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Abstract: This paper analyses the interplay between social structure and information exchange in two competing activities, crime and labor. We consider a dynamic model in which individuals belong to mutually exclusive two-person groups, referred to as dyads. Two individuals belonging to the same dyad hold a strong tie with each other, but each dyad partner can meet other individuals outside the dyad partnership, referred to as weak ties. Individuals learn about crime opportunities either through strong or weak ties and learn about jobs through employment agencies. There are multiple equilibria. If jobs are badly paid and/or crime is profitable, unemployment benefits have to be low enough to prevent workers for staying too long in the unemployment status because they are vulnerable to crime activities while, if people are well paid and/or crime is not profitable, unemployment benefits have to be high enough to induce workers to stay unemployed rather to commit crime because they are less vulnerable to crime activities. Social cohesion favors employment but also knits together delinquents into resilient clusters, and more deterrence effort is needed to reduce crime.

Keywords: social interactions, crime, labor market.

JEL Classification: A14, J40, K42.

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1 Introduction

It is well-recognized that friends and labor market opportunities have a strong impact on criminal behavior. For instance, the positive correlation between self-reported delinquency by adolescent and the number of delinquent friends is among the strongest and most consistent findings in the delinquency literature (Warr 1996, Matsueda and Anderson 1998).[1] There are also sizable and significant effects of unemployment (Raphael and Winter-Ebmer 2001), wages (Machin and Meghir 2004) and inequality (Bourguignon *et. al.* 2003) on crime. The aim of this paper is to propose a model that incorporates these two features and to derive policy implications.

The impact of labor market outcomes on crime has been modeled in different ways (see in particular the recent contributions of İmrohoroğlu *et. al.* 2000, Burdett *et. al.* 2003, and Verdier and Zenou 2004) but the role of friends and peers on crime has received so far less attention (exceptions include Sah 1991, Glaeser, Sacerdote and Scheinkman 1996, Calvó-Armengol and Zenou 2004, and Ballester *et. al.* 2004).

In the present paper, we focus on the role of social contacts on crime, and distinguish between weak and strong ties in the pattern social interactions. Following Granovetter (1973), we consider that the strength of a social tie corresponds to the duration of a relationship. We define as *strong tie* a social relationship between two agents that is repeated over time (for example members of the same family or very close friends) and as *weak tie* a transitory social encounter between two persons.[2]

To be more precise, we consider a model in which individuals belong to mutually exclusive two-person groups, referred to as *dyads*. Dyad members do not change over time so that two individuals belonging to the same dyad hold a *strong tie* with each other. However, each dyad partner can meet other individuals outside the dyad partnership, referred to as *weak ties* or random encounters. By definition, weak ties are transitory and only last for one period.

We then assume that individuals learn about crime opportunities by interacting with active criminals. These interactions can take the form of either strong or weak ties. The process through which individuals learn about crime behavior and opportunities results from a combination of a socialization process that takes place *inside* the family (in the case of strong ties) and a socialization process *outside* the family (in the case of weak ties). Bisin and Verdier (2000) refer to the former as *vertical* socialization and to the latter as *oblique* socialization. Both currently active criminals and potential criminals exert an influence over one another to commit offences by meeting each other. In contrast, we assume that individuals learn about job opportunities exclusively through

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[1]In economics, empirical evidence also suggests that peer effects are very strong in criminal decisions. For instance, Ludwig *et. al.* (2001) find that, in the U.S., relocating families from high to low-poverty neighborhoods (Moving To Opportunity program) reduces juvenile arrests for violent offences by 30 to 50 percent of the arrest rate for control groups. Chen and Shapiro (2003) and Bayer *et al.* (2003) find also strong peer effects in crime by investigating the influence that individuals serving time in the same facility have on the subsequent criminal behavior of offenders.

[2]Montgomery (1994) uses a similar model of weak and strong ties in the labor market.

employment agencies.

We analyze the flows of dyads between states and characterize the different steady-state equilibria. Four possible equilibria can emerge. Either individuals are all unemployed, or both criminals and unemployed, or both employed and unemployed or a mixture of these three states when criminals, employed and unemployed workers coexist. We characterize the ranges of exogenous parameter values for which each of those equilibria emerges. In some cases, multiple equilibria may coexist.

We then analyze how endogenous outcomes (crime, employment and unemployment) respond to variations of the exogenous parameters. First, when jobs are badly paid and/or crime is profitable, the aggregate crime level increases with the unemployment benefit. An optimal policy is thus to reduce the unemployment insurance. Note, indeed, that unemployed workers are more prone to enter in the crime business than employed workers. When the unemployment benefit is low, the opportunity cost of searching a job is reduced, and workers quickly find jobs. This reduction in unemployment spells reduces the workers exposure to crime opportunities, and aggregate crime decreases. Second, when workers are well paid and/or crime is not profitable case, the unemployment benefit has to be high enough to induce workers to stay unemployed longer rather than to commit crime.

This highlights the way unemployment is viewed in this model. It is not only the "waiting room" for employment (as it is usually perceived) but also for crime. Unemployed workers trade off the costs and benefits from becoming employed or a criminal. In a dynamic setting, the opportunity cost of searching for a good job becomes a crucial determinant of this trade-off. The impact of the unemployment insurance on crime thus depends on the relative values of being employed or criminal. Our analysis suggests that an optimal unemployment benefit policy should discriminate among the different characteristics of local labor markets.

Finally, we find that increasing punishment both reduces crime and increases employment, but has an ambiguous effect on unemployment. We also find that strong ties (i.e. social cohesion) are important to keep the employment rate high but have an ambiguous effect on crime.

2   The model

Consider a population of individuals of size one. Individual are either employed, unemployed, or involved in criminal activities.

Dyads   We assume that individuals belong to mutually exclusive two-person groups, referred to as *dyads*. We say that two individuals belonging to the same dyad hold a *strong tie* with each other. We assume that dyad members do not change over time. Equivalently, a strong tie is created once and for ever; it can never be broken. We can thus think of strong ties as links between members of the same family, or between very close friends.

Individuals can be in either of three different states: employed, unemployed or criminals. Dyads, which consist of paired individuals, can thus be in six different states,[3] which are the following:

(i) both members are employed −we denote by d_2 the number of such dyads;

(ii) one member is employed and the other is unemployed (d_1);

(iii) both members are unemployed (d_0);

(iv) one member is unemployed and the other is a criminal (d_{-1});

(v) both members are criminals (d_{-2});

(vi) one member is employed while the other is a criminal $(d_{\pm 1})$.

By assumption, we exclude this last type of dyad, in which one member is employed and the other is a criminal. This is explained below.

Aggregate state   Denoting by e_t, u_t and c_t respectively the employment rate, the unemployment rate, and the crime rate at time t, where $c_t, e_t, u_t \in [0, 1]$, we have:

$$\begin{cases} e_t = 2d_{2,t} + d_{1,t} \\ u_t = 2d_{0,t} + d_{1,t} + d_{-1,t} \\ c_t = 2d_{-2,t} + d_{-1,t} \end{cases} \tag{1}$$

The normalization condition can then be written as

$$e_t + c_t + u_t = 1 \tag{2}$$

or, alternatively,

$$d_{-2,t} + d_{-1,t} + d_{0,t} + d_{1,t} + d_{2,t} = \frac{1}{2} \tag{3}$$

Social interactions   Time is continuous and individuals live for ever. We assume that individuals randomly meet by pairs repeatedly through time. Matchings make take place between dyad partners or not. We assume that any given individual is matched with his dyad partner with probability $1 - \omega$, while with complementary probability ω he is matched randomly to any other individual in the population. We refer to matchings outside the dyad partnership as *weak ties* or random encounters. By definition, weak ties are transitory and only last for one period.[4]

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[3]The inner ordering of dyad members does not matter.

[4]Another way of interpreting ω and $1 - \omega$ is to view them as the time spend per period with weak and strong ties respectively. Indeed, if each individual has one unit of time per period then, on average, he spends ω units of time with one individual outside the dyad (weak tie) and $1 - \omega$ units of time with his dyad partner (strong tie).

Once again this highlights the way we view strong and weak ties. Each individual is born with a strong tie and will spend all his life with this person (think for example of a brother, a sister or a best mate). However, throughout his life, he will also meet other people that are not as close as his strong tie. We refer to them as weak ties or random encounters.

Within each matched pair, information is exchanged, as explained below.

Information transmission   At the beginning of each time period, currently unemployed workers hear of job vacancies at exogenous rate λ. All jobs and all workers are identical (unskilled labor). At the end of the current time period, employed workers lose their job at exogenous rate δ. At the beginning of each time period, current delinquents are aware of some criminal activity for the current time period at exogenous rate α. Delinquents pass this information on to their current matched partner. This could also be within a dyad or a random acquaintance. Thus, we assume that information about crime is essentially through friends and relative (i.e. strong and weak ties) whereas information about jobs is mainly through formal methods (without the help of any ties).

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad. Transitions depend on the labor market and crime turnover and on the nature of social interactions as captured by ω.

We assume that time periods are small enough so that at most one dyad partner hears of an exogenous offer per period and that both members of a dyad cannot change their status at the same time. For example, two unemployed workers cannot at the same time either find a job or become criminal, i.e. the probability assigned to a one-period transition from a d_0-dyad to either a d_2-dyad or a $d_{-2}-$dyad is zero. Similarly, two employed workers (d_2-dyad) or two criminals ($d_{-2}-$dyad) cannot become both unemployed, i.e. switch to a d_0-dyad, in just one period. Of course, this does not imply that a d_0-dyad can never become a d_2-dyad. It just means that a d_2-dyad cannot switch to a d_0-dyad in just one period. As we will see below, at least one period will be required. This applies to all the other dyads mentioned above.

Incentives   Within each time period, the material payoff is w for an employed worker, and b for an unemployed worker, where b refers to the unemployment insurance benefit. We assume that $w > b$.[5]

Unemployed workers decide between becoming a criminal, staying unemployed, or becoming employed. Individuals are forward-looking with respect to their future status when taking this decision, and anticipate the impact of current decisions on their future opportunities and payoffs. Yet, they are myopic with respect to the status of their current partner, which they treat as a default state.

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[5]We keep the wage w fixed throughout. A partial equilibrium approach renders the analysis more tractable without affecting qualitatively the results. See Burdett *et al.* (2003) for a general equilibrium analysis where the wage is endogenous.

