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Credit Card Debt Puzzles

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Abstract

Most US credit card holders revolve high-interest debt, often combined with substantial (i) asset accumulation by retirement, and (ii) low-rate liquid assets. Hyperbolic discounting can resolve only the former puzzle (Laibson et al., 2003). Bertaut and Haliassos (2002) proposed an 'accountant-shopper' framework for the latter. The current paper builds, solves, and simulates a fully-specified accountant-shopper model, to show that this framework can actually generate both types of co-existence, as well as target credit card utilization rates consistent with Gross and Souleles (2002). The benchmark model is compared to setups without self-control problems, with alternative mechanisms, and with impatient but fully rational shoppers.

JEL Classification: E210, G110.

Key Words: Credit cards, debt, self control, household portfolios.

1 Introduction

Almost three quarters of United States households had bank-type credit cards by 2001. As reported by Bertaut and Haliassos (2005), in all Surveys of Consumer Finances between 1983 and 2001, more than half of bank-type credit card holders revolve debt on their cards, while just under half of holders of such cards declare that they "hardly ever" pay off their card balance. The median interest rate paid by household that "hardly ever" pay off their card balance is 16 percent. About 98 percent of credit card revolvers actually write checks each month to cover less than the outstanding amount, i.e. they do not rely on automatic payment facilities. Thus, debt revolving cannot be attributed to ignorance, inertia, teaser interest rates or accident.

About 45 percent of card holders that revolved debt in 2001 actually had liquid assets greater than their card balance (and had at least $1,000 and at least one-half monthly income in liquid assets). Their median card balance was 1,000 dollars, while their median liquid assets were 8,000 dollars, and their median total financial assets were just over 40,000 dollars. The size of liquid assets among households in this group makes it unlikely that this co-existence of debt with assets can be attributed simply to a motive for holding transactions balances. The majority of households in the group claims only to "sometimes" pay off the balance in full, and about one-third admit to "hardly ever" paying off their card balance.

Moreover, despite these pronounced tendencies to revolve credit card debt, by the time they are included in the 55-64 age range, US households in this category exhibit median levels of total net worth of the order of 196,730 dollars in 2001. The surprising portfolio co-existence of high-interest credit card debt, low-interest liquid asset holdings, and substantial accumulation of assets for retirement is a challenge for the modern theory of saving and forms the focus of this paper.

Gross and Souleles (2002) used a unique, proprietary, administrative panel data set of thousands of individual credit card accounts from several different card issuers and identified these two co-existence puzzles. Laibson, Repetto, and Tobacman (2003) pointed out that, despite heavy credit card borrowing, US households in the 50 to 59 age range attain median ratios of total wealth (net of credit card debt, mortgages, and education loans) to annual total after-tax income of the order of 3.5.[1] They proposed a resolution of one puzzle, namely the co-existence of credit card debt with large retirement accumulations, while Bertaut and Haliassos (2002) proposed an

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[1] This is the average of the median ratios for this age group obtained for the following years of the Survey of Consumer Finances: 1983, 1989, 1992, and 1995.

explanation for the other puzzle (co-existence with liquid assets). Both were based on self-control considerations.[2]

Support for considering self-control explanations of saving and borrowing behavior can be found in a number of different sources. Information on attitudes of credit card holders in the 2001 *Survey of Consumers* shows that about forty percent of card holders perceive self-control problems emanating from credit cards and the possibility of overspending, although fewer are willing to admit that they personally face such problems (Durkin, 2002). Self-control considerations are stressed in the Marketing literature and in research on consumer psychology. Hoch and Loewenstein (1991) argue that self-control problems occur when the benefits of consumption occur earlier and are dissociated from the costs, as is the case with credit card spending. There is evidence suggesting that liquidity enhances both the probability of making a purchase and the amount one is willing to pay for a given item being purchased, over and above any effects due to relaxation of liquidity constraints (see Shefrin and Thaler, 1988; Prelec and Simester, 2001; and Wertenbroch, 2002). Indeed, Soman and Cheema (2002) present experimental and survey evidence to argue that consumers interpret their available credit lines as indications of future earnings potential when deciding their level of consumption expenditures. More generally, costly self-rationing as a means of self-control has been stressed in Marketing research and in economic research on mental accounting.[3] Interestingly, the idea that self-control considerations and "low-willpower techniques" are important in understanding saving behavior is now openly considered by policy makers.[4]

Laibson, Repetto and Tobacman (2003) focused on the co-existence of credit card debt and accumulation of assets for retirement. They showed that a standard consumption-saving model with credit cards and consumers discounting future utility exponentially could not account simultaneously for the proportion of households revolving debt on their credit cards and for the considerable pre-retirement ratios of total wealth to total after-tax, non-asset

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[2] Gross and Souleles (2002) had also hinted at the possibility of resolving the co-existence puzzles using self-control considerations.

[3] See Thaler (1980), Thaler and Shefrin (1981), Schelling (1992), Ariely and Wertenbroch (2002), Wertenbroch (1998, 2002).

[4] Vice Chairman of the Board of Governors of the Federal Reserve Roger W. Ferguson, Jr. remarked before the National Council on Economic Education that

> "...we will also need to rely on other models and frameworks [than the neoclassical] to help us better undrstand how people process information and make decisions. The work ... on the benefits of 'low-willpower' techniques to increase savings is an example in this regard."

(Washington, DC, May 13, 2002, quoted in Bertaut and Haliassos, 2002).

income (including income from transfers, bequests, and lump-sum windfalls). Intuitively, they showed that if a household, as commonly analyzed, valued the future so much as to accumulate such high levels of wealth for retirement, it could not be acting so impatiently as to tend to revolve debt on the credit card at anywhere close to the observed frequency.

Their proposed solution was a model of hyperbolic discounting. In such a model, the current incarnation of the self discounts the immediate future considerably, thus justifying high-interest credit card borrowing, but discounts the distant future by less than he anticipates his future incarnations to discount it. In other words, his current willingness to accumulate for retirement is greater than the willingness he expects to have at a later stage in his life. So, he decides to accumulate illiquid assets for retirement now, imposing heavy (in the limit, infinite) liquidation costs to the future selves who will be prone to act impatiently, and thus helping to ensure that enough assets will have been accumulated by the time of retirement. In this model, accumulation of illiquid assets can co-exist with credit card debt revolving, since hyperbolic consumers can act impatiently with respect to short-run objectives and patiently with respect to longer-run objectives. Credit card borrowing is intended for short-term consumption smoothing, while accumulation of illiquid assets is used as an instrument of self-control, in particular as an instrument of controlling future selves. As the authors mention, however, this model with its emphasis on temporally separated selves cannot simultaneously address the other unhappy co-existence, namely that of credit card debt with low-interest liquid assets.

This other co-existence was the focus of Bertaut and Haliassos (2002). Bertaut and Haliassos proposed a different self-control mechanism. Instead of assuming that the current self is attempting to control future selves, they considered self-control problems among contemporaneous selves. They proposed a breakdown of the household into an "accountant" self and a "shopper" self who act contemporaneously but who handle different decisions.[5] The very nature of credit cards separates the act of purchase from the act of payment. The accountant self handles payment of all bills and long-term financial planning for the household. The shopper self is the one who visits the stores, with credit card in hand, and who determines consumption expenditures on the basis of the unused credit line available to him. These two can be two spouses or two incarnations of the current self, one as a financial planner and the other as the consumer in a store tempted by the "buy

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[5]This is not to say that the accountant-shopper framework cannot accommodate both types of self-control problems: accountant-shopper self-control, and hyperbolic discounting with problems of controlling future selves.

now, pay later" mentality. If the shopper exhibits self-control problems, then the accountant can control the shopper by restraining the amount of unused credit line available to him.[6]

Instead of modeling explicitly the behavior of the shopper, Bertaut and Haliassos assumed a type of shopper behavior that is suggested by the empirical work of Gross and Souleles (2002). Using their administrative data set, Gross and Souleles found that exogenous line increases (i.e. those not requested by account holders themselves) lead credit card holders to return to the levels of utilization of the credit card limit observed prior to the line increase, normally within the space of four to five months. Households thus appear to have target utilization rates of their credit card limits that are independent of the size of these limits. Bertaut and Haliassos showed that, if shoppers are assumed to behave so as to maintain a constant ratio of credit card debt outstanding to the size of the credit line, then it does not make sense for the accountant to arbitrage between high-cost credit card debt and low-interest liquid accounts. Intuitively, if the accountant reduces holdings of illiquid assets by $1 in order to pay off credit card debt, then the shopper responds by charging $1 more to the credit card. The amount of credit card debt outstanding is unaffected, but the accountant has lost $1 of liquid assets in the process. Unless the accountant wants to engineer increases in consumption, he will not run down liquid assets to pay off credit card debt in this case. The authors provide household-level evidence consistent with such mechanisms, but neither solve nor simulate a full-blown model of an accountant-shopper pair.

The motivation for this paper is our conjecture that a fully-specified accountant-shopper model may be able to deliver resolution of both portfolio puzzles simultaneously. Our starting point is the observation that, in an accountant-shopper framework, the credit card balance ceases to be motivated mainly by intertemporal consumption smoothing considerations and is used instead as an instrument for controlling the consumption behavior of the shopper who is present contemporaneously with the accountant. Bertaut and Haliassos (2002) had illustrated, using a simple setup, that this allowed

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[6]Lehnert and Maki (2001) argued that the generous provisions of bankruptcy law in some states tend to raise the probability of holding simultaneously high amounts of credit card debts and liquid assets. This happens because households declaring bankruptcy are allowed to rescue some part of their assets, which they would not have available if they had used it to repay their credit card debt. Bertaut and Haliassos (2002) estimate that about 14 percent of households with liquid assets in excess of credit card debt meet the Lehnert-Maki asset-debt level requirements at the $3,000 threshold. Even if all of these are regarded as strategic defaulters, this still leaves 86 percent of debt revolvers to be explained with reference to other considerations.

substantial card debt to co-exist with low-interest liquid assets without violating arbitrage. However, this particular role for credit card debt also raises the possibility that an accountant who exhibits exponential discounting can choose simultaneously to accumulate sizeable assets for retirement and to revolve substantial amounts of debt in his effort to control the shopper. Thus, departing from the unitary household and distinguishing between an accountant and a shopper may account for both types of co-existence, even under exponential discounting.[7]

In this paper, we explore the validity of this conjecture by fully specifying an accountant-shopper model and then solving numerically variants of the model to examine robustness. Instead of assuming a particular type of shopper behavior that accords with the Gross and Souleles findings, we model the behavior of the shopper from first principles. This forces us to be explicit about the source of the self-control problem, to describe policy functions for the accountant and the shopper, and to perform simulations that show the extent of co-existence of credit card debt with liquid assets and with retirement accumulations.

The exercise of self-control is not costless: since the shopper has a self-control problem, the size of payments into the credit card account are not as large as would be dictated by the rate of interest on outstanding balances relative to returns on asset holding. These interest costs are likely to affect the policy functions and the time paths of consumption and asset accumulation relative to what would be observed in the absence of self-control problems. We contrast the accountant-shopper model with one in which there is no self-control problem and the household is fully rational. The monetary equivalents of utility costs of handling self-control problems through revolving credit card debt are derived.

Revolving card debt is not the only way to control spending by the impatient and unsophisticated shopper self. We focus on two other options: lowering the credit card limit; and switching to a debit card. We find that lifetime costs of revolving credit card debt relative to adopting these alternative approaches for solving the self-control problem are not sizeable and drop quickly as a proportion of household cash on hand. Thus, households could easily be deterred by information or other costs in adopting them, and households with moderate or large financial resources would be more likely to revolve debt rather than adopt the other two alternatives which require more information (e.g. knowledge of debit cards) or time-consuming actions

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[7]Of course, there is nothing in this line of thinking to preclude intoducing in addition hyperbolic discounting, if its use is otherwise motivated. We will not explore such variants in this paper, in order to avoid the extra complexity they entail.

(e.g., calculating and applying for reductions of the credit card limit or for issuance of debit cards linked to a separate and closely monitored account).

In our benchmark model, the shopper is not only more impatient than the accountant but also lacks full understanding of the financial consequences of his purchases. Given the unavoidable arbitrariness in modeling lack of sophistication, we also present results from a model variant where the shopper is fully rational but simply more impatient than the accountant. The main co-existence results survive, suggesting that they are driven by differential impatience between the contemporaneously present accountant and shopper and not by the particular assumption about lack of sophistication. However, we argue that the benchmark model may still be a more fruitful and realistic representation of the accountant-shopper interaction.

Section 2 describes the accountant-shopper model with credit cards and liquid assets and derives policy rules for the accountant and for the shopper. Section 3 presents simulation results. Section 4 compares the accountant-shopper model to a fully rational model without self-control problems, to one where the accountant foregoes the credit card and simply uses a debit card, and to another in which the accountant obtains a lower credit card limit. Results from a model variant with a rational but impatient shopper are also discussed. Section 5 offers concluding remarks. The solution algorithm and calibration details are described in Appendices.

2   The Benchmark Model

We depart from models of unitary households and allow each household to consist of two entities, that can be thought of either as two different people or as two 'selves'. We will call one entity the "accountant", who pays bills and handles household finances, and the other the "shopper" who makes purchases with a credit card, taking the available balance on the credit card as given. We will be using 'he' for both, to avoid gender interpretations.

In our formulation, the accountant and the shopper have different assigned roles. The accountant chooses asset holdings and the extent of debt repayment. The shopper chooses consumption for the household, taking as given the size of the unused credit line that is determined by the actions of the accountant who pays bills at the beginning of the period. The financially sophisticated accountant knows how the shopper's consumption is related to the resources available to him and takes this into account when determining the size of the free credit line. The shopper differs from the accountant in two respects. First, he is more impatient than the accountant, i.e. he discounts the future more than the accountant does. Secondly, he does not

perceive (or is not interested in) the full consequences of his current spending on the amount of resources that the accountant can afford to make available to him in the future. Thus, we model the self-control problem as arising from impatience and lack of financial sophistication in shopping decisions. It is, of course, possible to introduce additional problems for the accountant self, such as unpredictability in the behavior of the shopper. While such elements could characterize certain individuals, they are not necessary to generate co-existence of credit card debt with liquid assets and substantial retirement accumulations, as will be seen below.

2.1 The Accountant's Problem

The accountant derives current utility (felicity) from consuming an amount C_t of the consumption good. Consumption goods can be purchased (by the shopper) with a credit card,[8] and the amount of unsecured revolving debt on the credit card at the beginning of period t is denoted by B_t. Given that the accountant has survived to period $t-1$, he knows he has probability s_t of surviving in period t. With probability $1 - s_t$, the accountant dies, in which case he derives utility from the size of bequests of assets and of credit card debt.[9]

Expected lifetime utility of the accountant consists of current felicity and expected utility payoffs over the accountant's horizon, taking into account both life uncertainty and uncertainty regarding future state variables:

$$U(C_0, n_0) + E_0 \sum_{t=1}^{T+N-1} \beta^t \left(\prod_{j=1}^{t-1} s_j \right) [s_t U(C_t, n_t) + (1 - s_t) W(A_t, B_t)] \quad (1)$$

where β is the discount factor, E_t is the expectations operator conditional on information in period t, W is the utility from bequests, and n_t is the effective household size in period t, assumed exogenous and computed as $n = adults + \kappa \cdot children$. Felicity is of the constant relative risk aversion form:

$$U(C_t, n_t) = n_t \left[\frac{\left(\frac{C_t}{n_t} \right)^{1-\rho} - 1}{1 - \rho} \right] \quad (2)$$

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[8]Assuming that all consumption is charged on the credit card simplifies the model without restricting the overall level of consumption. For example, if the accountant desires consumption at a level that exceeds the full credit limit, then he can deposit enough in the credit card account to leave a positive balance, which can then be used by the shopper.

[9]For simplicity, we assume that the accountant and shopper die at the same time, which of course holds exactly when we refer to two selves.

At the beginning of each period t, the accountant receives the credit card bill showing the outstanding balance B_t. The accountant maximizes expected lifetime utility by choosing how much of the credit card balance to leave unpaid, $B_t - P_t$, where P_t denotes the payment into the credit card account. Following this, the shopper visits the stores and charges an amount C_t to the credit card. The transition equation governing the evolution of the credit card balance is

$$B_{t+1} = (B_t - P_t + C_t)\ R_t^c \tag{3}$$

where R_t^c is the credit card rate that applies to any outstanding balance between the beginning of period t and that of period $t+1$. The relationship between B_t and P_t determines the applicable interest rate:

$$R_t^c = \begin{cases} R^h & \text{if } P_t < B_t \\ \\ 1 & \text{if } P_t \geq B_t \end{cases} \tag{4}$$

If the accountant decides to revolve debt on the credit card, then he pays an amount P_t that falls short of the outstanding balance. The remaining balance, $B_t - P_t$, plus any new purchases C_t made by the shopper are charged the (high) credit card rate R^h. If, on the other hand, the accountant pays off the entire balance (or more), then new purchases by the shopper, C_t, are subject to a grace period and are charged zero rate of interest (a gross rate of 1).[10] We assume that the accountant knows the pattern of behavior displayed by the shopper, namely the policy function that relates the shopper's consumption level to the size of the unused credit line available to him, and that he internalizes this policy function when solving his problem.

The household faces a borrowing limit on its credit card, set by the bank and taken as given by the accountant:

$$B_t \leq \overline{B}_t \tag{5}$$

Following Laibson et al., we assume that this borrowing limit is related to the average labor income earned by the population group that has the same education as the accountant. Before retirement, it is a constant fraction of average labor income:

$$\overline{B}_t = \lambda \overline{Y}_t \tag{6}$$

At retirement, income drops instantaneously, but the credit card limit does not. We assume a simple adaptive process for the credit card limit after

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[10]Note also that if the accountant decides to pay more into the account than necessary to pay off the balance, this extra amount earns no interest. In this sense, its role in providing liquidity is equivalent to cash given to the shopper directly.

retirement:

$$\overline{B}_t = \lambda \left[\rho^{t-t^{Ret}} \, \overline{Y}_t^{BefRet} + (1 - \rho^{t-t^{Ret}}) \, \overline{Y}_t \right] \tag{7}$$

where $(t - t^{Ret})$ measures the time elapsed since retirement (in years), and $\overline{Y}_t^{BefRet}$ is average income immediately before retirement. We chose $\rho = 0.7$.

