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Abstract

The Kyoto protocol allows Annex I countries to deduct carbon sequestered by land use, land-use change and forestry from their national carbon emissions. Thornley and Cannell (2000) demonstrated that the objectives of maximizing timber and carbon sequestration are not complementary. Based on this finding, this paper determines the optimal selective management regime taking into account the underlying biophysical and economic processes. The results show that the net benefits of carbon storage only compensate the decrease in net benefits of timber production once the carbon price has exceeded a certain threshold value. The sequestration costs are significantly lower than previous estimates.

Key words: Kyoto protocol, forest management, selective logging, carbon sequestration, dynamic optimization.

JEL classification: C61, Q23, Q54.

1. Introduction

Many signatory countries of the Kyoto Protocol are currently analyzing the policy options available to them to comply with their country-specific CO_2 protocol targets. In order to make rational choices between the various options, the countries need to have precise information about the costs of each measure. This is also the case for Articles 3.3 and 3.4 of the protocol, where Annex 1 countries can deduct carbon sequestrated by land use, land-use change and forestry from their national carbon emissions. The protocol and later agreements in Marrakesh and Bonn (Interngovernmental Panel on Climate Change, 2001) explicitly mention changes in forest management, afforestation and reforestation within certain limits either in the Annex I country itself or in Non-Annex I countries in the form of clean development mechanisms.

The economic literature determines the cost of forest carbon sequestration in two different ways. One strand of the literature estimates the cost of forest carbon sequestration at the regional or national level (R. Lubowski et al., 2006, B. Sohngen and R. Mendelsohn, 2003, K. van't Veld and A. Plantinga, 2005). These studies focus on the cost of land-use changes given a predetermined tree harvesting regime. Hence, they do not determine an optimal forest management that maximizes timber and carbon sequestration benefits. Precisely this fact was considered in a second strand of the literature (J. Creedy and A.D. Wurzbacher, 2001, C. Huang and G. Kronrad, 2001, V. Tassone et al., 2004, G. C. van Kooten et al., 1995). These studies determined the optimal logging regime in the presence of timber and carbon sequestration benefits. However, the optimal logging regime was determined based either on purely biological indicators, such as the mean annual increment of the biomass, or on the Faustman Formula, and did not take into account the interdependencies between the formulated objectives. Forest management with respect to maximizing the net benefits of timber calls for growing a reduced number of high value trees, while the maximization of the net benefits of sequestered carbon requires maximizing the standing biomass. Consequently, the compatibility of these two maximization objectives is limited (J. Healey et al., 2000, G. C. van Kooten and E. Bulte, 1999). Thornley and Cannell (2000) demonstrated that there is no simple inverse relationship between the amount of timber harvested from a forest and the amount of carbon stored. They show that management regimes that maintain a continuous canopy cover and mimic, to some extent, regular natural forest disturbance are likely to achieve the best combination of high wood yield and carbon storage.[1] Since these findings are completely based on physical units, determining the optimal forest management regime that considers the net benefits from timber production and carbon sequestration remains an open question. To achieve this objective, correct modeling of the forest dynamics that allows for different management regimes is crucial. In particular, it should allow for modeling disturbances that range from management regimes without harvesting to clear cutting. Between these two extremes we have the selective logging regime.

The results of the study show that carbon sequestration costs for forests seem to be significantly lower for a change in the forest management regime than for a change in land use, i.e., a change from agriculture to forestry. In this respect the study suggests to reanalyze the role of forest carbon sequestration as a part of climate change policies within the context of a change in the management regime and not within a change in land-use.

1.1 Distinguishing features of this study

[1] Thornley and Cannell (2000) report that more carbon was stored in an undisturbed forest (35.2 kg C m^{-2}) than in any regime in which wood was harvested. Plantation management gave moderate carbon storage (14.3 kg C m^{-2}) and timber yield (15.6 m^3 ha^{-1} $year^{-1}$). Notably, the annual removal of 10 or 20% of woody biomass per year gave both a high timber yield (25 m^3 ha^{-1} $year^{-1}$) and high carbon storage (20 to 24 kg C m^{-2}). The efficiency of the latter regimes could be attributed (in the model) to high light interception and net primary productivity, but less evapotranspiration and summer water stress than in the undisturbed forest, high litter input to the soil giving high soil carbon and N_2 fixation, low maintenance respiration and low N leaching owing to soil mineral pool depletion.

Until recently the existing literature about the economics of carbon sequestration did not describe the growth of individual trees but rather the growth of the biomass of the entire forest. Hence, it assumes that the forest is not structured, i.e., all trees have the same age, and the number of trees is not relevant to the correct description of the biological growth process. Cunha e Sá, Costa-Duarte and Rosa (2007) extended the previous literature by basing their analysis on an age-structured model of forest growth. Every year trees pass from the current to the sequent age class. As each age class is associated with a particular biomass that increases with age, trees "grow" as they get older. Although their model is an important advancement, it does not establish a functional relationship between the biomass and age, but rather forms a tuple of different pairs of biomass and age. In this respect, their analytical results are limited to certain combination points of age and biomass, but do not fully describe the underlying biophysical processes as proposed in the literature on biological mathematics (N. Keyfitz, 1977, 1968). In particular it does not model the fact that the objectives of maximizing timber and carbon benefits are not fully complementary. Moreover, the structuring variable age cannot play the same role as the structuring size as it is an inadequate proxy of the size (L. Björklund, 1999). A study by Seymour and Kenefic (1998) showed that age only explains 30% of tree growth in size.

The literature on biological mathematics also demonstrates that the growth of each tree does not depend only on its own size but also on the tree density (trees per hectare) and the distribution of the size within the stand (N. Keyfitz, 1968). The last two aspects portray the competition between the individual trees for space, light and nutrients. Since the density and the intraspecific competition of the forest affect the biological growth of the individual trees, the number of planted trees should be determined endogenously and the intraspecific competition effect should be considered in the model. If the forest reproduces naturally, the density of the forest can be regulated by thinning, i.e., by the selective logging of younger trees. Xabadia and Goetz (2007) and Goetz, Xabadia and Calvo (2007) determined the optimal selective logging regime for timber production of a size-structured forest in which planting and intraspecific competition were taken into account, without, however, considering the net benefits from carbon sequestration.

The modeling of a size-structured forest can include the fact that the price of timber increases with the size of the trees because the timber of larger trees can be used for higher-value products, such as furniture. Consequently, the price of timber can be considered as a function of the size of the tree. This aspect is not only important for correctly determining the net benefits of timber production but also for accurately measuring the permanence of the sequestered carbon in the final product. The larger the tree is, the higher the value of the wood product which in turn is positively correlated with the permanence rate of the sequestrated carbon in the product. The size of the tree, the density of the stand and the intraspecific competition are therefore essential for an accurate description of the biological growth process of the trees and of the carbon cycle, and for the correct modeling of economic factors.