In the long-run, individual values for each possible dyad outcome are giveny by the following Bellman equations. Each Bellman equation is written for the individual with the first subscript. For example, V_{01} is the lifetime expected utility of an unemployed worker whose strong tie is currently employed, while V_{10} corresponds to the lifetime expected utility of an employed worker with an unemployed dyad partner:

$$rV_{11} = w + \delta \left(V_{01} - V_{11} \right) \tag{4}$$

$$rV_{01} = b + \lambda \left(V_{11} - V_{01} \right) \tag{5}$$

$$rV_{10} = w + \delta \left(V_{00} - V_{10} \right) \tag{6}$$

$$rV_{00} = b + \lambda\phi \left(V_{10} - V_{00} \right) + q(c)\psi \left(V_{-10} - V_{00} \right). \tag{7}$$

In these expressions, r is the interest rate, $q(c) = \omega c \alpha$ denotes the probability to hear from a crime opportunity from a weak tie and ϕ and ψ are the endogenous decisions to accept a job and a crime opportunity, respectively. There are binary values, that is, $\phi, \psi \in \{0, 1\}$, where $\phi = 1$ (resp. $\psi = 1$) when an unemployed worker accepts a job (resp. becomes a criminal), and $\phi = 0$ (resp. $\psi = 1$) otherwise.

We assume that $w > b$ so that $V_{11} > V_{01}$. We have:

$$rV_{-10} = g + p \left(F_{00} - V_{00} \right)$$

where g denotes the proceeds from crime, p the rate at which criminal are caught and

$$F_{00} = -f + V_{00},$$

where $f > 0$ is the penalty associated with jail. So basically, when a criminal is caught, he spends some time in prison $(-f)$ and then with probability 1 gets out. Observe that these equations are written under the assumption that, when a released criminal becomes automatically unemployed. Also, the time spent in prison is assumed to be sufficiently short so that the strong tie does not change his status.

Combining, we obtain:

$$rV_{-10} = g - pf \tag{8}$$

We also have:

$$rV_{0-1} = b + h(c)\psi_c \left(V_{-1-1} - V_{0-1} \right) \tag{9}$$

where $h(c) = (1 - \omega + \omega c)\alpha$ is the probability to hear from a crime opportunity either by a weak or a strong tie and V_{-1-1} is the expected payoffs from crime for a criminal associated with a criminal (α is the rate at which 'potential' criminals hear from a crime opportunity) and ψ_c is the probability to become a criminal because of a peer-pressure that favors conformism. More precisely, we assume that an unemployed worker associated with a criminal, always accepts a crime offer. As we have seen above (see equation (7)), this is not always true for an unemployed worker whose strong tie is also unemployed. In this case, he may or may not accept a crime offer. Similarly, he may or may not accept a job offer. The pressure for conformism also works in the labor market, and an unemployed worker hearing of a job opportunity and whose dyad partner is employed always accepts the job offer. We have:

$$rV_{-1-1} = g + g_c + p\left(F_{-1-1} - V_{-1-1}\right)$$

where g_c is the additional gain from conformism. Also:

$$F_{-1-1} = -f + V_{0-1}.$$

The expected lifetime utility of a criminal (whether his strong tie is unemployed or criminal) consists of today's expected gain αg (the rate α at which a criminal hears from a crime opportunity times the gain g of committing the crime) plus the expected probability to be caught αp times the expected utility loss. In our definition, a criminal (identified by -1) is not necessary someone who is committing a crime but someone who is actively (full time) searching for a crime opportunity and will commit a crime if he obtains the information on a crime opportunity (this occurs at rate α). As a result, this individual will never obtain a job since he is not looking for it. Observe that, contrary to a "criminal", an unemployed worker whose strong tie is unemployed (his utility is V_{00}) does not always accept a crime opportunity, only when $\psi = 1$.

Combining

$$rV_{-1-1} = g + g_c - pf + p\left(V_{0-1} - V_{-1-1}\right) \tag{10}$$

Observe also that strong ties and thus conformity matter a lot in this model. Indeed, as stated above, employed workers are incompatible with criminals, and vice-versa because strong ties influence each other. Indeed, conformism prevents individuals to switch to polar activities than that of strong ties. As a result, no employed-criminal dyad can ever form. Furthermore, the behavior of an unemployed worker is totally different whether his strong tie is employed, unemployed or criminal. Indeed, if an unemployed worker has a strong tie who is employed (dyad 01), he will always accept a job opportunity (see (5)) whereas if his strong tie is unemployed (dyad 00), he will accept a job opportunity with probability ϕ (see (7)). Similarly, an unemployed whose strong tie is criminal (dyad $0 - 1$) will always accept to commit a crime (see (9)) whereas the same unemployed whose strong tie is unemployed (dyad 00) will accept a crime opportunity with probability ψ (see (7)). All the assumptions are compatible with a model in which individual's behavior is strongly influenced by his friends and relatives (peers).

Flows of dyads between states   It is readily checked that the net flow of dyads from each state between t and $t+1$ is given by

$$
\begin{cases}
\dot{d}_{2,t} = \lambda d_{1,t} - 2\delta d_{2,t} \\
\dot{d}_{1,t} = 2\phi\lambda d_{0,t} - (\delta + \lambda) d_{1,t} + 2\delta d_{2,t} \\
\dot{d}_{0,t} = pd_{-1,t} - 2\left[\psi q(c_t) + \phi\lambda\right] d_{0,t} + \delta d_{1,t} \\
\dot{d}_{-1,t} = 2pd_{-2,t} - \left[p + h(c_t)\psi_c\right] d_{-1,t} + 2\psi q(c)d_{0,t} \\
\dot{d}_{-2,t} = -2pd_{-2,t} + h(c_t)\psi_c d_{-1,t}
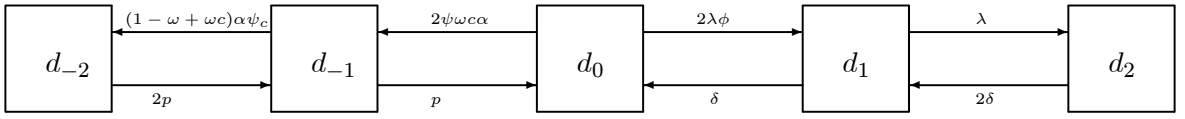\end{cases}
\tag{11}
$$

Graphically,

Figure 1

These dynamic equations reflect the flows across dyads. For instance, in the first equation, the variation of dyads composed of two employed workers ($\dot{d}_{2,t}$) is equal to the number of d_1-dyads in which the unemployed worker has found a job minus the number of d_2-dyads in which one the two employed has lost his job. In the last equation, the variation of dyads composed of two criminals ($\dot{d}_{-2,t}$) is equal to the number of $d_{-1}-$dyads in which the unemployed has become a criminal (through either his strong tie with probability $(1-\omega)\alpha$ or his weak tie with probability $\omega c\alpha$) minus the number of $d_{-2}-$dyads in which one the two criminals has been caught. All the other equations have a similar interpretation.

Observe that the assumption stated above that both members of a dyad cannot lose their status at the same time is reflected in the flows described by (11). Take for example the dyad d_{-2}. To switch to a d_0-dyad, it will take at least two periods. During the first period, there is a probability $2\alpha p$ for a $d_{-2}-$dyad to become a $d_{-1}-$dyad (indeed it has to be that only one of them has been caught, that is either the first or the second member of the $d_{-2}-$dyad; this occurs with probability $\alpha p + \alpha p = 2\alpha p$). Then, for the second period, there is a probability αp (since now there is only one member who is criminal) for a $d_{-1}-$dyad to become a d_0-dyad. What is crucial in our analysis is that members of the same dyad (strong ties) always stay together throughout their life. So, for example, if a $d_{-1}-$dyad becomes after some periods a d_1-dyad, the members of this dyad are exactly the same; they have just changed their status.

Observe also that the encounter of weak ties is "localized". Take for example an unemployed who belongs to a $d_{-1}-$dyad. The only random encounters (weak ties) he can meet is among the pool of criminals and unemployed, and therefore will only obtain information about crime but not job opportunities (this is our definition of active criminal). Similarly, an unemployed worker belonging

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to a d_1-dyad, can only randomly encounter (weak tie) unemployed or employed workers and thus can only obtain information about jobs but not criminal opportunities. Only unemployed workers belonging to a d_0-dyad (where both dyad members are unemployed) can have access through weak ties to both types of information, i.e. both crime and job opportunities. As a result, a criminal can only become employed through an intermediate period of unemployment, and vice-versa. The networks of criminals and employed workers are thus separated, except for unemployed workers who may receive information about both crime activities and job vacancies. From an informational viewpoint, the overlapping of crime and labor networks exists only through the dyads composed of two unemployed (d_0).

What we capture here is the fact that the type of random encounters is strongly influenced by the status of the partner in the dyad (conformism, as in Akerlof 1987). There is indeed a complementarity between strong and weak ties since someone's strong tie influences the nature of his relationship with random encounters. This is consistent with our assumption that criminals and employed workers are never partners and thus do not form strong ties with each other leading to $d_{\pm 1}-$dyads.

Taking into account (3), the system (11) reduces to a four-dimensional dynamic system in $d_{2,t}, d_{1,t}, d_{-1,t}$ and $d_{-2,t}$ given by:

$$\begin{cases} \dot{d}_{2,t} = \lambda d_{1,t} - 2\delta d_{2,t} \\ \dot{d}_{1,t} = 2\phi\lambda\left[1/2 - d_{-2,t} - d_{-1,t} - d_{1,t} - d_{2,t}\right] - (\delta + \lambda) d_{1,t} + 2\delta d_{2,t} \\ \dot{d}_{-1,t} = 2pd_{-2,t} - [p + h(c_t)\psi_c] d_{-1,t} + 2\psi q(c)\left[1/2 - d_{-2,t} - d_{-1,t} - d_{1,t} - d_{2,t}\right] \\ \dot{d}_{-2,t} = -2pd_{-2,t} + h(c_t)\psi_c d_{-1,t} \end{cases}$$

with, using (1),

$$\begin{cases} e_t = 2d_{2,t} + d_{1,t} \\ c_t = 2d_{-2,t} + d_{-1,t} \end{cases}$$

3 Steady-state equilibrium analysis

Steady-state dyad flows At a steady-state $(d_2^*, d_1^*, d_0^*, d_{-1}^*, d_{-2}^*)$, each of the net flow in (11) is equal to zero. Setting these net flows equal to zero leads to the following relationships:

$$d_2^* = \frac{\lambda}{2\delta}d_1^*, \qquad d_1^* = \frac{2\phi\lambda}{\delta}d_0^* \tag{12}$$

$$d_{-2}^* = \frac{(1 - \omega + \omega c^*)\psi_c \alpha}{2p}d_{-1}^*, \qquad d_{-1}^* = \frac{2\psi\omega c^* \alpha}{p}d_0^* \tag{13}$$

where

$$e^* = 2d_2^* + d_1^* \text{ and } c^* = 2d_{-2}^* + d_{-1}^* \tag{14}$$

$$d_0^* = \frac{1}{2} - d_2^* - d_1^* - d_{-1}^* - d_{-2}^* \tag{15}$$

Definition 1 *A steady-state equilibrium is a seven-tuple $(d_2^*, d_1^*, d_0^*, d_{-1}^* d_{-2}^*, u^*, c^*)$ such that equations (3), (12), (13), (14) and (15) are satisfied.*

The following result identifies all steady-state equilibria and provides conditions for their existence.