The evolution of stocks of the liquid asset, A, that offers a riskless interest rate R_t is given by

$$A_{t+1} = (A_t + Y_t - P_t) \; R_t \tag{8}$$

where Y_t is labor income received at the beginning of period t. At the beginning of each period t, the accountant observes the outstanding stock of liquid assets, A_t, and receives labor income, Y_t. He then decides whether and how much of this cash on hand to use in order to reduce (or pay off completely) the outstanding balance on the credit card. For each dollar of payment, P_t, the accountant is giving up the return R_t that he would obtain from the liquid asset and he thus lowers the amount of liquid assets available to him next period, A_{t+1}. However, as shown by equation (3), he also saves R_t^c and he lowers the credit card balance he will face next period, B_{t+1}. We abstract from explicit consideration of illiquid assets for simplicity.[11]

We introduce a borrowing constraint at the low riskless rate R:

$$A_t \geq 0 \;\; \forall \; t \tag{9}$$

This constraint means that the household can only borrow using the credit card if it is to finance purchases of the consumption good and economizes on the need to model alternative modes of financing. It will not be binding typically for households we focus on, namely those who have liquid assets but revolve credit card debt.

Income Y_t (called loosely "labor income") represents all after-tax income from transfers and wages, and its logarithm during working life ($20 \leq t \leq T$) behaves as

$$y_t = f^W(t) + u_t + v_t^W \tag{10}$$

where $f^W(t)$ is a cubic polynomial in age, u_t is a Markov process, and v_t^W is iid and normally distributed with mean 0 and variance $\sigma_{v,w}^2$. During retirement ($T \leq t \leq T + N$),

$$y_t = f^R(t) + v_t^R \tag{11}$$

where $f^R(t)$ is linear in age, and the shock is iid and normally distributed $N(0, \sigma_{v,R}^2)$. The parameters of the "labor income" processes are calibrated

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[11]In a preliminary version of the paper, we showed that portfolio co-existence of liquid assets, illiquid assets, and credit card debt can also be obtained in a model that allows for illiquid asset purchases. Unlike in the Laibson et al (2000) model, though, illiquid assets are not needed in order to exercise self control.

differently depending on the education category of the accountant, who is the household head for finances, but our benchmark calibration is reported for high-school graduates.

Thus, the state variables of the problem facing the accountant are the credit card balance B_t, liquid assets A_t, labor income Y_t, and the value of the Markov income shock u_t. The accountant's control variable is the amount of credit card debt he revolves, $B_t - P_t$.

2.2 The Shopper's Problem

In modeling the shopper, we ought to specify the source of his problem that the accountant attempts to rectify by restricting the size of the unused credit line. Clearly, if the shopper shared the same preferences with the accountant, perceived the same constraints and the same stochastic environment, and had full understanding of the consequences of his actions, there would be no point in separating the two entities and we would be back to the one-person framework that fails to account for the coexistence of credit card debt with liquid and illiquid assets. Indeed, we will contrast below the behavior of our benchmark accountant-shopper household to that of a standard, unitary household.

Research in Marketing, cited in the Introduction, suggests that access to a credit card makes it more likely that consumers will buy any given item and also more likely that they will be willing to pay a higher price for each item, even in the absence of liquidity constraints. The finding on the discrete choice of whether to buy or not, as well as the more general findings in Hoch and Loewenstein (1991), support the view that the separation of purchase from payment made possible by a credit card tends to make shoppers behave more impatiently than if they were forced to run down their cash balances or withdraw funds from an account. The finding on willingness to pay conditional on having decided to purchase the item is consistent with consumers failing to appreciate the full effect of additional consumption spending on household finances.

On the other hand, there is empirical evidence that credit card holders have reasonably good understanding of the interest rate and of the other main terms governing their credit card accounts, including how balances cumulate (Durkin, 2000). Moreover, the findings of Gross and Souleles (2002) suggest that households tend to have target utilization rates of credit lines and to leave a certain portion unused as a buffer to future shocks, rather than consuming today as much as possible. Thus, impatience needs to be distinguished from ignorance of credit card terms and from myopia.

In order to balance these considerations, we model the shopper's problem

as a combination of impatience and lack of full understanding of the financial consequences of current purchases. Specifically, we assume that the shopper has a higher rate of time preference than the accountant but otherwise has the same horizon and preferences as the accountant. Secondly, we formalize the notion that the shopper fails to take into account the full consequences of his actions by assuming that he fails to compute the optimal response of the accountant to the extra amount charged on the card. There are various ways to operationalize this notion. Here we assume that the shopper sees the payment process into the credit card account as varying with age but exogenous to his actions in the sense that it cannot be manipulated by his own spending decisions. Otherwise, the shopper would have an incentive to enter into a complicated game with the accountant that would not be in the spirit of a self-control problem faced by a shopper in a store.[12] We nevertheless impose some consistency on shopper beliefs, by requiring that the first two moments of the perceived payment process, at any given age, coincide with those of the unconditional distribution of payments at this age, taken over many series of earnings realizations (cf. Section 2.3). Note the implication that future payments are not perceived by the shopper to depend on past realizations of income shocks nor on assets of the particular household. The perceived distribution function is assumed to be uniform.[13]

Formally, the preferences of the shopper are assumed to take the form

$$U(C_0, n_0) + E_0 \sum_{t=1}^{T+N-1} \beta_S^t \left(\prod_{j=1}^{t} s_j \right) U(C_t, n_t) \tag{12}$$

where the subscript S refers to the shopper, and $\beta_S < \beta$ to allow for shopper impatience relative to the accountant. The extent of such relative impatience will be important for the extent of credit card debt revolving predicted by the model. The shopper's felicity is given by equation (2) that describes also the accountant's felicity.

Still, the shopper is not totally myopic in the sense of "spending like there is no tomorrow". If indeed the shopper were solving a one-period utility maximization problem subject to the available credit line, he would always be consuming up to the available credit limit, unlike what Gross and Souleles (2002) found. Assuming that the shopper has a multiperiod horizon

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[12] Later in the paper, we explore a model variant with a fully sophisticated shopper who enganges in strategic interactions with the accountant.

[13] Unreported experiments show that portfolio coexistence can also be generated by other assumptions regarding the perceived distribution of future payments, as long as these do not make the shopper substantially more pessimistic about future payments than what is warranted by household finances.

is not sufficient to prevent him from charging the card to the limit in each period. For example, if the shopper perceived absolutely no consequences of today's spending on future available credit line, then the problem would not be dynamic and the shopper would again be solving a sequence of one-period maximizations leading to full utilization of the credit line in each period (except in the uninteresting case of satiation).

We assume instead that the shopper's problem is dynamic. The shopper understands the terms of his account and he correctly perceives the credit card limit $\overline{B}_t$ given by equation (6), as well as the fact that the credit card balance increases when he charges an extra dollar on the credit card today, as shown in the transition equation (3). What the shopper fails to perceive in deciding current consumption is the true relationship between his consumption purchases today and the size of the payment the accountant will decide to make into the account tomorrow. Formally, equation (3) and the construction of the accountant's problem imply that the true effect of an extra dollar charged by the shopper today on resources available to him for consumption next period is

$$\frac{\partial \left(\overline{B}_{t+1} - B_{t+1} + P_{t+1} \right)}{\partial C_t} = -R_t^C + \frac{\partial P_{t+1}}{\partial B_{t+1}} \tag{13}$$

When the shopper charges an extra dollar on the credit card, he faces in reality two consequences: a higher outstanding balance for given payments by the accountant, equal to the gross interest rate on the credit card; and any change in the size of payments induced by this higher balance. Understanding the first consequence simply requires knowledge of the interest rate on the credit card, which the shopper is assumed to possess, consistent with empirical evidence. The second term is more involved and refers to the amount by which it is optimal for the accountant to adjust his payment into the credit card account in response to the extra purchase by the shopper. When picking out consumption items at the store, the shopper fails to internalize all the information about future state variables and constraints that shape the accountant's distribution of optimal responses and bases his actions instead on a uniform distribution of payments. This means that the perceived value of $\frac{\partial P_{t+1}}{\partial B_{t+1}}$ is zero.

Given our assumptions about the shopper's perception of the payment process, it is clear that the shopper has a buffer stock saving motive. The shopper sees the accountant's payments into the credit card account as exogenous and stochastic, and he chooses to leave a fraction of the credit card line unused as a buffer. Essentially, his problem is analogous to a savings problem with background income risk inducing precautionary saving (see, for example, Carroll, 1997).

2.3 Equilibrium in the Benchmark Model

We model the interaction between a sophisticated self that behaves according to the basic paradigm of economic theory, and another self, more impatient and less sophisticated. In the benchmark, the game is not played between two fully rational agents under asymmetric information or beliefs, but between a financially sophisticated leader (accountant) who is trying to double-guess and restrain an impatient, unsophisticated, though predictable shopper who behaves on the basis of simpler beliefs regarding future amounts of consumable resources. The accountant internalizes the responses of the shopper to different sizes of the unused credit line made available to him. The shopper does not take into account the full financial consequences of his consumption expenditures.

At the beginning of each period (month), the accountant decides how much to pay into the credit card account, and thus he determines the amount of consumable resources that the shopper has at his disposal in order to buy consumption goods. Given this amount of consumable resources, $\overline{B}_t - B_t + P_t$, and on the basis of his beliefs regarding future payments into the credit card account, the shopper solves his problem (described in section 2.2), and comes up with the policy function that relates optimal consumption, chosen by the shopper, to consumable resources, made available to him by the accountant. In our model, the financially sophisticated accountant knows how the shopper thinks and is able to compute the policy function of the shopper. He internalizes the shopper's policy function for consumption into his own problem, described in section 2.1. This effectively allows the accountant to decide the amount of payment into the credit card account taking into consideration the shopper's response to the size of consumable resources made available to him. In other words, the accountant is able to use payments into the credit card account to control fully (though not costlessly) the amount the shopper charges on the credit card. In equilibrium, the accountant generates the amount of consumption he deems optimal given the state variables and the shopper's policy function, and the shopper charges on the card as much as he deems optimal given the amount of consumable resources made available to him by the accountant and his beliefs regarding future resources.

To derive the equilibrium path of household consumption, assets, and debts we use the following iterative procedure:

1. We make an initial choice for the shopper's belief about the process of payments into the credit card account.

2. Based on this belief, we solve the shopper's problem. This can be done independently of the accountant's problem, and provides the shopper's

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consumption policy as a function of age and of the unused credit card line.

3. Based on the shopper's consumption policy, we solve the accountant's problem, which provides the accountant's payment policy as a function of age, cash on hand (assets net of credit card debt plus labor income) and credit card debt.

4. Using the policy functions together with the transition equations for assets and credit card debt, we simulate the model for many different realizations of the earnings shocks.

5. From the simulations, we obtain a new estimate of the unconditional distribution of the accountant's payments into the credit card account, that depends on age. This is used to update the shopper's beliefs about the payment process.

6. We iterate steps 2–5 until the shopper's perception of the payment process converges.

3   Calibration

In this section we present our choice for calibrating parameters and stochastic processes that are used in the numerical implementation of the model. Our calibration choices for a model based on time periods of one year are intentionally kept as close as possible to those of Laibson et al (2003) to facilitate comparisons of model predictions. Since we are modeling the month-to-month interaction between an accountant and a shopper, we need to calibrate a model where the time period is one month long (see Appendix).

3.1   Demographics

The household is assumed to consist of an accountant and a shopper of the same age that die simultaneously (regardless of whether we refer to two selves or to two people).[14] The accountant and the shopper live for up to 90 years,

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[14] This assumption essentially removes the need to distinguish between households that exhibit self-control problems and those characterized by interpersonal control issues. A direct consequence is that we abstract from issues that arise from differential probabilities of death and from single-parent households.

and face conditional probabilities of survival given by the 1998 United States Life Tables (National Vital Statistics Report, 2001).[15]

We use the computations of effective household size in Laibson et al (2003), using their estimated numbers of dependent adults (in addition to head and spouse) and of children based on the PSID.[16] Our benchmark runs are performed for high-school graduates, but two other education categories are also considered: high-school dropouts and college graduates. The estimated age profile of the number of children and the number of dependent adults (in addition to household head and spouse) differs by education category. College graduates tend to have smaller household size than others, and the number of their children peaks slightly later than in other education categories. Household size is computed as the sum of the number of adults and of the weighted number of children (under 17), with 0.4 used as the weight.

3.2 Incomes

Income is defined as after-tax non-asset income. It includes not only labor income, but also government transfers, bequests, and lump-sum windfalls. Separate income regressions are used for each of the three education categories, and for working and retired households of the same education category. For working households, the age polynomial is cubic, while for retired households it is linear. The regressions allow for age, separately for the number of each of the three components of the household (head and spouse, number of dependent adults, and number of children), cohort dummies (based on year of birth and split into five-year cohorts), and the unemployment rate in the household's state of residence (as a proxy for time effects).[17]

The stochastic part of annual pre-retirement income is modeled as the sum of a household fixed effect, an autoregressive process of order 1, and a purely transitory shock:

$$\xi_{it}^W = \zeta_i + u_{it} + \nu_{it}^W = \zeta_i + \alpha u_{it-1} + \varepsilon_{it} + \nu_{it}^W \tag{14}$$

The variances of ε and of ν^W, as well as the value of α, are estimated using the residuals from the regressions for income during working years and applying a weighted GMM procedure that minimizes the distance between the theoretical and the empirical first seven autocovariances of the change in the

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[15]These probabilities may differ somewhat from those used by Laibson et al, due to updating of the Life Tables.

[16]Their non-linear least squares regression estimates for the two separate equations are reported in their Table 3. The profile for each category is obtained by using these regression estimates and setting the errors equal to zero.

[17]The regression estimates used are in Table 4 of Laibson et al.

stochastic part, $E(\Delta \xi_t^W \Delta \xi_{t-k}^W)$. Different estimates are obtained and used for the three different education categories.[18]

The transitory noise in annual retirement income is modeled as the sum of a household fixed effect and of a purely transitory shock:

$$\xi_{it}^R = \vartheta_i + \nu_{it}^R \tag{15}$$

The variance of ν^R is estimated using the residuals from the regressions for income during retirement years.[19] The retirement age for high-school dropouts is set to 61, for high-school graduates to 63, and for college graduates to 65, based on mean ages observed in the data.

3.3   Assets and Debts

The credit card limit is assumed equal to 30 percent of the average annual income for the current year and for the education category to which the household belongs. The after-tax real gross interest rate on liquid assets, R, is set to 1.0375.This reflects higher than typical estimates of the riskless rate that are close to 1.01, to allow for the fact that households typically have better investment opportunities. Since we simplify by assuming that asset returns are riskless, we ought to think of the asset as a well-diversified portfolio of stocks and bonds. The real gross interest rate on revolving credit card balances, R^{CC}, is set to 1.1075, three percentage points below the mean debt-weighted real interest rate measured by the Federal Reserve Board, to allow for an effect of bankruptcy that is absent from our model (as from the Laibson et al benchmark model).

3.4   Preferences

As noted above, we postulate a felicity function that exhibits constant relative risk aversion. The degree of relative risk aversion, ρ, is set to 2, close to empirical estimates commonly used in the consumption-saving literature. The discount rates, β, for high-school dropouts, high-school graduates, and

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[18]The vector of estimates for $(\alpha, \sigma_\varepsilon^2, \sigma_{\nu W}^2)$ is $(0.881, 0.024, 0.041)$ for high-school dropouts; $(0.782, 0.029, 0.026)$ for high-school graduates; and $(0.967, 0.019, 0.014)$ for college graduates. In the numerical simulations, the household fixed effect is set to zero, and u_{it} is represented with a symmetric, two-state Markov process and symmetric transition probability p between the states $\{-\theta, +\theta\}$. This process matches the estimated variance and autocovariance of u_{it} if $\theta = \sqrt{\frac{\sigma_\varepsilon^2}{1-\alpha^2}}$ and $p = \frac{\alpha+1}{2}$.

[19]The estimates of $\sigma_{\nu R}^2$ for high-school dropouts, high-school graduates, and college graduates are, respectively, 0.077, 0.051, 0.042. In simulations, household fixed effects are set to zero.

college graduates are set, respectively, at 0.912, 0.944, and 0.945. These correspond to rates of time preference, $\delta = \frac{1}{\beta} - 1$, equal to 0.096, 0.059, and 0.058. We use the values for the discount factor chosen by Laibson et al. so that their model with exponential consumers (the one that does not introduce hyperbolic discounting and is therefore closer to ours) matches wealth-to-income ratios in the corresponding education group and in the (pre-retirement) age category 50 to 59. [20] The median ratios of net wealth to income to which these calculations refer are 2.5, 3.2, and 4.3, from lowest to highest education category.

We parameterize the accountant's bequest function as follows:

$$B(X_t) = (R - 1) \cdot \max\{0,\, X_t\} \cdot \frac{\alpha^{Beq} U_1(\overline{y}, 0, \overline{n})}{1 - \delta} \tag{16}$$

In our benchmark run, reported below, α^{Beq} is set to 1, as in Laibson et al (2003). $\overline{y}$ and $\overline{n}$ are the mean values for income and household size over the life cycle that apply to the education category of the household, and $U_1(\overline{y}, 0, \overline{n}) = \left(\frac{\overline{y}}{\overline{n}}\right)^{-\rho}$. This formulation of bequests essentially views the bequest recipient as having total consumption approximately equal to $\overline{y}$, effective household size equal to $\overline{n}$, and as consuming bequeathed wealth in the form of an annuity. Finally, Appendix B describes the adjustments made to convert the model from annual to monthly frequency.

