{-\lambda_n}] \tag{1}$$

where

$$\overline{U}_n = U_n + \frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n[1 - e^{-\lambda_n}]} u_n. \tag{2}$$

(1) implies the expected number of matches in month n is $\overline{U}_n$, the number of unemployed workers 'at-risk' in month n, times $[1 - e^{-\lambda_n}]$, which is the matching probability of an unemployed worker over the entire month. To see that $\overline{U}_n$ is an 'at-risk' measure, suppose $\lambda_n \to 0$ (i.e. each unemployed worker in month n matches very slowly). The definition of $\overline{U}_n$ (equation (2)) implies $\overline{U}_n \to U_n + 0.5u_n$. Given nobody finds work ($\lambda_n = 0$) then each newly unemployed worker in month n is, on average, unemployed in that month for exactly half of it (given the identifying assumption that newly unemployed workers enter the market at a uniform rate). Hence $U_n + 0.5u_n$ measures the average number of unemployed workers at risk over the whole month.

Suppose instead $\lambda_n \to \infty$; all workers immediately find work. (2) implies $\overline{U}_n \to U_n + u_n$. $U_n + u_n$ is the relevant at-risk measure of unemployment for this case as each unemployed worker who enters the market matches immediately. (2) therefore computes the 'at risk' measure of unemployment for any arbitrary matching rate $\lambda_n \geq 0$ (given data U_n, u_n), and $M_n = \overline{U}_n[1 - e^{-\lambda_n}]$ is then the predicted number of matches.

The above temporal aggregation argument also applies to vacancies. If vacancies match at average rate μ_n in month n and v_n describes the total inflow of new vacancies within the month, then the relevant at-risk measure for vacancies is

$$\overline{V}_n = V_n + \frac{e^{-\mu_n} - 1 + \mu_n}{\mu_n[1 - e^{-\mu_n}]} v_n, \tag{3}$$

and expected matches are given by

$$M_n = \overline{V}_n[1 - e^{-\mu_n}].$$

Note, this implies the identifying restriction

$$\overline{U}_n[1 - e^{-\lambda_n}] = \overline{V}_n[1 - e^{-\mu_n}], \tag{4}$$

as the expected number of workers who match must equal the expected number of vacancies that match.

Given data $\{U_n, V_n, u_n, v_n\}$, we estimate λ_n by adopting the functional form

$$\lambda_n = \lambda^M(\overline{U}_n, \overline{V}_n; \theta). \tag{5}$$

This specification says that the average re-employment rate λ_n in month n does not depend on the initial stocks U_n, V_n, but on the number of unemployed workers and vacancies who try to match over the entire month. The parameter set θ potentially contains time dummies to proxy for systematic variations in average search effort.

Given parameters θ and the data, equations (2)-(5) can be solved numerically for the 4 unknowns $\overline{U}_n, \overline{V}_n, \lambda_n$ and μ_n. Conditional on θ, the predicted number of matches is

$$M_n(\theta) = \overline{U}_n[1 - e^{-\lambda_n}], \tag{6}$$

where the identifying restriction (4) implies the matching rates of workers are consistent with the matching rates of vacancies.[4] Given data on actual matches, Section 5 provides maximum likelihood estimates of θ. The next section, however, derives the identifying equations for stock-flow matching.

3.2   Temporal Aggregation with Stock-Flow Matching.

Suppose now that during month n, proportion p_n of newly unemployed workers are on the short side and so match immediately (within the month), and all other unemployed workers are on the long side and match at average rate λ_n. Given (p_n, λ_n), the expected number of matches is

$$M_n = U_n[1 - e^{-\lambda_n}] + p_n u_n + \int_{t=0}^{1} (1 - p_n)u_n \left[1 - e^{-\lambda_n(1-t)}\right] dt.$$

The first term describes the number of workers in the original stock who match within the month, the second describes those who become unemployed but being on the short side of the market quickly re-match, the third describes those who do not find employment immediately but manage to find employment before the end of the month. Integration implies the temporally aggregated matching function

$$M_n = U_n[1 - e^{-\lambda_n}] + u_n(1 - p_n)\frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n} + p_n u_n.$$

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[4]In contrast, Gregg and Petrongolo (2005) do not compute these at risk measures and instead estimate (3) assuming $\lambda = \lambda^M(U_n, V_n; \theta)$ which ignores the matching effects due to the inflow of new vacancies.

In this case, the appropriate at-risk measure for unemployment is

$$\overline{U}_n = U_n + u_n(1 - p_n)\frac{e^{-\lambda_n} - 1 + \lambda_n}{\lambda_n[1 - e^{-\lambda_n}]}, \tag{7}$$

as the temporally aggregated matching function then reduces to

$$M_n = \overline{U}_n[1 - e^{-\lambda_n}] + p_n u_n.$$

Note, stock-flow matching implies matches can be decomposed into those workers who are on the long side of the market and who match slowly (at average rate λ_n) and those newly unemployed who are on the short side and match quickly (proportion p_n).

