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Abstract

This paper presents a theoretical study of the effects of globalization on risk sharing and welfare. We model globalization as a gradual and exogenous increase in the fraction of goods that are tradable. In the absence of frictions, globalization opens new goods markets and raises welfare. We assume, however, that countries cannot commit to pay their debts. Unlike the previous literature, and motivated by changes in the institutional setup of emerging-market borrowing, we also assume that countries cannot discriminate between domestic and foreign creditors when paying their debts. Although globalization still opens new goods markets, we find that it can also open or close some asset markets. The net effect on risk sharing and welfare of this process of creation and destruction of markets might be either positive or negative depending on a variety of factors that the theory highlights.

Keywords: globalization, risk sharing, sovereign risk, domestic markets, international markets.

JEL Classification: F34, F36, G15.

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This paper presents a theoretical study of the effects of globalization on risk sharing and welfare. We model globalization as a gradual and exogenous increase in the fraction of goods that are tradable. In the absence of frictions, globalization opens new goods markets and raises welfare. We assume, however, that countries cannot commit to pay their debts. Unlike the previous literature, and motivated by changes in the institutional setup of emerging-market borrowing, we also assume that countries cannot discriminate between domestic and foreign creditors when paying their debts. Although globalization still opens new goods markets, we find that it can also open or close some asset markets. The net effect on risk sharing and welfare of this process of creation and destruction of markets might be either positive or negative depending on a variety of factors that the theory highlights.^{1,2}

We study a world with two regions. The basic setup also has two periods, youth and old age, although sometimes we re-interpret it as a many-period model with an overlapping-generations structure. During youth, all individuals have identical preferences and own an ex-ante identical project that delivers output during old age. The return to this project is random and leads to ex-post differences in production bundles during old age. This provides a role for markets to help individuals pool or share production risks. Goods markets open during old age and allow individuals to trade commodities or goods, while asset markets open during youth and allow individuals to trade promises or assets. Naturally, asset markets can only open if governments enforce during old age the trades agreed upon during youth. We assume that governments choose enforcement policy so as to maximize the utility of the average or representative individual of their region. As usual, this leads governments to prefer different enforcement policies over time. During youth, governments would like to commit to enforce all payments during old age and allow domestic residents to reap all the gains from trade. But during old age, governments prefer not to enforce payments to foreign residents because they lower domestic consumption and welfare. This time-inconsistency problem is also known as sovereign risk in international economics.

There are two polar cases that deliver well-known results in this setup. The first one is the perfect commitment model. If governments can credibly commit during youth to enforce all payments

¹Whether globalization improves or worsens risk sharing is an old question in international economics. Newbery and Stiglitz (1984) provided a famous example in which asset markets are missing and globalization can reduce risk sharing and welfare. However, Dixit (1987, 1989a, and 1989b) showed that this cannot happen if the absence of asset markets, instead of simply assumed, is endogenously derived by introducing private information. Here we endogenize the absence of asset markets by introducing sovereign risk. We find that the result of Newbery and Stiglitz that globalization can reduce risk sharing and welfare still holds. Moreover, our result is even stronger since we also show that globalization can destroy asset markets. See also Levchenko (2005) and Rappoport (2005) for alternative approaches to this question.

²On the empirical side, there is substantial evidence that consumption volatility in emerging markets has increased (relative to output volatility) during the last decades, and that much of this increase can be attributed to globalization. (See Kose, Prasad, and Terrones (2003a and 2003b) and Prasad, Rogoff, Wei, and Kose (2003)). There is no hard evidence that we know of about consumption volatility at the individual level.

during old age, they will always choose to do so. In this case, asset markets are always open and there is perfect domestic sharing of all goods and perfect international sharing of tradable goods. Globalization is welfare-improving because it increases the fraction of goods that can be shared internationally. The other polar case is the perfect discrimination model without commitment. If governments choose during old age which payments to enforce, they will choose a discriminatory policy that enforces payments between domestic residents but does not enforce payments from domestic residents to foreign ones.³ In this case, asset markets are also open but geographically segmented. As a result, there is perfect domestic sharing of all goods but only imperfect international sharing of tradable goods. Globalization is welfare-improving again because it increases the fraction of goods that are shared internationally, even if imperfectly.⁴

We think that these polar cases leave behind the most interesting effects of globalization, namely, those on the workings of asset markets. To show this, we study here a third polar case in which governments have neither commitment nor the ability to discriminate between domestic and foreign creditors when enforcing payments. In this situation, asset markets are never geographically segmented if open, but some asset markets might be closed. This is the result of governments facing a trade-off when deciding whether to enforce payments that is absent in the other two polar cases. On the one hand, enforcement increases payments from domestic to foreign residents that lower domestic consumption and welfare. On the other hand, enforcement increases payments between domestic residents that contribute to domestic sharing of goods and therefore raise welfare. This trade-off determines the states of nature in which governments choose to enforce payments during old age and, therefore, the set of assets that can be traded during youth. In states of nature in which asset markets are open, there is perfect domestic sharing of all goods and perfect international sharing of tradable goods. In states of nature in which asset markets are closed, there is not only imperfect international sharing of tradable goods but also imperfect domestic sharing of all goods.

This enforcement trade-off provides a theory of asset market incompleteness based on sovereign risk that we exploit to study the effects of globalization. The most novel aspect of our analysis is that we show how globalization affects the degree of asset market incompleteness. In particular, globalization can either lead to the opening or closure of asset markets depending on its effect on the relative importance of domestic and international payments. Naturally, the opening of markets improves domestic sharing of all goods and international sharing of tradable goods, while

³Why would governments want to enforce payments between domestic residents? In equilibrium, payments are made by individuals that have a low marginal utility of consumption to individuals that have a high marginal utility of consumption and therefore raise the average utility of the region.

⁴International sharing of tradable goods is imperfect since it only takes place in goods markets. During old age, regions sell those tradable goods that are relatively abundant in their bundle and buy those that are relatively scarce.

the closure of markets does the opposite. Our theory also incorporates other, more standard, effects of globalization on risk sharing that also take place in models where the degree of asset market incompleteness is exogenous. In those states of nature in which asset markets are open, globalization leads to perfect international sharing of newly traded goods without affecting domestic sharing of goods. In those states of nature in which asset markets are closed, globalization still improves international sharing of tradable goods but, unlike in the other polar cases, globalization might now also improve or worsen domestic sharing of goods.

This paper is directly related to an extensive literature on sovereign risk that developed in response to the debt crises of the early 1980's in emerging markets. Without exception, this literature adopted the polar case of perfect discrimination.⁵ This choice was justified because this polar case provides a reasonably realistic description of the institutional setup of emerging markets in the late 1970's and 1980's. This was a period in which governments borrowed almost exclusively from foreign banks using syndicated loans, while the private sector was largely shut out from international financial markets. This institutional setup clearly facilitates ex-post discrimination, as governments can choose not to pay foreign banks without interfering with domestic asset trade.

But the institutional setup of emerging-market borrowing has changed dramatically in the 1990's and 2000's. Governments now borrow from abroad by selling bonds which are traded in increasingly deep secondary markets, while capital account liberalization now permits the private sector to access international financial markets directly or through an increasing variety of financial intermediaries. These changes have made it much more difficult for governments to discriminate ex-post.^{6,7} As a result, while the existing literature on sovereign risk is more relevant to understand

⁵See Eaton and Gersovitz (1981), Grossman and van Huyck (1988), Bulow and Rogoff (1989a and 1989b), Fernández and Rosenthal (1990), Atkeson (1991), Cole, Dow and English (1995), Cole and Kehoe (1997), Kletzer and Wright (2000), Kehoe and Perri (2002a), Wright (2002), and Amador (2003). See Eaton and Fernández (1995) for an excellent survey. All these papers assume there is perfect domestic risk sharing (i.e. each region contains a representative consumer) and ask when governments enforce international payments. Since the answer is 'never' in the two-period setup, these papers focus on reputational equilibria of the many-period version of the model.

⁶In an environment characterized by opaque financial intermediaries and deep secondary markets, governments' ability to discriminate between domestic and foreign holders of domestic debt is seriously limited. Governments typically do not know the nationality of the clients of banks, mutual funds and other financial intermediaries that hold domestic debt. And even if they knew, they might still not be able to control how these intermediaries distribute the losses from not enforcing payments among its domestic and foreign clients. Moreover, even in those cases in which asset trade is not intermediated, foreign creditors could still get repaid indirectly by selling their assets to domestic residents in secondary markets.

⁷In fact, governments might not even know how much domestic debt is held by each financial intermediary. For the case of public debt, the most recent issue of the IMF's Global Financial Stability Report (April 2006, pp. 95-96) literally says: "It is difficult to obtain complete data on the composition of investors in sovereign bonds. Unlike bilateral, multilateral, or bank loans, neither issuers nor other data gatherers publish comprehensive decompositions of commercial investors in emerging market sovereign debt. Some sovereigns have needed and compiled this information sporadically, for example, in the context of voluntary debt swaps or distressed debt restructurings, but the full composition of commercial investors in emerging markets debt is rarely known." The report goes on explaining the difficulties of obtaining this information and therefore justifying that "...only a handful of countries could provide detailed information on investor composition." Although the report focuses on public debt, it seems reasonable to assume that governments have even less information on the financial intermediaries that hold private debt.

emerging-market borrowing during the 1970's and 1980's, we believe that the results presented in this paper are more relevant to understand emerging-market borrowing in the 1990's and 2000's.

The rest of the paper is organized as follows. Section one presents the basic setup. Section two introduces sovereign risk. Section three studies the effects of globalization. Section four examines the role of externalities. Section five concludes. There are also three appendices that explain some technical aspects of the solution method, and extend the theory to the case in which governments can set borrowing limits and issue public debt.

1 A benchmark model of risk sharing

We consider a world in which all individuals are ex-ante identical since they all have the same preferences over different goods and they all have access to an ex-ante identical project. The return to this project is random and this generates ex-post differences in the quantity and types of goods produced by the different individuals. This creates a role for markets that can help individuals pool or share risks. In this section, we examine a situation in which these markets work well.

1.1 Preferences and technology

The world economy contains two regions: Home and Foreign, indexed by $j \in \{H, F\}$. Both regions have identical population size, normalized to 1. Let I^W be the set of inhabitants of this world, indexed by i , and let I^H and I^F be the sets of Home and Foreign residents, respectively. Naturally, $I^H \cup I^F = I^W$ and $I^H \cap I^F = \emptyset$. Let $j(i)$ denote the region where individual i resides. The world and its inhabitants last two periods, which we refer to as youth and old age. There is no uncertainty about youth, but there is uncertainty regarding old age. Let S be the set of all possible states of nature during old age. This set includes all the relevant aspects of the world economy that are not known during youth. We assume that, once realized, all individuals observe the state of nature. We denote by π_s the probability at youth of state $s \in S$ occurring during old age.⁸

There is a continuum of goods, indexed by $z \in [0, 1]$. A fraction τ of these goods can be transported between regions at negligible cost. We refer to these goods as “tradable.” The rest of the goods cannot be transported across regions and we refer to them as “nontradable.” The goods are indexed so that tradable goods correspond to low indices, i.e. $z \in [0, \tau]$, and nontradable goods correspond to high indices, i.e. $z \in (\tau, 1]$. When considering two alternative specifications, we shall say that the world is more globalized the higher τ is.

⁸With some abuse of language, we shall refer to π_s as the probability of state s even though for continuous state-spaces we are really referring to the probability density function.

Utility is derived only from old age consumption, and individuals are expected-utility maximizers. Let $c_{is}(z)$ be the quantity of good z consumed by individual i in state s . The objective function of individual i during old age is assumed to take the popular logarithmic form, i.e.

$$u_{is} = \int_0^1 \ln c_{is}(z) \cdot dz \quad \text{for all } s \in S \text{ and } i \in I^W, \quad (1)$$

while his/her objective function during youth is given by

$$U_i = \int_{s \in S} \pi_s \cdot u_{is} \quad \text{for all } i \in I^W. \quad (2)$$

A standard feature of dynamic decision problems is that the objective function of agents (individuals or governments) varies over time, as the state of nature is revealed. This gives rise to a standard time-inconsistency problem that plays a central role in this paper.

During youth, individuals build a project located in their own region. Projects deliver a bundle of goods during old age. We refer to this bundle as the production of the project of individual i or, for short, as the production of individual i . Let $y_{is}(z)$ be the production of good z by individual i in state s . To simplify notation, let $y_s^j(z) \equiv \int_{i \in I^j} y_{is}(z)$ for $j \in \{H, F\}$ be the regional average productions of good z in state s , while $y_s^W(z) \equiv \frac{1}{2} \cdot (y_s^H(z) + y_s^F(z))$ be the corresponding world average.

There is full symmetry between and within regions. First, if there exists a state s with $\pi_s = \pi$ and given sets of productions in Home $\{y_{is}(\cdot)\}_{i \in I^H} = \bar{Y}$ and in Foreign $\{y_{is}(\cdot)\}_{i \in I^F} = \underline{Y}$, then there exists a corresponding state s' with $\pi_{s'} = \pi$ and sets of productions in Home $\{y_{is'}(\cdot)\}_{i \in I^H} = \underline{Y}$ and in Foreign $\{y_{is'}(\cdot)\}_{i \in I^F} = \bar{Y}$. Second, for every pair of individuals i and i' residing in the same region, if there exists a state s with $\pi_s = \pi$ and given sets of productions in Home and Foreign in which $y_{is}(\cdot) = \bar{y}(\cdot)$, then there also exists a corresponding state s' with $\pi_{s'} = \pi$ and the same sets of productions in Home and Foreign in which $y_{i's'}(\cdot) = \bar{y}(\cdot)$. These assumptions imply that ex-ante productions are the same in both regions and for all individuals within a region. Of course, this need not be the case ex-post and this is why there are gains from trade.

In this world, markets allow individuals to transfer consumption across goods and across states of nature. Some trades might involve the exchange of goods during old age, while some others might involve the exchange of promises during youth to deliver goods during old age. We refer to the former as “goods” trade and the latter as “asset” trade. We start by considering the benchmark case of complete markets. As usual, by “complete” it is meant that the existing set of markets allows all pairs of individuals to carry out all mutually desired trades. There are many possible

ways of organizing markets that ensure that all valuable trades are carried out. For convenience, we consider a sequential formulation of markets: during youth there are asset (or forward) markets where individuals can trade promises to deliver one unit of the numeraire good in state s in any of the two regions; and during old age there are goods (or spot) markets where individuals can exchange the different goods. Intuitively, asset markets are used to distribute income across states of nature, while goods markets are used to distribute consumption across goods.⁹

As usual, it is useful to construct the competitive equilibrium recursively, going backwards in time. During old age, individuals take their income as given and choose how to distribute their consumption across goods so as to maximize utility. During youth, individuals choose how to distribute their income across states of nature so as to maximize their expected utility. We study each of these choices in turn.