4 Results

4.1 Policy Functions

Before we examine policy functions for our benchmark accountant-shopper model, it is instructive to start with a model with credit cards but without self-control considerations. In such a model, the single, "rational" self solves

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[20]Specifically, they choose the values that produces simulated median wealth-to-income ratios that are equal to the medians obtained from the Survey of Consumer Finances data. Total wealth in the data includes liquid assets (checking, savings, money market, and call accounts, CD's, bonds, stocks, mutual funds, cash, less credit card debt), and illiquid assets (IRA's, defined contribution pension plans, life insurance, trusts, annuities, vehicles, home equity, real estate, business equity, jewelry, furniture, antiques, home durables, less education loans). SCF data from 1983, 1989, 1992, and 1995 are used to derive medians for each year, and then the average of these medians is the number that is matched by the simulated model. Simulated total wealth is measured as net assets plus $\frac{Y}{24}$, where the last term is the size that average cash inventories would have if annual income were paid in 12 equal monthly instalments and if consumption were spread smoothly throughout the year.

the dynamic intertemporal optimization model for both controls, namely the level of consumption and the size of unused credit line to have available for purchases in the current period. The policy functions for both controls and for three different ages (25, 50, and 70 years) are plotted against the household's "cash on hand", defined as assets, minus credit card debt, plus labor income at the beginning of the period. Figure 1 shows that the policy function for consumption in such a model has a shape that looks quite similar to policy functions from models without credit cards. Since the minimum labor income level is positive and the minimum allowable asset level is zero, negative cash on hand can only be observed if the household starts the period with credit card debt outstanding. When this is the case, optimal consumption is fairly low, but it increases quite rapidly as cash on hand increases to turn positive. From then on, consumption rises with cash on hand, but its slope is well below the 45-degree line, implying saving. Again, consistent with standard consumption-saving models, households tend to increase their consumption through their working years, controlling for the available cash on hand. The policy function for consumption shifts downwards during retirement (here exemplified by the function at age 70), because of the substantial drop in non-asset income.

Figure 2 shows the corresponding policy function for the credit card balance after the payment at the beginning of the month, but before any new purchases have been charged on the card. When the rational accountant determines both consumption and payments into the credit card account, the balance is zero for our calibration settings, and no debt is revolved throughout the positive range of cash on hand and for all three different ages considered.[21] When the agent starts the period with negative cash on hand, i.e. with no assets and with more credit card debt than his income, he obviously continues to revolve debt.

These policy functions of a household without a self-control problem should be contrasted with Figures 3 and 4 that refer to our benchmark accountant-shopper model. If we compare the policy functions for consumption in Figures 1 and 3, the most striking difference is not in the levels of the policy functions but in their smoothness. Specifically, policy functions jump at the level of cash on hand where the accountant decides to fully pay back the credit card debt. This can be seen in Figure 4 that plots the policy function for credit card balance after the payment. Focusing on the

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[21]Of course, this is not true for any calibration settings: it is easy to pick parameter values such that it is optimal for the credit card holder to use the card for borrowing as well as for consumption. In those cases, however, the agent holds no assets. By contrast, for our benchmark settings the agent is a saver, and he does not revolve credit card debt because it charges higher interest than the assets he holds.

policy function for any given age, we find that there is a low range of cash on hand in which the accountant chooses to revolve credit card debt, despite the fact that he also holds assets that offer lower interest rates. This happens in order to discourage the shopper from overspending, and indeed comparison of Figures 1 and 3 shows that policy functions for consumption in the self-control model are remarkably similar to those for rational households over this range: revolving credit card debt proves to be a reliable method of self-control. Then, there is a range of cash on hand where the accountant sees no point in revolving credit card debt and lets the credit limit itself be the factor that restrains the shopper. Then, there is a third range of cash on hand, where the accountant wants to encourage more consumption than the credit card limit allows. In this region, the accountant deposits more funds in the credit card account than necessary to pay off any outstanding balance, and this shows up in our policy function as a negative (i.e., credit) balance on the card account. Although the particular method whereby the accountant encourages further consumption is an artifact of our assumption that all consumption has to be charged on the credit card, the key insight remains that at high ranges of cash on hand, the accountant does not choose to restrain the shopper but actually makes more resources available to him than the credit limit, so as to engineer the desired level of consumption.

Finally, Figures 5 and 6 show the policy functions for a third case, where the sophisticated accountant knows about debit cards and uses them to control the impatient, unsophisticated shopper. In this case, the accountant provides the shopper with a debit card linked to a special account, rather than giving him access to all his assets. At the beginning of each period, the accountant deposits into this account the sum that he deems appropriate so as to induce the desired level of consumption by the shopper. This setup is isomorphic to our benchmark credit card setup if we set the credit limit at zero, given that we allow for negative (i.e., credit) balances on the account. Figure 5 shows that the accountant is quite successful in inducing the optimal level of consumption.[22] Figure 6 shows the pattern of sums that the accountant makes available in the account linked to the debit card, as a function of cash on hand. We see that consumption tracks these sums quite closely at low levels of cash on hand, but falls well behind them as we move to larger amounts. The shortfall of consumption relative to sums made available is especially pronounced for the young and for the elderly retired households who prefer to maintain a buffer of unused resources. Given that such sums typically earn no or little interest, the need to elicit the right

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[22]Negative values of cash on hand are not allowed here, because the credit card limit is at zero and assets are not allowed to follow below zero.

consumption response from the shopper generates interest costs even for this way of handling the self-control problem.

4.2 Simulations

4.2.1 Benchmark Accountant-Shopper Model

We first report results for our benchmark run of high-school graduates, calibrated with the parameter values reported in Section 3.4 above. Figure 7 shows results for the simulated time path of averages of various assets and debts of accountant-shopper households. Households on average start accumulation of assets in their twenties, and accumulate substantial amounts up to retirement age when average asset holdings reach their peak just under $150,000. The peak average wealth-to-income ratio is of the order of 8.[23] This is reasonably close to empirical estimates based on survey data, despite the fact that we have not picked the rate of time preference so as to match this ratio in our model.[24] In addition to assets, the figure shows the evolution of credit card debt outstanding at the beginning of each period t, prior to the payment made by the accountant into the credit card account, as well after payment has been made (i.e., revolving debt). Average amounts of revolving credit card debt hover around the $2,500 level and co-exist with accumulation of assets bearing lower interest. Intuitively, this happens because accountant-shopper households on average use credit card debt to control the shopper and not to borrow against the future. Thus, they do not perceive a conflict between accumulating low-interest assets and revolving higher-interest credit card debt.

On the figure is also plotted the assumed evolution of the credit card limit. It is hump-shaped during working years, and it goes down during retirement because of the drop in incomes after retirement, to which credit card limits are linked. Comparison of the time path of credit limits relative to the simulated average amount of debt that is revolved given the corresponding limit reveals that accountant-shopper households choose on average to utilize only part of the credit line, maintaining an unused buffer in order to smooth future shocks to available resources. The existence of this buffer is consistent with empirical observation, as mentioned above. During retirement years, we note

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[23]This is computed as a ratio of averages from the simulations. Net wealth is assets minus credit card debt at the end of the year, plus $\frac{1}{24}$ of annual income to allow for average cash inventories used for consumption, analogous to the measure used by Laibson et al (2003).

[24]For example, Laibson et al. report in their Table 6 an average ratio of 6.51 for households between 50 and 59 years in the 1995 Survey of Consumer Finances, going to 15.3 in the 60 to 69 age group.

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that households on average tend not to revolve credit card debt but actually make more resources available to the shopper than the credit card limit, so as to boost consumption to the desired levels. This finding is consistent with the much lower tendency of retired households to revolve credit card debt compared to their younger counterparts that is observed in the data.

Figure 8 shows average consumption (decided by the shopper) as a fraction of average credit lines (made available to him by the accountant) over the life cycle. We see that on average consumption is a stable fraction of unused credit lines over the life cycle. Utilization of the credit limit (defined as the ratio of credit card debt to credit card limit) is on average constant during working years, despite exogenous changes in credit card limits. Of course, at the household rather than at the aggregate level, we find that each household responds to realizations of income shocks by changing the utilization of the credit card so as to smooth month-to-month consumption. This confirms the role of the unused credit line as a buffer to smooth income fluctuations. Utilization of the credit line drops to a lower level during retirement years, consistent with empirical observations.[25] This change in credit card behavior occurs in the model because the retired shopper is more reluctant to charge the card, and the accountant is happy to leave a larger fraction of the (now lower) credit card limit free to generate the desired level of consumption.

Figure 9 shows average consumption and average income realizations over the life cycle. We observe that our accountant-shopper households tend to smooth income fluctuations and to have a consumption path that fluctuates less than their income path. Our model is also consistent with a drop in consumption after retirement, as is typically observed in the data.

Consumption, averaged across households, tracks very well average payments into the credit card account. In fact, the extent of comovement of consumption and payments is so great that if one plots one against the other, one can fit almost a 45-degree line through them (see Figure 10). This is even true of consumption and payments for a single household (not shown). Thus, consumption by the shopper seems to be moving almost one-for-one with payments by the accountant into the credit card account. Recall that this is an equilibrium outcome and not behavior forced upon the shopper: indeed, the shopper typically leaves a fraction of consumable resources unused and is not constrained to tailor his consumption purchases to match the funds paid into the account in the current period. Following closely the

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[25]Bertaut and Haliassos (2002) found that the percentage of bank-type credit card holders who revolve credit card debt drops from 49.6 percent in the age category between 55 and 65 to 26.6 percent among those 65 and above. Median credit card debt for card holders who do not usually pay off credit card debt and who also hold substantial liquid assets drops correspondingly from $3,100 to $1,000.

pattern of payments made by the accountant is consistent with the shopper having a target utilization rate for the credit card and provides the key intuition for understanding why it is not optimal for the accountant to run down his assets continually in order to pay off the credit card balance: if he does so, the shopper will respond to such payments with higher consumption purchases and this will frustrate the attempt of the accountant to lower the credit card balance.

Thus, the benchmark accountant-shopper model with revolving debt can generate results consistent with a number of empirical observations: co-existence of revolving credit card debt with liquid assets and substantial retirement accumulation; a tendency to keep a fraction of the credit line unutilized as a buffer; and a reduced tendency of the elderly to revolve credit card debt. At the same time, accountant-shopper considerations do not seem to interfere with key properties of the model regarding more conventional issues, such as the presence of consumption smoothing, despite some effects on the policy function for consumption.

4.2.2   No Self-Control Problem

Simulations of the benchmark accountant-shopper model can be compared to those from alternative models, in a similar fashion as for policy functions above. Figure 11 shows average assets and credit card debt for a household that does not face a self-control problem but is otherwise identical to the benchmark household. The key difference is that an accountant who does not need to control the shopper does not revolve credit card debt.[26] This was also clear from the policy functions. Simulations contribute a comparison of asset accumulation under the two models. Comparing Figure 11 to Figure 7, we see that the overall pattern of accumulation and decumulation, including the peaks, is comparable in the two cases, but average asset holdings tend to be somewhat higher in the accountant-shopper case than in the case without self-control problem when comparing across the same age. This includes the size of bequest households leave on average, provided that they have survived to age 90. The average bequest level seems to be somewhat higher in the accountant-shopper model than in the model without self-control problem. All in all, however, the figures suggest that a need to control the shopper affects mostly credit card debt but has much smaller (positive) effects on asset accumulation relative to a model without self-control problems. This happens despite the interest costs that an accountant-shopper household has

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[26]The small positive amounts at low cash on hand result simply from the fact that we are matching averages to averages. No individual household accumulates assets and revolves debt at the same time.

to pay in order to control the shopper by revolving credit card debt.

4.2.3   Debit Card

Figure 12 shows average credit card debt and asset holdings for a household with a self-control problem but without a credit card. The accountant provides the shopper with a debit card linked to a special account and pays into the account so as to induce the shopper to consume the desired amounts. The Figure shows that, on average, the shopper does not exhaust the resources made available to him but retains a buffer in a manner analogous to the credit card case and the unused credit line. Thus, it would not be appropriate to think of this as a case in which no interest is sacrificed in the effort to control the shopper. The simulations show that, in order to handle the self-control problem, the accountant needs to forego interest by committing more resources to the low- (or zero-) interest account linked to the debit card than the shopper spends on average.

No clear ranking of average asset holdings is observed when we compare holdings at the same age across the benchmark model with self-control problems and the model with a debit card. In the simulations shown, debit card households leave somewhat bigger bequests on average than credit card households; but the reverse ranking is observed, say, for households aged 40.

Differences in policy functions and in average behavior are useful for understanding how alternative models perform. However, since differences across models or cases are multidimensional, such discussions do not shed light on how a household would choose among alternative ways of handling the self-control problem. It is to this question that we now turn.

4.3   Comparing Alternatives

It is clear from our results above that the behavior of a household facing a self-control problem is not the same as that of a household in which consumption choices and accounts are handled by the same person engaged in intertemporal maximization of expected utility. We have also noted differences between the benchmark model and models with debit cards or with a lower credit card limit. This section examines and compares the costs of facing a self-control problem and of handling it in three different ways: by revolving credit card debt, by asking for a lower credit limit, and by foregoing the use of a credit card but depositing sums in a (non-interest-bearing) account to which the shopper has access via a debit card or otherwise. In what follows, we compute differences in value functions at the start of economic life (age twenty), and express them in terms of equivalent sums of money

that would be necessary to give to the young household so as to make it indifferent between following each two alternatives over his lifetime.

Figure 13 plots these monetary costs as functions of the cash on hand available to the twenty-year old household. The monetary cost of facing a shopper-control problem and handling it by revolving credit card debt starts at under $6,000 for households with negligible sum of net wealth plus labor income, and roughly stabilizes under $8,000 for young households with more than about $30,000 of resources net of debts. The slight decline for rich households arises mainly from the tendency not to restrict the shopper when resources are abundant. This estimate of the amount of monetary cost to a twenty-year old household is not small enough to render the self-control problem negligible for households that do face it, except perhaps for those rich households that end up not having to restrain the shopper in most periods of their lives.

The other two curves show the monetary costs of exercising self-control by revolving credit card debt versus two alternatives that are in principle open to the household, namely lowering its credit card limit to equal 15 percent instead of 30 percent of average income in each period t; and replacing the credit card with a debit card linked to a special account. Lowering the limit would reduce costs for households in the range of net resources considered, though the benefits would be very small at low and high resource levels. Benefits are below the $2,000 mark for high-school graduates starting life with up to about $30,000 in resources net of debts. They are close to zero for those starting off at very low resource levels, and remain below $3,000 regardless of resources at age 20. Switching to a debit card is not necessarily better than revolving credit card debt. It proves to be worse for high-school graduates starting life with very limited resources, its benefits for better endowed households are comparable to those under reduced credit limits, and it is dominated by reduced credit limits for well-endowed households that start their economic life with $80,000 or more in net financial resources.

What do these results imply about the plausibility of self-control problems as a simultaneous explanation of the two credit card debt puzzles? Our simulations demonstrate that self-control problems of the accountant-shopper variety could lead to behavior that is otherwise puzzling but can be explained within the logic of our model. One may still wonder, however, whether revolving credit card debt would be the best way for a household to deal with its impatient and unsophisticated shopper. Our cost computations can shed some light on this question.

Our results imply that acquiring a debit card or halving credit card debt limits would produce either small benefits or even losses at the lower and the higher ends of net resources. Indeed, many households starting their

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economic life at age twenty are likely to find themselves with very limited resources, unless they receive an inheritance or major inter-vivos transfer. Even for households with intermediate initial wealth, the monetary cost of choosing forever a strategy of revolving credit card debt versus the other two strategies is rather small, of the order of about $2,000. The small size of this cost is more evident if we recognize that the two competing alternatives impose much heavier administrative and informational requirements on households than simply not paying off their entire credit card balance. For example, the administrative data of Gross and Souleles (2002) suggests that households do not typically ask for reductions in credit limits on existing accounts, while financial institutions are quite reluctant to initiate such reductions. Moreover, credit cards tend to be more widely known and used than debit cards, and many households may simply not be aware of this alternative. Revolving credit card debt is simpler, known to households as part of their credit card contract, and it avoids the costs of setting up debit cards and filing applications for reductions in the credit card limit. Finally, note that the alternative of reducing the limit below the average limit for all households in the relevant category, as we model it, may still generate revolving of credit card debt. Thus, some of the credit card debt revolvers we observe in the data may well be households that have already restricted their overall credit limits, e.g. by closing some credit card accounts or by refusing to accept additional credit card offers that other households in their demographic group tend to accept. All in all, revolving credit card debt does not appear to be a bad way to restrain a shopper, and it can explain two types of puzzling behavior among a significant subset of credit card holders.

4.4   Robustness check: sophisticated shopper

In our benchmark model above, we characterized the self control problem of the shopper by two aspects: greater impatience than the accountant (self), and a lack of full understanding of the financial consequences of credit card purchases. Modeling the latter aspect is not straightforward: once we leave the modeling discipline of perfect rationality, there is a large, perhaps infinite, set of possible specifications that entail limited understanding, and we would not like our main conclusions to depend narrowly on our precise choice of specification. In the course of writing this paper, we have tried a number of specifications of how the shopper perceives the consequences of his spending decisions on the payments that he receives from the accountant, and we find that the main conclusions of our model are quite robust to the details of the specifications. Fortunately, we can go beyond these checks to argue that the fundamental ingredients for generating both types of portfolio co-existence

that we set out to resolve are the breakdown of the unitary household into an accountant and a shopper, and the greater impatience of the shopper. In other words, we show that the main results survive in an accountant-shopper model where the shopper is assumed to be perfectly rational and to engage in a dynamic game with the accountant. While this rational framework is conceptually simpler, we argue that it may not be the most useful in modeling behavior of an impulsive shopper visiting the mall with credit card in hand.