The argument is symmetric for vacancies. Suppose proportion q_n of new vacancies match immediately (with the stock of longer term unemployed workers), and all other vacancies match at average rate μ_n. Define the at-risk measure for the stock of vacancies

$$\overline{V}_n = V_n + v_n\frac{e^{-\mu_n} - 1 + \mu_n}{\mu_n[1 - e^{-\mu_n}]}(1 - q_n). \tag{8}$$

The argument above implies expected matches satisfy

$$M_n = \overline{V}_n[1 - e^{-\mu_n}] + q_n v_n.$$

As in the random matching case, the expected number of unemployed workers who match must equal the expected number of vacancies that match. Stock-flow matching, however, implies two identifying restrictions. First as the stock of (longer term) unemployed workers matches with the inflow of new vacancies, we have

$$q_n v_n = (1 - e^{-\lambda_n})\overline{U}_n. \tag{9}$$

Second, newly unemployed workers (who match immediately) match with the stock of vacancies and so

$$p_n u_n = (1 - e^{-\mu_n})\overline{V}_n. \tag{10}$$

A closed form econometric structure is then obtained by specifying

$$\lambda_n = \lambda^{SF}(v_n, \overline{U}_n; \theta); \tag{11}$$

$$p_n = p^{SF}(u_n, \overline{V}_n; \theta) \tag{12}$$

which says that the stock of (longer-term) unemployed workers $\overline{U}_n$ matches with the inflow of new vacancies v_n, while proportion p_n of newly laid-off workers u_n successfully match with current vacancies $\overline{V}_n$.

Given these specifications for λ^{SF}, p^{SF}, the parameters θ and period n data, (7)-(12) jointly determine $(\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n, p_n, q_n)$. Expected matches are then

$$M_n(\theta) = \overline{U}_n(1 - e^{-\lambda_n}) + p_n u_n \tag{13}$$

and we use MLE techniques to estimate θ.

4 The data

Construction of the 'at risk' measures $\overline{U}_n, \overline{V}_n$ requires data which distinguish between flows (u, v) and stocks (U, V). Using inches of help-wanted advertisements to measure vacancies, as is the general procedure for the United States,[5] is not sufficient as there is no information on whether a particular job advertisement is new or is a re-advertisement. Job Center data, however, provides this information for the U.K. labor market.

The U.K. Job Center system is a network of government funded employment agencies, where each town/city typically has at least one Job Center. A Job Center's services are free of charge to all users, both to job seekers and to firms advertising vacancies. To be entitled to receive welfare payments, an unemployed benefit claimant in the United Kingdom is required to register at a Job Center.[6]

The vast majority of Job Center vacancy advertisements are for unskilled and semi-skilled workers. Certainly the professionally trained are unlikely to find suitable jobs there. Nevertheless, as the bulk of unemployment is experienced by unskilled and semi-skilled workers [rather than by professionals], it seems reasonable that understanding the determinants of re-employment hazard rates at this level of matching provides useful differentiating information between competing theories of equilibrium unemployment.

The data is a monthly time series running from September 1985 to December 1999, [172 observations]. The data record not only the number unemployed (U_n) and number of unfilled vacancies (V_n) carried over from the previous month in the United Kingdom, but also the number of new registered job seekers (u_n) and new vacancies (v_n) which register within each month n. The data also record the number of workers who leave unemployment, and the number of vacancies which are filled, and either series might be used as the measure of matches (M_n).

All data used are extracted from the Nomis databank and not seasonally adjusted. The series are plotted in Figures 2 and 3. To improve visual inspection of our data, the data

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[5]See Abraham (1987) for a description of U.S. vacancy data.

[6]Gregg and Wadsworth (1996) report that Job Centres are used by roughly 80-90 percent of the claimant unemployed, 25-30 percent of employed job seekers and 50 percent of employers.

series in these Figures are seasonally adjusted, while raw data are used in estimation. As also suggested by the Blanchard and Diamond (1989) data, Figure 3 establishes that the monthly vacancy outflow is very highly correlated with the inflow of new vacancies, and more weakly correlated with the vacancy stock. Correlation coefficients on raw data are 0.93 and 0.55 respectively. When only including vacancies which are filled, the correlation between filled and new vacancies becomes 10 times higher than that between vacancies filled and the vacancy stock (0.78 and 0.08 respectively). For the unemployed, the correlation coefficient between the inflow and the outflow is 0.63, and the one between the outflow and the stock is 0.54. Data also show a much higher turnover rate for vacancies than for the unemployed: the relevant monthly inflow/stock ratio being 0.15 for the unemployed and 1.12 for vacancies.