1.2 Goods markets

During old age, the state of nature is known and only goods markets are open. Let $p_s^j(z)$ be the price of one unit of good z in state s in region j . Let y_{is} be the value of the production of individual i in state s , i.e. $y_{is} \equiv \int_0^1 p_s^{j(i)}(z) \cdot y_{is}(z) \cdot dz$; and let x_{is} be the value of the assets held by individual i in state s . To simplify notation, let $y_s^j \equiv \int_{i \in I^j} y_{is}$ for $j \in \{H, F\}$ be the regional average values of production in state s , while $y_s^W \equiv \frac{1}{2} \cdot (y_s^H + y_s^F)$ is the corresponding world average. Also, let $x_s^j \equiv \int_{i \in I^j} x_{is}$ for $j \in \{H, F\}$ be the regional average values of assets in state s . We need not define the world average value of assets since assets are nothing but promises and the average value of these promises must be zero. With this notation, we can write the budget constraint of old individuals as follows:

$$\int_0^1 p_s^{j(i)}(z) \cdot c_{is}(z) \cdot dz \leq y_{is} + x_{is} \quad \text{for all } s \in S \text{ and } i \in I^W. \quad (3)$$

The budget constraint states that the value of consumption cannot exceed income, which in turn consists of the value of production plus the value of assets held.

⁹This sequential formulation of markets is sometimes referred to as a Radner equilibrium. The classic Arrow-Debreu equilibrium assumes instead that there is a set of forward markets during youth where individuals can trade promises to deliver one unit of *any* good in state s in any of the two regions. The Arrow-Debreu equilibrium minimizes the use of spot markets, while the sequential or Radner equilibrium minimizes the use of forward markets. If all markets work well, both equilibria deliver the same allocations. This equivalence breaks down however once we introduce sovereign risk in the next section. This type of risk negatively affects the functioning of forward markets, without affecting the functioning of spot markets. This provides incentives to minimize the use of forward markets and justifies our choice of equilibrium.

For goods markets to clear, we must impose these conditions:

$$\frac{1}{2} \cdot \int_{i \in I^W} c_{is}(z) = y_s^W(z) \quad \text{and} \quad p_s^H(z) = p_s^F(z) \equiv p_s^W(z) \quad \text{for all } z \in [0, \tau] \text{ and } s \in S, \quad (4)$$

$$\int_{i \in I^j} c_{is}(z) = y_s^j(z) \quad \text{for all } z \in (\tau, 1], s \in S, \text{ and } j \in \{H, F\}. \quad (5)$$

Equations (4) and (5) state that supplies of the different goods must equal their demands. For those goods that are tradable, international arbitrage ensures that the prices of a given good delivered at Home and Foreign are equalized. This international arbitrage does not operate for nontradable goods.

A competitive equilibrium during old age consists of a set of goods prices and quantities such that individuals maximize their utility –Equation (1)– subject to their budget constraint –Equation (3)– and goods markets clear –Equations (4) and (5). Note that the state variables of the old-age problem are individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{is}\}_{i \in I^W}$.

We show that the equilibrium exists and is unique by construction. It follows from individual maximization that consumption demands are given by $c_{is}(z) = \frac{y_{is} + x_{is}}{p_s^{j(i)}(z)}$ for all $i \in I^W$ and $z \in [0, 1]$. Substituting these demands into the market clearing conditions in Equations (4) and (5) we find that prices are given by $p_s^W(z) = \frac{y_s^W}{y_s^W(z)}$ for $z \in [0, \tau]$ and $p_s^j(z) = \frac{y_s^j + x_s^j}{y_s^j(z)}$ for $z \in (\tau, 1]$ and $j \in \{H, F\}$. Therefore, equilibrium consumption allocations are given by:

$$c_{is}(z) = \begin{cases} \frac{y_{is} + x_{is}}{y_s^W} \cdot y_s^W(z) & \text{if } z \in [0, \tau] \\ \frac{y_{is} + x_{is}}{\frac{y_s^j + x_s^j}{y_s^j(z)}} \cdot y_s^j(z) & \text{if } z \in (\tau, 1] \end{cases} \quad \text{for all } s \in S, \text{ and } i \in I^W. \quad (6)$$

Equation (6) shows how Home and Foreign residents distribute their consumption across the different goods. In particular, individuals share goods in proportions that are directly related to their incomes. For tradable goods, world production is shared according to world relative incomes. For nontradable goods, regional production is shared according to regional relative incomes. We can find individual incomes as a share of world income as follows:¹⁰

$$\frac{y_{is} + x_{is}}{y_s^W} = \int_0^\tau \frac{y_{is}(z)}{y_s^W(z)} \cdot dz + \frac{y_s^{j(i)} + x_s^{j(i)}}{y_s^W} \cdot \int_\tau^1 \frac{y_{is}(z)}{y_s^{j(i)}(z)} \cdot dz + \frac{x_{is}}{y_s^W} \quad \text{for all } s \in S \text{ and } i \in I^W, \quad (7)$$

and, integrating (7) over residents of each region, we also find regional incomes as a share of world

¹⁰To see this, substitute prices into the definition of y_{is} .

income:

$$\frac{y_s^j + x_s^j}{y_s^W} = \frac{1}{\tau} \cdot \left(\int_0^\tau \frac{y_s^j(z)}{y_s^W(z)} \cdot dz + \frac{x_s^j}{y_s^W} \right) \text{ for all } s \in S \text{ and } j \in \{H, F\}. \quad (8)$$

A region's income increases with its relative production of tradables and with its assets.¹¹

Equations (6), (7) and (8) provide a full description of the consumption allocation as a function of the state variables of this problem, i.e. individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{is}\}_{i \in I^W}$. Individual productions are determined by nature, but asset holdings are determined by trade during youth and we turn to this now.

1.3 Asset markets

During youth, only asset markets are open. Let q_s be the price of an asset that promises to deliver one unit of the numeraire in state s , and let x_{is} be the number of such assets held by individual i . Therefore, the budget sets of the young are characterized by:

$$\int_{s \in S} q_s \cdot x_{is} \leq 0 \text{ for all } i \in I^W, \quad (9)$$

$$x_{is} \geq -y_{is} \text{ for all } s \in S \text{ and } i \in I^W. \quad (10)$$

Equation (9) is the budget constraint and says that purchases of assets must be financed by corresponding sales of other assets, while Equation (10) is a solvency constraint that says that individuals can only issue promises that are backed by their own production. Naturally, during youth asset markets must clear:

$$\int_{i \in I^W} x_{is} = 0 \text{ for all } s \in S. \quad (11)$$

Equation (11) states that there is a zero net supply of all assets or promises.

A competitive equilibrium during youth consists of a set of asset prices and quantities such that individuals maximize expected utility –Equation (2)– subject to their budget and solvency constraints –Equations (9) and (10)– and asset markets clear –Equation (11). When maximizing their utility, individuals take as given how their individual consumption during old age depends on their individual asset holdings.

We show again that this equilibrium exists and is unique by construction. Note that log preferences imply that a young individual i will choose asset holdings $\{x_{is}\}_{s \in S}$ such that $y_{is} + x_{is} = \lambda_i^{-1} \frac{\pi_s}{q_s}$ where λ_i is the Lagrange multiplier associated with individual i 's budget constraint. Since all in-

¹¹Note that assets increase income more than one-to-one if $\tau < 1$. The reason is that assets shift purchasing power from foreign to domestic residents. This raises the demand for domestic nontradable goods relative to foreign ones. And this increases the value of domestic production relative to foreign. This additional effect of asset holdings on incomes is known as the “transfer problem.”

dividuals are ex-ante identical (preferences and technology) and have access to the same set of markets, they all have the same multiplier $\lambda_i \equiv \lambda$ for all $i \in I^W$. Integrating this expression over $i \in I^W$ and using the market clearing conditions in Equation (11) we find $\lambda^{-1} = \frac{q_s}{\pi_s} y_s^W$. As a result, we have:

$$x_{is} = y_s^W - y_{is} \quad \text{for all } s \in S \text{ and } i \in I^W. \quad (12)$$

Equation (12) provides the equilibrium asset holdings, i.e. $\{x_{is}\}_{i \in I^W}$. During old age income is always equally distributed within and between regions.

We have now a full description of the complete-markets equilibrium. For a given set of individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{is}\}_{i \in I^W}$, Equations (6), (7) and (8) describe the consumption allocation that come out of goods markets during old age. For a given set of individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$, Equation (12) describes the asset holdings that come out from asset markets during youth. We describe the welfare properties of this equilibrium next.

1.4 Domestic and international risk sharing with complete markets

Markets allow individuals to share production risks both within and between regions. We can provide a sharper description of how this happens by decomposing production, $y_{is}(z)$, as follows:

$$y_{is}(z) = \phi_{is}(z) \cdot \phi_s^{j(i)}(z) \cdot y_s^W(z) \quad \text{for all } z \in [0, 1], s \in S, \text{ and } i \in I^W, \quad (13)$$

where $\phi_{is}(z) \equiv \frac{y_{is}(z)}{y_s^{j(i)}(z)}$ and $\phi_s^{j(i)}(z) \equiv \frac{y_s^{j(i)}(z)}{y_s^W(z)}$ for $z \in [0, 1]$, $s \in S$, and $i \in I^W$ are the individual and regional components of production respectively. By construction, these components have a constant mean, i.e. $\int_{i \in I^j} \phi_{is}(z) = 1$ and $\frac{1}{2} \cdot (\phi_s^H(z) + \phi_s^F(z)) = 1$ for all $z \in [0, 1]$ and $s \in S$. We will refer to a (mean-preserving) spread in $\phi_{is}(z)$ and $\phi_s^{j(i)}(z)$ as an increase in individual and regional risk for good z respectively.

With these definitions at hand, we can use Equations (6) and (12) to find equilibrium consumption allocations:

$$c_{is}(z) = \begin{cases} y_s^W(z) & \text{if } z \in [0, \tau] \\ \phi_s^{j(i)}(z) \cdot y_s^W(z) & \text{if } z \in (\tau, 1] \end{cases} \quad \text{for all } s \in S, \text{ and } i \in I^W, \quad (14)$$

and plugging these consumption allocations in Equation (2), we obtain ‘ex-ante’ utility:¹²

$$U = \int_0^1 \left(\int_{s \in S} \pi_s \cdot \ln y_s^W(z) \right) \cdot dz + \int_\tau^1 \left(\int_{s \in S} \pi_s \cdot \ln \phi_s^{j(i)}(z) \right) \cdot dz \quad \text{for any } i \in I^W. \quad (15)$$

¹² All individuals enjoy the same ex-ante utility because of our symmetry assumptions.

Equations (14) and (15) provide a full description of consumption and welfare. There is perfect domestic sharing of all goods, but only perfect international sharing of tradable ones. Naturally, this is because it is not technologically possible to share nontradable goods across regions. Markets work well, but they cannot overcome technological constraints. In fact, it is straightforward to show that the complete-markets consumption allocations are ‘ex-ante’ Pareto efficient and strictly Pareto dominate all other symmetric consumption allocations.¹³

Not surprisingly, welfare increases with world production of all goods $y_s^W(z)$. Moreover, Jensen’s inequality shows that a mean-preserving spread in world production lowers welfare. Higher volatility in world production cannot be diversified away and must lead one-to-one to higher volatility in individual consumption. Since individuals are risk averse, they suffer from this.

A feature of the complete-markets equilibrium is that welfare is not affected by an increase in individual risk.¹⁴ Since there is perfect domestic sharing of all goods, the ‘ex-post’ distribution of production among individuals of the same region has no effects on individual consumption or welfare.

Welfare is not affected by an increase in regional risk on tradable goods either, but welfare is affected by an increase in regional risk on nontradable goods.¹⁵ Since there is perfect international sharing of tradable goods, the ‘ex-post’ distribution of tradable production between regions has no effects on consumption or welfare. Since transport costs preclude international sharing of nontradable goods, higher volatility of the regional component of their production must lead one-to-one to higher volatility in the consumption of these goods and this lowers ex-ante utility.

This discussion provides a short but comprehensive description of the complete-markets equilibrium. Goods and asset markets combine to allow individuals to share production risks. Given technological constraints to trade, this is an ideal world. But this is too rosy a picture of asset markets. There is a fundamental difference in the nature of goods and asset markets that the complete-markets model ignores. In goods markets individuals trade commodities for commodities, while in asset markets individuals trade promises for promises. Unlike commodities, promises are only valuable if individuals can commit to fulfill them later. We have assumed this implicitly in the previous analysis. In the next section we relax this assumption.

¹³Since we shall focus exclusively on symmetric consumption allocations throughout the paper, we refer to those in Equations (14) as “the” Pareto efficient consumption allocations, even though we recognize that there exist asymmetric allocations that are also Pareto efficient.

¹⁴To see this, simply note that the individual component of production is absent in Equations (14) and (15).

¹⁵To see the former, simply note that the regional component of tradable production is absent in Equations (14) and (15). To see the latter, use Jensen’s inequality to show that a mean-preserving spread in the nontradable component of regional production lowers ex-ante utility.

2 Sovereign risk

The feasibility of the complete-markets consumption allocation rests on society's ability to solve a standard time-inconsistency problem. Even though individuals would like to commit ex-ante to pay their debts, ex-post they have incentives not to do so and enjoy a higher level of consumption. Either old individuals are not maximizing their utility or their true utility cannot be fully represented by Equation (1). The standard way to think about the complete-markets model is as describing a world in which there is also a government that imposes an unbearable utility cost to those individuals that fail to pay their debts. In this situation, Equation (1) can be understood as representing utility only conditional on paying debts. The (very low) level of utility that results from not paying debts can be disregarded since it is never chosen in equilibrium.

Although recognizing the role that governments play in sustaining asset markets is a small step towards greater realism, it begs the question of why governments would always want to enforce payments. To the extent that governments care more about domestic residents than about foreign ones, they are subject to the same type of time-inconsistency problem that individuals are. Even though governments would like to commit ex-ante to enforce payments by domestic residents, ex-post they may have incentives to deviate to allow domestic residents to enjoy a higher level of consumption. This time-inconsistency problem of governments is usually referred to as sovereign risk, and the goal of this section is to analyze how it affects risk sharing and welfare.

2.1 The model with sovereign risk

We consider again the world economy described in section 1.1, but now we explicitly model governments and their role as enforcers of private contracts. There are two governments, a Home government which can enforce payments by residents of Home, and a Foreign government which can enforce payments by residents of Foreign. Ex-post, an individual only pays if his/her government forces him/her to pay. Governments only care about the utility of the residents of their region. In particular, they maximize the average utility of domestic residents, i.e. $v_s^j = \int_{i \in I^j} u_{is}$ for all $s \in S$ during old age and $V^j = \int_{i \in I^j} \pi_s \cdot U_i$ during youth for $j \in \{H, F\}$.