The model

We look at a model where the shopper understands *perfectly* the consequences of his acts on future credit card payments. The shopper has a different objective function than the accountant in the sense that he is more impatient, but he understands the behavior of the accountant, and knows that the accountant can use the credit card balance to restrain him. In choosing consumption, he plans strategically and chooses a consumption plan that elicits a favourable response of the accountant.

   If the shopper is impatient but otherwise perfectly rational, accountant and shopper enter into a dynamic game. The conceptual structure of the game is simple: there is a last period, since the death probability in the last month of year 90 is 1. Within each period, first decides the accountant, then the shopper. Both players share the same information. The action of the accountant, which is the payment P_t, depends on liquid assets A_t and credit card debt B_t at the beginning of the period. The action of the shopper, consumption C_t, depends on credit card debt after payment $B_t - P_t$, and liquid assets left after payment, $A_t - P_t$. The game can therefore be solved backwards:

1. In the last period (call it T), the shopper spends as much as the credit card limit allows. This determines the shopper's policy function $C_T(A_T - P_T, B_T - P_T)$.

2. In any period t, knowing the shopper's future policy functions $C_{t+i}(A_{t+i} - P_{t+i}, B_{t+i} - P_{t+i})$, $i = 0, 1, \ldots$, the accountant chooses his optimal payment $P_t(A_t, B_t)$.

3. In any period $t < T$, the shopper chooses his policy function $C_t(A_t - P_t, B_t - P_t)$ based on the future policy functions $P_{t+i}(A_{t+i}, B_{t+i})$, $i = 1, 2, \ldots$ of the accountant.

   The information regarding future periods is effectively contained in the accountant's value function $V_t(A_t - P_t, B_t - P_t)$ and the shopper's value

function $\tilde{V}_t(A_t, B_t)$, and the model can be solved by backward iteration in the standard way.

While the conceptual setup of the game is simple, the optimal strategies of the players look complicated, even bizarre. This should come as no surprise: in models of hyperbolic discounting, which share with this specification of the model the feature that they are games of players with different discount factors, policy functions are in general discontinuous and non-monotonic (see, for example, Laibson, Repetto, and Tobacman, 2003). In our model, we have an additional nonconvexity because the interest rate is higher when credit card debt is positive. This can only make matters worse. Solutions are usually smoother if there is some additional uncertainty, e.g., if both the accountant and the shopper do not know perfectly how the other behaves. Experimenting with this, we found that reasonable degrees of uncertainty generate too little smoothness for the increased complexity of the model they entail, and the essential results don't change. We therefore only present results for the basic model with a rational shopper that we have described above.

Results

We use the same benchmark calibration as in the text. There is no bequest motive. The rational benchmark refers to the case where both accountant and shopper have the same discount factor, $\beta = \beta_S$. Here, the self-control problem is characterized by $\beta_S < \beta$, and we look at the cases $\beta - \beta_S \in \{0.05, 0.1, 0.2\}$.

Figures 14–16 present the policy function of the accountant at three points of the life cycle, aged 20, 50, and 70 years, as a function of liquid assets. We assume that the beginning-of-period credit card debt is 5000, 5000 and 3000 USD, respectively. In each of the three cases, we see the smooth policy function of the rational household, but if the shopper is impatient, the optimal policy of the accountant is highly irregular.

Sometimes the accountant makes big payments (and then doesn't pay anything for a few month), but for most asset levels the accountant decides to roll over a significant amount of credit card debt. Simulation results are more illuminating. Figures 17 and 18 show sample averages of a simulation of 100000 households. Figure 17 gives the average credit card balance after payment. In the rational case, the balance is rather low, and it decreases over the life cycle as households accumulate assets, before it rises again at the very end of life (a consequence of the certain and finite life time). The higher the impatience of the shopper, the higher is the outstanding debt, on average. Figure 18 shows the fraction of all households with nonnegligible liquid assets

(bigger than 500 USD) which at the same time roll over some credit card debt (more precisely, it is the fraction of households with $B_t - P_t > 0$ and $A_t - P_t \geq 500$, divided by the fraction of households with $A_t - P_t \geq 500$). In the case of identical preferences between accountant and shopper (not shown in the graph), this fraction is obviously zero. We see that the fraction increases systematically with the (relative) impatience of the shopper. An additional impatience of 0.1 is more than sufficient to account for the incidence of such co-existence in the data, generating a fraction of 50 percent.

Figure 19 shows us that differential impatience between accountant and shopper is costly. It displays the value function of a household at the beginning of economic life (20 years), as a function of its liquid assets. As one would expect, for a given level of wealth, the value function decreases continuously in the impatience of the shopper. With impatience equal to 0.2, the household needs around 8000 USD of liquid assets to be as well off as a household with zero assets but no differential impatience.

Can this cost be easily avoided, for example by use of a debit rather than a credit card? Figure 20 compares the value function of the household without differential impatience between accountant and shopper with that of the impatient shopper household (level of impatience equals 0.2). If the household already starts out with a few thousand dollars of assets, so that the credit card is not much needed for consumption smoothing, then the household is better off with a debit card. However, the advantage is rather small unless initial assets are sizeable.

The results here are broadly consistent with the main results of the benchmark model, presented in Section 4, despite the absence of limited understanding by the shopper. Although the model with a sophisticated shopper avoids the hurdle of having to specify some departure from full sophistication, it also yields complicated policy functions that arise from the dynamic game between two highly sophisticated agents that are both implausible and hard to understand and analyze. Our benchmark model makes a simple assumption on the payment process that the shopper expects, which is broadly consistent with payment realizations in the simulation. Moreover, the particular choice of this assumption is not crucial for generating portfolio co-existence. Although the main point of the paper is to show that an accountant-shopper framework can deliver portfolio co-existence, we think that, all in all, lack of sophistication is not a bad accompanying assumption: the accountant, making decisions in a tranquil environment, acts in a more deliberate fashion than the shopper in the mall, who may be more interested in simply shopping than in strategic interaction with the accountant.

5   Concluding Remarks

The co-existence of substantial, low-interest liquid assets and high-interest credit card debt questions fundamental notions of arbitrage. The co-existence of credit card debt with substantial accumulation of assets for retirement makes households appear impatient with respect to some objectives and much more patient with respect to others. Households who revolve credit card debt appear to have target utilization rates of their credit card limits.

In this paper, we have presented a model that distinguishes between an accountant and a shopper function in the household and recognizes the potential role that credit card debt can play in disciplining the consumption behavior of the shopper. Low-interest assets can co-exist with higher-interest credit card debt, because if they were used to pay it off, the shopper would adjust his consumption behavior, frustrating the attempt of the accountant to lower credit card debt. Since credit card debt is not held simply because of impatience but has a role to play in moderating consumption, its existence is quite consistent with objectives to build up assets for retirement. Shoppers who are uncertain about their future ability to make purchases maintain an unused portion of their credit line as a buffer to future consumption. Accountants can exploit this behavior to control the amount of consumption by manipulating the size of the credit line that remains free. All in all, the accountant-shopper model is, to our knowledge, the first that is consistent with both types of puzzling portfolio co-existence observed in credit card debt data and with the existence of target utilization rates in credit cards. While it is conceivable that other models consistent with these three features may be produced in the future, it may be useful to think of the amount of revolving credit card debt not simply as an instrument of intertemporal consumption smoothing but also as a way to handle overspending that arises from the separation between purchases and payment afforded by credit cards.

A   Appendix

A.1   Outline of the Solution Algorithm

Benchmark model

The problem of the shopper can be written as a dynamic optimization problem in one state variable, namely the level of credit card balances after payment into the credit care account $(B - P)$. Since the shopper perceives the

accountants' payments to the credit card as exogenous, the problem is formally analogous to a savings problem with exogenous income. The difference to standard problems of this sort is that the present problem is not concave in general, because there is a jump in the interest rate at a credit card balance of zero. With higher wealth (lower credit card debt), the shopper faces a more favourable interest rate, and the standard conditions for a concave programming problem are not satisfied. We therefore solve the shopper's problem by dynamic programming, searching carefully for the optimal level of consumption. The problem is solved on a grid of 500 points. Between grid points, we approximate the value function by Schumaker's quadratic splines, which preserve concavity in those regions where the value function is concave, and are pretty robust otherwise. The solution to the shopper's problem yields a consumption function $C(B - P, t)$. The accountant knows this policy function for consumption and takes it as a basis for his decisions.

The accountant's problem is also solved by backward induction on a finite grid of state vectors. We define the value function at the beginning of the period *after* the realization of income shocks. We next describe the accountant's state space. The problem of the accountant has two state variables: liquid assets A and the credit card balance B. Note that, after the labor income shock is realized, the separation of liquid wealth into A and B plays a role only insofar as the accountant cannot increase credit card debt (the payment P cannot be negative). If we ignore this constraint, the desired level of credit card debt B^* after payment (corresponding to $B - P$) can be expressed as a function of the single state variable $\chi \equiv A + Y - B$. In the following algorithm we exploit this fact. In each period in the backward induction, we take the following steps:

- Compute $B^*(\chi)$

- Define the variable

$$\xi \equiv \begin{cases} 0 & \text{if } B^*(\chi) \leq B \\ (B^*(\chi) - B)/(B^*(\chi) + \chi) & \text{otherwise} \end{cases} \quad (17)$$

Note that $\chi > -B$ since $A + Y > 0$. ξ lies therefore between 0 and 1. The variable ξ is always 0 if the credit card debt at the beginning of the period (before the payment) is higher than the balance that the accountant wants to leave after payment. This should be the normal case, a $\xi > 0$ should only happen after big negative income shocks where the accountant wants to increase debt to make the shopper reduce consumption drastically.

With the above definitions, and given the function $B^*(\chi)$, there is a one-one mapping between $(A + Y, B)$ and (χ, ξ).

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- Compute the value function recursively on a rectangular grid of (χ, ξ)-points. Since ξ is normally equal to 0, the idea is that we can use very few grid points in ξ-direction and nevertheless get good accuracy.

- In the backward iteration we compute next period's value function off the grid by interpolation. For given value of ξ in the grid, compute $v(\chi, \xi)$ by Schumaker's quadratic spline interpolation in χ. Then interpolate piecewise linearly in ξ.

The results reported in the text use 400 points in χ-dimension and 5 points in ξ-dimension. Additional tests have shown that a further increase in the number of grid points has only small effects on the numerical results and does not change any qualitative conclusion of the paper.

The model with the sophisticated shopper

Since this version of the model is highly nonconvex, it seems unavoidable to discretize the action spaces of the players in order to get a reliable, if not necessarily very precise solution. Both the accountant's and the shopper's action can be seen as a choice of the credit card balance, the balance before and after consumption takes place, respectively. We therefore choose a discrete grid of possible values of B_t (the grid changes with age to account for the changing credit card limit). Liquid assets A are a continuous variable. The value function is stored at discrete points of A, and between grid points it is interpolated by shape-preserving quadratic splines. For the reported results, we have used a grid of 120 points for A and 400 points for B.

A.2   Conversion from Annual to Monthly Frequency

In this Appendix, we describe the adjustment that we made to convert the model from annual to monthly frequency. A parameter related to the monthly calibration is denoted by a tilde, a parameter without tilde refers to the annual calibration.

For the death probabilities we assume, for simplicity, that they are constant within a year. Therefore if $P_D(y)$ denotes the probability of surviving from year y to $y+1$, the probability $P_D(\tilde{y} : m)$ of surviving from month $y : m$ to the next month (which is $y : m + 1$ or $y + 1 : 1$ if $m = 12$), is equal to

$$P_D(\tilde{y} : m) = P_D(y)^{1/12}$$

The discount factor β and the interest rates R and R^{CC} are converted to monthly rates by the formula $\widetilde{\beta} = \beta^{1/12}$ etc.

We now turn to the adjustment of the parameters of the individual income processes. The deterministic part of log income is adjusted simply by substracting $\log 12$. More problematic is the adjustment of the stochastic part, since the shocks to these processes are i.i.d. For the parameters of the two-state Markov process governing the serially correlated income shock, u_{it}, we set $\tilde{\alpha} = \alpha^{1/12}$ and $\tilde{\sigma}_\epsilon^2 = \sigma_\epsilon^2/12$. These parameter choices then determine θ and p. We use a transitory income shock, ν, that is i.i.d. at the monthly level such that the variance of annual income is unchanged. This probably leads to a series of monthly income that is too volatile, but it is unavoidable unless we want to introduce serial autocorrelation in these income shocks, which is against the spirit of the annual income calibration.[27] We therefore adjusted the parameter by applying the formula

$$\tilde{\sigma}_\nu^2 = 12\sigma_\nu^2$$

for both working and retired population (i.e., for both ν^W and ν^R shocks). To understand this, note that σ_ν^2 is the shock to the *logarithm* of income. Since the standard deviation of income has to be divided by $\sqrt{12}$ when going from annual to monthly frequency, the standard deviation of income in percentage terms has to be multiplied by $\sqrt{12}$, which leads to the above formula.

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[27]Introducing serial correlation would also complicate the computation substantially because it implies introducing an extra continuous state variable.

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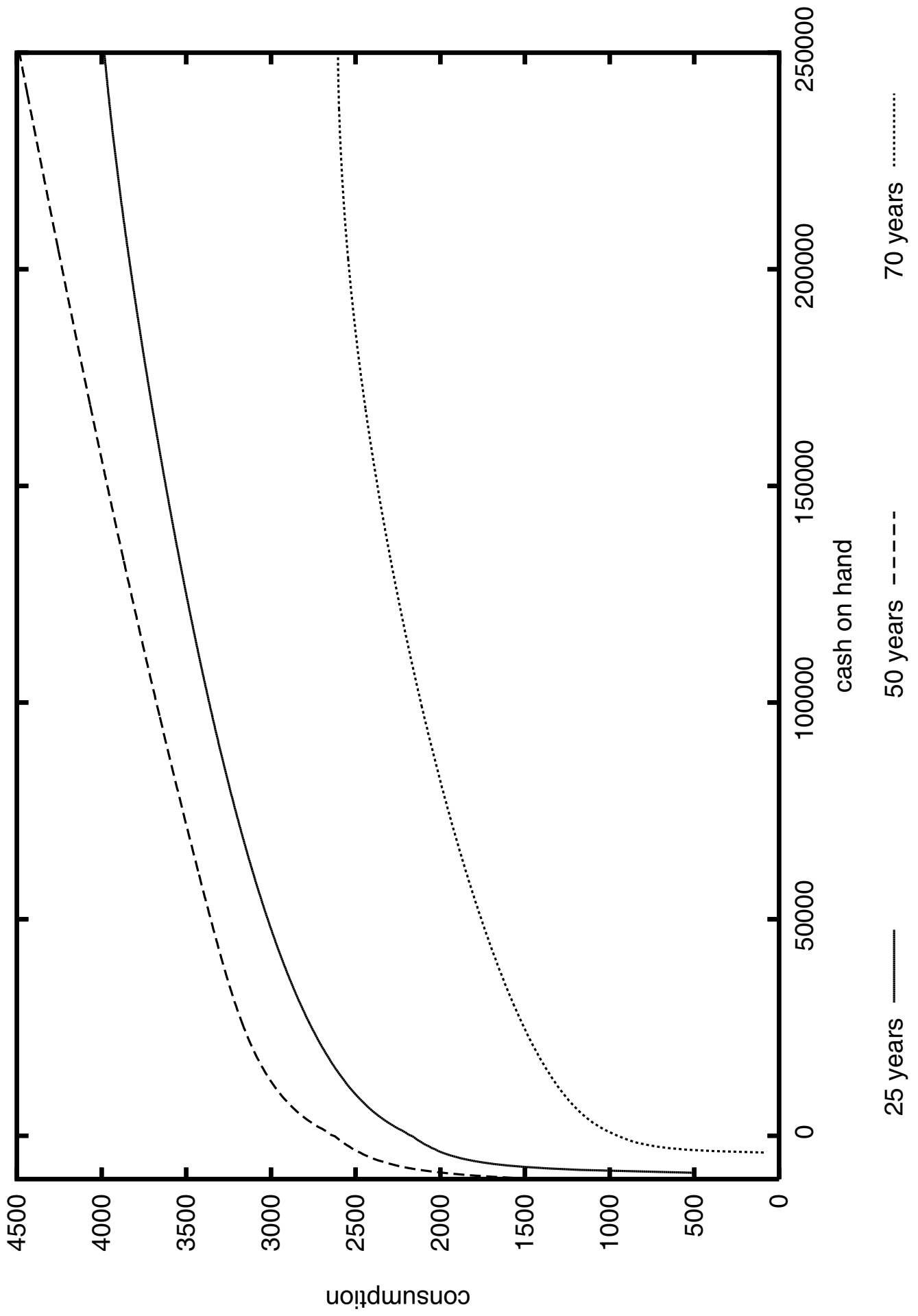
Figure 1: Policy Fuction for Consumption without Self-Control Problem