There are several data issues. First, we prefer to use unemployment outflow, rather than vacancy outflow, as our measure of matches (M_n). Given the high rate of vacancy turnover, where the average duration of a vacancy coincides with the length of the data period (one month), then having monthly vacancy inflows and outflows on the two sides of the regression equation potentially produces spurious regression results.[7]

Simple OLS estimates of the random matching function, using unemployment outflow as the left hand side variable, generates results which are reasonably consistent with the literature. In particular, estimating a log-linear matching function à la Blanchard and Diamond (1989) gives

$$\ln M_n \;=\; \begin{array}{cccc} -2.207 & + & 0.235\ln V_n & + & 0.809\ln U_n, \\ (3.819) & & (0.149) & & (0.197) \end{array} \tag{14}$$

where constant returns are not rejected $(F = 0.88)$ and $R^2 = 0.74$ (the regression includes both monthly and yearly dummies, with standard errors reported in brackets). These results are fairly close to those obtained by Pissarides (1986) on a similar log linear specification for the U.K.[8].

A second issue is that vacancies advertised at Job Centres are only a fraction of existing job openings. Gregg and Wadsworth (1996) report that Job Centres are used by roughly 50 percent of employers. As we use log-linear functional forms, this mismeasurement of total vacancies does not bias the results (apart from the constant term) as long as we assume the

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[7]Results (not reported here) which use vacancy outflow as the measure of matches provide even stronger support for stock flow matching.

[8]Note that in both equation (14) and Pissarides (1986) the vacancy elasticity is lower and the unemployment elasticity is higher than in the findings of Blanchard and Diamond (1989), who obtain estimates around 0.6 and 0.4, respectively. These differences are due to the different choice of dependent variable, see Petrongolo and Pissarides (2001, Section 4.2) for a discussion.
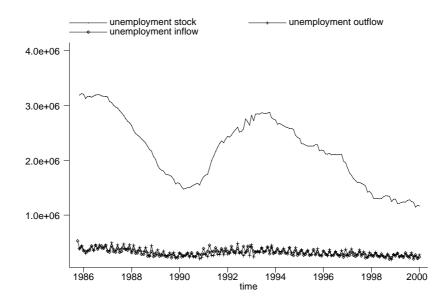
Figure 2: Monthly unemployment stock, inflow and outflow in Britain, September 1985-December 1999. Source: Nomis. Data seasonally adjusted.
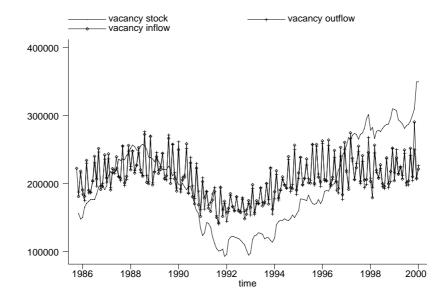

Figure 3: Vacancy stock, inflow and outflow in Britain, September 1985-December 1999. Source: Nomis. Data seasonally adjusted.

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fraction of vacancies advertised in U.K. Job Centers remains constant over time. Nevertheless, we rescale the vacancy measures so that the identifying restriction - that the measured number of vacancies which match is equal to the number of unemployed job seekers that find work - is not unreasonable. By constructing a series for total hires in the economy, we find that filled Job Center vacancies account, on average, for 44% of total new hires in the United Kingdom.[9] We therefore rescale both Job Center vacancy measures V_n, and v_n, by dividing through by 0.44. This rescaling, however, is largely cosmetic - the results are qualitatively identical without it.

The time series for the stocks of unemployment and vacancies are not stationary.[10] Indeed there is quite a literature on so-called shifting 'Beveridge curves', which hints at structural breaks in the long-run unemployment-vacancy relationship in Britain and elsewhere (see Jackman et al. 1990 for a multi-country study). The matching structure defined above describes short-run variations in matching rates due to short-run variations in labour market conditions. It cannot be used to explain long-run matching trends due to, say, changes in the composition of the workforce [more workers now attend higher education] or regional migration.

To focus on explaining the short-run variations on observed matching rates, an obvious approach is to include month and year dummies. Tables 1 and 2 report the regression results when year dummies are included. This approach has the disadvantage of generating discontinuous "jumps" at arbitrary discontinuity points, instead of a smooth long-run trend. In our second set of results, presented in Tables 3 and 4 in the Appendix, we do not include year dummies but detrend the data series instead, as already done in the matching literature by Yashiv (2000), by filtering all time series with a Hodrick-Prescott (1997) filter, with smoothing parameter equal 14400. To preserve series means we have added to the detrended series their sample averages. While data filtering fits a smooth long run trend through the data series instead of discontinuous jumps, including year dummies on raw data has the advantage of estimating structural breaks and matching function parameters simul-

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[9]Total hires can be proxied by $H_n = u_n + \Delta N_n$, where ΔN_n is the net change in aggregate employment and u_n is the inflow into unemployment in month n. If M_n^v denotes total vacancies filled in U.K. Job Centers, then M_n^v/H_n is the fraction of hires accounted for by Job Center matching. This statistic has average value 0.44 and has no discernible trend over the sample period.