Another important part of the carbon cycle is the carbon sequestered in the soil. The potential of forest soils to sequester carbon is well known (D. Rasse et al., 2001). According to Brown (1998) and to a study by the UNDP (2000), the capacity of forest soil to sequester carbon is superior to that of above-ground vegetation. Matthews (1993), for instance, reports that the change from agricultural land to forest land led, after 200 years, to an increase in soil carbon from 30 t C/ha initially to 70 t C/ha. The previous literature, with the exceptions of some studies (R. Lubowski, A. Plantinga and R. Stavins, 2006, G. McCarney et al., 2006, B. Sohngen and R. Sedjo, 2000), did not consider the dynamics of soil carbon. However, the consideration of selective logging requires the dynamics of soil carbon not to be modeled as a pure accumulation process, as was done in the previous studies, but as an accumulation and extraction process resulting from the continuous growth and harvest of the trees.

The paper is organized as follows. The next section describes the dynamics of a forest, and presents the economic decision problem. In Section 3 an empirical analysis is conducted to determine the optimal selective-logging regime of a privately owned forest with respect to timber and carbon sequestration, and the effects of carbon prices on the optimal management regime are analyzed. The conclusions are presented in Section 4.

2. The bioeconomic model

The dynamics of the bioeconomic model reflects, on one hand, the change in the density of the size-structured trees $x(t,l)$ over calendar time t and size l and, on the other hand, the change in the soil carbon $s(t)$ over time. Before we can specify the precise mathematical presentation of these two equations we need to introduce some notation.

The size of the tree is measured by the diameter at breast height (130 cm above the ground) and is denoted by l. The set of the admissible values of the variable l consists of the interval $[l_0, l_m]$, where l_0 corresponds to the minimum vital diameter of the tree and l_m is the maximum diameter that the trees can reach. However, as we are considering the case of a completely managed forest, no natural reproduction takes place, and all young trees have to be planted. Therefore, the parameter l_0 corresponds to the diameter of the trees when they are planted. The flux of logged trees is denoted $u(t,l)$, and the flux of planted young trees at time t with diameter l_0 by $p(t;\mathrm{l}_0)$. We assume that a diameter-distributed forest can be fully characterized by the number of trees and by the distribution of the diameter of the trees. In other words, space and the local environmental conditions of the trees are not taken into account.

As the density function $x(t,l)$ indicates the population density with respect to the structuring variable l at time t, the number of trees in the forest at time t is given by

$$X(t) = \int_{l_0}^{l_m} x(t,l)dl. \tag{1}$$

The term $E(t,l_0)$ presents environmental characteristics that affect the growth rate of the diameter of individual trees. In the absence of pests these environmental characteristics are given by the local conditions where the tree is growing, and by the neighboring trees. Since our model does not consider space, the term $E(t,l_0)$ exclusively presents the competition between individuals, i.e., the competition between individuals for space, light and nutrients. Environmental pressure can be expressed, for example, by the total number of trees, or the basal area of all trees of the stand.[2] Hence, a large (small) basal area of the stand signifies a high (low) intraspecific competition of the trees. This feature of the model is supported by Álvarez et al. (2003), who analyzed various indices to evaluate the effect of intraspecific competition on the individual growth rate of the trees and found that the basal area is the statistic that best explains the differences in diameter growth. Thus, the change in diameter of the tree over time is described by the function $g(E(t,l_0),l)$, i.e.,

$$\frac{dl}{dt} = g(E(t,l_0),l), \tag{2}$$

where the basal area $E(t, l_0)$ is determined based on the functional relationship between diameter and basal area, that is $E(t,l_0) = \int_{l_0}^{l_m} \frac{\pi}{4} l^2 x(t,l)dl$.

The instantaneous mortality rate, $\delta(E(t,l_0),l)$, describes the rate at which the probability of survival of an l-sized tree in the presence of intraspecific competition $E(t,l_0)$ decreases with time. Based on the well-known McKendrick equation for age-structured populations (A. McKendrick, 1926), the dynamics of a diameter-distributed forest can be described by the following partial integrodifferential equation as discussed by de Roos (1997) or by Metz and Diekmann (1986):

[2] The basal area is the area of the cross section of a tree measured at a height of 1.30 m above the ground. It is often used to indicate the tree density of the stand, where the sum of the basal area of all trees is normally expressed as square meters per hectare.

$$\frac{\partial x(t,l)}{\partial t}+\frac{\partial[g(E(t,l_0),l)x(t,l)]}{\partial l}=-\delta(E(t,l_0),l)x(t,l)-u(t,l), \quad t\in[0, T), \quad l\in[l_0, l_m], \tag{3}$$

subject to the boundary conditions $x(0,l) = x_0(l)$, $l\in[l_0, l_m]$, and $x(t,l_0) = p(t,l_0)$, $t\in[0, T)$. The first two terms of Equation (3) present the change in the tree density over time and diameter. The term $\partial(g(\cdot)x(\cdot))/\partial l = g\ \partial x/\partial l + \partial g/\partial l\ x$ takes into account the interdependence between diameter and time, i.e., it presents the temporal change in diameter multiplied by the change in tree density with respect to diameter plus the temporal change in diameter with respect to diameter multiplied by the tree density.[3] Thus, the forest dynamics are described by the flux of tree density with respect to diameter and time, which is equal to the terms of the right hand side of Equation (3), given by tree mortality and logging.

To consider, in addition, the net benefits of carbon sequestration in the economic model, it is necessary to analyze how the carbon content of the forest ecosystem changes over time. The change in the carbon content is given by $dz/dt = db/dt + ds/dt$, and depends on the change in carbon sequestered in the biomass captured by db/dt, and the change in the carbon content in the soil ds/dt .With respect to the dynamics of soil carbon, we denote the above-ground volume of the biomass of the forest by $V(t)=\int_{l_0}^{l_m}\gamma_0 l^{\beta}x(t,l)dl$, where the strictly positive parameters β and γ_0 have to be chosen according to the species of the tree and the empirical data at hand. Once function V is defined, we are in a position to specify the soil carbon dynamics, which are given by

$$\frac{ds(t)}{dt}=h\big(V(t),s(t)\big), \quad t\in[0, T), \quad s(0)=s_0. \tag{4}$$

[3] If we had chosen age as the structuring variable the function g would be constant and equal to 1 since $d(\text{age})/dt = g = 1$ and therefore the aging of the tree by one year corresponds to one year of calendar time.

Function h reflects to what extent the growth or harvest of trees and the current amount of soil carbon affect the change in the soil carbon over time.

The amount of sequestered carbon in the biomass is given by $b(t) = \gamma_1 \int_{l_0}^{l_m} l^{\beta} x(t,l) dl$.