25 years ———   50 years ‐ ‐ ‐ ‐   70 years ··········
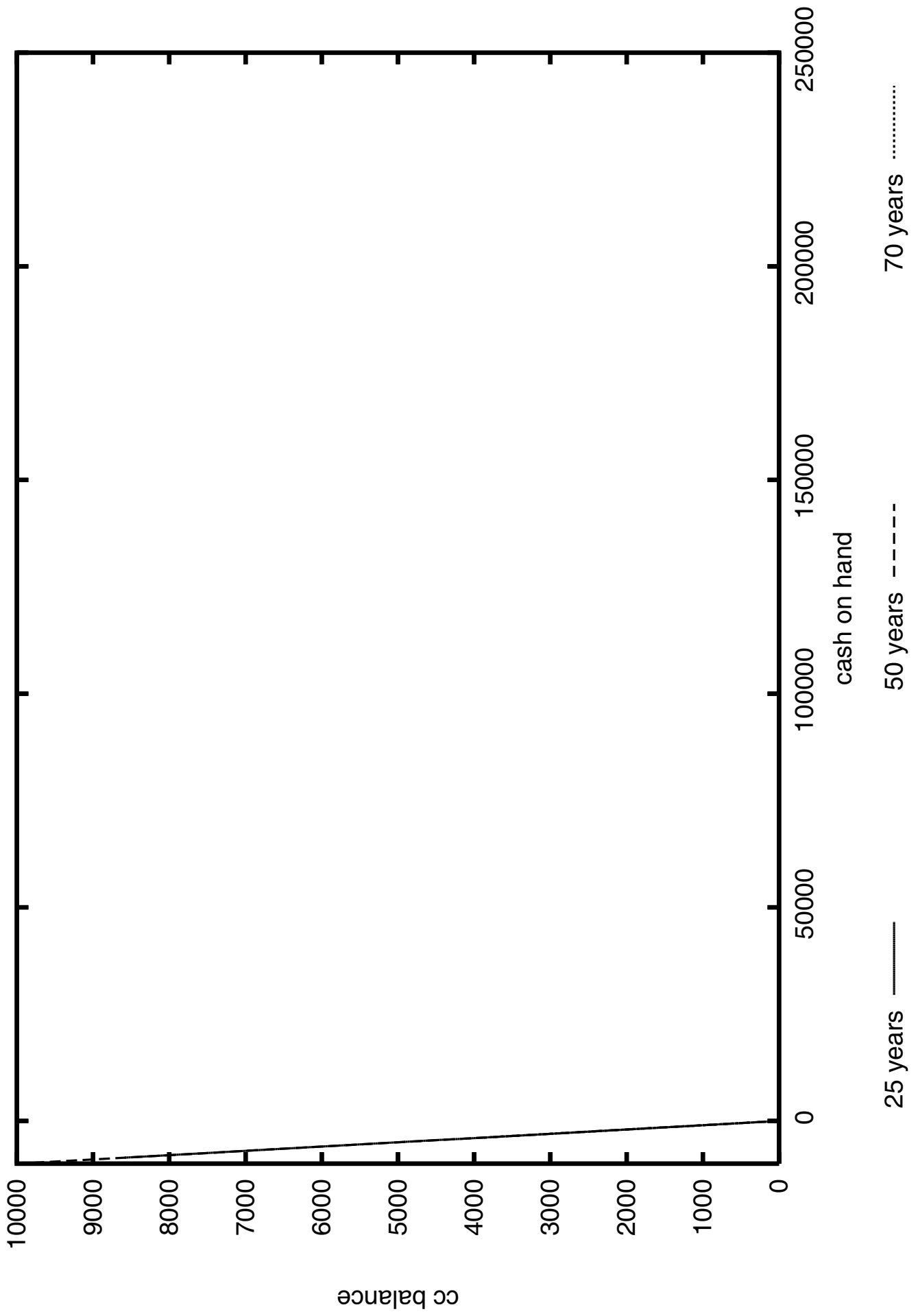
Figure 2: Policy Function for Credit Card Balance After Payment
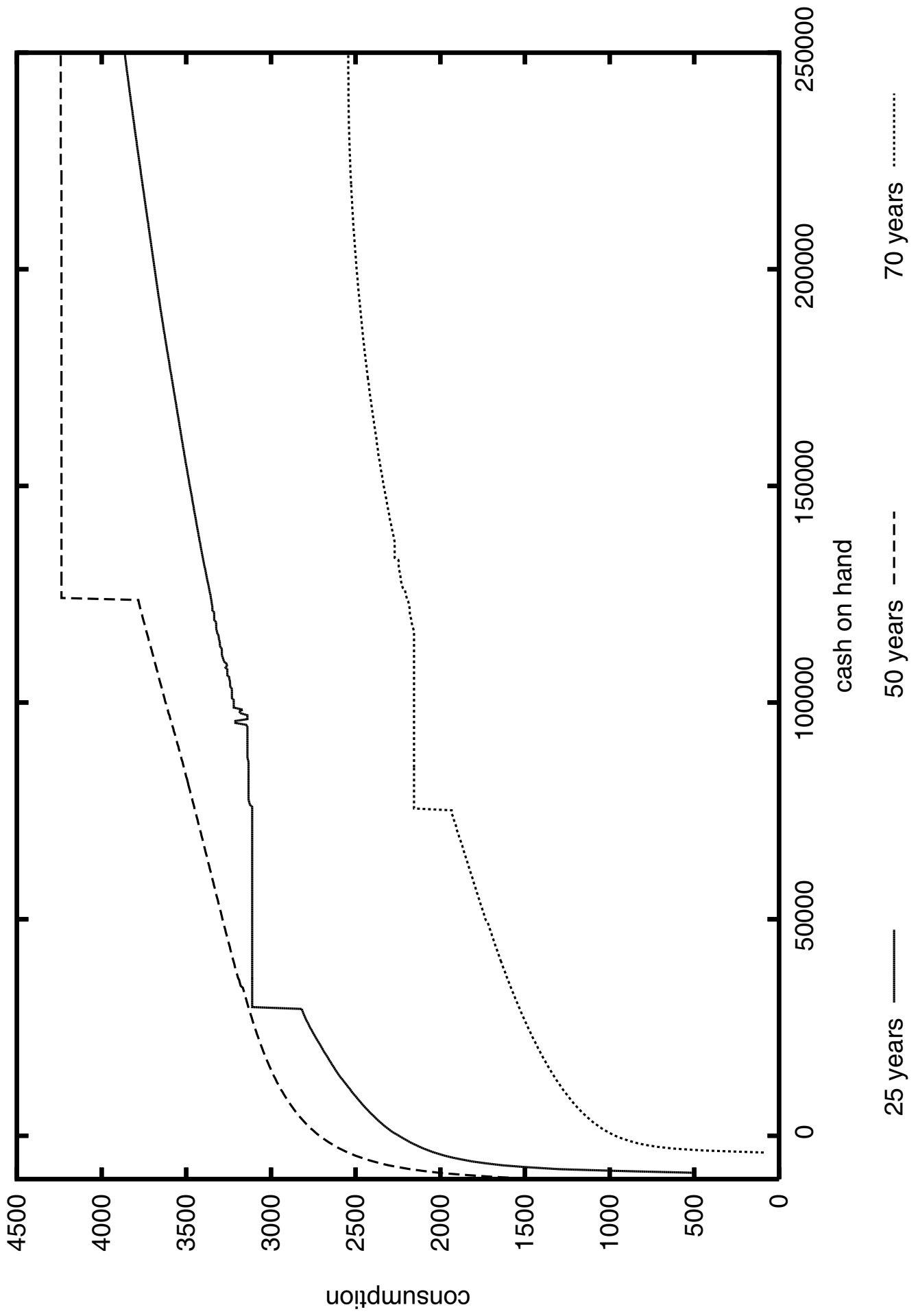
Figure 3: Policy Function for Consumption; Benchmark Model with Self-Control Problem

25 years ——— 50 years ‑ ‑ ‑ ‑ 70 years ·········

cash on hand
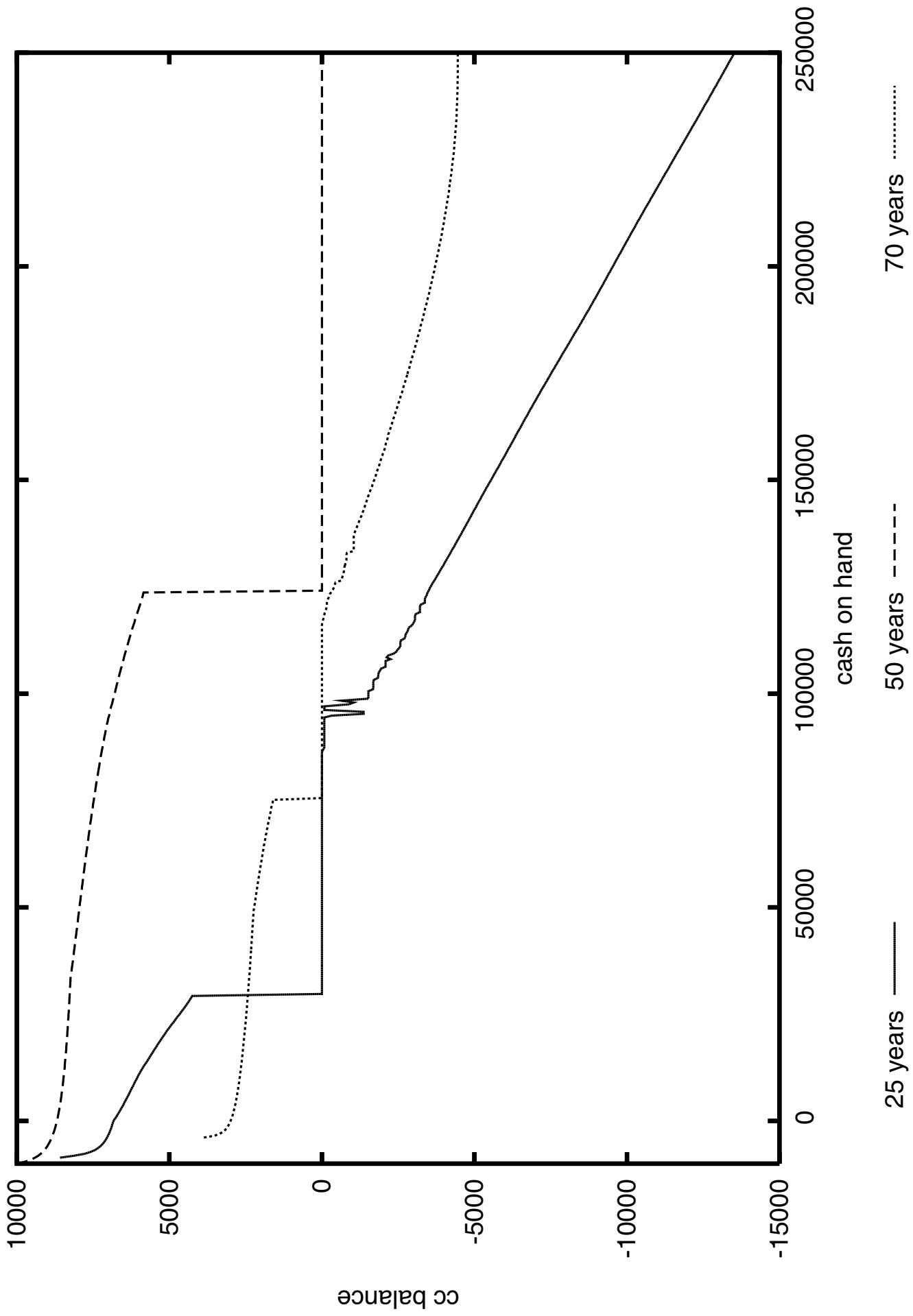
consumption

Figure 4: Policy Function for Credit Card Balance After Payment; Benchmark Model with Self-Control Problem

25 years —— 50 years – – – – 70 years ·········
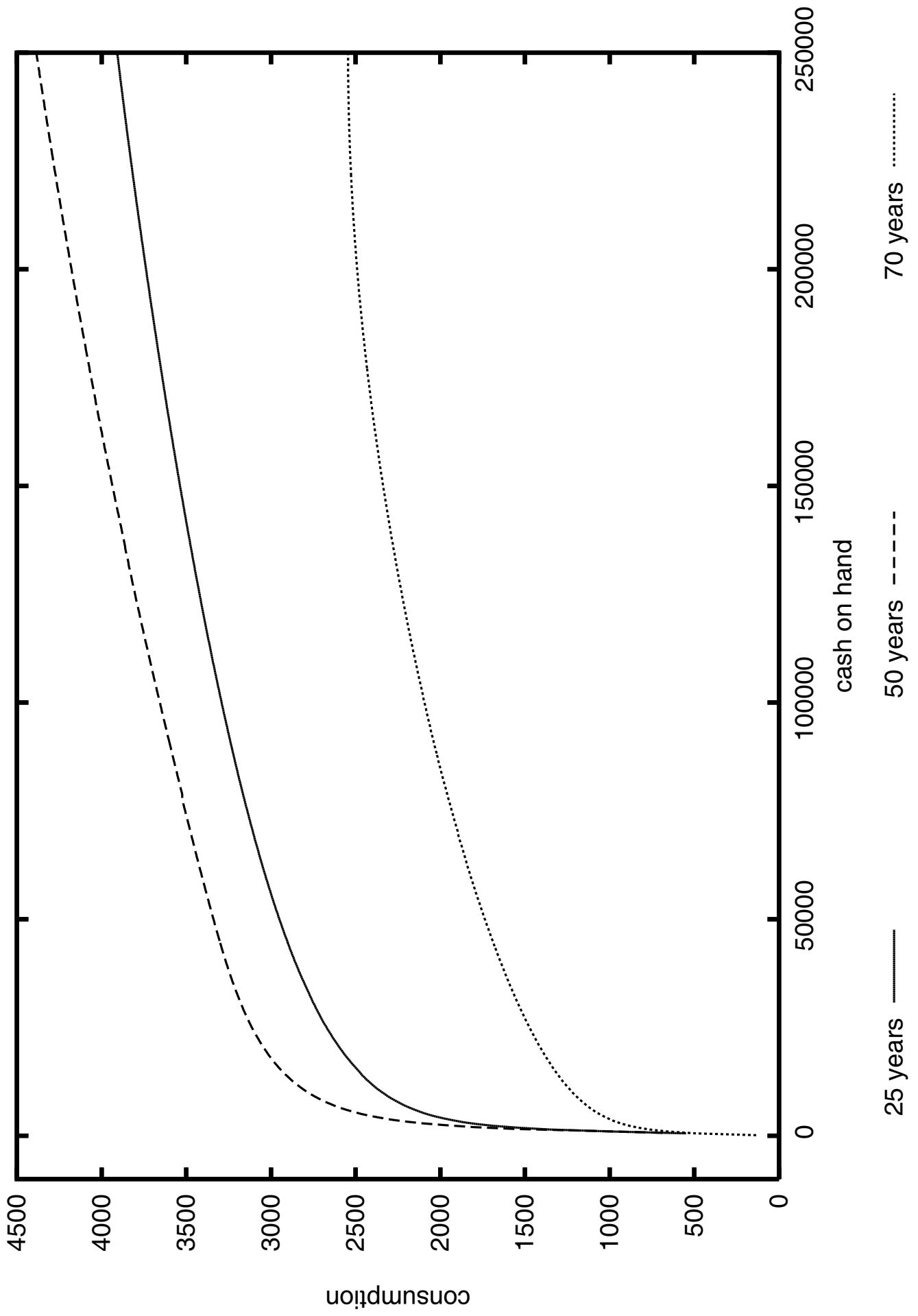
Figure 5: Policy Function for Consumption with Debit Card

25 years ———   50 years – – – –   70 years ·········
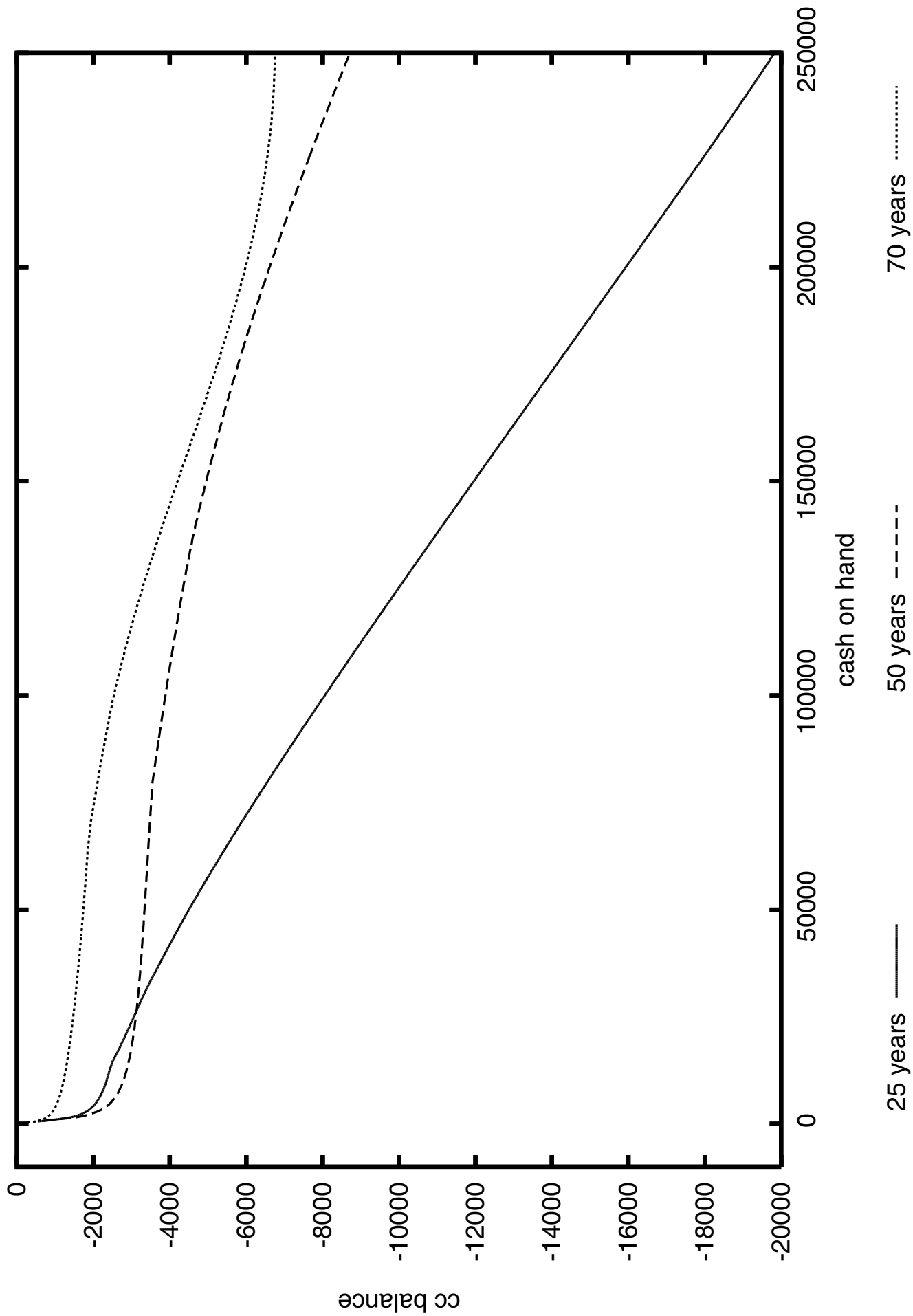
Figure 6: Policy Function for Credit Card Balance After Payment with Debit Card