If governments could credibly commit to enforce all payments before the state of nature is revealed, they would always choose to do so and all asset markets would be open. This is the extreme or polar case of perfect commitment.¹⁶ If governments have some choice over enforcement after the state of nature is revealed, they are tempted not to enforce payments to foreigners when

¹⁶With perfect commitment, the equilibrium would be identical to the complete-markets model and would therefore be fully described by Equations (6), (7), (8) and (12).

these are high enough. We ensure this temptation is always present by moving to the other extreme and assuming governments cannot commit to enforce at all:

Assumption 1. *LACK OF COMMITMENT: Governments simultaneously choose enforcement during old age after the state of nature has been revealed and before markets open.*

The effects of this lack of commitment depend crucially on the degree to which governments can discriminate among debtors when enforcing payments. Assume, for instance, that governments choose ex-post which particular payments to enforce so that they can fully discriminate between debtors when enforcing payments. This is the polar case of perfect discrimination without commitment. In the context of our model, this would imply that governments would never enforce any payment from a domestic resident to a foreign one. Asset markets would be geographically segmented and there would be no trade in assets between residents of different regions.¹⁷

If discrimination is less than perfect, lack of enforcement affects both domestic and international transactions and this creates new and interesting interactions between domestic and international risk sharing. We take a first step towards analyzing these interactions by going to the other polar case and assume that governments cannot discriminate among debtors. In particular, we assume:

Assumption 2. *NON-DISCRIMINATORY ENFORCEMENT: Governments choose whether to enforce all payments or none.*

This assumption is crucial for the results of this paper. Once again, we construct next the competitive equilibrium recursively going backwards in time.

2.2 Goods markets and enforcement

During old age, the state of nature is revealed, then governments enforce payments, and then goods markets open. Define $x_{j,is}$ as the assets held by individual i that pay in state s issued by residents of region j . Since governments now decide whether to enforce payments by their own residents independently, it is not sufficient to know the overall asset holdings of an individual, but also the residence of the issuer.

Unlike section 1.2, the budget constraints of old individuals must now reflect the fact that assets are worthless if there is no enforcement. That is, we must replace Equation (3) by the following

¹⁷With perfect discrimination without commitment, there would still be international trade in goods since such trade is arms' length and, thus, not affected by sovereign risk. In addition, domestic asset trade would still take place since, in equilibrium, this trade would result in payments from residents with low marginal utility to residents with high marginal utility. Enforcing these payments would raise the average utility of the region. Therefore, the equilibrium with perfect discrimination and without commitment is fully described by Equations (6), (7) and (8) with asset holdings $x_{is} = y_s^{j(i)} - y_{is}$ for all $s \in S$ and $i \in I^W$.

one:

$$\int_0^1 p_s^{j(i)}(z) \cdot c_{is}(z) \cdot dz \leq y_{is} + e_s^H \cdot x_{H,is} + e_s^F \cdot x_{F,is} \quad \text{for all } s \in S \text{ and } i \in I^W. \quad (16)$$

where e_s^j is an indicator variable that takes value one if government j enforces and zero otherwise.

Governments simultaneously choose whether to enforce payments or not so as to maximize the average utility of domestic residents. When considering their enforcement choice, governments take the actions of the other government as given. That is, enforcement decisions are the Nash equilibrium of a game between governments. Their best responses therefore satisfy:

$$e_s^j = \begin{cases} 1 & \text{if } v_s^j(\text{enforce}) > v_s^j(\text{not enforce}) \\ 0 & \text{if } v_s^j(\text{enforce}) < v_s^j(\text{not enforce}) \end{cases} \quad \text{for all } s \in S \text{ and } j \in \{H, F\}. \quad (17)$$

Note that when $v_s^j(\text{enforce}) = v_s^j(\text{not enforce})$, the government is indifferent between enforcing or not and both $e_s^j = 1$ and $e_s^j = 0$ are best responses. We define $E^j \subseteq S$ as the set of states in which government j decides to enforce payments for $j \in \{H, F\}$.

A competitive equilibrium during old age consists of a set of goods prices and quantities such that individuals maximize their utility –Equation (1)– subject to their budget constraint –Equation (16)–, governments enforce so as to maximize average utility of their region –Equation (17)– and goods markets clear –Equations (4) and (5). Once again, the state variables of this problem are individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{j,is}\}_{j \in \{H, F\}, i \in I^W}$.

To compute this equilibrium, replace

$$x_{is} = e_s^H \cdot x_{H,is} + e_s^F \cdot x_{F,is} \quad \text{for all } s \in S \text{ and } i \in I^W \quad (18)$$

in Equations (6), (7) and (8) to find the equilibrium consumption allocations as functions of enforcement decisions. Then, substitute these consumption allocations into the best responses of governments to find the equilibrium enforcement decisions as a function of the state variables of this problem, i.e. individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{j,is}\}_{j \in \{H, F\}, i \in I^W}$. Once again, asset holdings are determined during youth as we show next.

2.3 Asset markets

During youth, individuals trade in asset markets. The individual maximization problems are as in section 1.3, except that now agents can only sell securities which pay in states in which their

government enforces payments. Then, the budget sets in Equations (9) and (10) are replaced by

$$\int_{s \in S} (q_s^H \cdot x_{H,is} + q_s^F \cdot x_{F,is}) \leq 0 \text{ for all } i \in I^W, \quad (19)$$

$$x_{j(i),is} \geq -\hat{y}_{is} \text{ and } x_{-j(i),is} \geq 0 \text{ for all } s \in S \text{ and } i \in I^W, \quad (20)$$

where \hat{y}_{is} is now pledgeable income, defined as

$$\hat{y}_{is} = \begin{cases} y_{is} & \text{if } s \in E^{j(i)} \\ 0 & \text{if } s \notin E^{j(i)} \end{cases} \text{ for all } i \in I^W. \quad (21)$$

Equation (19) is the budget constraint. Equations (20) and (21) define the solvency constraint. They say that individuals cannot pledge income in states in which their government does not enforce payments.¹⁸ They also say that individuals cannot issue assets that are enforced by the government of the other region. The market clearing conditions for asset markets are now given by

$$\int_{i \in I^W} x_{j,is} = 0 \text{ for all } s \in S \text{ and } j \in \{H, F\} \quad (22)$$

Equation (22) simply states that there is a zero net supply of each country's assets.

A competitive equilibrium during youth consists of a set of asset prices and quantities such that individuals maximize expected utility –Equation (2)– subject to their budget and solvency constraints –Equations (19), (20) and (21)– and asset markets clear –Equation (22). Naturally, when maximizing their utility, individuals take as given how their individual consumption during old age depends on their individual asset holdings.

We restrict the analysis to symmetric equilibria.¹⁹ Define a coarse partition of states of nature based on sets of productions in Home and Foreign as opposed to individual productions. Abusing notation, we refer to the set of states $\{s \in S : \{y_{is}(\cdot)\}_{i \in I^H} = \bar{Y} \text{ and } \{y_{is}(\cdot)\}_{i \in I^F} = \underline{Y}\}$ as a single “state” characterized by regional sets of productions (\bar{Y}, \underline{Y}) . Given our assumption of symmetry within regions, each such “state” is composed of a large number of equiprobable states, one for each way in which these regional sets of productions can be distributed among residents within

¹⁸For example, a Home resident might want to sell assets that pay in a state, say s , in which his/her production is high in order to purchase assets that pay in states in which his/her production is low. However, if state s the Home government does not enforce payments, $s \notin E^H$, this resident will not pay his/her debts when state s materializes. Knowing this ex-ante, other agents would not be willing to purchase any assets that pay in state s from this Home resident. Therefore, Home production in state s is not pledgeable. Similarly, no agent would be willing to purchase assets from Foreign residents that pay in states in which the Foreign government does not enforce payments.

¹⁹Without loss of generality (see Appendix A), we also impose the restriction that there be no two-way international trade in the same asset. That is, either $\int_{i \in I^H} x_{F,is}$ or $\int_{i \in I^F} x_{H,is}$ is zero for all $s \in S$.

each region. Given our assumption of symmetry between regions, each state s characterized by sets of productions (\bar{Y}, \underline{Y}) has a corresponding symmetric state s' with the same probability and characterized by sets of productions (\underline{Y}, \bar{Y}) . We say that an equilibrium is symmetric if enforcement sets can be defined over this coarser partition of states and $(\bar{Y}, \underline{Y}) \subset E^H$ if and only if $(\underline{Y}, \bar{Y}) \subset E^F$. Throughout we focus on symmetric equilibria. This restriction is not without loss of generality, since the model also has asymmetric equilibria. But it delivers a high payoff in terms of tractability since it implies that residents in both regions have the same budget constraint multipliers λ during youth and we can therefore analyze pairs of symmetric states independently.

Typically, there are many symmetric equilibria. To see this, consider a pair of symmetric states. If individuals expect enforcement in both regions, it is possible (but not necessary) that asset trade be such that both regions enforce and validate individuals' expectations. If individuals expect non-enforcement in both regions, then there is no asset trade and the individuals' expectations are always a consistent belief even if it cannot be verified in equilibrium. Obviously, expectations play an important role in this world. But we do not emphasize this feature in what follows. Instead, we focus exclusively on the best symmetric equilibrium and we refer to it as “the” sovereign risk equilibrium. This equilibrium arises when individuals have the most optimistic expectations about enforcement and the maximum number of asset markets are open.

Appendix A provides a detailed description of how we construct this equilibrium. Basically, we do this by checking, for each pair of states, if there exists an equilibrium in which both governments enforce payments. If it exists, we choose it. Otherwise, we check whether there is an equilibrium in which at least one region enforces. If it exists, we choose it. Otherwise, we conclude that there is no enforcement. We find that the equilibrium constructed in this way sometimes contains states in which there is enforcement in one region but not in the other and, as a result, $E^H \neq E^F$. To streamline the exposition, in the main text we focus only on the case in which, in all states, there is enforcement either in both regions or in neither and $E^H = E^F \equiv E$. This case generates the following simple and intuitive closed-form solutions for equilibrium asset holdings:

$$x_{is} = \begin{cases} y_s^W - y_{is} & \text{if } s \in E \\ 0 & \text{if } s \notin E \end{cases} \quad \text{for all } i \in I^W \quad (23)$$

That is, income is equally divided among all individuals in those states in which asset markets are open. Naturally, there is no asset trade in those states in which asset markets are closed.

We have now a complete description of the sovereign risk equilibrium. For a given set of individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$ and asset holdings $\{x_{j,is}\}_{j \in \{H,F\}, i \in I^W}$, Equations (6), (7), (8),

(17), and (18) describe the consumption allocation that come out of goods markets during old age and Equation (17) determines the enforcement set. For a given set of individual productions $\{y_{is}(\cdot)\}_{i \in I^W}$, Equation (23) describes the asset holdings that come out from asset markets during youth. We describe the welfare properties of this equilibrium next.

2.4 Domestic and international risk sharing with sovereign risk

Sovereign risk destroys some asset markets, and this reduces domestic and international risk sharing.

The equilibrium consumption allocations are now given by:

$$c_{is}(z) = \begin{cases} y_s^W(z) & \text{if } z \in [0, \tau] \\ \phi_s^{j(i)}(z) \cdot y_s^W(z) & \text{if } z \in (\tau, 1] \end{cases} \quad \text{for all } s \in E \text{ and } i \in I^W. \quad (24)$$

$$c_{is}(z) = \begin{cases} \phi_{is} \cdot \phi_s^{j(i)} \cdot y_s^W(z) & \text{if } z \in [0, \tau] \\ \phi_{is} \cdot \phi_s^{j(i)}(z) \cdot y_s^W(z) & \text{if } z \in (\tau, 1] \end{cases} \quad \text{for all } s \notin E \text{ and } i \in I^W \quad (25)$$

where $\phi_{is} \equiv \int_0^\tau \phi_{is}(z) \cdot \frac{\phi_s^{j(i)}(z)}{\phi_s^{j(i)}} \cdot dz + \int_\tau^1 \phi_{is}(z) \cdot dz$ and $\phi_s^j \equiv \frac{1}{\tau} \cdot \int_0^\tau \phi_s^j(z) \cdot dz$. To interpret these expressions, note that Equations (7) and (8) imply that:

$$\frac{y_{is} + x_{is}}{y_s^W} = \begin{cases} 1 & \text{if } s \in E \\ \phi_{is} \cdot \phi_s^{j(i)} & \text{if } s \notin E \end{cases} \quad \text{for all } i \in I^W \quad (26)$$

That is, ϕ_{is} and ϕ_s^j measure the individual and regional components of incomes when there is no enforcement. By construction, these components have a constant mean, i.e. $\int_{i \in I^j} \phi_{is} = 1$ and $\frac{1}{2} \cdot (\phi_s^H + \phi_s^F) = 1$ for all $s \notin E$. In those states in which asset markets are open there are no individual and regional components to incomes because asset trade ensures perfect sharing of income risk. But this is not possible in those states in which asset markets are closed. Plugging the consumption allocations in Equations (24) and (25) into Equation (2), we obtain ex-ante utility:

$$U = \int_0^1 \left(\int_{s \in S} \pi_s \cdot \ln y_s^W(z) \right) \cdot dz + \int_\tau^1 \left(\int_{s \in S} \pi_s \cdot \ln \phi_s^{j(i)}(z) \right) \cdot dz + \int_{s \notin E} \pi_s \cdot \tau \cdot \ln \phi_s^{j(i)} + \int_{s \notin E} \pi_s \cdot \ln \phi_{is} \quad \text{for any } i \in I^W. \quad (27)$$

Finally, it follows from Equation (17) that the enforcement set is given by:

$$E = \left\{ s \in S : - \int_{i \in I^R} \ln \phi_{is} \geq \tau \cdot \ln \phi_s^R \right\}, \quad (28)$$

where R is the rich region in the corresponding state.

Equations (24), (25), (27) and (28) provide a full description of consumption and welfare. Now there is imperfect domestic sharing of all goods and imperfect international sharing of tradable goods. This is because individuals are forced to choose consumption baskets worth no more than their production bundle in those states in which the corresponding asset market is closed. The sovereign-risk consumption allocations are therefore ‘ex-ante’ Pareto inefficient. This is shown in Equation (27) which differs from (15) by the third and fourth integrals. Jensen’s inequality shows that these two integrals are negative. The third integral reflects the welfare loss from not being able to perfectly share tradable goods between regions, while the fourth integral reflects the welfare loss from not being able to perfectly share all goods within regions.

The complete-markets equilibrium can now be re-interpreted as the special case of the sovereign-risk equilibrium in which the enforcement set contains all states of nature, i.e. $E = S$, and markets are complete. In general, however, the enforcement set is smaller than the set of all states, i.e. $E \subset S$, and markets are incomplete. The number of asset markets that are closed and therefore the inefficiency created by sovereign risk depends on individual and regional income risk. A mean preserving spread in ϕ_{is} in the rich region increases the loss in average utility that results from a breakdown in domestic payments, increasing government incentives to enforce and therefore the size of the enforcement set. A mean preserving spread in ϕ_s^j raises the gains in average utility that result from not paying debts to foreigners, reducing incentives to enforce and therefore the size of the enforcement set.²⁰

The sovereign-risk equilibrium shares some features with the complete-markets equilibrium. For instance, in both equilibria welfare increases with world production of any good but decreases with a mean-preserving spread in world production of any good. Also, in both equilibria welfare decreases with an increase in regional risk on nontradable goods. Moreover, the intuitions behind these results are exactly the same in both equilibria since neither world production nor the regional component of the production of nontradables affect the size of the enforcement set.