Calculating $b(t)$ is based upon the above-ground volume of the forest and the constant γ_1. The ratio γ_1 / γ_0 reflects the carbon content per cubic meter of the biomass and is more or less constant throughout the growth process of the trees. Before stating the complete bioeconomic model we need to define the function $B(x(t,l),u(t,l))$. This presents the net benefits of timber production as a function of the standing and the harvested trees, with $B_u > 0$ and $B_x < 0$, where a subscript of a variable with respect to a function indicates the partial derivative of the function with respect to this variable. Finally, we denote the price of carbon by ρ_1 and the cost of planting young trees by ρ_2.

The decision problem (DP) of the manager or forest owner is to find the optimal trajectories of the control variables $u(t,l)$ and $p(t,l_0)$. Determining the optimal trajectories can determine, in turn, the optimal trajectories of the state variables $x(t,l)$ and $s(t)$, and variables $b(t)$, $E(t,l_0)$ and $V(t)$. We assume that the owner or manager of the forest maximizes the joint net benefits of timber production and carbon sequestration denoted by J. Currently, the literature distinguishes between two main accounting methods for carbon. In the first one (flow method) payments are based on the flux of carbon in the ecosystem within a period of time. That is, if there is a net storage of carbon $db(t)/dt + ds(t)/dt > 0$ within this time period, forest owners receive compensation. However, if the carbon released as a consequence of logging activities is greater than the carbon sequestered within this time period, forest owners have to pay. The second accounting method (stock method) is based on "carbon equivalent units", where forest owners are paid per period of time according to the amount of carbon they maintain sequestered during this period. This approach is suitable if information is available about the carbon stock but not about the flux. As our model accounts for the flow of carbon sequestered in the biomass and the soil, we opted for the first approach.

The economic benefits of carbon sequestration in the presence of a strictly positive discount rate reside in the fact that carbon is sequestered at time *t* and released at a later point of time. The sequestration occurs initially in the tree and once the tree is cut the carbon remains sequestered in the wood product. Therefore, we have two types of carbon parking, first in the tree and second in the timber product. Unfortunately, the carbon flow *db/dt* only takes into account the carbon sequestered in the tree, not in the wood product. According to the definition of *db/dt* the sequestered carbon is released immediately after the tree has been cut. Thus, the term *db/d*t does not take into account the fact that the sequestered carbon continues to be sequestered during the lifetime *PE*(*l*) of the wood product (permanence time). It can be generally assumed that *PE*(*l*) of the wood product, produced from timber of a tree with size *l*, increases with *l*, i.e., *PE*'(*l*)> 0.

When trees are cut the amount of sequestered carbon is reduced. In the case of an instantaneous release of carbon at time *t*, the discounted monetary costs of this reduction would be given by $u_{b_1} = \int_{l_0}^{l_m} e^{-rt} \rho_1 \gamma_1 l^{\beta} u(t,l) dl$. However, since the sequestered carbon is not released immediately, but gradually over *PE*(*l*) years, according to the release function $\omega(PE(l))$, the discounted monetary costs are given by $u_{b_2} = \int_{l_0}^{l_m} \left(\int_0^{PE(l)} e^{-r(t+\tau)} (\omega(PE(l)) d\tau \right) \rho_1 \gamma_1 l^{\beta} u(t,l) dl$. The term in the inner brackets takes into account the fact that the postponed release of the sequestered carbon leads to lower payments in comparison with payments resulting from an immediate release of the sequestered carbon after the tree is logged. Let $\upsilon(l) = \int_0^{PE(l)} e^{-r\tau} \omega(PE(l)) d\tau \in [0,1]$ denote the share of the sequestered carbon in the biomass that is released from the wood product if all carbon releases are expressed in terms of year t. This allows the amount of the released carbon in terms of year t to be calculated. Therefore, the longer the carbon is sequestered in the wood product, the lower the discounted amount of the released carbon.

A comparison of the payment for carbon releases shows that $u_{b_1} - u_{b_2} = e^{-rt}\rho_1\gamma_1\int_{l_0}^{l_m}\left(1-\upsilon(l)\right)l^{\beta}u(t,l)dl = e^{-rt}\rho_1 u_c$. Thus the term $u_c = \gamma_1\int_{l_0}^{l_m}\left(1-\upsilon(l)\right)l^{\beta}u(t,l)dl$ can be incorporated in the carbon flow equation leading to $db(t)/dt + u_c + ds(t)/dt$. Collecting the previously introduced elements, the decision problem takes the form of:

$$\max_{u(t,l),p(t)} J = \int_0^T e^{-rt}\left\{\int_{l_0}^{l_m} B(x(t,l),u(t,l))dl + \rho_1\left[\frac{db(t)}{dt} + u_c(t) + \frac{ds(t)}{dt}\right] - \rho_2 p(t)\right\}dt \qquad \text{(DP)}$$

under the restrictions

$$\frac{\partial x(t,l)}{\partial t} + \frac{\partial[g(E(t),l)x(t,l)]}{\partial l} = -\delta(E(t),l)x(t,l) - u(t,l), \qquad t\in[0,T),\ l\in[l_0,l_m],$$

$$E(t) = \frac{\pi}{4}\int_{l_0}^{l_m} l^2 x(t,l)dl,$$

$$\frac{ds(t)}{dt} = h\left(V(t),s(t)\right), \quad t\in[0,T), \qquad s(0) = s_0,$$

$$V(t) = \int_{l_0}^{l_m}\gamma_0 l^{\beta}x(t,l)dl,$$

$$b(t) = \gamma_1\int_{l_0}^{l_m} l^{\beta}x(t,l)dl,$$

$$u_c(t) = \gamma_1\int_{l_0}^{l_m}(1-\upsilon(l))l^{\beta}u(t,l)dl,$$

$$x(0,l) = x_0(l), \quad l\in[l_0,l_m], \qquad x(t,l_0) = p(t), \quad t\in[0,T),$$

$$0 \le u(t,l) \le u_{\max}(t,l), \qquad 0 \le p(t) \le p_{\max}(t),$$

where the parameter l_0 has been dropped from the presentation of functions E and p to simplify the notation.

As the decision problem (DP) is based on one partial integrodifferential equation, Equation (3), and one ordinary integrodifferential equation, Equation (4), the previously presented necessary conditions for maximizing the function J cannot be applied. Hritonenko et al. (2007) establish the necessary extremum conditions in the form of a maximum principle for this type of decision problem and discuss its characteristics.