25 years ——— 50 years ‑ ‑ ‑ ‑ 70 years ⋯⋯⋯
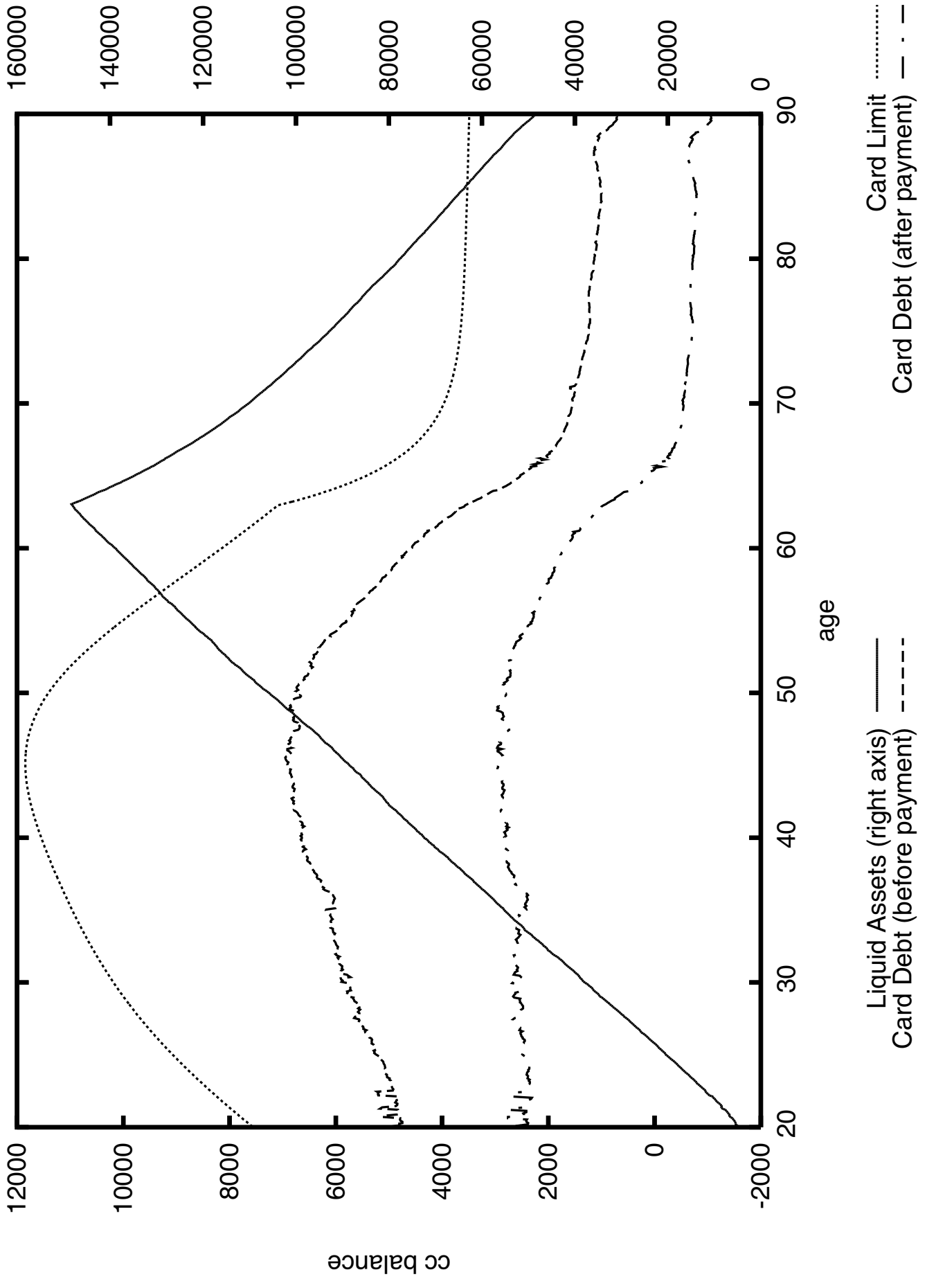
Figure 7: Assets and Debts; Averages, Benchmark Model
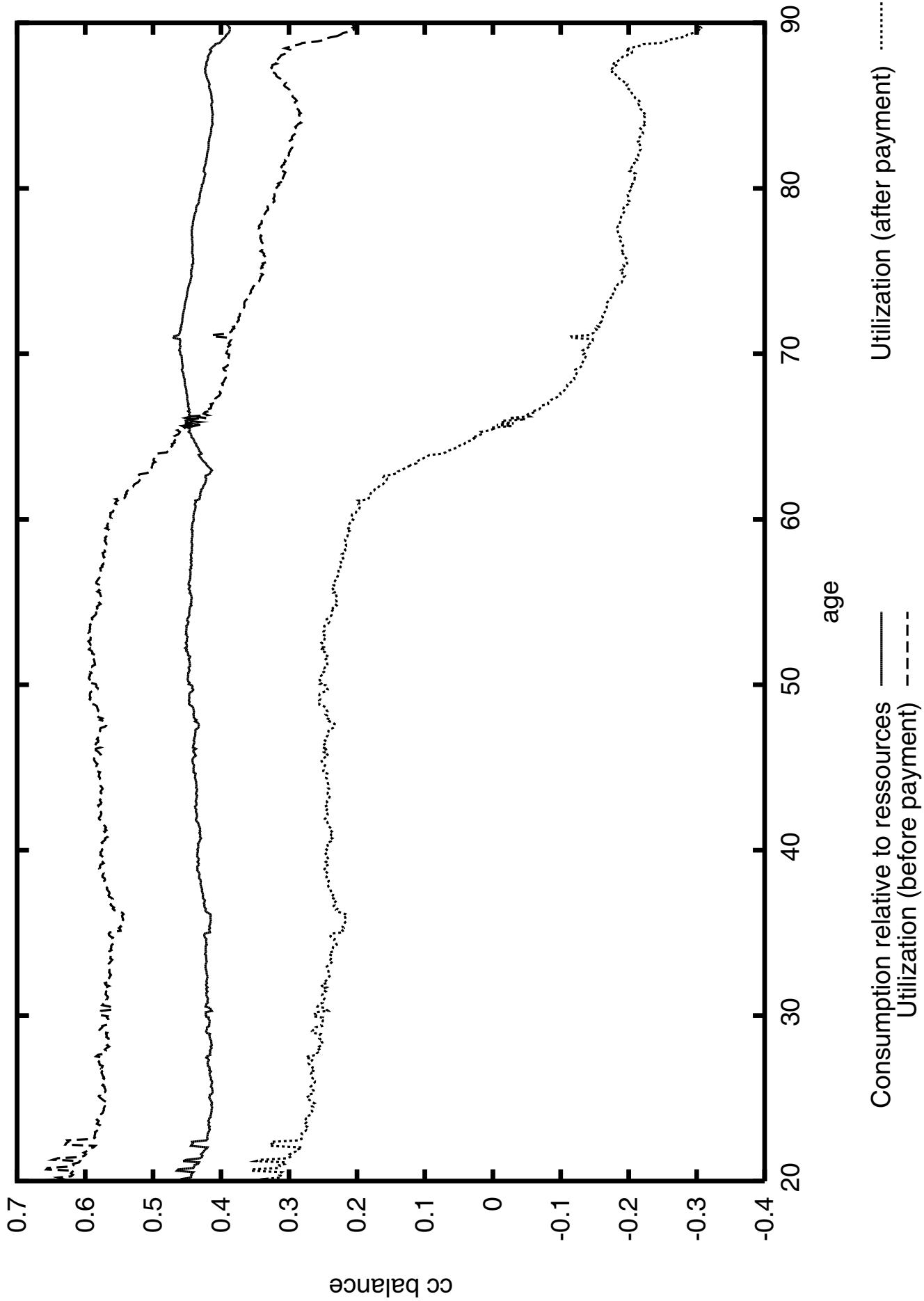
Figure 8: Utilization of the Credit Card Limit and Consumption as a Fraction of Unused Credit Line; Averages, Benchmark Model
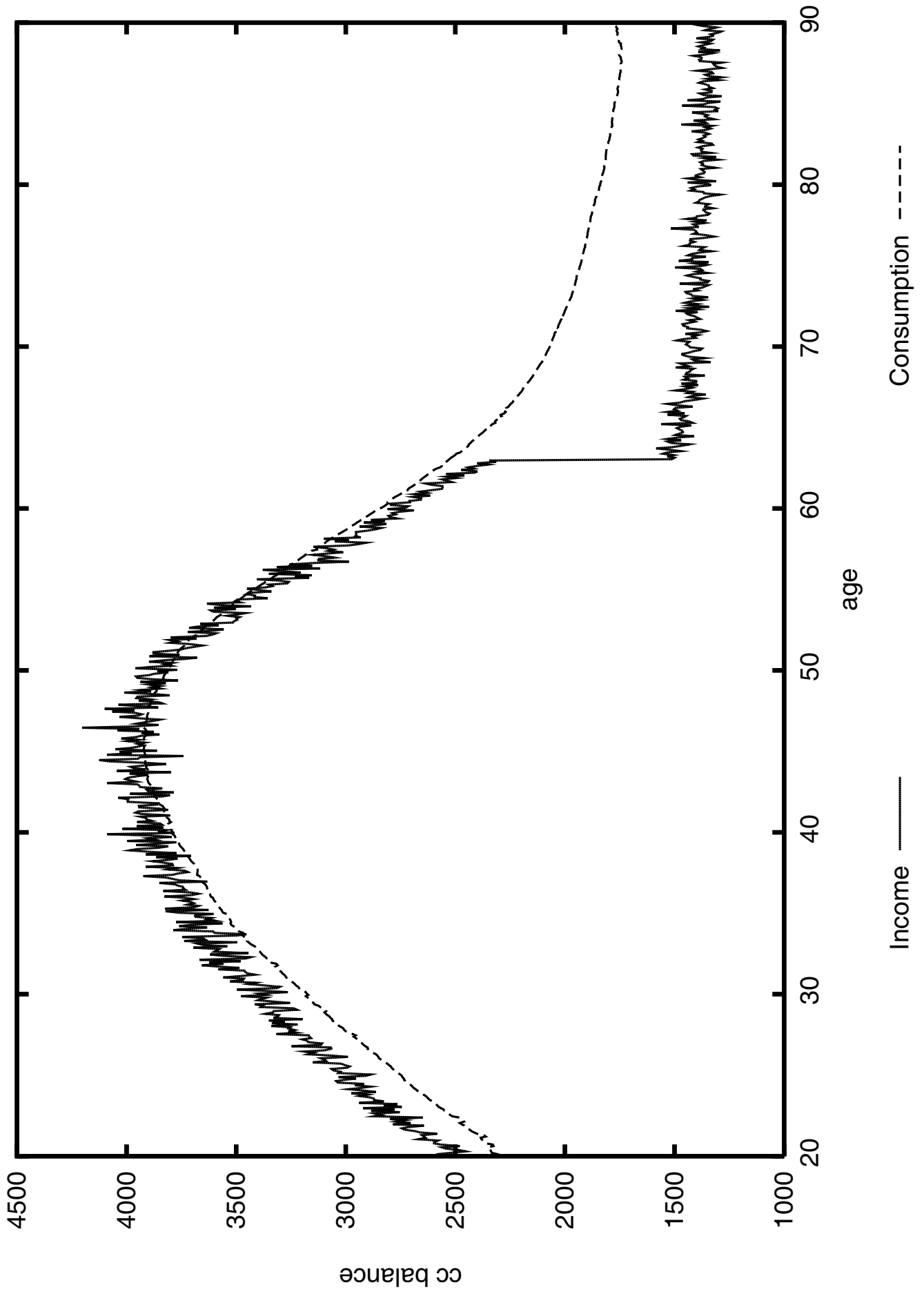
Figure 9: Income and Consumption; Averages, Benchmark Model
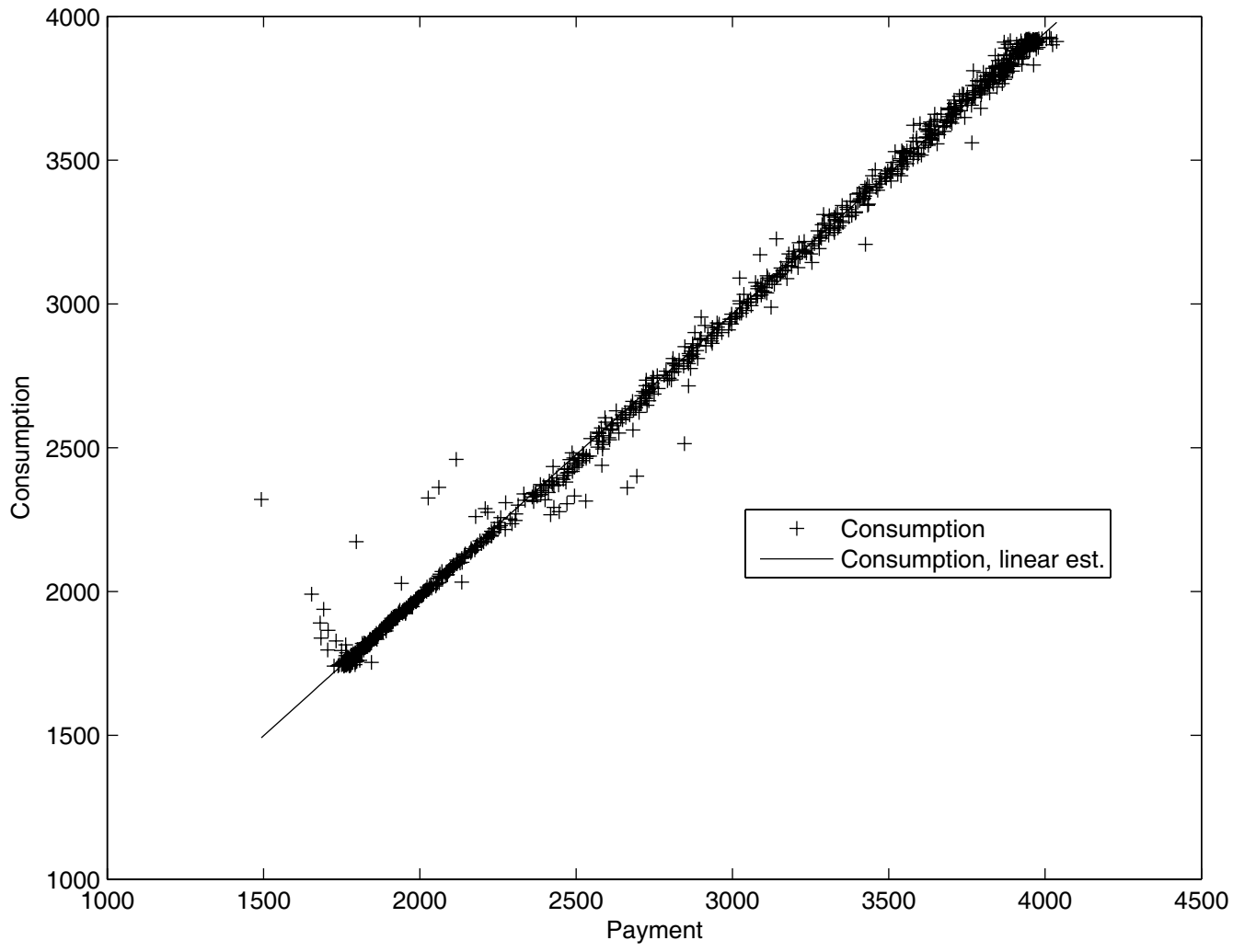
Figure 10: Consumption versus Payments; Averages, Benchmark Model
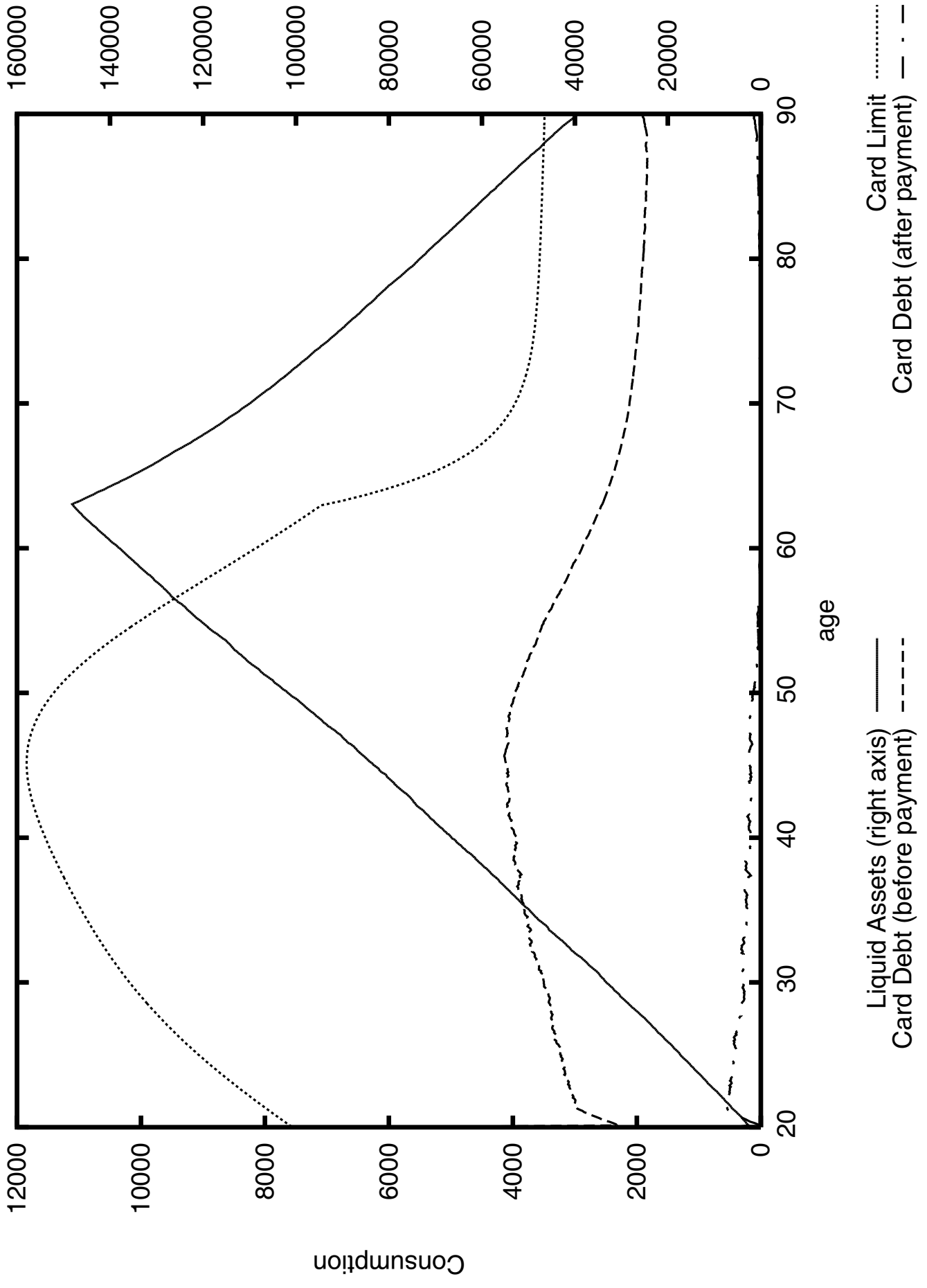
Figure 11: Assets and Debts; Averages, Model without Self-control Problem
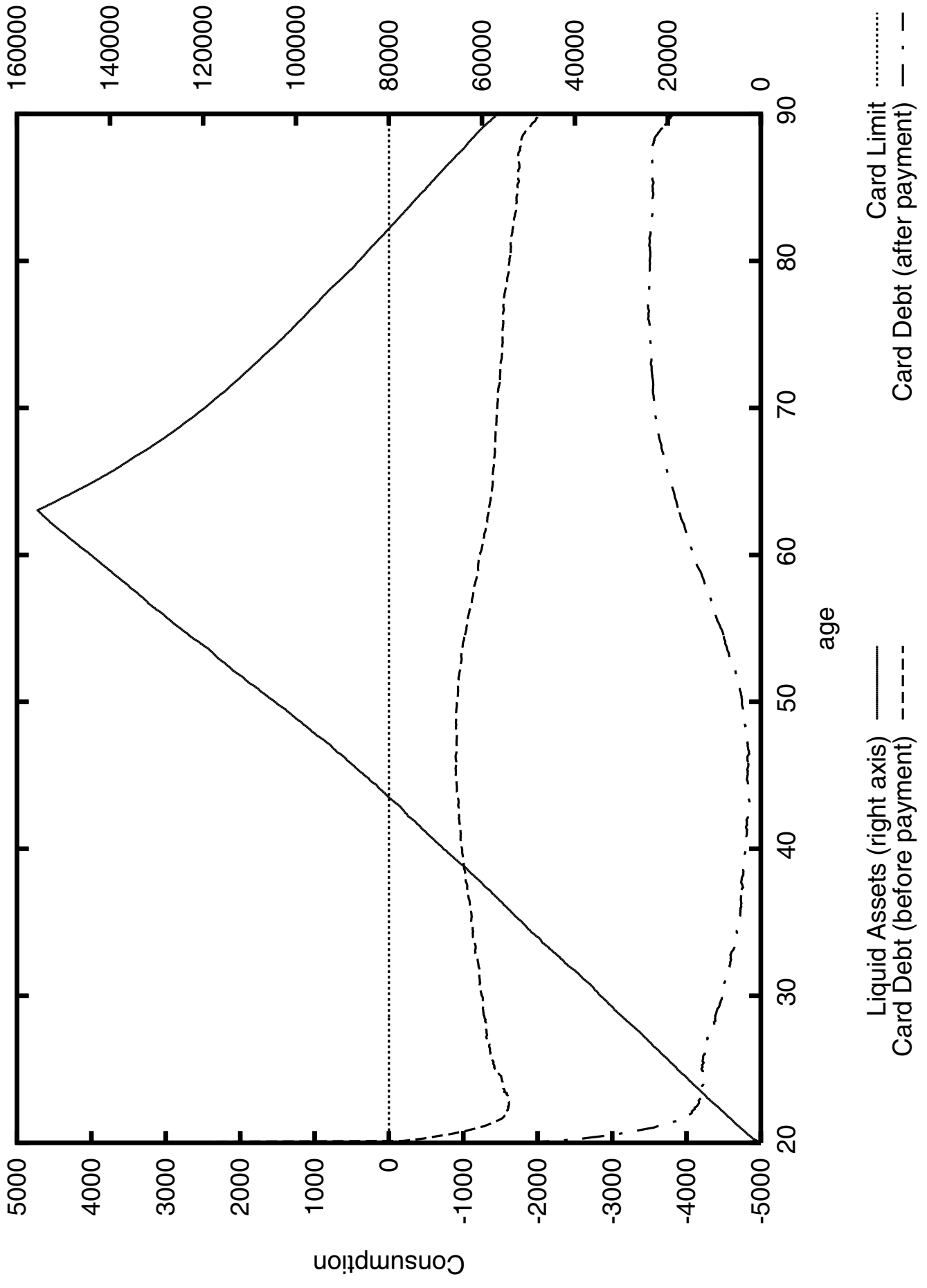
Figure 12: Assets and Debts; Averages, Debit Card Model
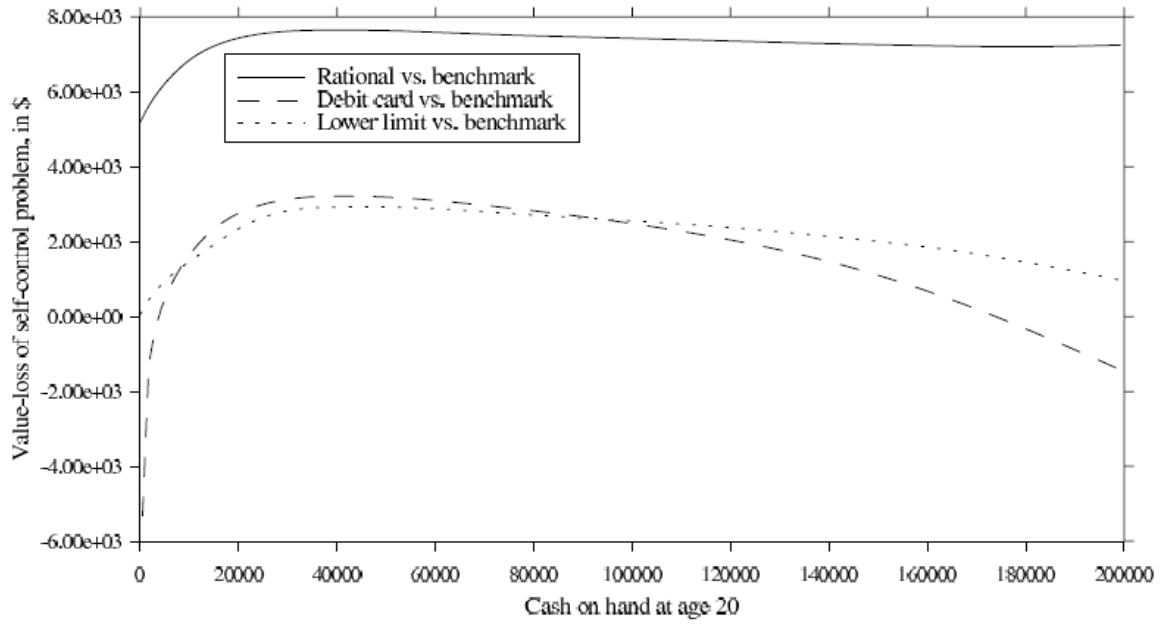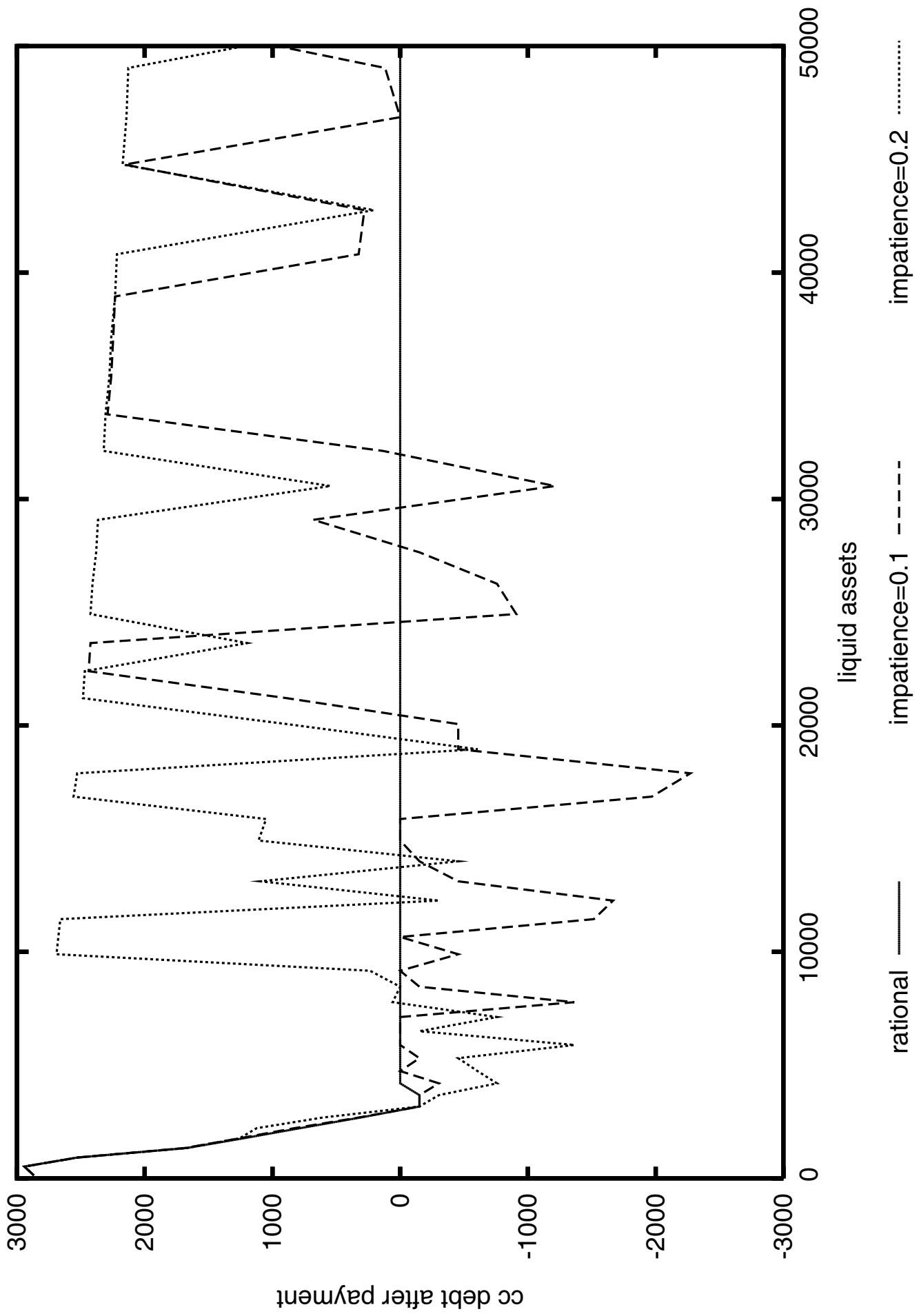
Figure 13: Comparison value losses
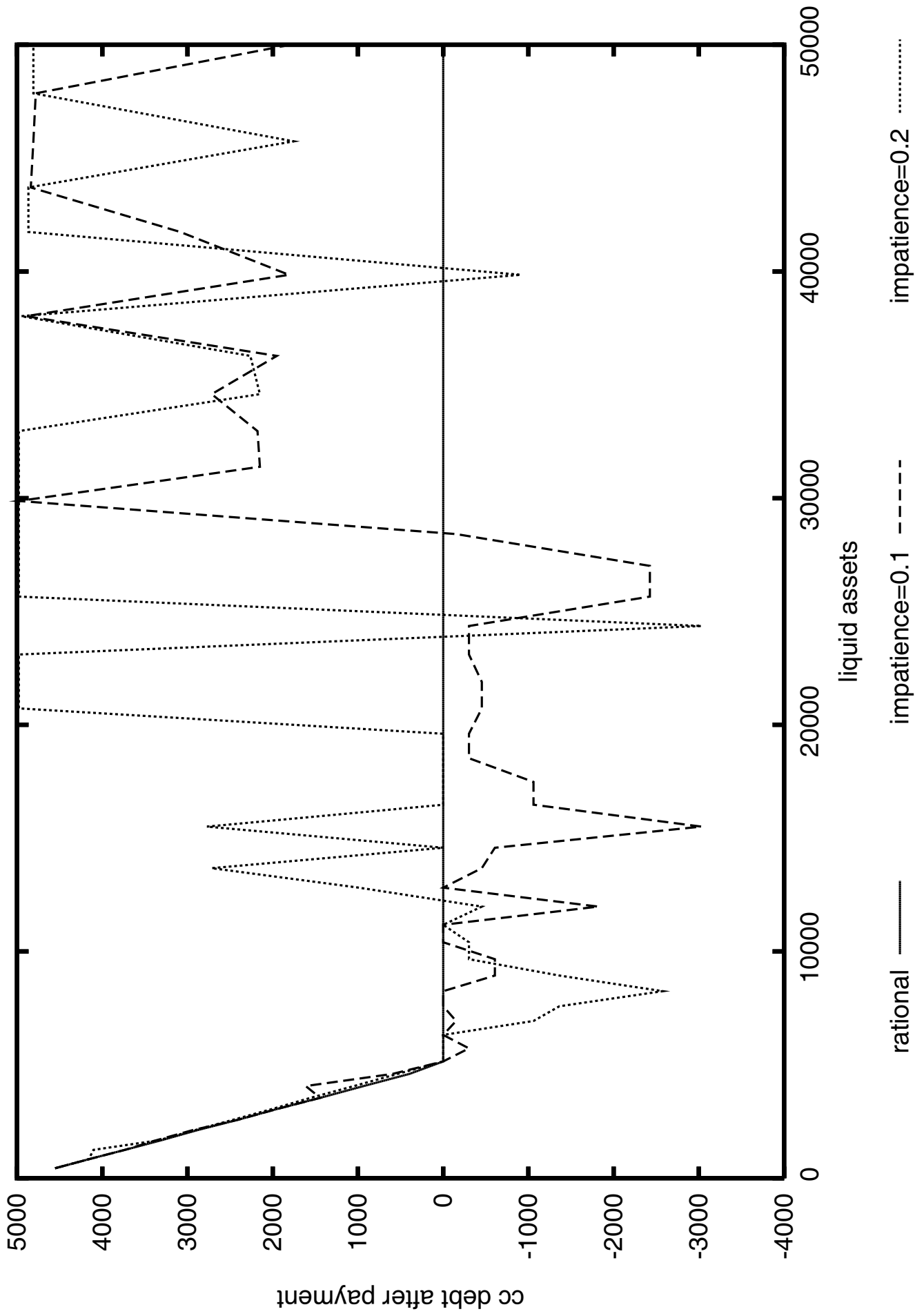
Figure 14: Sophisticated shopper model: Policy function of accountant at age 20: credit card debt after payment, when debt before payment = 5000 USD
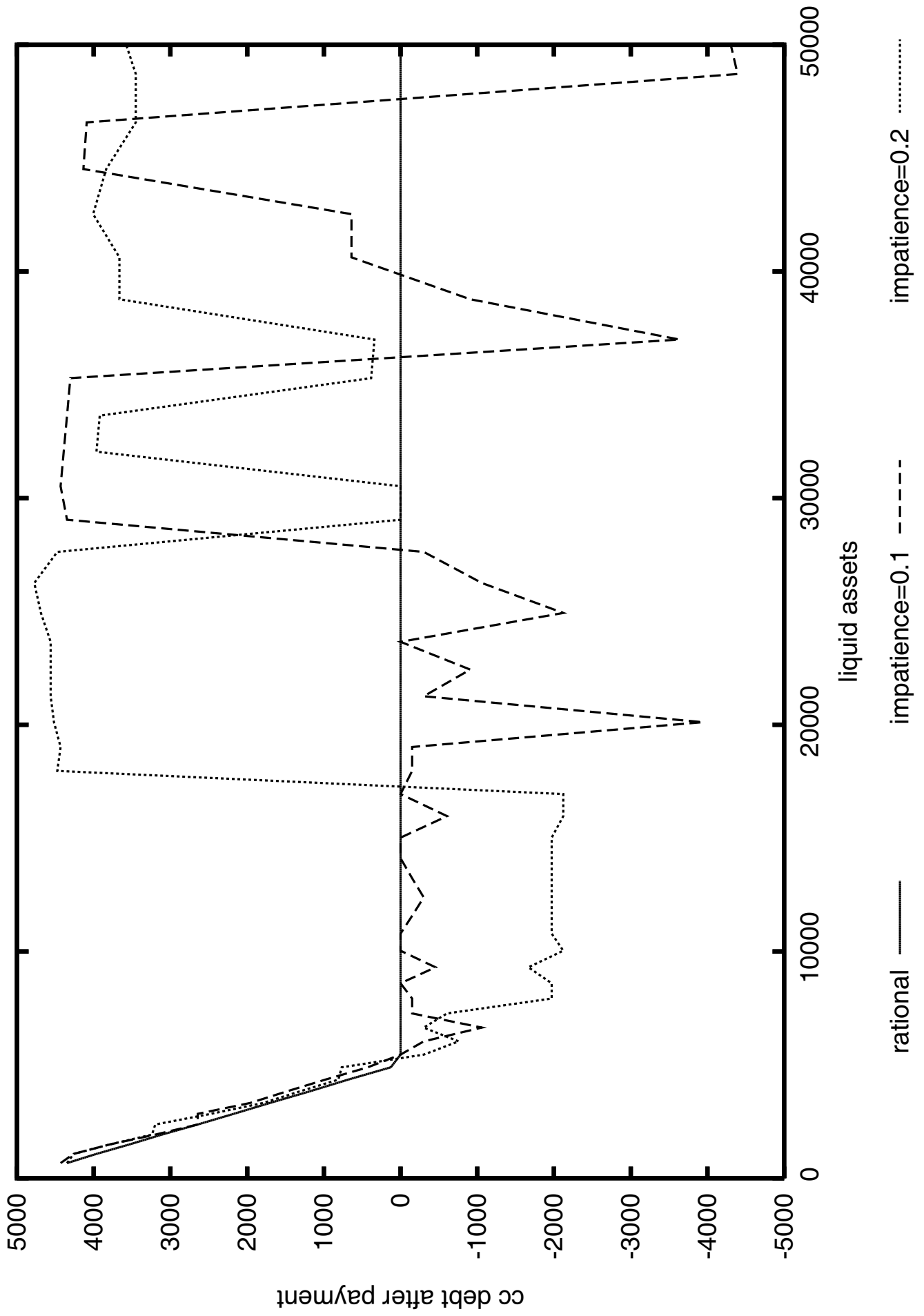
Figure 15: Sophisticated shopper model: Policy function of accountant at age 50: credit card debt after payment, when debt before payment = 5000 USD