A different approach is to note that if vacancy outflow described total U.K. matches, then $\lambda_n = M_n^v/U_n$ would be the average exit rate out of unemployment in month n, and hence $1/\lambda_n = U_n/M_n^v$ would be the average expected duration of unemployment [measured in months]. Computing this statistic implies an average duration of unemployment of around 14 months. In contrast, the actual average duration of unemployment for this period is around 6.5 months. This ratio, [6.5]:[14] equals 0.46, and so suggests that Job Center vacancies account for 46% of the total in the U.K..

[10]ADF statistics (with 4 lags) are -1.181 and -0.806, respectively, against a 5% critical value of -3.12.

taneously, allowing for possible correlation between shift variables and other right-hand side variables.

As one would expect, the estimates using the filtered data imply predictions which at times drift away from actual matches, but do a good job at reproducing the short-run fluctuations. In contrast, the estimates using non-filtered data and year dummies do not explain the short-run fluctuations so well, but do not drift so much from the actual series. Most of the discussion that follows focusses on the non-filtered data with year dummies. At the end we discuss the results using filtered data instead and shall establish that the estimates and insights are qualitatively identical.

5   Results

5.1   Random Matching

Given some initial parameters θ_0, then for each observation $n = 1, ..., 172$, we solve numerically (2), (3), (4) and (5) for $\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n$. Predicted matches for each n are then $M_n(\theta_0) = \overline{U}_n[1 - \exp(-\lambda_n)]$. Assuming residual errors are Normally distributed, a maximum likelihood estimator for θ is obtained using a standard hill-climbing algorithm.

Given the identifying restrictions for random matching, Table 1 describes the MLE results using various functional forms for $\lambda_n = \lambda^M(.)$. As the data is not seasonally adjusted, all estimated equations include monthly dummies, which turn out to be jointly significant in all specifications.[11]

Column 1 assumes the standard Cobb-Douglas specification

$$\lambda_n = \exp\left[\alpha_0 + \alpha_1 \ln \overline{V}_n + \alpha_2 \ln \overline{U}_n\right]$$

plus time dummies. The coefficients on (time-aggregated) vacancies and unemployment have the expected sign and are significantly different from zero. Estimated matching elasticities around 0.5 are very much in line with the previous matching function estimates (see Petrongolo and Pissarides 2001), and constant returns to scale in the matching function are not rejected, given a virtually zero Wald test statistics on the restriction $\alpha_1 = -\alpha_2$. The extremely low value of this test statistic, however, together with a non-significant constant term in λ_n makes one doubt that the elasticities on $\overline{V}_n$ and $\overline{U}_n$ are separately identified. In Column 2 we impose constant returns to scale: the constant term is now precisely determined, and the goodness of fit remains unchanged. In both Columns 1 and 2 the predicted value of λ_n is consistent with an expected unemployment duration just below 6 months (computed as

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[11]The exact specification used for predicted matches is $M_n(\theta) = \overline{U}_n[1 - \exp(-\lambda_n)]+$ dummies.

Table 1: Estimation results under random matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.152 (3.055)	-1.120 (0.108)	-1.276 (2.366)	-1.217 (1.641)	-0.710 (0.058)
	$\ln \overline{V}_n$	0.539 (0.129)	0.524 (0.064)	-0.347 (0.142)	-	-
	$\ln v_n$	-	-	0.980 (0.090)	0.694 (0.058)	0.673 (0.035)
	$\ln \overline{U}_n$	-0.534 (0.120)	-0.524^a	-0.597 (0.086)	-0.657 (0.074)	-0.673^a
Log-likelihood		-0.03582	-0.03593	-0.01424	-0.01567	-0.01610
R^2		0.865	0.864	0.946	0.941	0.939
CRS^b		0.001	-	0.047	0.102	-
monthly dummies $= 0^c$		150.2	181.0	176.3	184.4	204.4
yearly dummies $= 0^d$		32.2	37.0	120.0	100.1	97.3
ADF^e		-7.520	-4.523	-4.047	-4.065	-4.043
Sample averages:						
λ_n		0.189	0.184	0.156	0.176	0.177
$1/\lambda_n$		5.6	5.8	6.8	6.1	6.1
μ_n		0.828	0.800	0.682	0.758	0.761
$1/\mu_n$		1.4	1.4	1.8	1.5	1.5

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $1/\lambda_n$ and $1/\mu_n$, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$, $\ln v_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% singificance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

the sample average of $1/\lambda_n$), which is roughly in line with the actual unemployment duration during the sample period (6.5 months).