But the sovereign risk equilibrium differs from the complete-markets equilibrium in that welfare depends on both individual risk and regional risk on tradable goods. This dependence can be quite complex but can always be analyzed as the sum of two different effects. For a given enforcement set, higher volatility in individual and regional tradable production cannot be diversified away in those states in which asset markets are closed and must lead one-to-one to higher volatility in individual

²⁰One must be careful when studying the effects of individual and regional risk for a given good. It is possible that a mean-preserving spread in $\phi_{is}(z)$ benefits disproportionately poor individuals and reduces the enforcement set. Similarly, it is also possible that a mean-preserving spread in $\phi_s^j(z)$ benefits disproportionately the poor region and increases the enforcement set.

consumption in those states. This first effect of increases in individual and regional risk always lowers welfare. But higher volatility in individual and tradable production also affect the size of the enforcement set. An increase in individual risk tends to increase the enforcement set and this increases welfare. Therefore, the first and second effects tend to work against each other in the case of individual risk. An increase in regional risk for tradables tends to reduce the enforcement set and this lowers welfare. Therefore, the first and second effects tend to reinforce each other in the case of regional risk on tradable goods.

The sovereign-risk equilibrium provides a rich description of international trade in assets. Lack of commitment or trust destroys asset markets and constitutes an impediment to trade. Individuals cannot sell enough assets to finance the purchase of other assets that would protect them from the risks they face. Therefore, this is less than an ideal world given technological constraints to trade. Sovereign risk generates interesting interactions between domestic and international risk sharing. The more domestic risk sharing is needed, the more asset markets are open and the more international risk sharing is possible. After all, it is the fear to destroy domestic risk sharing that induces governments to enforce international payments and thus sustain asset markets. Similarly, the more international risk sharing is needed, the more asset markets are closed and the less domestic risk sharing is possible. After all, it is the temptation to default on foreigners that induces governments not to enforce payments and thus destroy asset markets. We use these interactions next to provide a novel account of the effects of globalization.

3 The effects of globalization

Since globalization is a dynamic process, we now re-interpret the model as describing the life of a typical generation in a world with overlapping generations. The world has many generations, $t = 0, 1, \dots, T(\leq +\infty)$. Generation t members are born at time t , with a project that pays at $t + 1$. They maximize expected utility from consumption at $t + 1$. At time t they trade in assets to diversify their production risk. Generation t members cannot trade assets with members of different generations: at time $t + 1$, they are old and the best they can do is to consume all of their income; at time t , the only other living generation is generation $t - 1$, but since this generation is old they are not willing to trade assets either. As a result, individuals diversify their production risk as much as they can by trading assets with other members of the same generation. The process of globalization consists of an increase over time of τ . In particular, we assume $\tau_0 = 0$, $\tau_{t+1} \geq \tau_t$,

and $\lim_{t \rightarrow T} \tau_t = 1$.²¹

We measure the gains from globalization in terms of consumption. More formally, define $G(\tau) \equiv U(\tau) - U(0)$. A generation born in autarky would be indifferent between experiencing growth in world production (of all goods in all states) by a factor $\exp\{G(\tau)\}$ and experiencing an increase in the fraction of traded goods from 0 to τ . It follows from Equation (27) that:

$$G(\tau) = - \int_{s \in E} \pi_s \cdot \int_0^\tau \ln \phi_s^{j(i)}(z) \cdot dz - \int_{s \notin E} \pi_s \cdot \int_0^\tau \ln \left(\frac{\phi_s^{j(i)}(z)}{\phi_s^{j(i)}} \right) \cdot dz + \int_{s \notin E} \pi_s \cdot \ln \phi_{is} \text{ for any } i \in I^W. \quad (29)$$

Equation (29), together with Equation (28), provides a full description of the gains from globalization. In autarky, sovereign risk is not a problem and all asset markets are open. There is perfect domestic sharing of all goods, but technological constraints to trade prevent international sharing. As a result i 's consumption of good z fluctuates across states following regional production. Globalization removes technological constraints to trade but also creates sovereign risk that leads to the closing of asset markets. In those states in which asset markets are open, i.e. $s \in E$, globalization allows perfect international sharing of tradable goods without affecting domestic sharing. This gain is captured by the first term in Equation (29). In those states in which asset markets are closed, i.e. $s \notin E$, globalization allows imperfect international sharing of tradable goods, but it reduces domestic sharing of all goods. The second and third terms in Equation (29) capture this gain and loss from globalization. In this section, we study how all of these forces combine to determine the dynamic effects of globalization on risk sharing and welfare.²²

3.1 Globalization without terms-of-trade effects

Globalization cannot affect individual or regional production bundles since we have assumed that $\{y_{is}(\cdot)\}_{i \in I^W}$ is taken as exogenous to the analysis. Therefore, the relative values of individual and regional production, i.e. $\frac{y_{is}}{y_s^{j(i)}}$ and $\frac{y_s^j}{y_s^W}$, can only be affected by globalization through changes in

²¹We focus on equilibria of this many-period model in which the present actions of governments and/or individuals are independent of their past actions. In this case, the consumption and welfare of each generation is identical to that of the two-period model of section 2, and is fully described by Equations (24), (25), (27) and (28). It is well known that the many-period model might also have additional equilibria in which governments and/or individuals condition their current actions to their past actions.

²²With perfect commitment, all asset markets would be open and all the gains from globalization would come from being able to perfectly share a larger fraction of goods, i.e. $G(\tau) = - \int_{s \in S} \pi_s \cdot \int_0^\tau \ln \phi_s^{j(i)}(z) \cdot dz \geq 0$ for any $i \in I^W$. With perfect discrimination without commitment, asset markets would be geographically segmented and the gains from globalization would come from being able to imperfectly share a larger fraction of goods, i.e. $G(\tau) = - \int_{s \in S} \pi_s \cdot \int_0^\tau \ln \left(\frac{\phi_s^{j(i)}(z)}{\phi_s^{j(i)}} \right) \cdot dz \geq 0$ for any $i \in I^W$. Both of these polar cases therefore yield a smooth and conventional picture of globalization gradually increasing welfare. The reason is that globalization opens new goods markets without affecting asset markets.

goods prices. When this is the case, we say that globalization has terms-of-trade effects.

It is useful to start the analysis with the case in which globalization has no terms-of-trade effects, i.e. the case in which changes in τ do not affect $p_s^j(z)$ for any $z \in [0, 1]$, $s \in S$, and $j \in \{H, F\}$. This case is a useful benchmark because the relative incomes of individuals and regions are not affected by globalization. That is, in this case globalization only affects risk sharing and welfare because it permits more (or less) trade, and not because it changes the amount of trade that is required for risk sharing. We ensure that globalization has no terms-of-trade effects by assuming that both regions produce the same bundle of goods and, as a result, there are no gains from international goods trade:

$$\phi_s^j(z) = \phi_s^j \text{ for all } z \in [0, 1], s \in S, \text{ and } j \in \{H, F\}. \quad (30)$$

This condition implies that $\frac{\partial \phi_{is}}{\partial \tau} = 0$ and $\frac{\partial \phi_s^j}{\partial \tau} = 0$, that is, individual and regional incomes are not affected by globalization even in those states in which asset markets are closed. (Of course, incomes are never affected by globalization in those states in which asset markets are open, as shown in Equation (26)). Moreover, we can then write the gains from globalization as follows:

$$G(\tau) = - \int_{s \in E} \pi_s \cdot \tau \cdot \ln \phi_s^{j(i)} + \int_{s \notin E} \pi_s \cdot \ln \phi_{is} \text{ for any } i \in I^W. \quad (31)$$

For a given enforcement set, $G(\tau)$ is non-decreasing in τ . In those states in which asset markets are open, i.e. $s \in E$, globalization permits international sharing in a growing fraction of goods. In those states in which asset markets are closed, i.e. $s \notin E$, globalization does not affect domestic and international sharing of goods.

But the enforcement set is itself a non-increasing function of τ . To see this, consider a pair of symmetric states $\{s, s'\}$. The top panel of Figure 1 shows the benefit and cost of enforcement in these states (see Equation (28)). While the benefit of enforcement does not depend on τ , the cost of enforcement is proportional to τ . If individual risk is not too high, i.e. $-\int_{i \in I^R} \ln \phi_{is} < \ln \phi_s^R$; there exists a threshold $\tau_s^*(= \tau_{s'}^*)$ such that, if $\tau \leq \tau_s^*$ both asset markets exist, but if $\tau > \tau_s^*$ both asset markets are missing. This threshold is obtained by equating the cost and benefit of enforcement,

$$\tau_s^* = \frac{- \int_{i \in I^R} \ln \phi_{is}}{\ln \phi_s^R} \text{ for all } s \in S. \quad (32)$$

This threshold is increasing in individual risk, but decreasing in regional risk. This is a direct implication of the already familiar trade-off behind enforcement decisions. If $\tau_s^* > 1$, globalization never closes the market for assets that pay in state s . If $\tau_s^* < 1$, globalization closes this market on

the first date in which $\tau_t > \tau_s^*$ and it never reopens again.

Our symmetry assumptions allow us to study the contribution to overall welfare of each pair of symmetric states separately. The bottom panel shows how the contribution of a pair of states s and s' changes as globalization proceeds. Assume $\tau_s^* < 1$ and let t_s^* be the generation such that $\tau_{t_s^*} \leq \tau_s^* < \tau_{t_s^*+1}$. All generations born at date $t \leq t_s^*$ open the asset markets for this pair of states. Therefore globalization allows international sharing on a growing number of goods and increases the contribution of this pair of states to welfare. But this also requires growing payments between regions in these states. When generation t_s^* arrives, these payments would have grown too large and the temptation to default would have been irresistible. Since individuals anticipate this, the asset markets for this pair of states close. This eliminates all international sharing of tradable goods and worsens domestic sharing of goods. As a result, the contribution to welfare of this pair of states drops discretely to a level that is below that of autarky. All the generations born at dates $t > t_s^*$ share this low level of welfare in this pair of states.

It is now straightforward to use the theory to provide an account of the effects of globalization. This is shown in Figure 2. Assume there are many pairs of symmetric states $S = \{(s_1, s'_1), (s_2, s'_2), \dots, (s_P, s'_P)\}$. Let τ_p^* be defined as above for the pair of states (s_p, s'_p) . In a given date t , asset markets exist for the pair of states (s_p, s'_p) if and only if $\tau_t \leq \tau_p^*$. Without loss of generality, we order pairs of symmetric states according to τ_p^* , i.e. $\tau_1^* \leq \tau_2^* \leq \dots \leq \tau_P^*$.

The possible effects of globalization on welfare are illustrated in the three panels of Figure 2.²³ Assume that there exists some (s_p, s'_p) such that $\tau_p^* < 1$ and, for these pairs, let t_p^* be the period such that $\tau_{t_p^*} \leq \tau_p^* < \tau_{t_p^*+1}$. All generations born in date $t \leq t_1^*$ benefit from globalization because all asset markets are open and globalization enlarges the set of goods that are shared internationally. At $t = t_1^*$, the asset markets corresponding to the pair of symmetric states (s_1, s'_1) close leading to a reduction in both domestic and international sharing in these states. This leads to a discrete loss of welfare that persists forever since these asset markets never re-open. All generations born in dates $t_1^* < t < t_2^*$ benefit from further globalization as, once again, it enlarges the set of goods that can be shared internationally. Note however that this effect is smaller than in earlier generations because the newly tradable goods cannot be shared in the pair of states (s_1, s'_1) . At $t = t_2^*$, the asset markets corresponding to the pair of symmetric states (s_2, s'_2) close and this leads to another discrete and persistent loss of welfare. After this, subsequent generations benefit from further globalization until the following pair of asset markets close. And this process continues until the world is fully globalized.

²³The jagged line shows the case of a finite number of states (as explained in the text) while the smooth line shows the limiting case in which the number of states approaches infinity.

This special case illustrates the interplay between two effects of globalization on welfare. On the one hand, globalization removes technological constraints to trade and improves international risk sharing in those states in which asset markets remain open. On the other hand, globalization creates sovereign risk and destroys domestic and international risk sharing in those states in which asset markets close. The top panel of Figure 2 shows the case in which the balance of these effects is always positive and welfare increases monotonically with globalization. The middle panel shows the opposite case in which the balance of these effects is negative and welfare falls monotonically with globalization. Finally, the lower panel shows a case in which the balance of these effects changes sign many times and the effects of globalization on welfare are not monotonic.

3.2 Terms-of-trade effects

If condition (30) fails globalization creates changes in the terms of trade that affect the relative values of individual and regional production, i.e. $\frac{y_{is}}{y_s^{j(i)}}$ and $\frac{y_s^j}{y_s^W}$. This substantially complicates the analysis of globalization since individual and regional incomes are now affected by increases in τ in those states in which asset markets are closed, i.e. $\frac{\partial \phi_{is}}{\partial \tau} \neq 0$ and $\frac{\partial \phi_s^j}{\partial \tau} \neq 0$.

For a given enforcement set, $G(\tau)$ no longer needs to be non-decreasing in τ , as shown by Equation (29). Increases in τ still permit international sharing of a larger fraction of goods in all states and this raises welfare. But now, in addition, globalization affects domestic sharing of goods in those states in which the corresponding asset market is closed, i.e. $s \notin E$. For instance, a change in the terms of trade that increases individual risk worsens domestic sharing of goods and lowers welfare.

Terms-of-trade effects also have implications for the shape of the enforcement set, as shown by Equation (28). Without terms-of-trade effects we found that, for any pair of symmetric states $\{s, s'\}$, enforcement only takes place at low values of τ . This was because the cost of enforcement grows proportionally with globalization, while the benefit of enforcement is not affected by globalization. But this need not be the case if globalization creates terms-of-trade effects. For instance, it is possible that terms-of-trade effects reduce regional risk sufficiently fast that the cost of enforcement falls with globalization. Or it could also be possible that terms-of-trade effects increase individual risk and the benefit from enforcement grows with globalization. Enforcement in a pair of symmetric states $\{s, s'\}$ might now change many times with globalization.

3.2.1 Regional terms-of-trade effects

An interesting special case is that in which all individuals within a region produce the same bundle of goods and, as a result, there are no gains from domestic goods trade:

$$\phi_{is}(z) = \phi_{is} \text{ for all } z \in [0, 1], s \in S, \text{ and } i \in I^W. \quad (33)$$

When this condition applies, we have that $\frac{\partial \phi_{is}}{\partial \tau} = 0$. That is, globalization changes the relative incomes of regions but it does not change the relative incomes of individuals within the same region.