The necessary conditions for a maximum yield

$\partial J/\partial u(t,l) \le 0$ *at* $u^*(t,l)=0$, $\partial J/\partial u(t,l) \ge 0$ at $u^*(t,l)=u_{\max}(t,l)$,

$\partial J/\partial u(t,l)=0$ *at* $0<u^*(t,l)<u_{\max}(t,l)$ for almost all (a.a.) $t\in[0, T)$, $l\in[l_0, l_m]$; (5)

$\partial J/\partial p(t) \le 0$ *at* $p^*(t)=0$, $\partial J/\partial p(t) \ge 0$ *at* $p^*(t)=p_{\max}(t)$, $\partial J/\partial p(t)=0$ *at* $0<p^*(t)<p_{\max}(t)$, a.a. $t\in[0, T)$, (6)

$$\frac{\partial x(t,l)}{\partial t} + \frac{\partial [g(E(t),l)x(t,l)]}{\partial l} = -\delta(E(t),l)x(t,l) - u(t,l), \tag{7}$$

$$x(0,l) = x_0(l), \quad l\in[l_0, l_m], \quad x(t,l_0) = p(t), \quad t\in[0, T), \tag{8}$$

$$\frac{\partial \lambda(t,l)}{\partial t} + g(E(t),l)\frac{\partial \lambda(t,l)}{\partial l} = [r + \delta(E(t),l)]\lambda(t,l) - B_x(x(t,l),u(t,l)) + \frac{\pi}{4}l^2\gamma(t) - r\gamma_1 l^{\beta}\rho_1 + F(t,l), \tag{9}$$

$$\lambda(T,l) = 0, \quad l\in[l_0, l_m], \qquad \lambda(t,l_m) = 0, \quad t\in[0, T), \tag{10}$$

$$\zeta(t) = \rho_1 + \int_t^T e^{-r(\tau - t)}\frac{\partial}{\partial s} h\left(V(\tau), s_0 + \int_0^{\tau} \frac{ds(\xi)}{d\xi} d\xi \right)\zeta(\tau)d\tau, \tag{11}$$

where $\lambda(t,l)$ and $\zeta(t)$ are unknown dual variables, and

$$F(t,l) = \gamma_0 l^{\beta}\zeta(t)\frac{\partial}{\partial V} h\left(V(t), s_0 + \int_0^{t} \frac{ds(\xi)}{d\xi} d\xi \right), \tag{12}$$

$$\gamma(t) = \int_{l_0}^{l_m} \{-\frac{\partial[g_E(E(t),l)x(t,l)]}{\partial l} - \delta_E(E(t),l)x(t,l)\}\lambda(t,l)dl \,. \tag{13}$$

The variables $\lambda(t,l)$ and $\zeta(t)$ present the in-situ value of trees and sequestered carbon in the soil, respectively.

Equations (5) and (6) provide, for an interior solution, the following necessary conditions.

$$e^{-rt}B_u(x(t,l),u(t,l)) = \lambda(t,l),\ (a.a.)\ t\in[0, T),\ l\in[l_0, l_m];\ \text{and} \tag{14}$$

$$-e^{-rt}\rho_2(t) = \lambda(t,l_0),\ t\in[0,T). \tag{15}$$

According to Equation (14) the discounted net benefits of logging have to be equal to the in-situ value of the trees for almost all t and l along the optimal path. Likewise, Equation (15) states that the discounted cost of planting a young tree has to be identical to the in-situ value of a young tree along the optimal trajectory. Equation (7) describes the dynamics of the forest as discussed above. However, due to the brevity of this exposition, the discussion is not repeated here. The subsequent line presents the boundary conditions of the state variable x., i.e., the initial diameter distribution of the trees and the inflow of newly planted trees. Equation (9) indicates that the changes in the value of a standing tree (in situ value) over time and diameter have to correspond to the changes in the value if the tree were cut. In this respect the interpretation of Equation (9) can be seen as a generalization of the Faustmann Formula. The change in the in-situ value over diameter in Equation (9) is a composite expression given by $g(E(t),l)\partial\lambda(t,l)/\partial l$ and reflects the physical growth in diameter over time evaluated by the change in the in-situ value over diameter. The right-hand side of Equation (9) indicates that if we had cut trees, the earned interest of the invested money from the sale of these trees would have been $r\lambda$ and the value of the trees that had not died naturally should have been $\delta\lambda$. Likewise, if we had cut trees the maintenance cost of the remaining trees, $-B_x > 0$, would diminish, which would contribute positively to the net benefits. In addition, the expression ${}^{\pi}\!/_{4}\, l^2\gamma(t)$ would reflect the monetary value of the improved conditions for the growth of the remaining trees due to the decrease in intraspecific competition if we had cut trees. Moreover, if we had cut trees we would have foregone the interest paid for the money received for the sequestering carbon that corresponds to the cut trees, leading to a decrease in net benefits given by the term $-r\gamma_1 l^{\beta}\rho_1$. Finally, if we had cut trees the volume of the forest would decrease, leading to a non-positive change in the carbon content of the soil. The monetary value of this change is given by the term $F(t,l)$.The transversality conditions of the decision problem are stated in Equation (10). Equation (11) provides the in-situ value of soil carbon which is given by the price of sequestered carbon plus the discounted value of future changes in soil carbon as its level increases or decreases.

As the decision problem (DP) is based on one partial integrodifferential equation, Eq. (3), and one ordinary integrodifferential equation, Eq. (4), the first order conditions include a system of partial integrodifferential equations. Given the complexity of the resulting equations, an analytical solution of the first order conditions cannot be obtained, and it is necessary to resort to numerical techniques to obtain a solution to the forest manager's decision problem.

To solve a distributed optimal control problem numerically, available techniques such as the gradient projection method (V.M. Veliov, 2003) or the method of finite elements may be appropriate (E. Calvo and R. Goetz, 2001). However, all of these methods require programming complex algorithms that are not widely known. Therefore, we propose a different method called the Escalator Boxcar Train, EBT, used by de Roos (1988) to describe the evolution of physiologically-structured populations. He has shown that this technique is an efficient integration technique for structured population models. In contrast to the other available methods, the EBT can be implemented with standard computer software such as

GAMS (General Algebraic Modeling System), used for solving mathematical programming problems.

Applying the EBT allows the partial integrodifferential equations of problem (DP) to be transformed into a set of ordinary differential equations which are subsequently approximated by difference equations. Besides a brief presentation of the EBT method, Goetz, Xabadia and Calvo (2007) show how this approach can be extended to account for optimization problems by incorporating decision variables. To transform the decision problem (DP), we first divide the range of diameter into n equal parts, and define $X_i(t)$ as the number of trees in the cohort i, being i = 0, 2, 3,…, n, that is, the trees whose diameter falls within the limits l_i and l_{i+1} are grouped in the cohort i. Likewise, we define $L_i(t)$ as the average diameter, $U_i(t)$ as the number of cut trees within cohort i, and $P(t)$ as the number of planted trees in cohort 0.

However, before the decision problem can be solved, all functions have to be specified. While the specification of the benefit and cost functions are straightforward, the growth and mortality functions have to be estimated, based on data generated by a biophysical forest growth simulator. For this purpose we employed the bio-physical simulation model GOTILWA (Growth Of Trees Is Limited by Water).[4] This not only simulates the development of the trees, it also simulates the amount of sequestered carbon in the biomass and in the soil. The next section describes the simulation of forest growth by GOTILWA in detail, and presents the solution to the forest manager's decision problem, i.e., the optimal short- and long-run trajectories of the logging and planting decisions and the evolution of the standing trees in the forest.