Proposition 1 *There are four different steady-state equilibria:*

(i) *There exists a full-unemployment steady-state $\mathcal{U}$ with $c^* = e^* = 0$ when either $\phi = \psi = 0, \psi_c \in \{0, 1\}$, or $\phi = 0, \psi = \psi_c = 1$.*

(ii) *There exists a crime-free steady-state equilibrium $\mathcal{E}$ with $c^* = 0$ and $e^* > 0$ when either $\phi = 1, \psi = 0, \psi_c \in \{0, 1\}$ or $\phi = \psi = \psi_c = 1$.*

(iii) *There exists a no-employment steady-state equilibrium $\mathcal{C}$ with $e^* = 0$ and $c^* > 0$ when $\phi = 0, \psi = \psi_c = 1$ and $p < X(\omega) = \frac{1}{2}[\omega\alpha + \alpha\sqrt{\omega(4 - 3\omega)}]$.*

(iv) *There exists a mixed steady-state equilibrium $\mathcal{M}$ with $e^* > 0$ and $c^* > 0$ when $\phi = \psi = \psi_c = 1$ and $p < X_\theta(\omega)$, where $X_\theta(\omega)$ is a uniquely defined function $X_\theta : [0, 1] \to \mathbb{R}_+$.*

Proof. See Appendix 1.

This proposition states that four steady-states equilibria may emerge. At $\mathcal{U}$, the economy is populated only with dyads of two unemployed agents, that is, $d_0^* = 1/2$. At $\mathcal{E}$, there are three types of dyads, d_0^*, d_1^* and d_2^*. These two equilibria can exist for any value of p. This is not true for the other equilibria. Consider for instance the no-employment equilibrium $\mathcal{C}$. This steady-state arises for low enough values of the probability to be caught p. Indeed, when p is too high, crime is reduced and employment becomes positive. So when p is large enough, the employment rate is strictly positive at steady-state. The range $(0, X(\omega))$ of admissible values for p increases with ω, the frequency of random encounters. When ω increases, individuals diversify their set of contacts, and the number of available sources for crime information widens. Crime is now present at equilibrium with higher levels of punishment. The intuition is the same for the mixed equilibrium $\mathcal{M}$.

Observe that our dynamic system has multiple steady-state equilibria. When $\phi = 0$ and $\psi = 1$, both $\mathcal{U}$ and $\mathcal{C}$ can coexist as long as $p < X(\omega)$. When $\phi = \psi = 1$, both $\mathcal{E}$ and $\mathcal{M}$ can coexist if $p < X_\theta(\omega)$.

Unemployed workers' decisions   We now endogeneize the parameters ϕ and ψ. We solve the Bellman equations given in section 2. First, observe that (4) and (5) can be solved separately. We easily obtain:

$$V_{11} = w\frac{(r+\lambda)}{(r+\lambda)^2 - \delta\lambda} + \frac{\delta}{(r+\lambda)^2 - \delta\lambda}b$$

$$V_{01} = w\frac{\lambda}{(r+\lambda)^2 - \delta\lambda} + \frac{(r+\lambda)}{\left[(r+\lambda)^2 - \delta\lambda\right]}b$$

with

$$V_{11} - V_{01} = \frac{w-b}{r+\lambda+\delta}$$

Second, observe that (9) and (10) can also be solved separately. We easily obtain:

$$V_{-1-1} - V_{0-1} = \frac{g + g_c - pf - b}{r + p + \psi_c h(c^*)}$$

where

$$h(c^*) = (1 - \omega + \omega c^*)\,\alpha$$

Thus

$$\psi_c \begin{cases} = 1 & \Leftrightarrow & V_{-1-1} \geq V_{0-1} & \Leftrightarrow & g + g_c \geq pf + b \\ = 0 & \Leftrightarrow & V_{-1-1} < V_{0-1} & \Leftrightarrow & g + g_c < pf + b \end{cases}$$

As it can be seen, all the results are independent of ϕ and ψ. This is because of our assumption of conformism according to which individuals are strongly influenced by their strong ties in their decisions.

We now solve for the last block of equations (6), (7) and (8), which will give us the different values of ϕ and ψ. By solving these three equations, we obtain:

$$V_{00} = \frac{b + \lambda\phi\frac{w}{r+\delta} + q(c)\psi\frac{g-pf}{r}}{r + \lambda\phi\frac{r}{r+\delta} + q(c)\psi}$$

$$V_{10} = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}V_{00}$$

$$V_{-10} = \frac{g - pf}{r}$$

This implies that

$$\psi_c \begin{cases} = 1 & \Leftrightarrow & V_{-1-1} \geq V_{0-1} & \Leftrightarrow & g + g_c \geq pf + b \\ = 0 & \Leftrightarrow & V_{-1-1} < V_{0-1} & \Leftrightarrow & g + g_c < pf + b \end{cases} \tag{16}$$

$$\phi = \begin{cases} = 1 & \Leftrightarrow & V_{10} \geq V_{00} & \Leftrightarrow & w_{10} \geq rV_{00} \\ = 0 & \Leftrightarrow & V_{10} < V_{00} & \Leftrightarrow & w_{10} < rV_{00} \end{cases} \tag{17}$$

$$\psi = \begin{cases} = 1 & \Leftrightarrow \quad V_{-10} \geq V_{00} \quad \Leftrightarrow \quad g - pf \geq rV_{00} \\ = 0 & \Leftrightarrow \quad V_{-10} < V_{00} \quad \Leftrightarrow \quad g - pf < rV_{00} \end{cases} \tag{18}$$

This is very intuitive. An unemployed worker whose strong tie is unemployed will accept a job opportunity if the current wage is large enough compare to the intertemporal utility of being unemployed. Instead, he will accept a crime opportunity when the current net expected crime gain is larger than the discounted intertemporal utility of being unemployed. These conditions are similar to the ones found in the search literature (see in particular Burdett *et al.* 2003) since w_{10} and γ correspond to the reservation wage and crime gain respectively.

Denote by $c^*(1,1)$ and $c^*(0,1)$ the equilibrium crime levels when $\phi = \psi = 1$ and $\phi = 0$ and $\psi = 1$, respectively. We have the following result:

Proposition 2 *Let $w > b$ and $p < \min\{X(\omega), X_\theta(\omega)\}$.*

Suppose first that $w > g - pf$. We distinguish two cases:

- (i) *if $b \leq -\frac{\lambda}{r+\delta}w + \left(1 + \frac{\lambda}{r+\delta}\right)(g - pf)$, there exists a unique (mixed) steady-state equilibrium $\mathcal{M}$.*

  (ii) *if $b > -\frac{\lambda}{r+\delta}w + \left(1 + \frac{\lambda}{r+\delta}\right)(g - pf)$, there exists a unique (crime-free) steady-state equilibrium $\mathcal{E}$.*

Assume now that $w \leq g - pf$. Then,

- (i) *if $b < w + [w - (g - pf)]\omega\alpha c^*(0,1)/r$, there exists a unique (mixed) steady-state equilibrium $\mathcal{M}$.*

  (ii) *if $w + [w - (g - pf)]\omega\alpha c^*(0,1)/r \leq b < w + [w - (g - pf)]\omega\alpha c^*(1,1)/r$, there are multiple equilibria since both the mixed and the no-employment steady-state equilibria $\mathcal{M}$ and $\mathcal{C}$ co-exist.*

  (iii) *if $w + [w - (g - pf)]\omega\alpha c^*(1,1)/r \leq b$, there exists a unique (no-employment) steady-state equilibrium $\mathcal{C}$.*

Proof. See Appendix 1.

Figures 2a and 2b illustrate the complete characterization of the steady-state equilibria.[6]

[*Insert Figure 2a and 2b here*]

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[6]Observe that in Figure 2b the slopes of the two equations are positive. Indeed, since $c^*(0,1)$ and $c^*(1,1)$ are a negative function of p (see Proposition 3), the slopes are respectively given by

$$\frac{\omega\alpha}{r}\left[f\,c^*(1,1) + [w - (g - pf)]\frac{\partial c^*(1,1)}{\partial p}\right]$$

and

$$\frac{\omega\alpha}{r}\left[f\,c^*(0,1) + [w - (g - pf)]\frac{\partial c^*(0,1)}{\partial p}\right]$$

This proposition gives the main results of this paper. First, multiple equilibria exist when $w \leq g - pf$, that is both/either people are badly paid and/or crime is profitable. Second, it predicts that, when $w \leq g - pf$, the level of crime should be in general higher than when $w > g - pf$, that is both/either people are well paid and/or crime is not profitable. Third, it gives very different policy implications depending on the values of the wage w, the proceeds from crime g and the punishment pf. Indeed, this proposition says that, when $w \leq g - pf$, the unemployment benefit has to be low enough to prevent workers for staying too long in the unemployment status (d_0) because they are vulnerable to crime activities: they are more likely to take crime opportunities since the returns from crime are relatively high. On the contrary, when $w > g - pf$, the unemployment benefit has to be high enough to induce workers to stay unemployed rather to commit crime because they are less vulnerable to crime activities.