For a given enforcement set, $G(\tau)$ is non-decreasing in τ like the previous section. In those states in which asset markets are open, i.e. $s \in E$, globalization allows perfect international sharing of a larger fraction of goods. In those states in which asset markets are closed, i.e. $s \notin E$, globalization does not affect domestic sharing but allows imperfect international sharing of a larger fraction of goods. But unlike the previous section now the enforcement set can take a variety of different shapes. Since condition (33) ensures that $\frac{\partial \phi_{is}}{\partial \tau} = 0$, we still have that the benefit of enforcement is independent of τ . But now the cost of enforcement need not be proportional to τ . If globalization increases regional risk, the cost of enforcement will increase more than proportionally with τ . If globalization reduces regional risk, the cost of enforcement will increase less than proportionally with τ and might even fall. Whether globalization increases or reduces regional risk depends on whether the marginal tradable good contains more regional production risk than the average tradable one.²⁴

To develop further intuition, consider a pair of states in which one region's production bundle is abundant in low-index goods, while the other region's production bundle is abundant in high-index goods. In particular, assume that $\phi^R(z) \equiv \phi^R(L) > 1$ for all $z \in [0, 0.5]$, and $\phi^R(z) \equiv \phi^R(H) = 2 - \phi^R(L)$ for all $z \in (0.5, 1]$. The top panel of Figure 3 shows that the cost of enforcement grows proportionally with τ until $\tau = 0.5$, but then starts declining and reaches zero when $\tau = 1$. Naturally, the benefit of enforcement is independent of τ since condition (33) applies. If individual risk is not too high, i.e. $-\int_{i \in I^R} \ln \phi_{is} < 0.5 \cdot \ln \phi^R(L)$; enforcement takes place at low and high levels of globalization, but not at intermediate levels. The threshold values at which enforcement changes are labelled τ_s^* and τ_s^{**} .

The contribution of this pair of states to welfare at different stages of globalization is shown in the bottom panel of Figure 3. Let t_s^* and t_s^{**} be the generations such that $\tau_{t_s^*} \leq \tau_s^* < \tau_{t_s^*+1}$ and $\tau_{t_s^{**}} \leq \tau_s^{**} < \tau_{t_s^{**}+1}$, respectively. Let also $t_s^{0.5}$ be the generation such that $\tau_{t_s^{0.5}} \leq 0.5 < \tau_{t_s^{0.5}+1}$. All

²⁴To see this, note that differentiating the definition of ϕ_s^j we find that $\frac{\partial \phi_s^R}{\partial \tau} = \frac{1}{\tau} \cdot \left(\phi_s^R(\tau) - \frac{1}{\tau} \cdot \int_0^\tau \phi_s^R(z) \cdot dz \right)$.

generations born at date $t \leq t_s^*$ open the asset markets for this pair of states. Therefore globalization allows international sharing on a growing number of goods and increases the contribution of this pair of states to welfare. But this also requires growing payments between regions in these states. When generation t_s^* arrives, these payments would have grown too large and the temptation to default would have been irresistible. Since individuals anticipate this, the asset markets for this pair of states close. This eliminates all international sharing of tradable goods and reduces domestic sharing of all goods. The contribution of this pair of states to welfare drops discretely to a level that is below that of autarky. All the generations born in $t \in [t_s^*, t_s^{0.5})$ share this very low level of welfare. Generations born in $t \in [t_s^{0.5}, t_s^{**})$ benefit from globalization. Although asset markets remain closed, goods markets now allow imperfect international sharing of tradable goods. Note also that changes in the terms-of-trade raise the relative income of the poor region and reduce the payments that would be required to achieve perfect international sharing of tradable goods. When generation t_s^{**} arrives, these payments are low enough and enforcement is possible again. Asset markets re-open and both domestic and international sharing is reestablished. This leads to a discrete increase in welfare. For $t \geq t_s^{**}$, asset markets are always open and globalization enlarges the fraction of goods that can be shared internationally.²⁵

3.2.2 Individual terms-of-trade effects

If we relax condition (33), globalization creates changes in goods prices that not only affect the relative incomes of regions but also the relative incomes of individuals within a region. Rather than performing a long and tedious discussion of this general case, we shall illustrate the new forces at work using a simple modification of the previous example. Instead of assuming that all individuals within a region produce the same bundle of goods, assume now that half of the residents produce only low-index goods while the other half produces only high-index goods. Namely,

$$\phi_i(z) = \begin{cases} 2 \text{ for } z \in [0, 0.5] \text{ and } 0 \text{ for } z \in (0.5, 1] & \text{with prob. } 0.5 \\ 0 \text{ for } z \in [0, 0.5] \text{ and } 2 \text{ for } z \in (0.5, 1] & \text{with prob. } 0.5 \end{cases} \quad \text{for all } i \in I^j \text{ and } j \in \{H, F\}.$$

Note that, in this example, full domestic sharing of all goods is achieved in autarky without asset trade, since the value of the production bundle of all the residents of a region is the same. That is, the value of asset markets in autarky is zero.

The top panel of Figure 4 shows the benefit and cost of enforcement both as functions of τ .

²⁵Note that asset markets are not used when globalization has been completed since the value of production in each region is the same in all states. The model of Cole and Obstfeld (1991) can be re-interpreted as the limiting case of this example in which $\phi^R(L) \rightarrow 2$ and $\tau \rightarrow 1$.

The cost of enforcement is as in the previous case, since it only depends on regional risk. But the benefit of enforcement now depends on τ since changes in τ affect individual risk. This benefit starts at zero when $\tau = 0$ since asset markets are not used in autarky. Globalization does not create any international goods trade in this example when $\tau \leq 0.5$ and, as a result, the relative prices of high- and low-index goods are not affected in this range. Without terms-of-trade effects, the benefit of enforcement continues being zero throughout this range. When $\tau > 0.5$, regions start to trade goods and terms-of-trade effects start to kick in. In particular, international trade in goods raises the prices of low-index goods relative to high-index ones. This reduces domestic risk sharing and increases the benefit of enforcement (this also reduces the cost of enforcement, as explained in the previous subsection). As τ increases, terms-of-trade effects grow stronger and the benefit of enforcement increases. There is therefore a threshold level τ_s^* such that there is enforcement only for $\tau \geq \tau_s^*$.

The bottom panel shows how the contribution to welfare of this pair of states changes with globalization. Generations born in $t \leq t_s^{0.5}$ are not affected by globalization. There is no enforcement but goods prices are such that there is perfect domestic sharing of all goods. As discussed above, there is no international sharing of tradable goods. Globalization has two opposing effects on the welfare of generations born in $t \in [t_s^{0.5}, t_s^*)$. On the one hand, globalization improves sharing of tradable goods between regions. On the other hand, globalization worsens domestic sharing of nontradable goods.²⁶ In this range, the negative effect of globalization on domestic sharing raises the benefit of enforcement. Also, the cost of enforcement declines as the same terms-of-trade effects that increase individual risk also reduce regional risk. When generation t_s^* arrives, the benefit of enforcement has increased enough and the cost of enforcement has decreased enough that enforcement becomes possible again and asset markets open. At this point there is a discrete increase in welfare. All generations born after t_s^* open asset markets and benefit from globalization as it enlarges the fraction of goods that can be shared internationally.

These examples show that terms-of-trade effects alter sometimes quite dramatically the relationship between globalization and asset market incompleteness. Without terms-of-trade effects, asset markets are open only in the early stages of globalization. But Figures 3 and 4 show situations in which asset markets are open only at intermediate and only at later stages of globalization, respectively. The theory therefore captures a rich set of interactions between globalization and the workings of asset markets.

²⁶This negative effect of globalization on domestic sharing of goods was first noted by Newbery and Stiglitz (1984).

4 Externalities

The cornerstone of the theory developed above is the trade-off that governments face when deciding their enforcement policy. On the one hand, enforcement increases payments from domestic to foreign residents that lower domestic consumption and welfare. On the other hand, enforcement increases payments between domestic residents that improve domestic sharing of goods and therefore raise welfare. This trade-off determines the states of nature in which governments choose to enforce payments during old age and, therefore, the set of assets that can be traded during youth.

In our setup, the closure of markets constitutes a failure and this is the direct result of two externalities. The first one is that governments do not take into consideration how their enforcement decisions affect foreigners. This externality leads governments not to enforce payments even though in old age the domestic gains from not enforcing are always below the foreign costs. It might seem therefore that the closure of markets could be avoided if governments could pay each other ex-post to enforce. Section 4.1 shows that this is not the case however.

The second externality is that individuals do not take into consideration how their choice of asset holdings affects the enforcement decision of their government. This externality leads individuals to borrow so much from abroad during youth that governments prefer not to enforce in old age. One might think that imposing borrowing limits would solve this problem and avoid the closure of markets. Section 4.2 shows that this is only partially true and that, in any case, allowing the government to set optimal borrowing limits has surprisingly little effect on the picture of globalization developed in the last couple of sections.

4.1 Renegotiating the debt

We have assumed throughout that governments decide enforcement policies non-cooperatively and do not take into account how their decisions affect foreigners. This policy externality leads markets to close whenever the costs of making foreign payments are higher than the benefit of keeping domestic payments. At first sight, this might seem an easy problem to solve. After all, the gains that the rich region obtains from not enforcing are always smaller than the losses that the poor region suffers. Allowing regions to renegotiate their debts ex-post should therefore ensure that there is always enforcement. Unfortunately, this argument is wrong and we explain next why.

Consider a pair of symmetric states for which we concluded there is no enforcement in the best symmetric equilibrium. We reached this conclusion by contradiction. Assume individuals expect enforcement, then asset trade would be as in the complete-markets model and the government of the rich region would have an incentive not to enforce. Therefore, individuals cannot expect

enforcement by the rich region. Assume then that the rich region does not enforce but the poor region does. But then there would be payments from the rich residents of the poor region to the poor residents of the rich region and the poor region would have an incentive not to enforce. Therefore, individuals cannot expect enforcement by the poor region either. This leads us to the conclusion that the only possible outcome is that individuals do not expect enforcement. In this case, there is no asset trade and expecting governments not to enforce is a consistent “off-equilibrium” belief.

This argument does not formally consider the possibility that regions cooperate during old age and the question now is whether this omission matters or not. To be concrete, assume the poor region can make a transfer to the rich region in exchange of enforcement. To raise revenue for this transfer, the poor region levies lump-sum taxes on its residents. Ex-post, the poor region would be willing to offer a transfer to the rich region that is as large as the payments that its residents are due. But the transfer need not be as large for it to work. Since not enforcing destroys valuable domestic payments, the value of enforcing for the rich region is less than the foreign payments it saves by not enforcing. This observation could lead us to conclude that enforcement should be the only outcome of any efficient (and also some inefficient) ex-post bargaining between regions.

But this conclusion would be unwarranted, since it assumes that free or unobstructed asset trade during youth can lead to imperfect sharing ex-post. To see this, consider again a pair of states for which we decided there is no enforcement in the best symmetric equilibrium. Assume now that individuals were to expect that there is enforcement in old age and that this enforcement requires a transfer. But then asset trade would not be as in the complete-markets model. Anticipating the transfer, domestic residents would now feel richer and sell more assets to foreign residents which now feel poorer. In particular, asset trade would be such that fully offsets the transfer and achieves perfect domestic sharing of all goods and perfect international sharing of tradable goods *after* the transfer has been paid. But we know already that in this case the rich region has an incentive not to enforce and therefore individuals cannot expect that the transfer be enough to induce the rich region to enforce.

This argument shows that the expectation of a debt renegotiation cannot sustain the opening of asset markets, and leads us to conclude that allowing ex-post cooperation between regions does not affect the equilibrium of the model.

4.2 Dealing with overborrowing

We have assumed throughout that individuals are atomistic and do not take into consideration how their choice of asset holdings affects the enforcement decision of their government.²⁷ This overborrowing externality leads markets to close because otherwise regions would borrow so much that there would be no enforcement. In principle, the solution to this problem consists of imposing limits to foreign borrowing. But this solution is not available if government policy cannot discriminate between asset holders, as we have assumed throughout the paper. The same reasons (opaque financial intermediaries and deep secondary markets) that impede governments to discriminate between asset holders when enforcing payments also impede them to discriminate between asset holders when imposing borrowing limits. We therefore assume that governments can only impose borrowing limits that are non-discriminatory.

Assume governments limit private borrowing using asset-specific issuance rights. Governments choose the number of issuance rights for each asset, denoted \bar{x}_s^j for $s \in S$ and $j \in \{H, F\}$, which they distribute equally among their domestic residents. During youth agents trade in both issuance rights and assets, under the constraint that their issuance is limited by the issuance rights they hold. Therefore, the budget sets during youth in Equations (19) and (20) are replaced by

$$\int_{s \in S} \left(q_s^H \cdot x_{H,is} + q_s^F \cdot x_{F,is} + \iota_s^{j(i)} \cdot \left(\bar{x}_{is} - \bar{x}_s^{j(i)} \right) \right) \leq 0 \text{ for all } i \in I^W, \quad (34)$$

$$x_{j(i),is} \geq -\min \{ \hat{y}_{is}, \bar{x}_{is} \} \text{ and } x_{-j(i),is} \geq 0 \text{ for all } s \in S \text{ and } i \in I^W, \quad (35)$$

where \bar{x}_{is} denotes the state- s issuance rights held by individual i after the market for issuance rights closes, ι_s^j denotes the price of state- s issuance rights in country j , and we used the fact that before the market for issuance rights opens individual i holds $\bar{x}_s^{j(i)}$ state- s issuance rights. In addition to the market clearing conditions for assets in Equation (22) we now also have market clearing conditions for issuance rights, which are given by

$$\int_{i \in I^j} \bar{x}_{is} = \bar{x}_s^j \text{ for all } s \in S \text{ and } j \in \{H, F\}. \quad (36)$$

A competitive equilibrium with borrowing limits during youth consists of a set of asset prices and quantities such that individuals maximize expected utility –Equation (2)– subject to their budget and solvency constraints –Equations (34), (35) and (21)– and asset markets clear –Equations (22) and Equation (36). Naturally, when maximizing their utility, individuals take as given how their

²⁷This externality is well known in the literature on sovereign risk. For recent discussions of the problem, see Caballero and Krishnamurthy (2001), Tirole (2003), Kehoe and Perri (2002b), and Wright (2006).

individual consumption during old age depends on their individual asset holdings.

In Appendix B we analyze the equilibrium with optimal borrowing limits and show that it contains three types of states. In those states in which there is enforcement even in the absence of borrowing limits, it is optimal not to impose binding limits and the price of issuance rights is zero in both regions. In the remaining states, it is possible (but not necessary) that borrowing limits in the rich region allow enforcement and trade. In those states in which this happens, we have that the price of issuance rights is positive in the rich region and asset markets are open. Paying issuance rights introduces a wedge between the ex-post incomes of borrowers and lenders and, even though asset markets are open, both domestic and international sharing of goods is imperfect. In those states in which there is no borrowing limit that would lead to enforcement and trade, the price of issuance rights is zero in both regions and asset markets remain closed.