3. Empirical analysis

[4] This program has been developed by C. Gracia and S. Sabaté, University of Barcelona, Department of Ecology and CREAF (Centre de Recerca Ecològica i Aplicacions Forestals), Autonomous University of Barcelona, respectively.

The purpose of the empirical analysis is to initially determine the selective-logging regime that maximizes the discounted private net benefits from timber production and carbon sequestration of a stand of Pinus sylvestris over a time horizon of 200 years. Thereafter, the analysis concentrates on the main aspect of the empirical study by establishing to what extent a change in the price of carbon affects the optimal logging regime. For this end we specify the parameters and the functions in Section 3.1, and we analyze the optimal management regime in Section 3.2.

3.1. Data and specification of functions

The net benefit function of the economic model, $B(x(t,l),u(t,l))$, consists of the net revenue from the sale of timber at time *t*, minus the costs of maintenance, which comprise clearing, pruning and grinding the residues. The net revenue is given by the sum of the revenue of the timber sale minus logging costs defined as: $\left[\sum_{i=0}^{n}\left(\rho\left(L_i(t)\right)-vc\right)tv\left(L_i(t)\right)mv\left(L_i(t)\right)U_i(t)\right]-\left[mc\left(X(t)\right)\right]$

where $X(t)=\sum_{i=0}^{n}X_i(t)$. The terms in the first square brackets denote the sum of the revenue of the timber sale minus the cutting costs of each cohort *i*, and the term in the second square bracket, $mc\left(X(t)\right)$, accounts for the maintenance costs. The parameter $\rho(L_i)$ denotes the timber price per cubic meter of wood as a function of the diameter; $tv(L_i)$ is the total volume of a tree as a function of its diameter; $mv(L_i)$ is the part of the total volume of the tree that is marketable; *vc* is variable cutting cost, and *fc* is the fixed cutting cost. Timber price per cubic meter was taken from a study by Palahí and Pukkala (2003), who analyzed the optimal management of a *Pinus sylvestris* forest in a clear-cutting regime. They estimated a polynomial function given by $\rho(L)=Min\left[-23.24+13.63\sqrt{L},\,86.65\right]$, which is an increasing and strictly convex function, for a diameter lower than 65cm. At L = 65 the price reaches its maximum value, and for L>65 it is considered constant. Data about costs were provided by the consulting firm Tecnosylva, which elaborates forest management plans throughout Spain. The logging cost comprises logging, pruning, cleaning the underbrush, and collecting and removing residues, and it is given by 15 € per cubic meter of logged timber. According to the data supplied by Tecnosylva, the maintenance cost function is approximated by $mc(X(t))$, and is given by $mc(X(t)) = 44.33 + 0.0159X + 0.0000186X^2$. The planting cost is linear in the amount of planted trees and is given by $C(P) = 0.73P$. The thinning and planting period, Δt, is set at 10 years, which is a common practice for a *Pinus sylvestris* forest (I. Cañellas et al., 2000).

To proceed with the empirical study, various initial diameter distributions of a forest were chosen. These distributions were specified as a transformed beta density function $\theta(l)$ since it is defined over a closed interval and allows a great variety of distinct shapes of the initial diameter distributions of the trees to be defined (W. Mendenhall et al., 1990). The initial forest consists of a population of trees with diameters within the interval 0 cm ≤ l ≤ 50 cm. The density function of the diameter of trees, $\theta(l;\gamma;\varphi)$, is defined over a closed interval, and thus the integral

$$\int_{l_1}^{l_{1+1}} \theta(l;\gamma,\varphi)\,dl \tag{16}$$

gives the proportion of trees lying within the range $[l_i, l_i+1)$. We defined l_0 = 0 and l_m = 80 as minimum and maximum diameters of the tree, respectively. For the simulation of the forest dynamics, we concentrate on the diameter interval [0, 50], because thereafter the growth rate of the trees is very small. This interval was divided into 10 subintervals of identical length. In this way, the diameters of the trees of each cohort differ at most by 5 cm, and the size of the trees of each cohort can be considered as homogeneous.

To determine the forest and soil dynamics, the growth of a diameter-distributed stand of *Pinus sylvestris* without thinning was simulated with the bio-physical simulation model GOTILWA. This model simulates growth and mortality and explores how the life cycle of an individual tree is influenced by the climate, the characteristics of the tree itself and environmental conditions. The model is defined by 11 input files specifying more than 90 parameters related to the site, soil composition, tree species, photosynthesis, stomatal conductance, forest composition, canopy hydrology and climate. We simulated the growth of the forest for over 300 years based on the previously specified initial diameter distributions. After that, the growth process practically comes to a halt so the time period of 300 years is sufficient to determine the factors affecting the growth of the trees and carbon sequestration processes in the biomass and the soil.

The data generated from the series of simulations allow function $g(E, L_i)$, which describes the change in diameter over time, to be estimated. This type of function was specified as a von Bertalanffy growth curve (L. von Bertalanffy, 1957), generalized by Millar and Myers (1990), which allows the growth rate of the diameter to vary with environmental conditions specified as the total basal area of all trees whose diameter is greater than that of the individual tree. The precise specification of the function is given by $g(E, L_i) = (l_m - L_i)(\beta_0 - \beta_1 \cdot E)$. The exogenous variables of this function are diameter at breast height (L_i) and basal area (E) provided by GOTILWA. The parameters β_0 and β_1 are proportionality constants and were estimated by OLS. The estimation yielded the growth function $g(E, L_i) = (80 - L_i)(0.0070177 - 0.000043079E)$. Other functional forms of $g(E, L_i)$ were evaluated as well, but they explained the observed variables to a lesser degree. As GOTILWA only allows the survival or death of an entire cohort to be simulated, but not the survival or death of an individual tree, it was not possible to obtain an adequate estimation of function

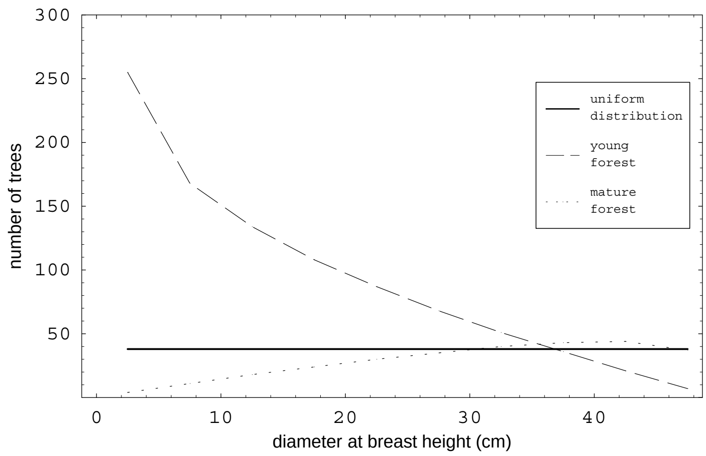
$\delta(E, L_i)$, which describes the mortality of the forest. Nevertheless, information provided by the company Tecnosylva suggests that the mortality rate in a managed forest can be considered almost constant over time and independent of the diameter. Thus, according to the data supplied by Tecnosylva, $\delta(E, L_i)$ was made constant over time and equal to 0.01 for each cohort.