This highlights the way unemployment is viewed in this model. It is not only the "waiting room" for employment (as it is usually perceived) but also for crime. So, it may be that, in some cases (here when $w > g - pf$), it is a good thing to encourage workers to stay unemployed whereas in other cases (here when $w \leq g - pf$), the contrary prevails because of crime opportunities that these individuals may accept. This result also sheds some lights on the crucial debate around the unemployment benefit policy (see for example the survey by Atkinson and Micklewright, 1991). Some think that it has to be low enough to induce workers to accept low-paid jobs whereas others postulate that it has to be high enough to help unemployed workers to survive. Here the implications are different since labor and crime activities affect each other. So basically it is only in areas where job opportunities are low (badly paid) and crime profitable (for example drug activities) that one has to set a low level of unemployment benefit. In this context, since b is viewed here as an opportunity cost, the main role of the planner is to control the time spent in the "waiting room".

Finally, one may observe the role of weak and strong ties in policies. As can be seen from Figure 2a, when $w > g - pf$, weak or strong ties do not affect the equilibria. However, when $w \leq g - pf$ (Figure 2b), societies with different levels of fragmentation react differently to same policy interventions. For example, when ω decreases, society is formed of highly cohesive small groups, and policies need to be tougher because of the intra-dyad correlation that generates clusters of unemployed and criminals.

4 Comparative statics

Let us perform some comparative statics analysis for the steady-state equilibrium $\mathcal{M}$.

There are thus both positive since $w \leq (g - pf)$. Furthermore, the intercepts are such that

$$w + (w - g)\,\omega\alpha c^*(1,1)/r > w + (w - g)\,\omega\alpha c^*(0,1)/r$$

since $w \leq (g - pf)$ and $c^*(0,1) > c^*(1,1)$ (see Lemma 3 in the Appendix).

Proposition 3 *Let $w > b$ and $p < \min\{X(\omega), X_\theta(\omega)\}$. Fix ω. Then, for the steady-state equilibrium $\mathcal{M}$,*

(i) *An increase in p, raises both the number of d_0^*-dyads and the employment rate e^*. If we further assume that $p > \alpha$, then an increase in p reduces the crime rate c^*. The effect on the unemployment rate u^* is ambiguous.*

(ii) *An increase in δ or a decrease in λ increases the crime rate c^* but reduces the employment rate e^* and the number of d_0^*-dyads.*

Proof. See Appendix 2.

It is easy to verify that these comparative statics results also hold for the equilibrium $\mathcal{C}$, which is a particular case of $\mathcal{E}$. Let us comment the results. First, when punishment p increases (recall that p is *not* a probability but a rate so its value is between 0 and $+\infty$), dyads d_{-2}^* and d_{-1}^* are destroyed at a faster rate since criminals are caught more often and thus the number of d_0^*-dyads increases. As a result, the relative chance to find a job is higher (since individuals are more often in a d_0^*-dyad) and the employment rate e^* rises. Second, in order to have the standard effect of punishment on crime, a sufficient condition is needed, that is $p > \alpha$. Indeed, when p increases two forces are at work. There is direct negative effect on crime since dyads d_{-2}^* and d_{-1}^* are destroyed at a faster rate but there is an indirect positive effect on crime because the increase of d_0^*-dyads implies that individuals return more quickly to criminal activities. When $p > \alpha$, i.e. the crime opportunities are relatively less frequent compared to punishment, then the first effect dominates the other and p has a negative impact on c^*. Third, since $u^* = 2d_0^* + d_1^* + d_{-1}$, increasing p has an ambiguous effect on u^* because it reduces the $d_{-1}-$dyads and increases both the d_0^* and d_1^*-dyads. Finally, not surprisingly an increase in the job destruction rate δ or a decrease in the job acquisition rate λ reduces the employment rate e^* and increases the crime rate c^*. This result is however interesting since it links the business cycle to crime rates so that in booms (downturns) crime is reduced (increased). This is a well-documented fact (Freeman 1999 is a good survey).

Let us now investigate the impact of weak and strong ties on crime and unemployment. We have:

Proposition 4 *For the steady-state equilibrium $\mathcal{M}$, if $p > \alpha$, then an increase in ω (weak ties) reduces the employment rate e^* and the number of d_0^*-dyads. Furthermore, for any $p > 0$,*

$$\frac{\partial c^*}{\partial \omega} \gtreqless 0 \Leftrightarrow \frac{\alpha + p}{p} \gtreqless \frac{p}{2\alpha\omega d_0^*} + \omega\frac{\alpha^2\omega d_0^*}{p^2 - \alpha^2\omega d_0^*}$$

Proof. See Appendix 2.

The first result is interesting since it suggests that strong ties (social cohesion) are important for keeping the employment rate high if $p > \alpha$. Indeed, when the time spent with strong ties

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$(1 - \omega)$ is long and the crime opportunities are relatively less frequent compared to punishment, then when criminals are caught they do stay unemployed for a long time (since the number of d_0^*-dyads increases) and thus their chance to find a job increases. Observe also that weak ties (and not strong ties) are the ones that pull d_0-dyads to $d_{-1}-$dyads. In other words, when two individuals are in a d_0-dyad (i.e. both of them are unemployed), the only way for one of them to become criminal is to hear about a criminal opportunity through weak ties (peers). This is why, social cohesion is good here, because it pulls d_0-dyads to d_1-dyads. So, starting from a d_0-dyad, there are two stages to become criminal. First, one of the unemployed has to hear about a criminal opportunity through his weak ties, and then when the dyad becomes a $d_{-1}-$dyad, the unemployed is more likely to become criminal because because he can hear from a crime opportunity both from his strong tie and weak ties.

The second result of this proposition is that the effect on crime is ambiguous. Indeed, when for example ω decreases, the society is formed of highly cohesive small groups (more strong ties) and individuals tend to imitate each other. So groups tend to cluster and if there are already a lot of criminals then crime would augment. However, depending on the policies (on p or b), ω can affect positively or negatively crime, as expressed by the condition stated in the proposition. In order to illustrate this result, we have run a simple numerical simulation. We interpret a time period of unit length to be one month. We set the separation rate δ to 0.04 and the job acquisition rate λ to 0.2, implying that the average durations of employment and unemployment are about two years and five months respectively. Furthermore, the rate α at which individual hear about crime is set to 0.2, implying an average duration time before a crime opportunity arises of about five months.

In figure 3, we have calculated $d_0^*(p)$ from (27) and then plugging this value in the following equation:

$$\frac{\alpha + p}{p} = \frac{p}{2\alpha\omega d_0^*} + \omega \frac{\alpha^2 \omega d_0^*}{p^2 - \alpha^2 \omega d_0^*}$$

We thus obtained p as a function of ω. Figure 3 displays this result by varying ω from 0.01 to 0.99. The line obtained corresponds to the locus of points such that $\partial c^*/\partial \omega = 0$. First, it is easy to see that this line is decreasing, implying that the value $\omega^*(p)$ for which $\partial c^*/\partial \omega = 0$, decreases with p. Second, for a given p, an increase in weak ties ω increases the chance that $\partial c^*/\partial \omega < 0$ is reached and the higher the initial p the quicker $\partial c^*/\partial \omega < 0$ is reached. This means that, apart of unemployment benefits, a planner can play with both p and ω to reduce crime. If the planner sets a high level of punishment p, then it is better that it enhances weak ties by dismantling social cohesion among criminals. If on the contrary, p is set at a low level, then the reverse prevails and strong ties should be enhanced.
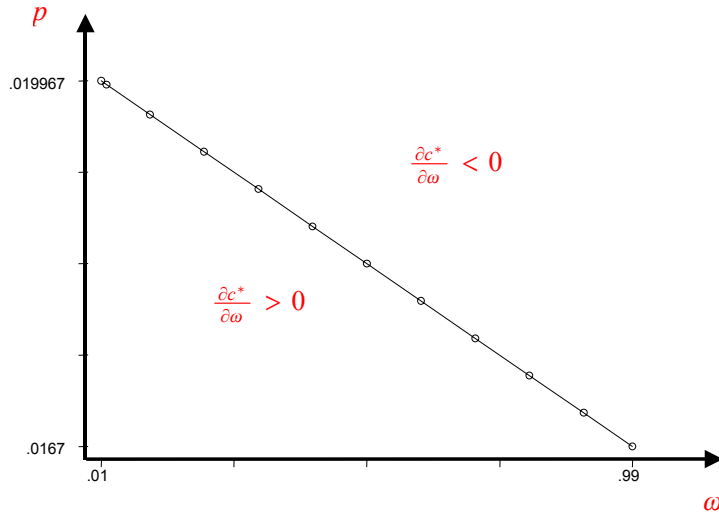
Figure 3

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Appendix 1: Steady-State Equilibrium Analysis

Proof of Proposition 1: We establish the proof in two steps. First, we characterize the set of steady-state equilibria. We then provide conditions for their existence.

Lemma 1 *There exists at most four different steady-state equilibria: (i) a full-unemployment equilibrium $\mathcal{U}$ such that $c^* = e^* = 0$, (ii) a crime-free equilibrium $\mathcal{E}$ such that $c^* = 0$ and $e^* > 0$, (iii) a no-employment equilibrium $\mathcal{C}$ such that $c^* > 0$ and $e^* = 0$, and (iv) a mixed equilibrium $\mathcal{M}$ such that $c^* > 0$ and $e^* > 0$.*

Proof. By combining (12) to (14), we easily obtain:

$$e^* = \frac{2\lambda(\lambda + \delta)\phi}{\delta^2}d_0^* \tag{19}$$

$$c^* = [(1 - \omega + \omega c^*)\psi_c\alpha + p]\frac{2\psi\omega c^*\alpha}{p^2}d_0^* \tag{20}$$

We consider four different cases.

(i) If $\phi = \psi = 0$, $\psi_c \in \{0, 1\}$, then $c^* = e^* = 0$, and equations (19) and (20) are satisfied. Furthermore, using (12) and (13), this implies that $d_1^* = d_2^* = d_{-1}^* = d_{-2}^* = 0$ and, using (3), we have $d_0^* = 1/2$. This is referred to as equilibrium $\mathcal{U}$. Observe that in this case, the value of ψ_c does not matter because there is no crime.