Comparing these results with those obtained using OLS (see the data description section above) finds that the estimated vacancy coefficient is much larger (0.52 rather than 0.24), is highly significant and the fit is much improved ($R^2 = 0.86$ rather than 0.74). Ignoring temporal aggregation implies a significant downward bias in the vacancy coefficient. The reason is that the initial vacancy stock V_n is a poor proxy for the total number of vacancies at risk over the month. For example, equation (3) with $\mu = 0.8$ (the mean value of μ_n estimated in Column 2) implies at risk measure $\overline{V}_n = V_n + 0.57 v_n$. This 'at risk' weighting (around one half) reflects that the entire stock of vacancies is 'at risk' from the very start of the month, while new vacancies only enter the market gradually during the month. The average monthly inflow to stock ratio, v_n/V_n, is large (equal to 1.12) which implies V_n is a poor proxy for $\overline{V}_n$. Further, as the unemployment outflow is highly correlated with the vacancy inflow during the month, correcting for temporal aggregation bias results in a much better fit and a higher estimated vacancy coefficient.

Column 3 is a test of an overidentifying restriction - that random matching implies the matching rate of individual workers does not depend directly on the inflow of new vacancies. Column 3 asks whether including the flow of new vacancies as an added explanatory variable for λ_n improves the fit. In fact the fit is not only much improved, it is important to note that the vacancy stock coefficient becomes wrong signed. Column 4 drops the vacancy stock term and the fit is essentially unchanged. In both Columns 3 and 4 constant returns in the matching function are not rejected, and this restriction is again imposed in Column 5.

Lagos (2000) provides a useful perspective for this result. He shows that even with stock flow matching at the micro-level, aggregation over locations may yield, in a *steady state,* aggregate matching behavior which appears consistent with a standard random matching function. Column 2 establishes that a standard random matching function fits the aggregate data well ($R^2 = 0.86$) and implies constant returns. But the identifying equations are not consistent with the 'out-of-steady-state' matching dynamics. The next set of results establish that the stock-flow identifying equations with spatial mismatch explain better the observed time series variation in unemployment outflow.

5.2 Stock-Flow Matching

Given some initial parameters θ_0, then for each observation n we solve numerically (7)-(12) for $\overline{U}_n, \overline{V}_n, \lambda_n, \mu_n, p_n, q_n$. Predicted matches, $M_n(\theta_0)$ are given by (13). A standard hill climbing algorithm then identifies the MLE for θ.

Assuming errors are Normally distributed, the results for stock-flow matching are reported in Table 2 under alternative specifications for $\lambda_n = \lambda^{SF}(.)$ and $p_n = p^{SF}(.)$. Recall that in contrast to random matching, stock flow matching implies λ_n depends on the vacancy inflow and not on the stock of vacancies. The pure job queueing hypothesis in addition predicts $p_n = 0$.

Column 1 adopts the functional form

$$\lambda_n = \exp\left(\alpha_0 + \alpha_1 \ln \overline{V}_n + \alpha_2 \ln v_n + \alpha_3 \ln \overline{U}_n\right)$$

while p_n is estimated as a constant parameter, and constrained to be non-negative, i.e. $p_n = \exp(\beta_0)$. Consistent with stock flow matching, Column 1 in Table 2 finds that λ is driven by the inflow of new vacancies, and that the vacancy stock effect is insignificant [and wrong-signed]. Column 2 drops the vacancy stock from the specification of λ_n and re-estimates. The results establish that the exit rates of the longer term unemployed, λ_n, are driven by the inflow of new vacancies with an estimated elasticity around 0.7, with crowding out by other unemployed job seekers. Further, the pure job queueing hypothesis is rejected - the matching probability of the newly unemployed, p_n, is around 0.4, and is significantly different from zero, with a standard error of 0.059.[12]

Columns 3-5 consider a more general specification for

$$p_n = \exp\left(\beta_0 + \beta_1 \ln \overline{V}_n + \beta_2 \ln v_n + \beta_3 \ln u_n\right)$$

while leaving the specification of λ_n as in column 2, which is consistent with the identifying assumptions. Unfortunately the parameter estimates only converge when we impose constant returns on the estimation routine; i.e. set $\alpha_2 + \alpha_3 = 0$ and $\beta_1 + \beta_2 + \beta_3 = 0$.[13] We are therefore unable to test for constant returns to matching. When imposing constant returns in columns 3-5, we still get a positive p_n, but no variables seem to explain it well. Column 4 is the 'stock-flow' specification, that p depends on the vacancy stock but not the inflow, but the vacancy variable is wrong signed and insignificant. The results for p are therefore a little disappointing, but we note throughout that the estimates for λ are robust to these variations.

As all specifications provide an identical fit, Column 2, being the most parsimonious, is the most preferred. It implies that around 40% of newly unemployed workers quickly find work. The exit rates of the longer-term unemployed, λ, are driven by the inflow of new vacancies with crowding out by other competing job seekers. The overall fit ($R^2 = 0.96$) is

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[12]Using the delta method: $var(p_n) = \exp(2*\beta_0)var(\beta_0) = 0.003$.