The value of the tree volume parameters, $tv(L_i)$, has also been estimated using the data generated with GOTILWA. The tree volume is based on the allometric relation $tv(L)=0.00157387L^{1.745087}$. A study by Cañellas et al. (2000) provides information that allows the marketable part of the tree volume, $mv(L_i(t))$, to be estimated as a function of the diameter. The marketable part of the timber volume of each tree is an increasing function of the diameter and is given by $mv(L)=0.699+0.0004311L$.

In addition, the data obtained from the GOTILWA simulations allowed us to estimate the change in soil carbon over time as a function of the forest tree volume and the amount of soil carbon. The estimation yielded the function $h=(212.12-s(t))(-0.0322+0.0003385\ V(t))$. Finally we assumed that the sequestered carbon in the biomass is released linearly, i.e., $\omega(PE(l))=1/PE(l)$ and $PE(l)=PE$. In accordance with the literature (G. C. van Kooten and E. Bulte, 1999), we assumed that the carbon is sequestered up to 10 years, i.e., $PE=10$. Hence we have $\upsilon=\int_0^{10} e^{-0.02\tau}/10\ d\tau=0.91$. This permanent time corresponds to timber which is used in fast decaying wood products such as paper and cardboard (D.W. McKenney et al., 2004). In the case in which the timber is used in slow decaying wood products such as lumber the permanent time may increase to 80-100 years (J.K. Winjum et al., 1998). To analyze the different situations, the results of our analysis are obtained for permanent times which range from 0 to 80 years.

For a given initial diameter distribution of the trees, and given specifications of the economic and biophysical functions of the model, a numerical solution of the decision problem (DP) can be obtained. To proceed with the empirical study, three different initial diameter distributions of a forest were chosen. They were obtained by varying the parameters γ and φ of the beta density function, and are depicted in Figure 1. They stand for a young forest, a mature forest, and a forest in which trees are distributed uniformly over the diameter range. The initial basal area of all three distributions is identical and equal to 25m^2/ha so that the resulting optimal management regimes can be compared.

Figure 1. Initial distributions used in the study



3.2. Analysis of the optimal selective-logging regime

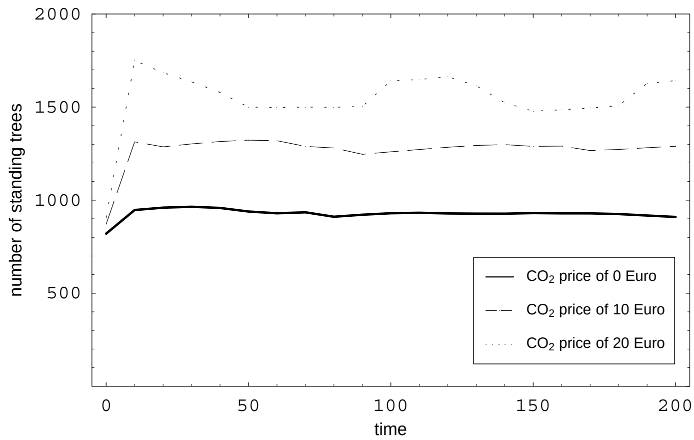
In this part of the empirical study we determine the optimal management regime of a size-distributed forest and examine how the optimal regime is affected by the carbon price on the market. The optimizations were carried out with the CONOPT3 solver available in the GAMS optimization package. For a given initial distribution, the numerical solution of the problem determines the optimal trajectories of the decision variables, logging, U_i, and planting, P, and of the state variables, number of trees, X_i, and diameter, L_i. Consequently, it also determines the value of the economic variables, such as the net revenue from timber sales, logging costs, planting costs, maintenance costs and revenues from carbon storage. All optimizations were carried out on a per-hectare basis.

The empirical analysis begins by calculating the optimal selective-logging regime for the young forest distribution, given an initial basal area of 25 m^2/ha, and a discount rate of 2%.[5] Figure 2 depicts the evolution of the number of standing trees and the carbon in the forest over time for different levels of the carbon price. It shows that the price of sequestered carbon, expressed in terms of CO_2, has a significant influence on the optimal selective-logging regime. When the price increases from zero (base case) to 20 €/CO_2 ton, it is optimal to increase the investment in the forest. Therefore, the number of planted trees increases from 202 to 888 in the first period, and the number of logged trees decreases from 115 to 28.[6] As a result, the number of standing trees in the forest increases, especially in the initial periods of the time horizon, and the amount of carbon sequestered increases in the long run (see Figure 2a). For a price of 0 €/CO_2 ton, the amount of carbon in the forest ecosystem at the end of the planning horizon, including soil carbon and the carbon stored in the above-ground and below ground biomass, is 109.5 tons/ha. However, it increases to 200.5 tons/ha when the carbon price is 20 €/CO_2 ton, that is, the carbon stored in the ecosystem in the long run is almost doubled (see Figure 2b).

[5] The choice of a discount rate of only 2% has been justified by several authors who analyzed optimal forest management (Palahí, M. and Pukkala, T., 2003, and Trasobares, A. and Pukkala, T., 2004).
[6] These results demonstrate how important it is to include planting as a separate decision variable.

Figure 2: Variation in the evolution of the main variables over time for different levels of the carbon price

2a)



2b)

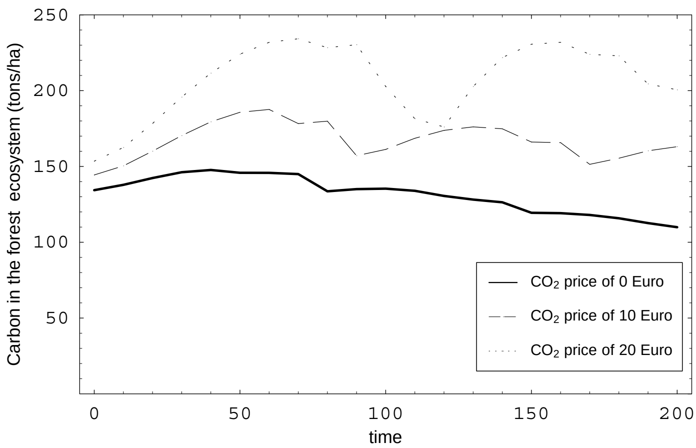
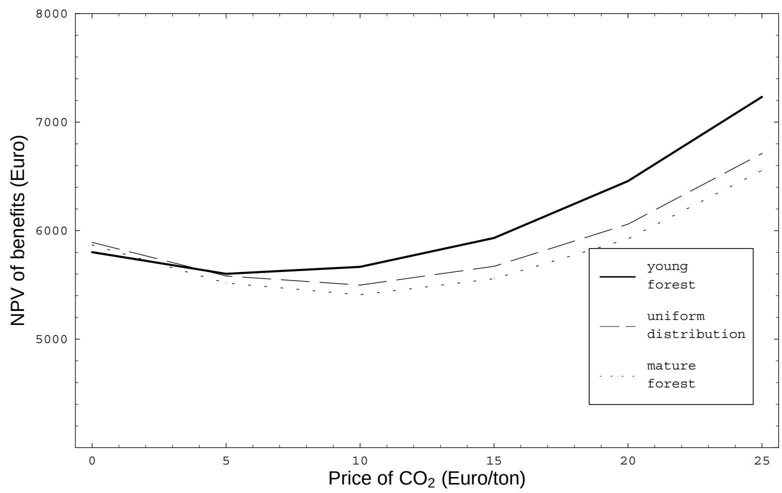