The effects of globalization with optimal borrowing limits are illustrated in Figure 5. The left two panels refer to the case of no terms-of-trade effects illustrated in Figure 1. The top panel shows the optimal issuance rights price in the rich region as a function of τ (this price is always zero in the poor region), which we denote $\iota_s^*(\tau)$. For $\tau \leq \tau_s^*$, borrowing limits are not needed for enforcement to take place so $\iota_s^*(\tau) = 0$. In addition, for τ sufficiently higher than τ_s^* borrowing limits are not useful either since the issuance rights price would need to be so high for enforcement to take place that no resident of the rich region would sell assets anyway. The optimal issuance rights prices are positive only for values of τ that are a bit above τ_s^* . The effects of globalization on welfare for this pair of symmetric states is shown in the bottom panel. These effects are quite similar to those in the absence of borrowing limits. The difference is that when generation $t_s^* + 1$ arrives, instead of asset trade disappearing the rich region imposes borrowing limits that lead to a positive issuance rights price $\iota_s^*(\tau_{t_s^*+1})$. Although asset markets remain open, there is imperfect domestic and international sharing of goods. Each new generation requires higher issuance rights prices to keep enforcement. Conditional on issuance rights prices and enforcement, globalization improves international sharing of newly tradable goods. However, domestic sharing of goods and international sharing of inframarginal tradable goods worsen as a result of higher issuance rights prices. The net effects of globalization on welfare are ambiguous. At some point, enforcement is impossible even with borrowing limits, so the price of issuance rights fall to zero and globalization eliminates all domestic and international sharing of goods. Borrowing limits delay the date in which enforcement breaks down.

The middle two panels refer to the case illustrated in Figure 3 in which there are only regional terms-of-trade effects. The top panel shows that the optimal issuance rights price is positive only for values of τ that are a bit above τ_s^* , and also for values of τ that are a bit below τ_s^{**} . The bottom

panel shows that the effects of globalization on welfare for this pair of symmetric states are not qualitatively affected by borrowing limits. Borrowing limits delay the time at which enforcement breaks down and bring forward the time at which enforcement reappears. The right two panels refer to the case illustrated in Figure 4 in which there are regional and individual terms-of-trade effects. The top panel shows that the optimal issuance rights price is positive only for values of τ that are a bit below τ_s^* . Once again, the bottom panel shows that the effects of globalization on welfare for this pair of symmetric states are not qualitatively affected by borrowing limits. Borrowing limits simply bring forward the time at which enforcement appears.

This discussion shows that borrowing limits, though welfare-improving, have little effect on the picture of globalization we presented in section 3.²⁸

5 Final Remarks

This paper has developed a novel theory of endogenous asset market incompleteness based on sovereign risk. The key departure from previous theory is our assumption that governments cannot discriminate between domestic and foreign creditors when choosing enforcement. Previous theory had instead assumed that governments can perfectly discriminate between domestic and foreign creditors. The results presented in this paper show that this choice of assumption shapes the analysis in a fundamental way:

- If a country can perfectly discriminate between domestic and foreign creditors, sovereign risk keeps all asset markets open but geographically segmented. Domestic asset trade is not affected by sovereign risk and can only be limited by other financial market imperfections. International asset trade is not feasible unless the country can offer some collateral.²⁹ Increases in collateral should improve international risk sharing, without affecting domestic risk sharing. Reductions in trade costs improve the functioning of goods markets without affecting the functioning of asset markets, and always raise welfare.³⁰

²⁸In Appendix C, we allow governments to shut down private asset markets and directly control the amount of borrowing by issuing public debt. We show that the outcome with optimal public debt is better than the outcome without ex-ante government intervention and worse than the outcome with optimal borrowing limits. We also show that public debt has little effect on the picture of globalization.

²⁹We use here a broad concept of collateral. This concept includes the narrow view of collateral, i.e. the value of the real assets that foreign creditors can directly seize in the event of default. It also includes the effects of default penalties, i.e. the value of the payments that foreign creditors can elicit through the threat of applying penalties of various sorts. Conceptually, narrow collateral and penalties play the same role in the theory and therefore makes sense to treat them jointly.

³⁰The trade theorist will immediately recognize that this statement needs a few words of qualification. In the model of this paper, globalization with perfect discrimination would raise the ex-ante welfare of all regions because they are ex-ante identical and a reduction in trade costs has no ex-ante terms-of-trade effects. With asymmetric regions, it is possible to construct examples in which there are ex-ante terms-of-trade effects that lead some regions

- If a country cannot discriminate between domestic and foreign creditors, sovereign risk closes some asset markets but keeps those that are open global. Even in the absence of other financial market imperfections, domestic asset trade is limited. Even in the absence of collateral, some international asset trade is possible. Increases in collateral should improve both international and domestic risk sharing. Reductions in trade costs improve the functioning of goods markets but might either improve or worsen the functioning of asset markets, and the effects on welfare can go either way.

Therefore, our assumption of no discrimination: (i) provides a new explanation for why countries can borrow abroad; (ii) shows that there are crucial interactions between domestic and international asset trade; and (iii) accounts for much richer effects of globalization on risk sharing. In addition, the assumption of no discrimination seems to us more realistic than the previous one of perfect discrimination. Although reality surely lies somewhere between these two polar cases, we argued in the introduction that the institutional setup of emerging-market borrowing during the 1990's and 2000's is one in which governments find it difficult to discriminate between domestic and foreign creditors. In any case, it seems also evident that the main theoretical results of this paper would still apply even if governments have some ability to discriminate between creditors and only vanish in the polar case of perfect discrimination.

There are, at least, two important directions in which the theory presented here can be extended. First, the theory abstracts from other types of financial frictions, for instance, those resulting from asymmetric information and transaction costs. These frictions are likely to affect the governments' incentives to enforce payments, since they may have different effects on the size of domestic and international payments. It would be interesting to analyze the interactions between sovereign risk and these other types of financial frictions. Second, the theory ignores the role of reputation in sustaining cooperative behavior. Although the role of reputation has been thoroughly analyzed in previous literature for the case of perfect discrimination, it seems likely that new results would arise if we allowed for reputational equilibria in our setting. For example, if a country cannot discriminate between domestic and foreign creditors, an increase in the incentive to keep a good reputation is likely to improve both international and domestic risk sharing. Furthermore, globalization is likely to affect enforcement, market incompleteness, and welfare not only as a result of its effects on the within-period trade off analyzed in this paper, but also through intertemporal interactions absent when there is perfect discrimination.

to gain and some to lose as a result of globalization. But even in this case, the world as a whole would still gain from globalization and, as a result, there would always exist a set of (international) ex-ante transfers that ensure that globalization leads to a Pareto-improvement. See Ventura (2005) for a comprehensive analysis of the effects of globalization (as modelled here) on trade, growth and welfare.

6 References

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7 Appendix A: Construction of the sovereign risk equilibrium

In this appendix, we construct the sovereign risk equilibrium used in the text. With complete markets, there are equilibria that share the same prices and quantities, but differ in the distribution of assets among individuals. This multiplicity is clearly irrelevant since it does not matter whose assets an individual holds. With sovereign risk, the distribution of assets may be relevant since it can affect the governments’ incentives to enforce payments ex-post. To simplify the exposition, we impose the condition that there be no state in which Home residents receive payments from Foreign and Foreign residents receive payments from Home. That is, either $\int_{i \in I^H} x_{F, is}$ or $\int_{i \in I^F} x_{H, is}$ is zero for all $s \in S$. This restriction is without loss of generality since it can be easily shown that if a given allocation can be supported as an equilibrium in which this condition is not satisfied, then

this allocation can also be supported as an equilibrium in which this condition is satisfied.

It follows from the symmetry assumption that we can analyze pairs of symmetric states independently. For each pair of symmetric states s and s' there are three possible symmetric enforcement levels: (i) both regions enforce: $s \in E^H \cap E^F$ and $s' \in E^H \cap E^F$; (ii) one region enforces: either $s \in E^F - E^H$ and $s' \in E^H - E^F$, or $s \in E^H - E^F$ and $s' \in E^F - E^H$; and (iii) no region enforces: $s \notin E^H \cup E^F$ and $s' \notin E^H \cup E^F$. We construct the best symmetric equilibrium and this is the one in which enforcement levels are as high as possible. To find this equilibrium, we take each pair of symmetric states s and s' and follow three steps:

STEP 1: We check whether in equilibrium both regions can enforce payments simultaneously.³¹ Assume this is the case. Then, asset holdings are as in the complete-markets model and consumptions are given by Equation (14). Using these consumption allocations and the fact that utility is logarithmic, we find that the enforcement condition is given by:

$$-\int_{i \in I^j} \ln \left(\frac{y_{is}^{N^j} + x_{-j, is}}{y_s^{j, N^j} + x_{-j, s}^j} \right) \geq \tau \cdot \ln \left(\frac{y_s^{j, N^j} + x_{-j, s}^j}{y_s^{W, N^j}} \right) \quad \text{for all } j \in \{H, F\},$$

where y^{N^j} stands for the value of income in case of unexpected non-enforcement by the government of region j . The left hand side measures the loss in average utility that results from a breakdown in domestic risk sharing in region j , while the right hand side measures the gains in average utility that result from not paying debts to foreigners. The left hand side is nonnegative for both regions, while the right hand side is zero for the poor (or creditor) region and positive for the rich (or debtor) region. Therefore, the poor region has no incentive to deviate. Has the rich region an incentive to deviate? Let R be the rich region. Since nobody in this region holds assets issued by residents of the poor region, i.e. $x_{P, is} = 0$ for all $i \in I^R$, individual and regional incomes of the rich region if it deviates are obtained by setting $x_{is} = 0$ in Equations (7) and (8). If, given these values of productions, the Equation above holds we conclude that the government of the rich region enforces payments. In this case, $s \in E^H \cap E^F$ and $x_{is} = y_s^W - y_{is}$ for all $i \in I^W$. Otherwise, we move to the next step.

STEP 2: We check whether the poor region enforces payments, even though the rich region does not. Assume this is the case. Since the rich region does not enforce payments, there are some residents of this region that would like to sell assets but cannot do so. Typically, there are also some “poor” residents of the rich region that purchase assets from “rich” residents of the poor

³¹Since states s and s' are symmetric, we just perform these steps on state s .

region. Therefore, the rich region becomes the creditor while the poor region becomes the debtor. Let R and P be the rich and poor regions. Then, we have that asset holdings are given by

$$x_{is} = \begin{cases} \max \{y_s^P + x_s^P - y_{is}, 0\} & \text{if } i \in I^R \\ y_s^P + x_s^P - y_{is} & \text{if } i \in I^P \end{cases}$$

and the market clearing condition in Equation (11). These asset holdings imply that there is full risk sharing among those individuals for which the solvency constraint is not binding. This includes all residents of the poor region and the “poor” residents of the rich region. The “rich” residents of the rich region are forced to consume all of their production. Substituting these asset holdings into Equations (6), (7) and (8), we obtain incomes and consumption allocations. Moreover, this allows us to write the enforcement condition for the poor region as:

$$- \int_{i \in I^P} \ln \left(\frac{y_{is}^{NP}}{y_s^{P,NP}} \right) \geq \tau \cdot \left[\ln \left(\frac{y_s^{P,NP}}{y_s^{W,NP}} \right) - \ln \left(\frac{y_s^P + x_{P,s}^P}{y_s^W} \right) \right].$$

Once again, the left hand side measures the loss in average utility that results from a breakdown in domestic risk sharing in the poor region, while the right hand side measures the gains in average utility that result from not paying debts to residents of the rich region. Both the left and right hand sides are nonnegative. Since residents of the rich region cannot sell assets, individual and regional incomes of the poor region if it deviates are obtained by setting $x_{is} = 0$ in Equations (7) and (8). If, given these values of productions, the condition above holds, we conclude that $s \in E^P - E^R$ and asset holdings are determined as described above. Otherwise, we move to the next step.

STEP 3: If we arrive to this step, it means that none of the regions enforce payments and we conclude that $s \notin E^H \cup E^F$ and $x_{is} = 0$ for all $i \in I^W$ and $j \in \{H, F\}$. We then obtain incomes and consumption allocations by substituting these asset holdings into Equations (6), (7) and (8).

This procedure delivers the best equilibrium. This follows from two observations. First, the enforcement level in a given pair of states does not affect enforcement or welfare in any other pair of states. This is because we focus on symmetric equilibria and in all of them the relative wealth of individuals is the same. Second, the welfare in any pair of states increases with the enforcement level. This is because there are gains from trade and the larger the number of markets the more of these gains individuals reap.

We can generate other symmetric equilibria by switching the order in which we perform the three steps above. For instance, moving step one to the end and then alternating between starting the procedure in steps two and three generates equilibria in which there is at least one missing

market. Or moving step two to the end and then alternating between starting the procedure in steps one and three generates equilibria in which there are either two open markets or none.³²

8 Appendix B: Borrowing limits

In this appendix, we analyze the equilibrium with optimal borrowing limits. Instead of finding the optimal issuance rights directly, we first find the optimal issuance rights prices $\{\iota_s\}_{s \in S}$. Given these prices and resulting asset issuance, the optimal issuance rights are given by

$$\bar{x}_s^j = \int_{i \in I^j} \max\{0, -x_{is}\} \text{ for all } s \in S \text{ and } j \in \{H, F\}.$$

As before, we can analyze pairs of symmetric states independently. To simplify the analysis of the effects of globalization and make it comparable to those in Section 3, we assume that either there is enforcement in both regions or there is not enforcement in either region. We only consider cases in which the issuance rights prices in the poor region are $\iota_s^P = 0$. This condition will be satisfied at the optimum because it is the government of the rich region which might have ex-post incentives not to enforce payments. So let us denote the issuance rights prices in the rich region by ι_s . Equilibrium in asset markets in state s is characterized by

$$x_{is} = \begin{cases} (1 + \iota_s) \cdot (y_s^P + x_s^P) - y_{is} & \text{if } (1 + \iota_s) \cdot (y_s^P + x_s^P) < y_{is} \\ 0 & \text{if } y_s^P + x_s^P \leq y_{is} \leq (1 + \iota_s) \cdot (y_s^P + x_s^P) \\ y_s^P + x_s^P - y_{is} & \text{if } y_{is} < y_s^P + x_s^P \end{cases} \quad \text{if } s \in E \text{ for all } i \in I^R, \quad (37)$$

$$x_{is} = y_s^P + x_s^P - y_{is} \quad \text{if } s \in E \text{ for all } i \in I^P, \quad (38)$$

and $x_{is} = 0$ if $s \notin E$ for all $i \in I^W$; and the asset market clearing condition $x_s^R + x_s^P = 0$. These conditions imply that, when there is enforcement, the richest residents of the rich region make payments to the poorest residents of the rich region and to the residents of the poor region. Borrowing limits introduce a wedge between the ex-post incomes of individuals in these two groups.