(ii) If $\phi = 1$, $\psi = 0$, $\psi_c \in \{0, 1\}$, then $c^* = 0$ and $e^* > 0$, and equation (20) is satisfied. Using (13), $c^* = 0$ implies that $d_{-1}^* = d_{-2}^* = 0$. Finally, using (3), (12) and (14), we easily obtain:

$$d_2^* = \frac{1}{2}\left(\frac{\lambda}{\delta + \lambda}\right)^2, \qquad d_1^* = \frac{\delta\lambda}{(\delta + \lambda)^2}, \qquad e^* = \frac{\lambda}{\delta + \lambda} \tag{21}$$

where

$$d_0^* = \frac{1}{2}\left(\frac{\delta}{\delta + \lambda}\right)^2 \tag{22}$$

This is referred to as equilibrium $\mathcal{E}$. Observe again that in this case, the value of ψ_c does not matter because there is no crime.

(iii) If $\phi = 0$ and $\psi = \psi_c = 1$, then $e^* = 0$ and equation (19) is satisfied.

Two cases must be considered.

First, $c^* = 0$, this is an equilibrium since (20) is satisfied and we are back to case (i) where $c^* = e^* = 0$. This is equilibrium $\mathcal{U}$.

Second, $c^* > 0$. Using (12), $\phi = 0$ implies that $d_1^* = d_2^* = 0$. Finally, using (3), (12) and (14), we easily obtain:

$$d_{-2}^* = \frac{(Z + c^*)c^*}{B^2}d_0^*, \qquad d_{-1}^* = \frac{2c^*}{B}d_0^*, \qquad c^* = \frac{B^2}{2d_0^*} - B - Z$$

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where

$$Z = \frac{1 - \omega}{\omega} \text{ and } B = \frac{p}{\omega\alpha}$$

and where d_0^* is given by the solution to $\Phi(d_0^*) = 0$, where

$$\Phi(d_0^*) = -\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 \tag{23}$$

This is referred to as equilibrium $\mathcal{C}$.

(iv) If $\phi = \psi = \psi_c = 1$.

Two cases must be considered.

First, $c^* = 0$, this is an equilibrium since (20) is satisfied and we are back to case (ii) where $c^* = 0$ and $e^* > 0$. This is equilibrium $\mathcal{E}$.

Second, $c^* > 0$. Equations (19) and (20) are satisfied. By combining (12) to (14), we obtain

$$d_2^* = \frac{\theta}{2}d_1^*, \qquad d_1^* = 2\theta d_0^* \tag{24}$$

$$d_{-2}^* = \frac{(Z + c^*)\, c^*}{B^2}d_0^*, \qquad d_{-1}^* = \frac{2c^*}{B}d_0^* \tag{25}$$

$$e^* = 2\theta\,(1+\theta)\, d_0^*, \qquad c^* = \frac{B^2}{2d_0^*} - B - Z \tag{26}$$

where

$$\theta = \frac{\lambda}{\delta}$$

and where d_0^* is given by the solution to $\Phi_\theta(d_0^*) = 0$, where

$$\Phi_\theta(d_0^*) = \left[\theta\,(2+\theta) - \frac{Z}{B}\right]d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 \tag{27}$$

∎

Lemma 2

(i) *The equilibrium* $\mathcal{U}$ *always exists whenever* $\phi = \psi = 0$, $\psi_c \in \{0,1\}$ *or whenever* $\phi = 0$ *and* $\psi = \psi_c = 1$.

(ii) *The equilibrium* $\mathcal{E}$ *always exists whenever* $\phi = 1$, $\psi = 0$, $\psi_c \in \{0,1\}$ *or whenever* $\phi = \psi = \psi_c = 1$.

(iii) *The equilibrium* $\mathcal{C}$ *exists whenever* $\phi = 0$ *and* $\psi = \psi_c = 1$ *if and only if* $p < X(\omega)$, *where* $X(\omega) = \frac{1}{2}[\omega\alpha + \alpha\sqrt{\omega(4 - 3\omega)}]$ *with* $X : [0,1] \to [0,1]$ *increasing, concave, with* $X(0) = 0$ *and* $X(1) = 1$.

(*iv*) *The equilibrium* $\mathcal{M}$ *exists whenever* $\phi = \psi = \psi_c = 1$ *if and only if* $p < X_\theta(\omega)$, *where* $X_\theta(\omega)$ *is a uniquely defined function* $X_\theta : [0,1] \to \mathbb{R}_+$, *with* $X_\theta(0) = 0$ *and* $X_\theta(1) = \sqrt{1 - \theta(2 + \theta)}$ *if* $\theta(2 + \theta) \leq 1$ *and* $X(1) = 0$, *otherwise.*

Proof.

(*i*) Straightforward from Lemma 1.

(*ii*) Straightforward from Lemma 1 and by observing that in both cases ($\phi = 1$ and $\psi = 0$ or $\phi = \psi = 1$), $0 < d_0^* < 1/2$.

(*iii*) From Lemma 1, we have seen that equilibrium $\mathcal{C}$ only exists when $\phi = 0$ and $\psi = 1$. However, we have to check that $c^* > 0$ and $0 < d_0^* < 1/2$. This implies that equilibrium $\mathcal{C}$ exists if and only if there exists $\Phi(d_0^*) = 0$ for some $0 < d_0^* < 1/2$. The function $\Phi(d_0^*)$, defined by (23), is a second order polynomial in d_0^* that has two different roots with product equal to $-Z/B (B/2)^2 < 0$. Therefore, there exists a unique positive solution to $\Phi(d_0^*) = 0$. Given that $\Phi(0) = (B/2)^2 > 0$, this solution is smaller than $1/2$ if and only if

$$\Phi(1/2) = \frac{1}{4}\left[B^2 - (1 + Z) - \frac{Z}{B}\right] = \frac{1}{4}(1 + \frac{1}{B})(B^2 - B - Z) < 0.$$

Denote by $x(\omega)$ the unique positive solution to $x^2 - x - Z = 0$. Then, $d_0^* < 1/2$ if and only if $B < x(\omega)$, which is equivalent to $p < \omega\alpha x(\omega)$. We set $X(\omega) = \omega\alpha x(\omega)$. With some algebra, we obtain:

$$X(\omega) = \frac{1}{2}[\omega\alpha + \alpha\sqrt{\omega(4 - 3\omega)}]$$

Observe that $d_0^* < 1/2$ guarantees that $c^* > 0$.

(*iv*) From Lemma 1, we have seen that equilibrium $\mathcal{M}$ only exists when $\phi = \psi = \psi_c = 1$. Here also, we have to check that $c^* > 0$ and $0 < d_0^* < 1/2$. This implies that equilibrium $\mathcal{M}$ exists if and only if there exists $\Phi_\theta(d_0^*) = 0$ for some $0 < d_0^* < 1/2$. The function $\Phi_\theta(d_0^*)$, defined by (27), is a second order polynomial in d_0^*. We have that $\Phi_\theta(0) = (B/2)^2 > 0$, $\Phi_\theta'(0) = -(1 + Z)/2 < 0$ and

$$\Phi_\theta''(d_0^*) \gtreqqless 0 \Leftrightarrow p \gtreqqless \frac{(1 - \omega)\alpha}{\theta(2 + \theta)}$$

Let us consider two cases. If $p \leq \frac{(1-\omega)\alpha}{\theta(2+\theta)}$, then $\Phi_\theta(d_0^*)$ is a decreasing and concave function and has thus a unique positive root. If $p > \frac{(1-\omega)\alpha}{\theta(2+\theta)}$, then $\Phi_\theta(d_0^*)$ is a strictly convex function with $\Phi_\theta(0) > 0$ and $\Phi_\theta'(0) < 0$. But since we impose below that $\Phi_\theta(1/2) < 0$, there is also a unique positive root for $d_0^* < 1/2$. Therefore, there always exists a unique positive solution to $\Phi_\theta(d_0^*) = 0$ whatever the value of p. Given that $\Phi_g(0) > 0$, this solution is smaller than $1/2$ if and only if

$$\Phi_\theta(1/2) = \frac{1}{4}\left[B^2 - (1 + Z) - \frac{Z}{B} + \theta(2 + \theta)\right] < 0$$

This is equivalent to

$$B^3 - [1 + Z - \theta(2 + \theta)]B - Z < 0$$

Let us consider the three-degree polynomial

$$f(x) \equiv x^3 - [1 + Z - \theta(2 + \theta)]x - Z = 0$$

Let us show that it has a unique positive root $x_\theta(\omega)$. Two cases must be considered. If $1 + Z < \theta(2 + \theta)$, then $f'(x) > 0$ and $f(0) = -Z < 0$. Thus there is one positive root and two complex conjugates. If $1 + Z > \theta(2 + \theta)$, then $f''(x) > 0$ for $x \in \mathbb{R}^+$ and $f(0) = -Z < 0$. As a result, there are three real roots but only one is positive. Thus, we have shown that this three-degree polynomial has a unique positive root denoted by $x_\theta(\omega)$. As a result, $d_0^* < 1/2$ if and only if $B < x_\theta(\omega)$, which is equivalent to $p < \omega\alpha x_\theta(\omega) = X_\theta(\omega)$. Observe that $d_0^* < 1/2$ guarantees that $c^* > 0$. ▮

Proposition 1 then follows from the two previous lemmata. ■

Proof of Proposition 2:

We have first the following lemma.

Lemma 3 *By comparing equilibria $\mathcal{C}$ and $\mathcal{M}$, we have*

$$c^*(0,1) > c^*(1,1)$$

Proof.

Using the proof of Lemma 1 above, one can see that

$$c^*(0,1) = \frac{B^2}{2d_0^*(0,1)} - B - Z \text{ and } c^*(1,1) = \frac{B^2}{2d_0^*(1,1)} - B - Z$$

so that

$$\frac{\partial c^*(0,1)}{\partial d_0^*(0,1)} < 0 \text{ and } \frac{\partial c^*(1,1)}{\partial d_0^*(1,1)} < 0 \tag{28}$$

For $c^*(0,1)$, $d_0^*(0,1)$ is defined by (23), i.e.

$$\Phi(d_0^*(0,1)) = -\frac{Z}{B}(d_0^*(0,1))^2 - \frac{(1+Z)}{2}d_0^*(0,1) + \left(\frac{B}{2}\right)^2$$

whereas for $c^*(1,1)$, $d_0^*(1,1)$ is defined by (27), i.e.

$$\Phi_\theta(d_0^*) = \left[\theta(2+\theta) - \frac{Z}{B}\right](d_0^*(1,1))^2 - \frac{(1+Z)}{2}d_0^*(1,1) + \left(\frac{B}{2}\right)^2$$

Observe that

$$\Phi_\theta(d_0) = \theta(2+\theta)d_0^2 + \Phi(d_0)$$

so that

$$\Phi_\theta(d_0^*(1,1)) = 0 = \theta(2+\theta)(d_0^*(1,1))^2 + \Phi(d_0^*(1,1))$$

This implies that $\Phi\left(d_0^*(1,1)\right) < 0$. But since $\Phi'(d_0) < 0$ and $\Phi\left(d_0^*(0,1)\right) = 0$, we have that $d_0^*(1,1) > d_0^*(0,1)$. This in turn implies that $c^*(0,1) > c^*(1,1)$ because of (28). ∎

Let us now determine the values of (17) and (18) for each steady-state equilibrium.