Table 1 presents a more detailed analysis of the numerical results for the case where net benefits only originate from timber (price of 0 €/CO_2 ton) and the case where the price is 10 €/CO_2 ton. In particular, it is important to analyze the discounted sum of the net benefits obtained from forest management. In the remaining part of the paper we will refer to it as the NPV of the benefits for short. One can observe that when carbon sequestration does not form part of the management objective, the NPV of the benefits over 200 years is about 5800 €/ha. However, when the benefits and costs of carbon sequestration are incorporated in the formulation of the decision problem, the NPV of the benefits decreases to 5666 €/ha. This result can be explained by the fact that the forest owner needs to pay when the net carbon flux is negative. Hence, the discounted sum of carbon revenues, given a price of 10 €/CO_2 ton, does not compensate the lower net benefits obtained from the timber sale and the higher maintenance costs resulting from a change in the forest management regime. This is a very significant result, since it implies that forest owners are not likely to enter the carbon market, i.e., they will most probably not adapt their management regime to increase the sequestered carbon of the stand.

Table 1 here

To analyze to what extent this result can be generalized, we conducted a sensitivity analysis for the three considered initial distributions of the forest, and for different levels of the price ranging from 0 to 25 €/CO_2 ton. Figure 3 depicts the NPV of the different optimization scenarios. The figure corroborates the previously obtained result by showing that the NPV of the benefits of forest management decreases with an increase in the price from 0 to 10 €/CO_2 ton (to 15 €/CO_2 ton in the case of the uniform and mature forest distributions). A further increase in the CO_2 price produces an increase in the NPV of benefits. Only when the price reaches 15 €/CO_2 ton (20 €/CO_2 ton in the case of the uniform and mature forest distributions) do forest owners have sufficient incentive to adapt a management regime that favors carbon sequestration.

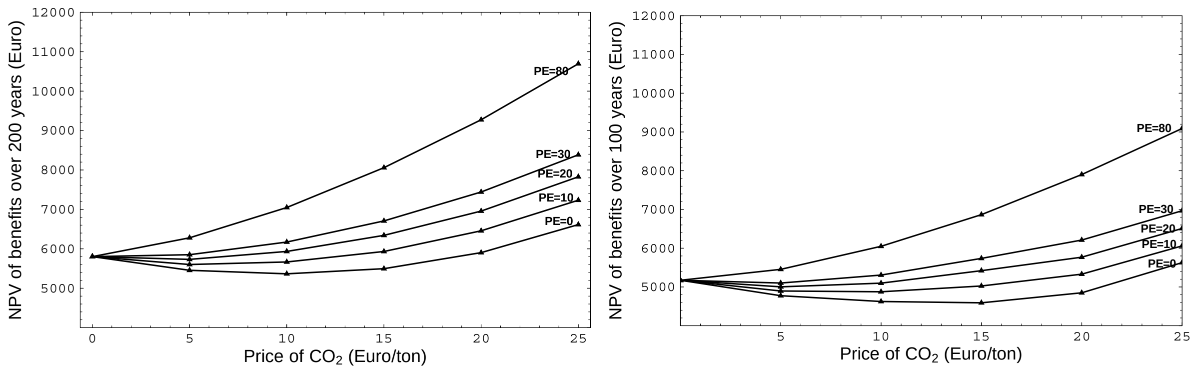
Figure 3: NPV of the benefits over 200 years as a function of the CO_2 price for different initial distributions of the forest



Obviously, one of the key elements of the obtained results is the permanence time, *PE*, of the sequestered carbon in the wood products since it determines the amount that the forest owner has to pay for the release of carbon once the trees have been logged. To analyze this point in more detail, we conducted a sensitivity analysis with respect to *PE* for the young distribution. Figure 4 depicts the results for the case in which the entire sequestered carbon is released immediately once the trees are logged (*PE*=0), and for the cases in which it is released gradually over 10, 20, 30 and 80 years.

The figure shows that an increase in the CO_2 price may lead to a substantial increase in the NPV of benefits if the release of carbon into the atmosphere is very slow. For instance, when the carbon is released into the atmosphere over 70 years, a price of 25 €/CO_2 ton leads to an increase in the NPV of the benefits of 78% compared to the base case. Thus, if carbon is stored long enough in the wood products, there is no trade-off between carbon and timber management. In contrast, when the permanence time and CO_2 price are sufficiently low, maximizing the net benefits from timber and carbon management leads to conflicting objectives. In the extreme case where it is assumed that all carbon is released immediately after logging, only a price of 20 €/CO_2 ton suffices to compensate the loss in timber benefits. As shown in Figure 2a), incorporating the carbon sequestration benefits of the forest management regime requires an initial investment in the forest by planting a large amount of trees and by decreasing the number of logged trees in the first periods. This investment is only retrieved in the long-run. Thus, the negative effects are exacerbated and the losses of the NPV of the benefits over 100 years are even greater than the losses of the NPV over 200 years.

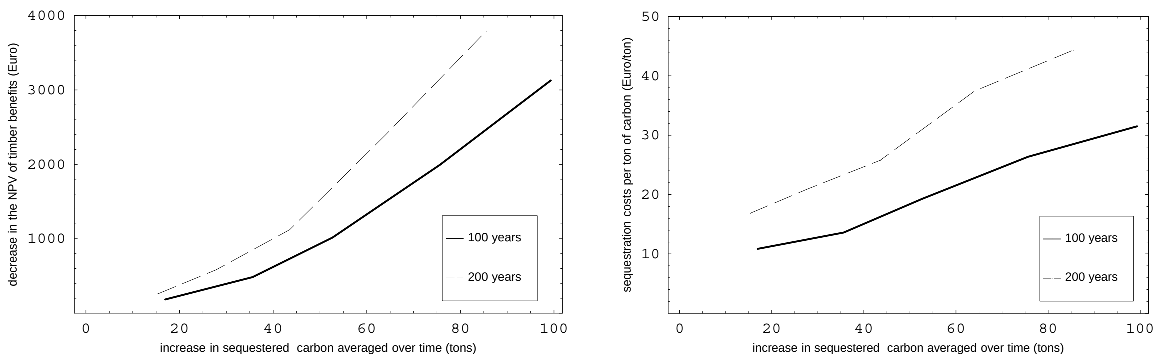
Figure 4: Net present value of the benefits of forest management over 200 years as a function of the CO_2 price for different levels of the permanence time, *PE*.