[13]This perhaps reflects a multi-collinearity problem between $\overline{V}_n$ and v_n.

Table 2: Estimation results under stock-flow matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.286 (3.411)	-1.311 (2.330)	-1.039 (0.143)	-1.011 (0.112)	-0.994 (0.191)
	$\ln \overline{V}_n$	-0.159 (0.152)	—	—	—	—
	$\ln v_n$	0.840 (0.052)	0.724 (0.094)	0.746 (0.048)	0.731 (0.040)	0.734 (0.056)
	$\ln \overline{U}_n$	-0.677 (0.124)	-0.717 (0.095)	-0.746^a	-0.731^a	-0.734^a
$\ln p_n$	constant	-0.932 (0.150)	-0.848 (0.125)	-0.996 (0.153)	-1.107 (0.174)	-1.124 (0.203)
	$\ln \overline{V}_n$	—	—	-0.193 (0.176)	-0.236 (0.182)	
	$\ln v_n$	—	—	-0.111 (0.249)	—	-0.161 (0.354)
	$\ln u_n$			0.304^a	0.236^a	0.161^a
log-likelihood		-0.01039	-0.01067	-0.01053	-0.01061	-0.01091
R^2		0.961	0.960	0.960	0.960	0.959
CRS^b		0.0002	0.002	—	—	—
monthly dummies $= 0^c$		276.6	300.7	304.5	294.3	334.1
year dummies $= 0^d$		90.8	100.2	132.9	128.7	121.8
ADF^e		-5.382	-5.435	-5.412	-5.474	-8.668
Sample averages:						
λ_n		0.102	0.102	0.117	0.122	0.124
p_n		0.393	0.428	0.316	0.289	0.297
$(1 - p_n)/\lambda_n$		6.4	6.1	6.3	6.3	6.1
μ_n		0.273	0.300	0.235	0.214	0.219
q_n		0.442	0.438	0.498	0.522	0.529
$(1 - q_n)/\mu_n$		2.7	2.5	3.3	3.4	3.0

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $(1 - p_n)/\lambda_n$ and $(1 - q_n)/\mu_n$, respectively. No. observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

also better than all specifications in Table 1, even those that (inconsistently) include vacancy inflow.

5.3   The Results Using HP Filtered Data.

We quickly discuss Tables 3 and 4 which describe the results when the data is first passed through an HP filter and the identifying equations estimated without year dummies. The results are qualitatively identical. For the random matching case (Table 3), Column 2 accepts constant returns to matching and estimates a slightly higher vacancy coefficient (0.64). Column 3 is the over-identifying test which says that the vacancy inflow term should have no significant impact on λ. As before including vacancy inflow results in a much better fit and the vacancy stock term becomes wrong signed. Omitting the vacancy stock variable does not affect the goodness of fit.

Table 4 estimates stock flow matching. As in Table 2, the estimates of λ are robust across all specifications and implies that the stock of longer term unemployed workers are waiting for new suitable vacancies to come onto the market. As in Table 2, no variables seem to explain p well, and Column 2 is again the preferred specification.

In fact the results described in Column 2, Table 4, are interesting for several reasons. Over this data period, the average completed spell of unemployment was 6.5 months. Further Coles at al (2003) report that the average uncompleted spell of unemployment across the stock of unemployed workers had a mean value of around 14 months. Note, Column 2 estimates mean $\lambda = 0.07$. Hence conditional on remaining unemployed, it predicts an average spell of unemployment $1/\lambda = 14$ months. Column 2 also implies that conditional on becoming unemployed, the mean expected duration of unemployment is $(1 - p)/\lambda = 6.8$ months. A third surprising feature is that it predicts mean $q = 0.31$ which is also on the button; Coles and Smith (1998) report that approximately 30% of new vacancies are filled on the first day of being posted. Finally, note the estimated intercept has value exp(-1.2)=0.30, which suggests functional form $\lambda \simeq 0.3v/U$; i.e. the stock of longer term unemployed workers match with around 30% of all new vacancies, which is consistent with the prediction that 30% of new vacancies match immediately.