Figure 16: Sophisticated shopper model: Policy function of accountant at age 70: credit card debt after payment, when debt before payment = 5000 USD
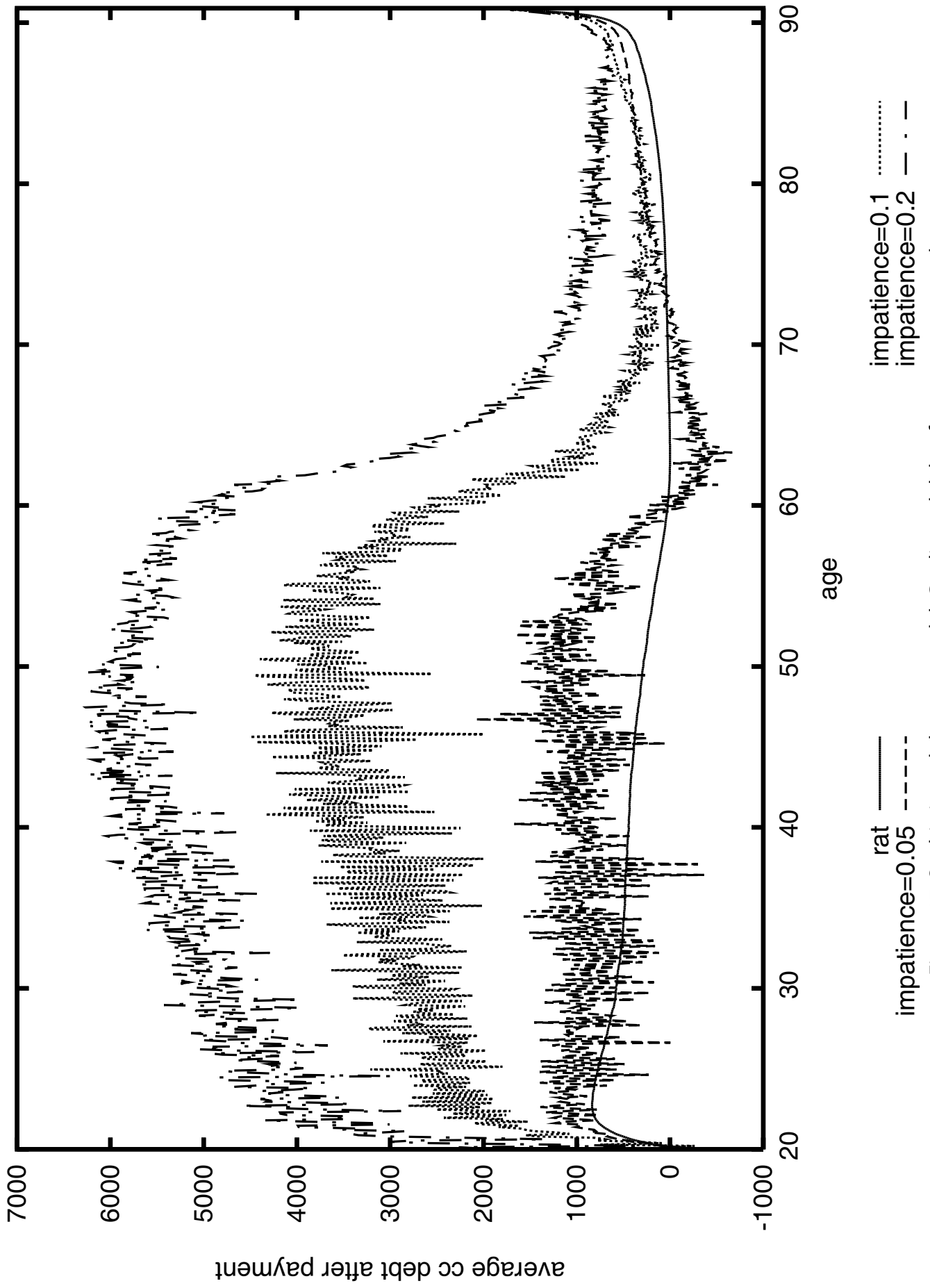
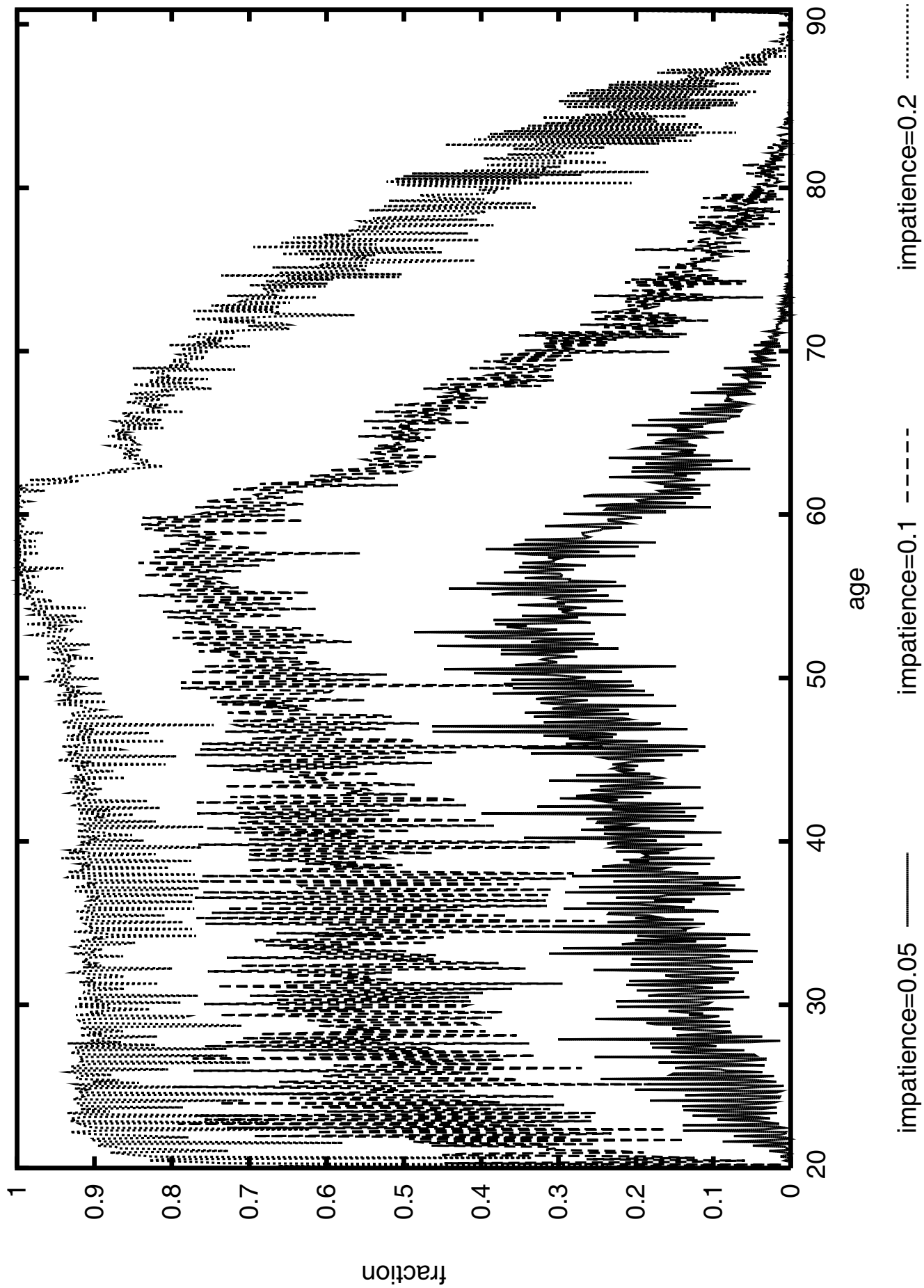Figure 17: Sophisticated shopper model: Credit card debt after payment, sample