Equilibrium $\mathcal{U}$: It exists whenever $\phi = 0$. In this case, $V_{00} = b/r$ and thus the condition for $\phi = 0$ is

$$w < rV_{00} \Leftrightarrow b > w$$

which is impossible since we have assumed that $w_{10} > b$. As a result, there does not exist an equilibrium $\mathcal{U}$.

Equilibrium $\mathcal{E}$: $\phi = 1$ and $\psi = 0$. In this case,

$$V_{00} = \frac{b + \lambda\frac{w}{r+\delta}}{r + \lambda\frac{r}{r+\delta}}$$

and thus the conditions for $\phi = 1$ and $\psi = 0$ are:

$$\begin{cases} w \geq rV_{00} & \Leftrightarrow & b \leq w \\ g - pf < rV_{00} & \Leftrightarrow & b > -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g - pf) \end{cases}$$

Equilibrium $\mathcal{C}$: $\phi = 0$ and $\psi = 1$. In this case,

$$V_{00} = \frac{b + \omega\alpha c^*(0,1)\frac{g-pf}{r}}{r + \omega\alpha c^*(0,1)}$$

and thus the conditions for $\phi = 0$ and $\psi = 1$ are:

$$\begin{cases} w < rV_{00} & \Leftrightarrow & b > w\left[1 + \omega\alpha c^*(0,1)/r\right] - (g - pf)\,\omega\alpha c^*(0,1)/r \\ g - pf \geq rV_{00} & \Leftrightarrow & b \leq g - pf \end{cases}$$

Thus, the condition is:

$$w\left[1 + \omega\alpha c^*(0,1)/r\right] - (g - pf)\,\omega\alpha c^*(0,1)/r < b \leq g - pf$$

which implies that $w < g - pf$.

Equilibrium $\mathcal{M}$: $\phi = \psi = 1$. In this case,

$$V_{00} = \frac{b + \lambda\frac{w}{r+\delta} + \omega\alpha c^*(1,1)\frac{g-pf}{r}}{r + \lambda\frac{r}{r+\delta} + \omega\alpha c^*(1,1)}$$

and thus the condition for $\phi = \psi = 1$ are:

$$\begin{cases} w \geq rV_{00} & \Leftrightarrow & b \leq w\left[1 + \omega\alpha c^*(1,1)/r\right] - (g - pf)\,\omega\alpha c^*(1,1)/r \\ g - pf \geq rV_{00} & \Leftrightarrow & b \leq -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g - pf) \end{cases}$$

Thus the condition is

$$b \leq \min\left\{w\left[1 + \omega\alpha c^*(1,1)/r\right] - (g - pf)\,\omega\alpha c^*(1,1)/r, -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g - pf)\right\}$$

Now we consider two cases:

(1) $w > g - pf$. Then
- if $b > -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g-pf)$, Equilibrium $\mathcal{E}$ exists.
- if $w\left[1 + \omega\alpha c^*(0,1)/r\right] - (g-pf)\omega\alpha c^*(0,1)/r < b \leq g - pf$, Equilibrium $\mathcal{C}$ exists. But for this condition exist it has to be that $w\left[1 + \omega\alpha c^*(0,1)/r\right] - (g-pf)\omega\alpha c^*(0,1)/r < g - pf$, which is equivalent to:

$$w < g - pf$$

Thus if $w > g - pf$, Equilibrium $\mathcal{C}$ does not exist.

Finally, for equilibrium M to exist we must have

$$b \leq \min\left\{w\left[1 + \omega\alpha c^*(1,1)/r\right] - (g-pf)\omega\alpha c^*(1,1)/r, -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g-pf)\right\}$$

If $w > g - pf$, then

$$w\left[1 + \omega\alpha c^*(1,1)/r\right] - (g-pf)\omega\alpha c^*(1,1)/r > -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g-pf)$$

and thus the condition is

$$b \leq -\frac{\lambda}{r+\delta}w + \left(\frac{r+\delta+\lambda}{r+\delta}\right)(g-pf)$$

(2) $w < g - pf$. Then
- if $[w - (g-pf)]\omega\alpha c^*(0,1)/r + w < b \leq g - pf$, then Equilibrium $\mathcal{C}$ exists.
- if $b > g - pf - [w - (g-pf)]\frac{\lambda}{r+\delta}$, Equilibrium $\mathcal{E}$ exists. But since

$$g - pf - [w - (g-pf)]\frac{\lambda}{r+\delta} > g - pf \text{ and } w > b$$

Equilibrium $\mathcal{E}$ cannot exist.
- if $b \leq [w - (g-pf)]\omega\alpha c^*(1,1)/r + w$ Equilibrium $\mathcal{M}$ exists.

Since $c^*(1,1) < c^*(0,1)$, we have

$$[w - (g-pf)]\omega\alpha c^*(0,1)/r + w < [w - (g-pf)]\omega\alpha c^*(1,1)/r + w$$

The result follows. ∎

By using these values and Lemma 3, Proposition 2 follows. ∎

Appendix 2: Comparative Statics Analysis

Proof of Proposition 3

(i) Let us show that

$$\frac{\partial d_0^*}{\partial p} > 0$$

The equilibrium crime level in equilibrium $\mathcal{M}$ is defined by (26), i.e.

$$c^* = \frac{B^2}{2d_0^*} - B - Z$$

where d_0^* is given by the solution to $\Phi_\theta(d_0^*) = 0$, where $\Phi_\theta(d_0^*)$ is given by (27), i.e.

$$\Phi_\theta(d_0^*) = \left[\theta\left(2+\theta\right) - \frac{Z}{B}\right] d_0^{*2} - \frac{(1+Z)}{2} d_0^* + \left(\frac{B}{2}\right)^2$$

Observing that $B = p/\alpha\omega$, it is equivalent to analyze the variation of d_0^* and c^* with respect to B than with p. By totally differentiating (27), we easily obtain:

$$\frac{\partial d_0^*}{\partial B} = -\frac{\left(Z/B^2\right) d_0^{*2} + B/2}{2\left[\theta\left(2+\theta\right) - Z/B\right] d_0^* - (1+Z)/2} > 0$$

Observe that the denominator is negative because d_0^* is the unique positive positive root to $\Phi_\theta(d_0^*)$, which was guaranteed by

$$\Phi_\theta'(d_0^*) = 2\left[\theta\left(2+\theta\right) - Z/B\right] d_0^* - (1+Z)/2 < 0$$

This implies that

$$\frac{\partial d_0^*}{\partial p} > 0$$

Let us now show that

$$\frac{\partial c^*}{\partial p} < 0 \text{ for } p > \alpha$$

For that, let us calculate the partial derivative of c^* with respect to B. Let us show that B^2/d_0^* is a decreasing function of B when $\theta\left(2+\theta\right) \in \left[\frac{Z}{2B}, \frac{Z}{B}\right]$. For this rewrite the condition $\Phi_\theta(d_0^*) = 0$ (dividing by d_0^{*2}) as

$$\Psi\left(\frac{B^2}{d_0^*}, B\right) = \left[\theta\left(2+\theta\right) - \frac{Z}{B}\right] - \frac{(1+Z)}{2B^2}\frac{B^2}{d_0^*} + \frac{1}{4B^2}\left(\frac{B^2}{d_0^*}\right)^2 = 0$$

Let us analyze the function where $X \equiv B^2/d_0^*$:

$$\Psi(X, B) = \left[\theta\left(2+\theta\right) - \frac{Z}{B}\right] - \frac{(1+Z)}{2B^2}X + \frac{1}{4B^2}X^2 \tag{29}$$

We have that $\Psi(0, B) = \theta(2 + \theta) - \frac{Z}{B} < 0$ and $\Psi'_X(X, B) = -\frac{(1+Z)}{2B^2} + \frac{1}{2B^2}X$ and $\Psi''_{XX}(X, B) = \frac{1}{2B^2} > 0$. The function $\Psi(X, B)$ is convex in X with a negative minimum at $X = 1 + Z$. Thus a solution X^* of $\Psi(X, B) = 0$ is such that $X^* > 1 + Z$ and $\Psi'_X(X^*, B) > 0$. By totally differentiating (29), we obtain:

$$\frac{\partial X^*}{\partial B} = -\frac{\Psi'_B(X^*, B)}{\Psi'_X(X^*, B)}$$

while

$$\Psi'_B(X^*, B) = \frac{Z}{B^2} + \frac{(1+Z)}{B^3}X^* - \frac{1}{2B^3}X^{*2}$$
$$= \frac{1}{B^2}\left[Z + \frac{(1+Z)}{B}X^* - \frac{1}{2B}X^{*2}\right]$$

Substituting $\Psi(X^*, B) = 0$, one gets

$$\Psi'_B(X^*, B) = \frac{1}{B^2}\left(Z + 2B\left[\theta(2+\theta) - \frac{Z}{B}\right]\right)$$
$$= \frac{1}{B^2}(-Z + 2B[\theta(2+\theta)])$$

Thus

$$\frac{\partial X^*}{\partial B} = -\frac{\Psi'_B(X^*, B)}{\Psi'_X(X^*, B)} < 0$$

and B^2/d_0^* is a decreasing function of B. It follows from

$$c^* = \frac{B^2}{2d_0^*} - B - Z$$

that

$$\frac{\partial c^*}{\partial B} = \frac{\partial(\frac{B^2}{2d_0^*})}{\partial B} - 1$$
$$= -\frac{\frac{1}{B^2}(-Z + 2B[\theta(2+\theta)])}{-\frac{(1+Z)}{2B^2} + \frac{1}{2B^2}X^*} - 1$$
$$= -\frac{(-Z + 2B[\theta(2+\theta)])}{-\frac{(1+Z)}{2} + \frac{1}{2}X^*} - 1$$
$$= \frac{(Z - 2B[\theta(2+\theta)]) + \frac{(1+Z)}{2} - \frac{1}{2}X^*}{-\frac{(1+Z)}{2} + \frac{1}{2}X^*}$$