The only sample mean Column 2 fails to predict adequately is the average duration of a vacancy. The sample average is between 3 and 4 weeks, but the final row of Column 2 predicts average duration $(1-q)/\mu = 2.2$ months. At first sight, this seems to be an important failure of the model. Note, however, that all specifications in Tables 1-4 overestimate this statistic (see the bottom rows in each Table). A potential explanation for this is that approximately 1/3 of all vacancies are withdrawn unfilled from Job Centres [measured as the difference

Table 3: Estimation results under random matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.130 (3.045)	-1.122 (0.090)	-1.290 (2.352)	-1.222 (1.896)	-0.826 (0.083)
	$\ln \overline{V}_n$	0.610 (0.135)	0.637 (0.089)	-0.269 (0.142)	-	-
	$\ln v_n$	-	-	0.856 (0.101)	0.700 (0.072)	0.680 (0.057)
	$\ln \overline{U}_n$	-0.612 (0.134)	-0.637^a	-0.559 (0.103)	-0.671 (0.105)	-0.680^a
Log-likelihood		-0.03616	-0.03618	-0.02225	-0.02297	-0.02327
R^2		0.663	0.663	0.793	0.786	0.783
CRS^b		0.00005	-	0.030	0.047	-
monthly dummies $= 0^c$		153.9	169.8	117.9	126.0	132.4
ADF^d		-6.160	-3.703	-3.358	-3.364	-3.362
Sample averages:						
λ_n		0.155	0.155	0.148	0.149	0.149
$1/\lambda_n$		6.5	6.5	6.9	6.8	6.8
μ_n		0.623	0.623	0.588	0.592	0.592
$1/\mu_n$		1.6	1.6	1.7	1.7	1.7

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $1/\lambda_n$ and $1/\mu_n$, respectively. No. Observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$, $\ln v_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% significance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% singificance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

Table 4: Estimation results under stock-flow matching

		1	2	3	4	5
$\ln \lambda_n$	constant	-1.252 (4.198)	-1.201 (3.301)	-1.063 (0.322)	-1.323 (0.236)	-1.342 (0.577)
	$\ln \overline{V}_n$	-0.199 (0.198)	$-$	$-$	$-$	$-$
	$\ln v_n$	0.852 (0.142)	0.792 (0.125)	0.815 (0.072)	0.889 (0.094)	0.854 (0.124)
	$\ln \overline{U}_n$	-0.676 (0.170)	-0.810 (0.173)	-0.815^a	-0.889^a	-0.854^a
$\ln p_n$	constant	-0.636 (0.116)	-0.600 (0.108)	-0.829 (0.208)	-0.603 (0.134)	-0.597 (0.235)
	$\ln \overline{V}_n$	$-$	$-$	-0.151 (0.284)	-0.012 (0.139)	
	$\ln v_n$	$-$	$-$	-0.318 (0.404)	$-$	-0.090 (0.395)
	$\ln u_n$			0.469^a	0.012^a	0.090^a
log-likelihood		-0.01290	-0.01277	-0.01275	-0.01287	-0.01274
R^2		0.880	0.881	0.881	0.880	0.881
CRS^b		0.006	0.006	$-$	$-$	$-$
monthly dummies $= 0^c$		251.3	283.6	248.6	260.4	324.0
ADF^d		-3.870	-3.867	-3.882	-3.772	-7.289
Sample averages:						
λ_n		0.071	0.068	0.097	0.068	0.070
p_n		0.529	0.549	0.348	0.542	0.517
$(1 - p_n)/\lambda_n$		6.8	6.8	6.9	7.0	7.1
μ_n		0.310	0.322	0.208	0.317	0.312
q_n		0.321	0.308	0.441	0.307	0.317
$(1 - q_n)/\mu_n$		2.3	2.2	2.9	2.3	2.3

Notes. Monthly data not seasonally adjusted. Dependent variable: vacancies filled at U.K. Job Centres (adjusted). All specifications include monthly and yearly dummies. Estimation method: non-linear least squares. Heteroskedatic-consistent standard errors (White 1980) are reported in brackets. Predicted unemployment and vacancy durations are computed as sample averages of $(1 - p_n)/\lambda_n$ and $(1 - q_n)/\mu_n$, respectively. No. observations: 171. Source: NOMIS.

a. Coefficient constrained to equal the value reported.
b. Wald test, distributed as $\chi^2(1)$, of the hypothesis that the sum of the coefficients on $\ln \overline{V}_n$ and $\ln \overline{U}_n$ is zero. Critical value at 5% significance level: $\chi^2(1) = 3.841$.
c. Wald test, distributed as $\chi^2(11)$, of the hypothesis that monthly dummies are jointly zero. Critical value at 5% singificance level: $\chi^2(11) = 19.675$.
d. Wald test, distributed as $\chi^2(14)$, of the hypothesis that yearly dumies are jointly zero. Critical value at 5% significance level: $\chi^2(14) = 23.685$.
e. ADF statistics (four lags) for the presence of a unit root in the estimated residuals. Critical value at 5% significance level: -2.23.

between vacancy outflow and vacancies filled]. To correct our estimates for the average duration of a vacancy, suppose that vacancies are withdrawn exogenously according to a Poisson rate s. Further suppose that $sV = (1/3)v$, so that on average one third of all vacancies are withdrawn unfilled. As the sample average $v/V = 1.12$, this suggests $s \approx 0.37$. We now use $s = 0.37$ to correct the estimates of the average duration of a vacancy by replacing the estimated exit rate μ with the gross exit rate $\mu + s$.