Whether or not there is enforcement in state s depends on asset holdings, which in turn depend on borrowing limits. Let $x_{is}(\iota_s)$ be the amount of assets individual i purchases when issuance rights prices are ι_s , if all individuals expect enforcement. Then $x_s^P(\iota_s) \equiv \int_{i \in I^P} x_{is}(\iota_s)$ is a decreasing function of ι_s . This is because, as ι_s increases, both the set of richest residents in the rich region

³²Following this procedure until we have tried all possible orderings allows us to construct all symmetric equilibria except for those in which the rich region enforces but the poor region does not. If we added an additional step in which we checked whether the rich region enforces payments while the poor region does not, the procedure would generate the entire set of symmetric equilibria.

who want to sell assets and the amount of assets that each such resident wants to sell decreases. It also follows that $x_s^P(\iota_s)$ is continuous in ι_s and that there exists a high enough ι_s such that $x_s^P(\iota_s) = 0$. Let us define $\bar{\iota}_s \equiv \min \{\iota_s : x_s^P(\iota_s) = 0\}$, which we call the prohibitive issuance rights price for state s . Let ι_s^E be the set of issuance rights prices such that enforcement takes place, namely

$$\iota_s^E \equiv \{\iota_s : v_s^R(\text{enforce}) \geq v_s^R(\text{not enforce}) \text{ when } x_{is} = x_{is}(\iota_s) \text{ for } i \in I^W\} \text{ for all } s \in S.$$

Let the optimal issuance rights price be denoted ι_s^* . Then $[\bar{\iota}_s, \infty) \subseteq \iota_s^E$ and, since the optimal issuance rights prices are those that maximize asset trade, $\iota_s^* \leq \bar{\iota}_s$. The optimal issuance rights price will be such that there is enforcement in all states except in those in which the issuance rights prices need to be so large for enforcement that no issuance takes place (i.e. $\iota_s^E = [\bar{\iota}_s, \infty)$ and $x_{is}(\bar{\iota}_s) \geq 0$ for all $i \in I^R$). The optimal issuance rights prices are then given by

$$\iota_s^* \equiv \begin{cases} \min \{\iota_s^E\} & \text{if } \min \{x_{is}(\min \{\iota_s^E\}) : i \in I^R\} < 0 \\ 0 & \text{if } \min \{x_{is}(\min \{\iota_s^E\}) : i \in I^R\} \geq 0 \end{cases} \text{ for all } s \in S.$$

Clearly, $\iota_s^* = 0$ for those states in which there was enforcement in the equilibrium without borrowing limits, and $\iota_s^* \in [0, \bar{\iota}_s]$ for the other states. If $\min_{i \in I^R} \{x_{is}(0)\} < x_s^P(0)$, then when $\iota_s = \bar{\iota}_s$ there are no payments to residents of the poor region while there are payments from the richest residents of the rich region to the poorest residents of the rich region. As a result, the government of the rich region strictly prefers to enforce payments. By continuity, $\iota_s^* < \bar{\iota}_s$. As a result, if $\min_{i \in I^R} \{x_{is}(0)\} < x_s^P(0)$ ex-ante utility is strictly higher with optimal borrowing limits than without them. In addition, there is some international sharing of goods since $x_s^P(\iota_s^*) > 0$. If $\min_{i \in I^R} \{x_{is}(0)\} \geq x_s^P(0)$, then when $\iota_s = \bar{\iota}_s$ there are neither payments to residents of the poor region nor payments to the poorest resident of the rich region. Whether or not there exists an issuance rights price $\iota_s < \bar{\iota}_s$ such that there is enforcement depends on the distribution of individual shocks in the rich region and the fraction of goods that are tradable τ . In all cases, even with optimal borrowing limits sovereign risk still leads to imperfect domestic and international sharing of goods.

9 Appendix C: Public debt

In this appendix, we allow governments to address the problem of private overborrowing by replacing private borrowing with public debt. Assume that at the beginning of youth governments issue

contingent public debt. Governments choose the number of public bonds contingent on each state of nature, denoted $b_{j,s}$ for $s \in S$ and $j \in \{H, F\}$, which they distribute equally among their domestic residents. Assume for now that governments prohibit agents from trading private assets. During old age, instead of an enforcement choice, governments choose whether to repay their debt or not. If a government chooses to repay, it imposes non-distortionary lump-sum taxes on its domestic residents $t_{j,s} = b_{j,s}$. If a government chooses not to repay, taxes $t_{j,s} = 0$. The governments' budget constraints during old age are then given by

$$t_{j,s} = e_{j,s} \cdot b_{j,s} \text{ for all } s \in S \text{ and } j \in \{H, F\}, \quad (39)$$

where e_s^j is an indicator variable that takes value one if government j repays its debt and zero otherwise. We assume that governments cannot discriminate among domestic residents when imposing taxes and cannot discriminate between domestic and foreign residents when repaying its debt. During youth, agents trade in a market for government debt, selling bonds that pay in states in which their production is high and buying bonds that pay in states in which it is low.

The budget sets during youth in Equations (19), (20), and (21) are replaced by

$$\int_{s \in S} (q_s^H \cdot b_{H,is} + q_s^F \cdot b_{F,is}) = \int_{s \in S} q_s^{j(i)} \cdot b_{j(i),s} \text{ for all } i \in I^W, \quad (40)$$

where $b_{j,is}$ denotes the number of public bonds that pay in state s issued by government j held by individual i after the market for public debt closes, q_s^j denotes the price of public bonds that pay in state s issued by government j , and we used the fact that before the market for public debt opens individual i holds $b_{j(i),s}$ bonds that pay in state s . The market clearing conditions in Equation (22) are replaced by

$$\int_{i \in I^W} b_{j,is} = b_{j,s} \text{ for all } s \in S \text{ and } j \in \{H, F\}. \quad (41)$$

Finally, the budget constraints of old individuals reflect both payments from holding public debt and taxes. We thus replace Equation (16) with

$$\int_0^1 p_s^{j(i)}(z) \cdot c_{is}(z) \cdot dz \leq y_{is} + e_s^{j(i)} \cdot (b_{j(i),is} - b_{j,s}) + e_s^{-j(i)} \cdot b_{-j(i),is} \text{ for all } s \in S \text{ and } i \in I^W, \quad (42)$$

where we used the fact that governments can default on their debts and also the governments' budget constraint in Equation (39) to find taxes.

Note that a higher level of public debt leads in principle to more risk sharing (conditional on repayment) since it allows individuals to make higher payments in more states. At one extreme, if

$b_{j,s} = 0$ residents of region j cannot make any payment in state s . At the other extreme, if $b_{j,s}$ is very high residents of j can make very high payments. Public debt issuance thus plays a similar role as issuance rights in the previous section.

As before, we can analyze pairs of symmetric states independently. To simplify the analysis of the effects of globalization and make it comparable to those in Section 3, we will assume that either both governments repay or neither does. With some abuse of notation, we will denote the set of states in which both governments repay as E . We will only consider cases in which the government of the poor region issues enough public debt so that their residents are unconstrained in the amount of payments they make during old age. This condition will be satisfied at the optimum because it is the government of the poor region which might have ex-post incentives not to repay its debt. Equilibrium in the market for government debt that pays in state s is characterized by

$$b_{is} = \begin{cases} 0 & \text{if } y_s^P + b_s^P + (b_{R,s} - b_{P,s}) \leq y_{is} \\ y_s^P + b_s^P + (b_{R,s} - b_{P,s}) - y_{is} & \text{if } y_{is} < y_s^P + b_s^P + (b_{R,s} - b_{P,s}) \end{cases} \quad \text{if } s \in E \text{ for all } i \in I^R, \quad (43)$$

$$b_{is} = y_s^P + b_s^P - y_{is} \quad \text{if } s \in E \text{ and } i \in I^P, \quad (44)$$

and b_{is} undetermined if $s \notin E$ for all $i \in I^W$; and the asset market clearing condition $b_s^R + b_s^P = b_{H,s} + b_{F,s}$. To make the outcome more transparent, note that consumption levels are given by

$$c_{is} = \begin{cases} y_{is} - b_{R,s} & \text{if } y_s^P + b_s^P + (b_{R,s} - b_{P,s}) \leq y_{is} \\ y_s^P + (b_s^P - b_{P,s}) & \text{if } y_{is} < y_s^P + b_s^P + (b_{R,s} - b_{P,s}) \end{cases} \quad \text{if } s \in E \text{ for all } i \in I^R, \quad (45)$$

$$c_{is} = y_s^P + (b_s^P - b_{P,s}) \quad \text{if } s \in E \text{ for all } i \in I^P, \quad (46)$$

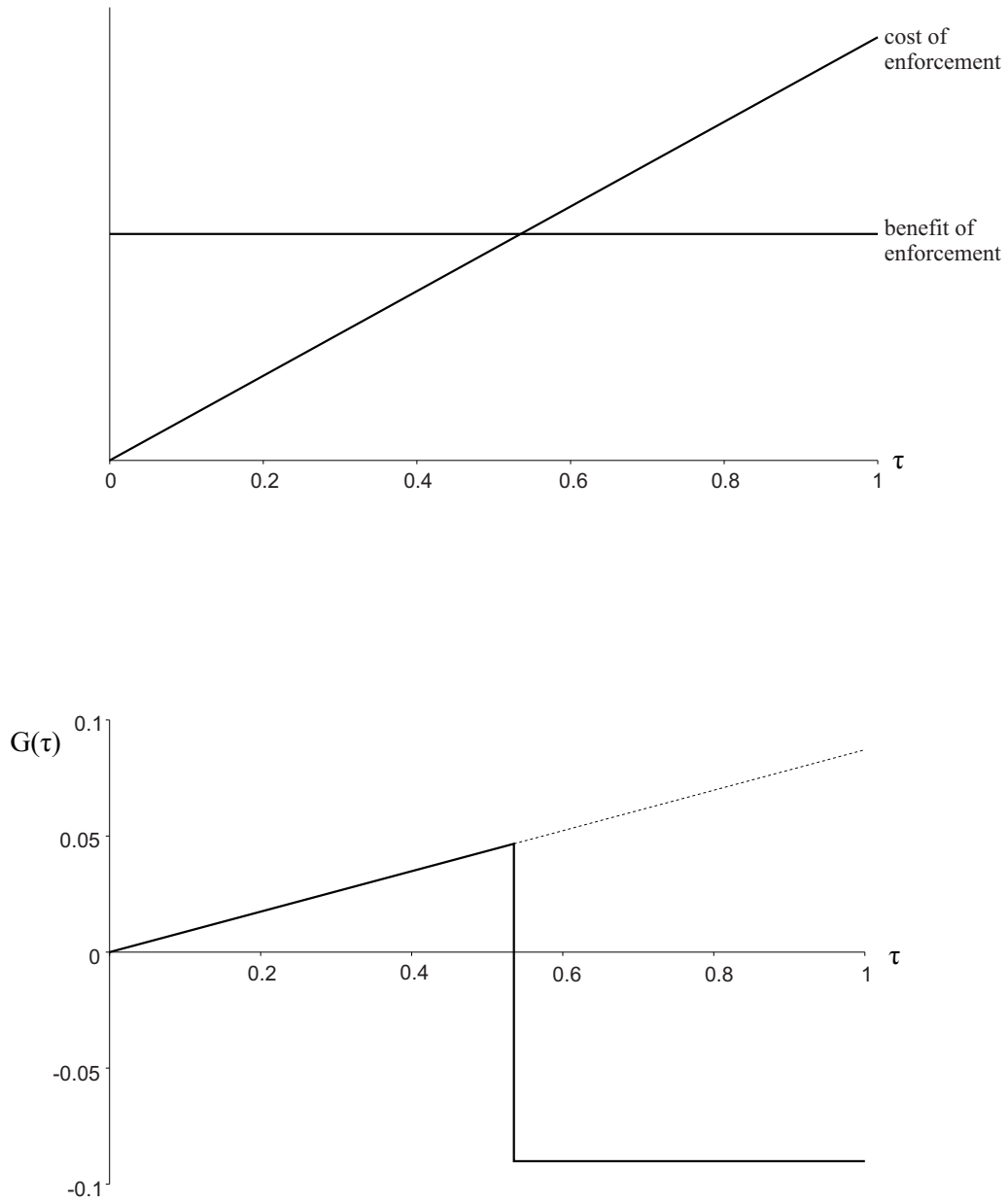
and $c_{is} = y_{is}$ if $s \notin E$ for all $i \in I^W$. This shows that the richer residents of the rich region make payments (through taxes) to the poorer residents of the rich region and to the residents of the poor region.

In principle, we could do a full analysis of the case of optimal public debt policy. However, it is easy to see that in terms of sharing of goods and welfare it falls in between the cases of no ex-ante policies and optimal borrowing limits. With respect to the case of no ex-ante policies, it is easy to see that for states in which there would be enforcement with private debt the same outcome can be obtained by setting $b_{j,s}$ high enough (in particular, as high as the largest payment a domestic resident would have made in that state). In addition, in states in which there would not be enforcement, it is sometimes possible to get some asset trade with public debt. With respect to

the case of optimal borrowing limits, in states in which enforcement is gained with both optimal public debt and optimal borrowing limits the outcome with optimal public debt is no better and in general worse than the outcome with optimal borrowing limits. This is because with optimal borrowing limits payments are made by the richest residents of the rich region (and, among these, the higher the value of production the higher the payment). This is reflected in the fact that consumption levels are equalized among the richest residents of the rich region. With optimal public debt, however, residents are constrained in the payments they can make by the size of taxes. This constraint binds for the richest residents and that is why consumption levels are not equalized among them. In addition, since the resulting domestic sharing of goods is worse with public debt, governments have fewer incentives to tax and pay the debt held by domestic residents and, thus, there are some states in which enforcement is gained with optimal borrowing limits but not with optimal public debt.