Thus the sign of $\frac{\partial c^*}{\partial B}$ is equivalent to the sign of the numerator (the denominator is positive as $\Psi'_X(X^*, B) > 0$) and therefore $\frac{\partial c^*}{\partial p} < 0$ (dividing by B) is equivalent to :

$$\left(\frac{Z}{B} - 2[\theta(2+\theta)]\right) + \frac{(1+Z)}{2B} - \frac{1}{2B}X^* < 0$$

or

$$X^* > 1 + Z + 2B\Omega \tag{30}$$

with $\Omega = \frac{Z}{B} - 2[\theta(2 + \theta)]$. The inequality (30) is satisfied when $\Omega \leq 0$ as we know that $X^* > 1 + Z$ and therefore $\frac{\partial c^*}{\partial B} < 0$. Suppose now that $\Omega > 0$. Then (30) is equivalent to

$$\Psi(1 + Z + 2B\Omega, B) < 0$$

or

$$-\Omega - \theta(2 + \theta) - \frac{(1 + Z)}{2B^2}(1 + Z + 2B\Omega) + \frac{1}{4B^2}(1 + Z + 2B\Omega)^2 < 0$$

or

$$-\Omega - \theta(2 + \theta) - \frac{(1 + Z)^2}{2B^2} - \frac{(1 + Z)}{B}\Omega + \frac{(1 + Z)^2}{4B^2} + \frac{(1 + Z)\Omega}{B} + \Omega^2 < 0$$

or

$$-\Omega - \theta(2 + \theta) - \frac{(1 + Z)^2}{4B^2} + \Omega^2 < 0$$

Now, when $p' = p/\alpha > 1$, one gets $\frac{Z}{B} = \frac{1 - \omega}{p'} < 1$ and therefore $\Omega = \frac{Z}{B} - 2[\theta(2 + \theta)] < \frac{Z}{B} < 1$, implying $\Omega^2 - \Omega < 0$ and thus

$$-\Omega - \theta(2 + \theta) - \frac{(1 + Z)^2}{4B^2} + \Omega^2 < 0$$

This in turn implies that

$$\frac{\partial c^*}{\partial B} < 0 \text{ when } p' > 1$$

or

$$\frac{\partial c^*}{\partial p} < 0 \text{ when } p > \alpha$$

Let us now show that

$$\frac{\partial e^*}{\partial p} > 0$$

By (26), we have

$$e^* = 2\theta(1 + \theta)d_0^*$$

so

$$\frac{\partial e^*}{\partial p} = 2\theta(1 + \theta)\frac{\partial d_0^*}{\partial p} > 0$$

(ii) By totally differentiating (27), and observing that $2\theta(1 + \theta) = 2\lambda/\delta + 2\lambda^2/\delta^2$, we easily obtain:

$$\frac{\partial d_0^*}{\partial \delta} = \frac{d_0^{*2}\left(\frac{2\lambda}{\delta^2} + \frac{4\lambda^2}{\delta^3}\right)}{2[\theta(2 + \theta) - Z/B]d_0^* - (1 + Z)/2} < 0$$

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$$\frac{\partial d_0^*}{\partial \lambda} = -\frac{d_0^{*2}\left(\frac{2}{\delta} + \frac{4\lambda}{\delta^2}\right)}{2\left[\theta\left(2 + \theta\right) - Z/B\right]d_0^* - \left(1 + Z\right)/2} > 0$$

since

$$\frac{\partial\left[2\theta\left(1 + \theta\right)\right]}{\partial\delta} = -\left(\frac{2\lambda}{\delta^2} + \frac{4\lambda^2}{\delta^3}\right) < 0$$

$$\frac{\partial\left[2\theta\left(1 + \theta\right)\right]}{\partial\lambda} = \frac{2}{\delta} + \frac{4\lambda}{\delta^2} > 0$$

Thus, by totally differentiating c^* and e^* in (26), we have:

$$\frac{\partial c^*}{\partial\delta} = -\frac{B^2}{2\left(d_0^*\right)^2}\frac{\partial d_0^*}{\partial\delta} > 0$$

$$\frac{\partial c^*}{\partial\lambda} = -\frac{B^2}{2\left(d_0^*\right)^2}\frac{\partial d_0^*}{\partial\lambda} < 0$$

$$\frac{\partial e^*}{\partial\delta} = 2\frac{\partial\left[\theta\left(1 + \theta\right)\right]}{\partial\delta}d_0^* + 2\theta\left(1 + \theta\right)\frac{\partial d_0^*}{\partial\delta} < 0$$

$$\frac{\partial e^*}{\partial\lambda} = 2\frac{\partial\left[\theta\left(1 + \theta\right)\right]}{\partial\lambda}d_0^* + 2\theta\left(1 + \theta\right)\frac{\partial d_0^*}{\partial\lambda} > 0$$

Finally, for the unemployment rate u^*, we get:

$$\begin{aligned}
u &= 2d_0^* + d_1^* + d_{-1}^* = 2(1 + \theta)d_0^* + \frac{2c^*}{B}d_0^* \\
&= 2(1 + \theta)d_0^* + 2\left(\frac{B^2}{2d_0^*} - B - Z\right)\frac{d_0^*}{B} = 2(1 + \theta)d_0^* + 2\left(\frac{B}{2} - d_0^* - Z\frac{d_0^*}{B}\right) \\
&= 2\left[\theta - \frac{Z}{B}\right]d_0^* + B
\end{aligned}$$

The derivative of u with respect to B is then ambiguous as

$$\frac{\partial u}{\partial B} = 1 + 2\left[\theta - \frac{Z}{B}\right]\frac{\partial d_0^*}{\partial B} + \frac{Z}{B^2}d_0^*$$

Given that $\theta\left(2 + \theta\right) < Z/B$, it follows that $\theta < Z/B$ and in the above expression the term $2\left[\theta - \frac{Z}{B}\right]\frac{\partial d_0^*}{\partial B}$ is negative so we cannot sign unambiguously the derivative $\partial u^*/\partial B$ ∎

Proof of Proposition 4

Let us calculate $\frac{\partial d_0^*}{\partial\omega}$ and show that

$$\frac{\partial d_0^*}{\partial\omega} < 0 \text{ for } p > \alpha$$

By totally differentiating (27), using the notation $p' = p/\alpha$, we obtain:

$$\frac{\partial d_0^*}{\partial\omega} = -\frac{\frac{1}{p'}d_0^{*2} + \frac{1}{2\omega^2}d_0^* - \frac{(p')^2}{2\omega^3}}{2\left[\theta\left(2 + \theta\right) - Z/B\right]d_0^* - \left(1 + Z\right)/2}$$

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Since the denominator is negative, the sign of $\frac{\partial d_0^*}{\partial \omega}$ is the same as the one of

$$\Xi(d_0^*) \equiv \frac{1}{p'} d_0^{*2} + \frac{1}{2\omega^2} d_0^* - \frac{p'^2}{2\omega^3}$$

We have

$$\Xi(0) = -\frac{p'^2}{2\omega^3} < 0$$

$$\Xi'(d_0^*) = \frac{2}{p'} d_0^* + \frac{1}{2\omega^2} > 0 \text{ and } \Xi''(d_0^*) = \frac{2}{p'} > 0$$

$$\Xi(1/2) \equiv \frac{1}{4p'} + \frac{1}{4\omega^2} - \frac{p'^2}{2\omega^3}$$

Let us show that $\Xi(1/2) < 0$. This is equivalent to

$$f(\omega) \equiv \omega^3 + p'\omega - 2p'^3 < 0$$

It is easy to see that

$$f(0) < 0 \ , \ f'(\omega) > 0, \ f''(\omega) > 0$$

and

$$f(1) = 1 + p' - 2p'^3 = \left(1 - p'\right)\left(2p'^2 + 2p' + 1\right) < 0^7$$

when $p' > 1$ or $p > \alpha$. As a result,

$$\omega^3 + p'\omega - 2p'^3 < 0 \ , \ \forall \omega \in [0, 1]$$

We have thus shown that

$$\frac{1}{p'} d_0^{*2} + \frac{1}{2\omega^2} d_0^* - \frac{p'^2}{2\omega^3} < 0 \ , \ \forall d_0^* \in [0, 1/2]$$

This implies that

$$\frac{\partial d_0^*}{\partial \omega} = -\frac{\frac{1}{p'} d_0^{*2} + \frac{1}{2\omega^2} d_0^* - \frac{p'^2}{2\omega^3}}{2\left[\theta\left(2+\theta\right) - Z/B\right] d_0^* - \left(1 + Z\right)/2} < 0 \text{ for } p > \alpha$$

Let us now show that

$$\frac{\partial e^*}{\partial \omega} < 0 \text{ for } p > \alpha$$

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[7]Indeed

$$g(p) \equiv -2p^2 + 2p - 1 < 0 \ , \ \forall p \in [0, 1]$$

since $g(0) = g(1) = -1$, $g(1/2) = -1/2$, $g''(p) < 0$ and

$$g'(p) = 0 \Leftrightarrow p = 1/2$$

In other words, the graph of this polynomial is always belows the horizontal axis.