Figure 18: Sophisticated shopper model: Fraction of households with liquid assets > 500 USD which roll over credit card debt, sample average
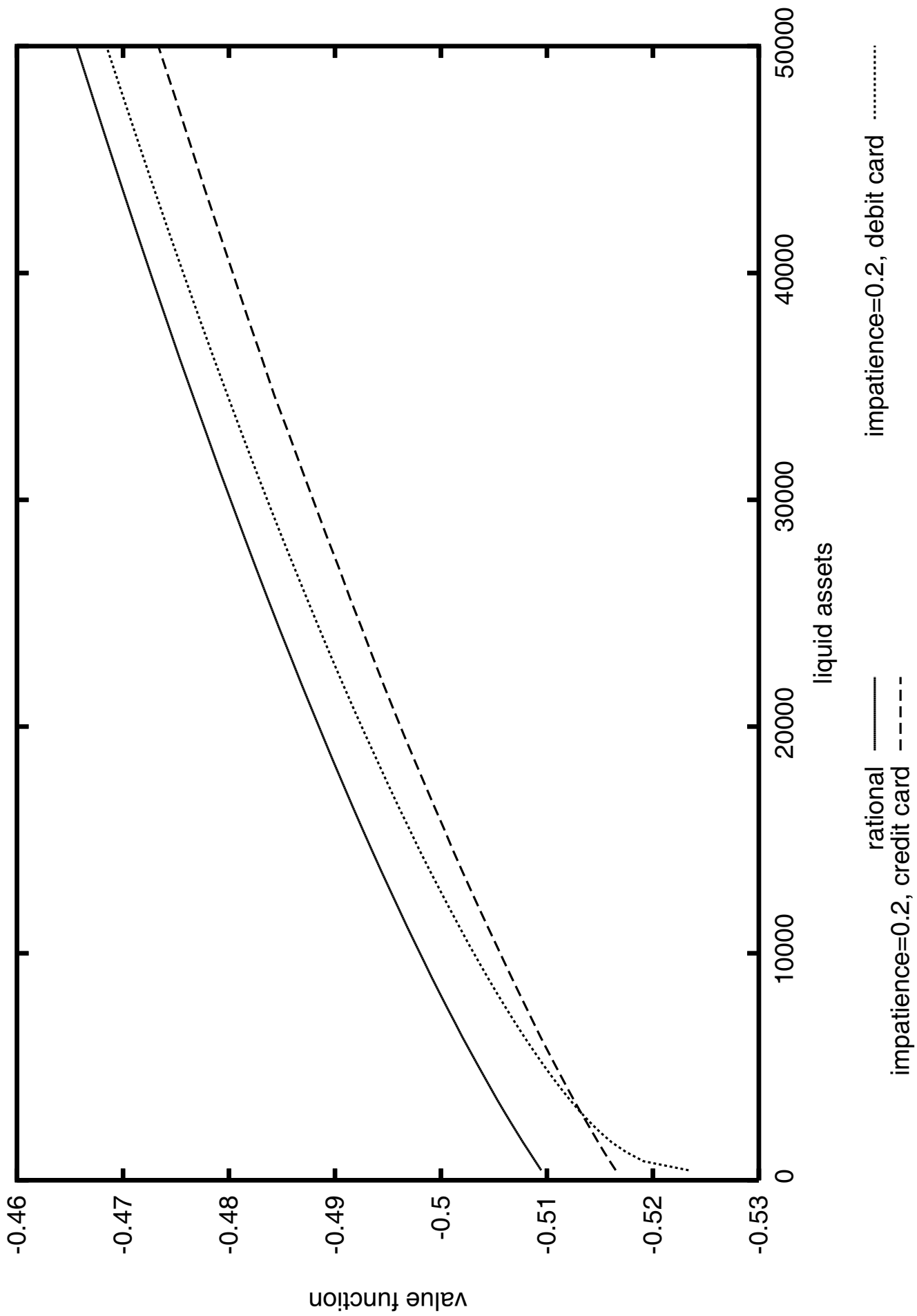
impatience=0.05 ——— impatience=0.1 — — — impatience=0.2 ·········

Figure 19: Sophisticated shopper model: Value functions at beginning of life, with $B = 0$

Legend:
- rational ———
- impatience=0.2, credit card – – – –
- impatience=0.2, debit card ········

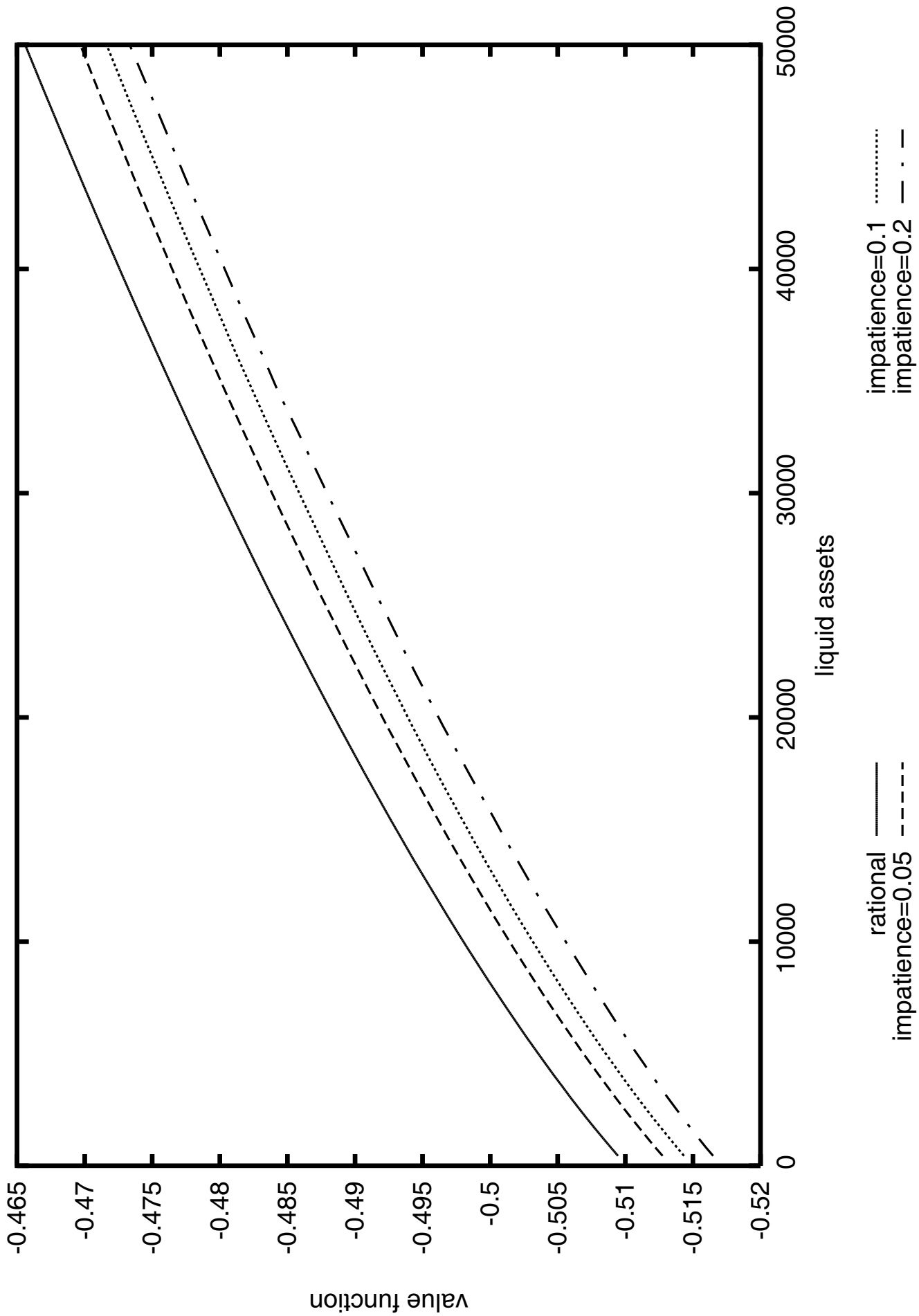Axis labels: liquid assets (x-axis), value function (y-axis)

Figure 20: Sophisticated shopper model: Value functions at beginning of life, with $B = 0$