For example, Column 2 in Table 1, representing random matching, finds $\mu = 0.8$ which, uncorrected, predicts an average vacancy duration $1/\mu = 1.4$ months, but corrected implies $1/(\mu + s) = 0.85$ which is then consistent with the sample average. Similarly, column 2 in Table 2, testing for stock-flow matching, predicts an average vacancy duration $(1-q)/\mu = 2.5$ months. The correction implies expected duration $(1-q)/(\mu+s) = 0.84$ months. Column 4 in Table 2 seems to do the most badly, predicting an average vacancy duration of 3.4 months, but when corrected yields $(1-q)/(\mu+s) = 0.82$ months. Finally, this correction for Column 2 in Table 4 implies an average vacancy duration of one month, which is slightly high but is clearly consistent with the sample mean.

6   Conclusion

As described in the Introduction, Lagos (2000) considers an equilibrium trading framework where matching at the micro-level implies stock-flow matching, but aggregation over locations and a restriction to *steady state* implies aggregate matching appears consistent with a standard random matching function. Our results are consistent with that view. The random matching function fits the aggregate data reasonably well and constant returns to matching are accepted. The random matching equations, however, are not consistent with observed 'out-of-steady-state' matching dynamics. Instead stock-flow matching with spatial mismatch provides a better explanation for the observed time series variations in unemployment outflow.

The stock-flow matching specification in Table 4, column 2 predicts (i) around 30% of new vacancies are filled immediately, (ii) the longer-term unemployed experience average unemployment spells of around 14 months and (iii) the average completed spell is around 7 months. All of these results are consistent with the empirical means. The estimates of λ are particularly interesting. Table 4, column 2 suggests $\lambda \simeq 0.3v/U$, which implies that the longer-term unemployed chase new vacancies as they come onto the market (job queues), and that around 30% of those new vacancies provide suitable work opportunities for the long term unemployed. This is clearly consistent with the mean prediction that 30% of new

vacancies are immediately snapped up by unemployed job seekers. Estimated $p \approx 0.5$ and is also highly significant. Although unemployment rates for this period were rather high (and average unemployment spells were long), this estimate implies that for the period 1986-99 in the U.K., on average around half of newly unemployed workers were able to find work quickly, the other half became long term unemployed.

Perhaps the major contribution of this paper is that the matching behavior of the longer term unemployed is robustly identified. As in Shapiro and Stiglitz (1984), estimates imply the long term unemployed have to wait for suitable new vacancies to come onto the market. This insight is important for policy purposes. There is a large optimal unemployment insurance (UI) literature which argues that unemployment benefit payments should be reduced with unemployment duration to encourage greater search effort (e.g. Shavell and Weiss (1979), Layard et al (1991), Frederiksson and Holmlund (2001) but also see Hopenhayn and Nicolini (1997), Cahuc and Lehmann (2000). But with pure crowding out in a job queueing framework, creating incentives to increase job search effort is undesirable. As in the taxi market analogy, encouraging customers to fight harder for each taxi is counterproductive.

We do not claim that our results are conclusive - the evidence on the matching behavior of workers on the short-side is weak. This is perhaps not surprising - rather than match immediately, one might expect that workers on the short-side take a little time to locate their preferred vacancy and start work. A preferable econometric structure might be that proportion p of entrants match at rate λ_s and $1 - p$ match at rate λ_L where $\lambda_s > \lambda_L$ (e.g. Lancaster and Nickell (1980), Heckman and Singer (1984)). Unfortunately such a structure cannot be identified on the data used here.[14] Nevertheless, suppose instead that workers on the short side match quickly but not immediately, say around one month $\lambda_s = 1$. Fitting the average duration of a completed spell of unemployment (6.5 months) and the average uncompleted spell of unemployment (14 months) in a steady state requires values

$$1 - p \simeq 0.4 \text{ and } \lambda_L \simeq 1/15.$$

For these parameter values, 40% of entrants become long term unemployed (i.e. match at rate λ_L) and steady state implies that 91% of workers in the unemployment stock are long term unemployed. The average completed spell is then 0.4[1]+0.6[15]=6.6 months while the average uncompleted spell is [0.09][1]+[0.91][15]=13.7 months. A possible interpretation of the overidentifying test for the random matching function (Tables 1 and 3, column 3) is that

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[14]The complication is that workers on the short side may then match with the inflow of new vacancies. As types (short or long) are not observed, this generates non-linear compositional dynamics which are not identified on the data.

it shows the majority of workers in the unemployment stock match with the inflow of new vacancies.

The stock-flow matching equations instead identify the data by setting $\lambda_s = \infty$ and Column 2, Table 4 then estimates

$$1 - p = 0.45 \text{ and } \lambda_L = 1/14.7$$

Although this approach provides little information on the matching behavior of the short-term unemployed, the estimates for λ_L are seemingly robust.

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