Using (26), we have:

$$e^* = 2\theta\left(1+\theta\right)d_0^*$$

and thus

$$\frac{\partial e^*}{\partial\omega} = 2\theta\left(1+\theta\right)\frac{\partial d_0^*}{\partial\omega} < 0$$

Let us calculate

$$\frac{\partial c^*}{\partial\omega}$$

Using (26), we have:

$$c^* = \frac{B^2}{2d_0^*} - B - Z = \frac{p'^{\,2}}{2\omega^2 d_0^*} - \left(\frac{1+p'}{\omega}\right) + 1$$

Thus

$$\frac{\partial c^*}{\partial\omega} = -\frac{p'^{\,2}}{2\omega^3 d_0^{*2}}\left(2d_0^* + \omega\frac{\partial d_0^*}{\partial\omega}\right) + \frac{1+p'}{\omega^2}$$

and it is easy to see that

$$\begin{aligned}
\frac{\omega}{d_0^*}\frac{\partial d_0^*}{\partial\omega} &= -\frac{\frac{\omega}{p'}d_0^{*2} + \frac{1}{2\omega}d_0^* - \frac{p'^{\,2}}{2\omega^2}}{2\left[\theta\left(2+\theta\right) - Z/B\right]d_0^{*2} - (1+Z)\,d_0^*/2}\\[2mm]
&= -\frac{\frac{\omega}{p'}d_0^{*2} + \frac{1}{2\omega}d_0^* - \frac{p'^{\,2}}{2\omega^2}}{2\left[\theta\left(2+\theta\right) - Z/B\right]d_0^{*2} - \frac{1}{2\omega}d_0^*}\\[2mm]
&= -\frac{\frac{\omega}{p'}d_0^{*2} + \frac{1}{2\omega}d_0^* - \frac{p'^{\,2}}{2\omega^2}}{\frac{1}{\omega}d_0^* - \frac{p'^{\,2}}{2\omega^2} - \frac{1}{2\omega}d_0^*}\\[2mm]
&= \frac{\frac{\omega}{p'}d_0^{*2} + \frac{1}{2\omega}d_0^* - \frac{p'^{\,2}}{2\omega^2}}{\frac{p'^{\,2}}{2\omega^2} - \frac{1}{2\omega}d_0^*} = -1 + \frac{\frac{\omega}{p'}d_0^{*2}}{\frac{p'^{\,2}}{2\omega^2} - \frac{1}{2\omega}d_0^*}\\[2mm]
&\quad -\frac{p'}{2\omega d_0^{*2}}\left(2d_0^* + \omega\frac{\partial d_0^*}{\partial\omega}\right) + \frac{1+p'}{p'}
\end{aligned}$$

$$\frac{\partial c^*}{\partial\omega} \gtrless 0 \Leftrightarrow \frac{1+p'}{p'} \gtrless \frac{p'}{2\omega d_0^*}\left[2 + \frac{\omega}{d_0^*}\frac{\partial d_0^*}{\partial\omega}\right] = \frac{p'}{2\omega d_0^*}\left[1 + \frac{\frac{\omega}{p'}d_0^{*2}}{\frac{p'^{\,2}}{2\omega^2} - \frac{1}{2\omega}d_0^*}\right]$$

$$\Leftrightarrow \frac{1+p'}{p'} \gtrless \frac{p'}{2\omega d_0^*} + \omega\frac{\omega d_0^*}{p'^{\,2} - \omega d_0^*}$$
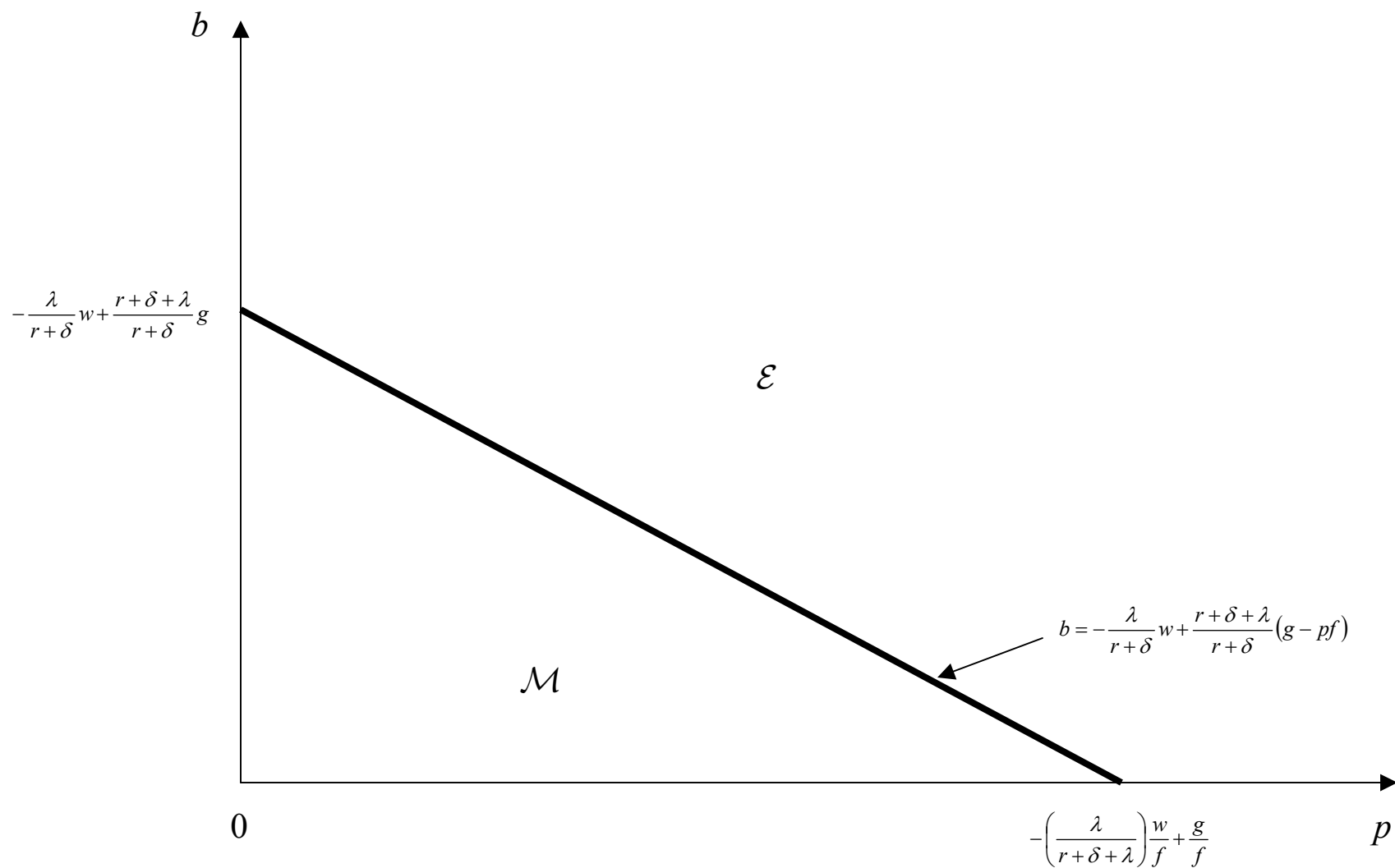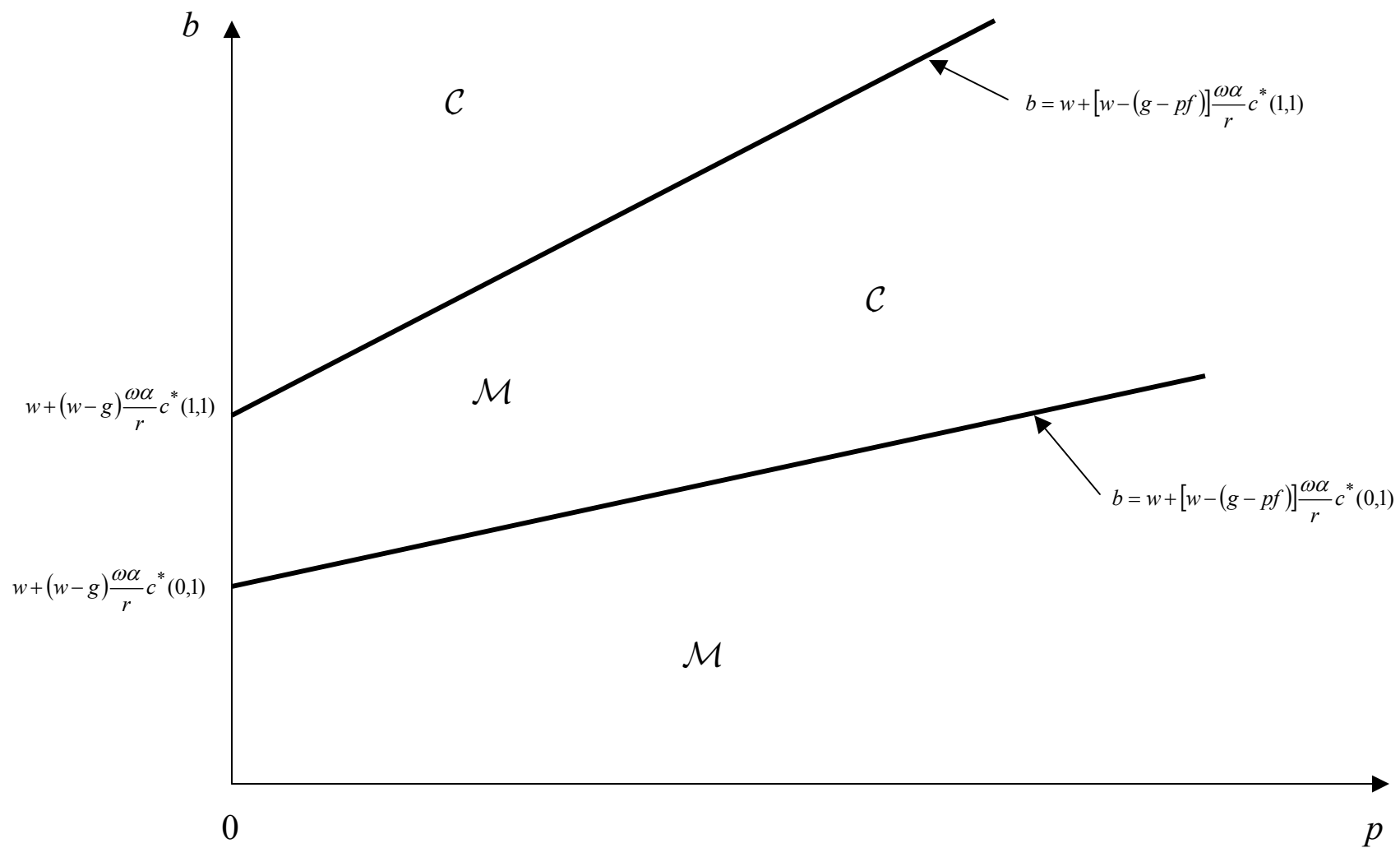
∎

Figure 2a: Steady-State Equilibria when $w > g - pf$

Figure 2b: Steady-State Equilibria when $w \leq g - pf$