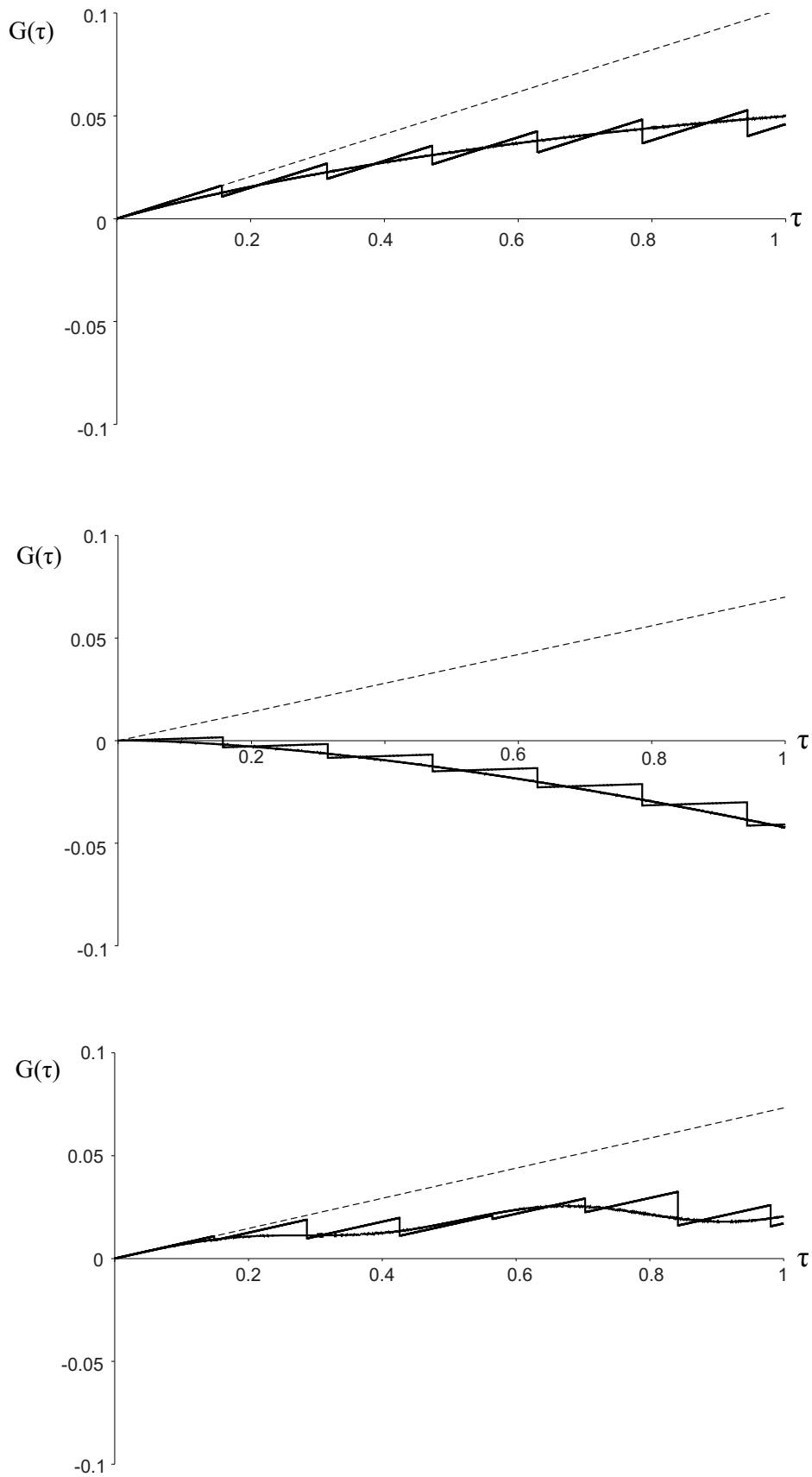
Finally, is it possible to obtain a better outcome by allowing private markets to operate in parallel with public debt markets? The answer is no. First, in states in which borrowing needs to be constrained for enforcement to take place agents cannot expect enforcement of private contracts and repayment of government debt. If they did, the outcome would be as in the complete markets case and, thus, ex-post governments would prefer not to enforce private payments and/or not to repay government debt. Second, in these states agents cannot expect enforcement of private contracts and no repayment of public debt for the same reason. So even if governments allowed private markets, they would be irrelevant in states in which borrowing needs to be constrained. In other states, optimal public debt can be partly or even totally replaced by private assets without affecting the fact that the outcome is as in the complete markets case.

Figure 1



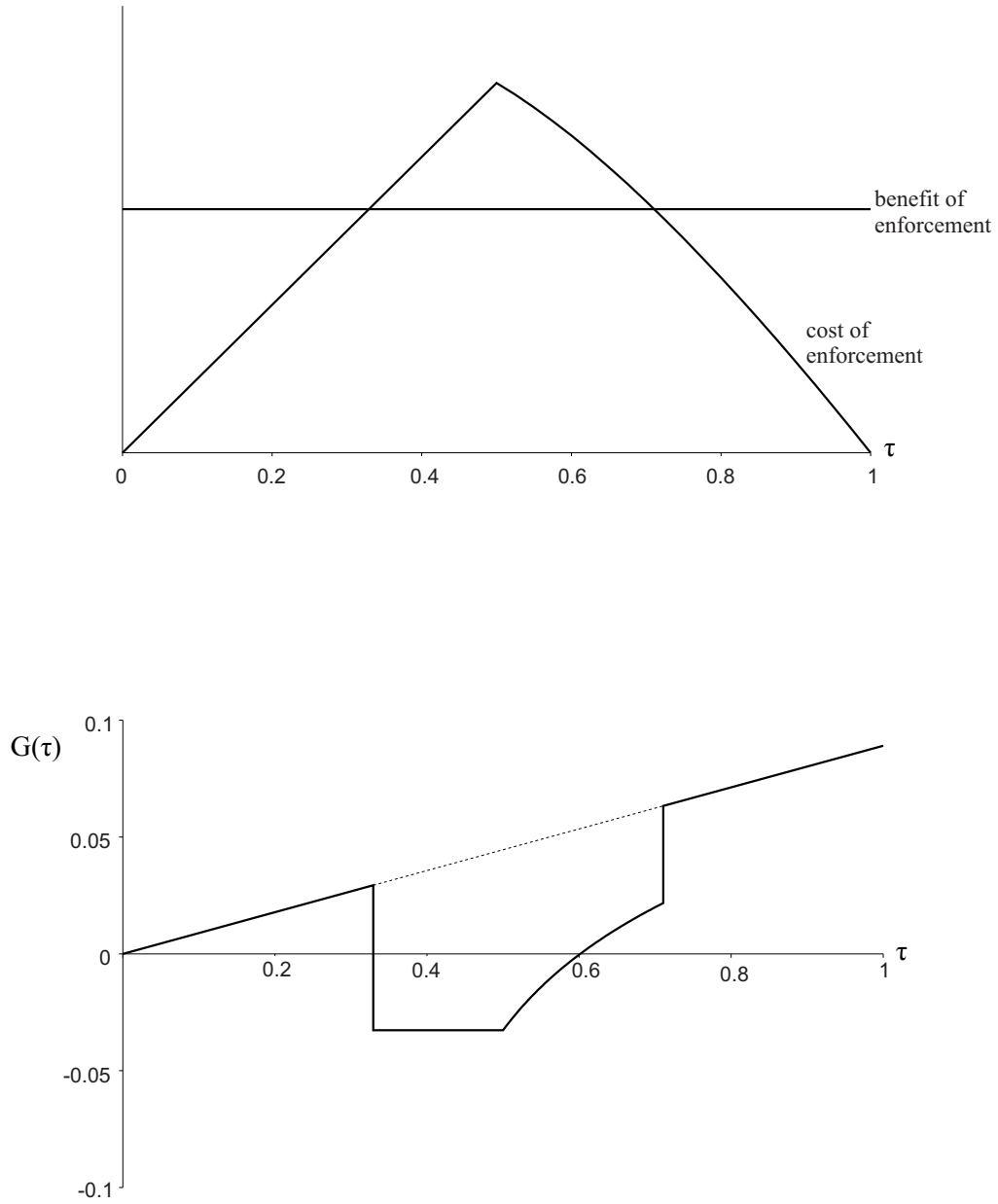
This case is constructed with one pair of symmetric states and no terms-of-trade effects. The regional component of production is such that $\phi^R(z) = 1.4$ and $\phi^P(z) = 0.6$ for all $z \in [0,1]$. The individual component of production satisfies $\phi_{is}^R(z) = 1.55$ for half of the residents in R and $\phi_{is}^R(z) = 0.45$ for the other half. There is no individual risk in the poor region.

Figure 2



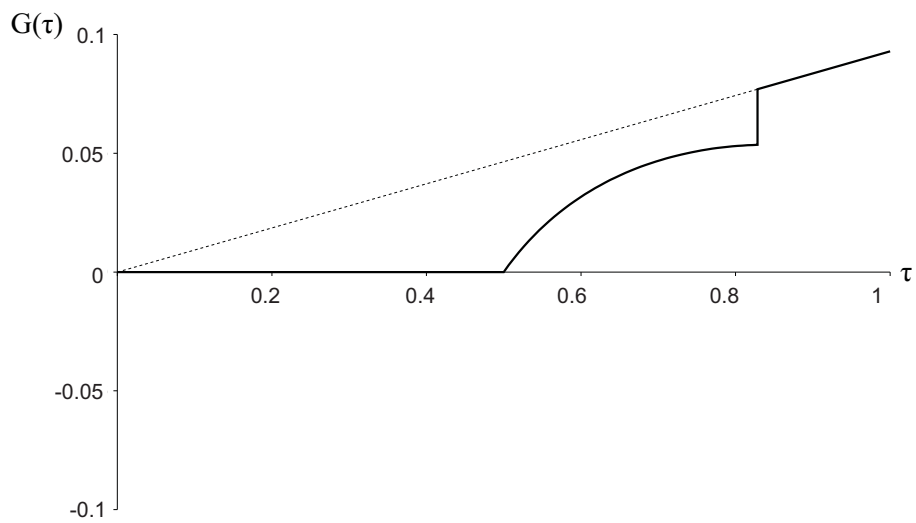
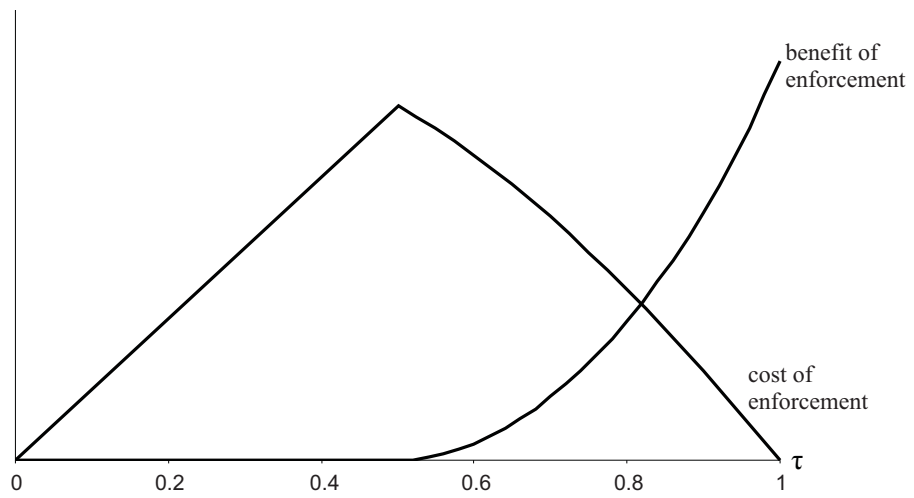
These panels illustrate the effects of globalization on welfare with many pairs of symmetric states. The top panel uses uniformly distributed pairs of states (14 for the jagged line and 20,000 for the smooth line) with a large mass of states satisfying $\tau_p^* > 1$. The middle panel is obtained by setting $\tau_p^* < 1$ for a large number of states. The bottom panel is constructed with the same number of states as before but distributed according to a sinusoidal probability density function.

Figure 3



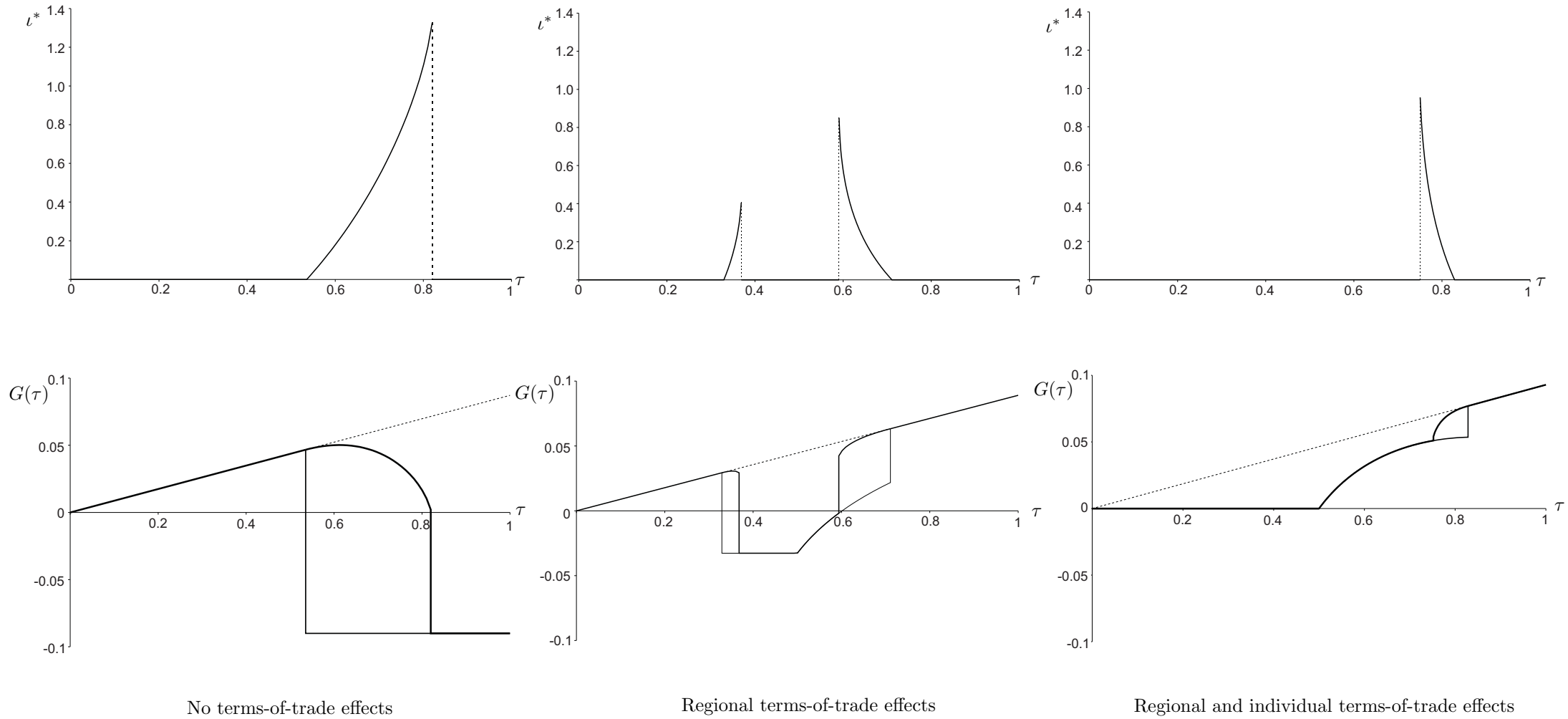
This case illustrates the behavior of the model with regional terms-of-trade effects and one pair of symmetric states. The regional component of production is characterized by $\phi^R(L)=1.58$. The individual component of production satisfies $\phi_{is}^R(z)=1.51$ for half of the residents in R and $\phi_{is}^R(z)=0.49$ for the other half. There is no individual risk in the poor region.

Figure 4



This case illustrates the behavior of the model with regional and individual terms-of-trade effects and one pair of symmetric states. The regional component of production is characterized by $\phi^R(L)=1.58$. The individual component of production in the rich region is specified in section 3.2.2. There is no individual risk in the poor region.

Figure 5



This figure replicates the examples presented in figures 1, 3 and 4 with optimal borrowing limits