Our calculations show that an increase in the carbon price induces management regimes that augment the sequestered carbon in the biomass and soil. However, the increase in the sequestered carbon goes along with a decrease in the aggregate discounted net benefit from timber production. The relation between the decrease in the aggregate discounted net benefits of timber production and the increase in sequestered carbon over a time horizon of 100 and 200 years is presented in Figure 5a. It also shows that the decrease in the aggregate discounted net benefits of timber is more pronounced over 100 years than over 200 years. This result can be explained by the fact that a large share of the timber net benefits can only be obtained after 100 years because only then have the trees achieved the optimal logging diameter. The decrease in discounted net benefits of timber production results from an increase in the carbon price which in turn leads to a change in the management regime that favors biomass production at the cost of timber production. Note that carbon payments do not affect timber net benefits and as such they do not have to be considered for the graphs of Figure 5a. Moreover, Figure 5a allows the costs of sequestered carbon to be calculated per ton, presented in Figure 5b. This figure shows that the costs of sequestered carbon per ton are between 11 and 44 € (3 – 12 €/ton of CO_2). Based on an extensive literature review and a meta analysis, van Kooten and Sohngen (2007) state that the carbon sequestration costs in Europe range between 120 and 710 €/tC.[7] In this respect the sequestration costs presented in this study are clearly at the lower end of previous studies. It may seem that the results of our study cannot be compared directly with the results of the previous literature since previous studies predominantly determined the carbon sequestration costs for afforested land. Consequently, the opportunity costs of the land have to form part of the carbon sequestration costs. However, since we are considering a change in the management regime of an already standing forest, the opportunity costs of land affect the aggregate discounted net benefit of timber but not a change in these benefits. In other words, the opportunity costs of the land are independent of the forest management regime and therefore do not need to be taken into account when calculating the costs for additionally sequestered carbon.

Figure 5: Carbon sequestration costs as a function of the increment in the sequestered carbon.

a) Total costs b) Unitary costs



[7] Van Kooten and Sohngen present their results in US$ which we converted into € based on an exchange rate of 1.45 €/$.

4. Conclusions

In this paper we present a theoretical model that allows us to determine the optimal management of a diameter-distributed forest. The theoretical model can be formulated as a distributed optimal control problem where the control variables and the state variable depend on two arguments: time and diameter of the tree. As the resulting necessary conditions of this problem include a system of partial integro-differential equations, the problem cannot be solved analytically. For this reason, a numerical method is employed. The Escalator Boxcar Train technique can be used to solve a distributed optimal control problem by transforming the independent argument, diameter, into a state variable of the system that evolves over time. In this way, the partial integro-differential equation is decoupled into a system of ordinary differential equations, converting the distributed optimal control problem into a classic optimization problem that can be solved by utilizing standard mathematical programming techniques.

An empirical analysis is conducted to determine the optimal selective-logging regime, that is, the selective logging scheme that maximizes the present value of net benefits from timber production and carbon sequestration of a privately owned Pinus sylvestris forest, and to evaluate how the optimal management of the forest is affected by variation in the market prices of carbon sequestration. The study is characterized by the fact that the complex growth process of the trees and the storage of carbon in the soil and in the biomass are taken into account.

The results of our calculations show that an increase in the carbon price leads to an increase in the number of trees in the forest and to an increase in the carbon sequestered in the forest ecosystem. However, the results demonstrate that the net benefits from carbon storage only compensate the decrease in net benefits from wood production once the carbon price has exceeded a certain threshold value. This threshold can be high depending on how long it is assumed the carbon is sequestered in the wood products. In the extreme case in which the forest owner does not have to pay for the release of carbon, it is worthwhile to sequester additional carbon for any given positive price of CO_2. However, when it is assumed that the carbon is released into the atmosphere in a proportional manner over 10 years, even a price of 10 €/CO_2 ton cannot make carbon storage profitable, and therefore the forest manager will not change the selective logging scheme. Hence, the results demonstrate that correctly assessing carbon cycle is crucial for correctly determining the optimal forest and carbon management regime. Most importantly, the results of this study suggest that the carbon sequestration costs are lower for a change in the forest management regime than for a change in land use.

Table 1: Optimal Selective-Logging Regime of a Young Stand

Year	Number of trees	Planted trees	Logged trees	Logged volume (m^3/ha)	Net revenue from timber sale (€/ha)(b)	Maintenance cost (€/ha)	Planting cost (€/ha)	Carbon in the forest (tons/ha)	Carbon revenue (€/ha)	Net benefit (€/ha)	Discounted net benefit (€/ha)
Considering only timber (price =0 €/CO_2 ton)											
0	820	202	115	98.78	3211.29	-699.04	-121.19	134.35	-	2391.06	2391.06
10	947	107	67	42.19	1127.83	-760.98	-64.33	137.87	-	302.51	248.17
20	959	119	85	49.74	1274.32	-767.42	-71.48	142.35	-	435.41	293.02
30	964	132	105	57.78	1430.57	-769.90	-79.13	146.17	-	581.55	321.06
40	958	144	129	68.06	1642.03	-766.57	-86.52	147.66	-	788.93	357.30
50	939	137	154	79.06	1874.82	-756.83	-82.11	145.82	-	1035.88	384.86
60	929	133	137	70.17	1662.47	-752.11	-79.52	145.77	-	830.84	253.23
70	935	163	118	71.39	1872.41	-754.87	-97.50	144.95	-	1020.04	255.04
80	909	121	179	108.66	2850.05	-741.91	-72.45	133.57	-	2035.69	417.54
90	921	129	100	60.31	1576.15	-747.78	-77.07	135.02	-	751.29	126.41
100	929	136	111	66.69	1738.22	-751.93	-81.17	135.31	-	905.12	124.94
Discounted sum over 200 years					10370.58	-4101.38	-467.94		-		5801.26
Considering timber and carbon sequestration (price =10 €/CO_2 ton)											
0	871	502	64	63.87	2229.01	-723.23	-300.35	144.39	-623.24	582.20	582.20
10	1313	52	50	40.29	1236.46	-973.62	-31.41	150.39	251.83	483.26	396.44
20	1286	113	67	48.95	1433.91	-956.29	-67.89	160.09	394.36	804.08	541.12
30	1302	130	84	57.70	1622.53	-966.37	-77.68	170.35	421.81	1000.29	552.23
40	1315	148	104	66.65	1807.70	-974.72	-88.51	179.43	385.66	1130.13	511.83
50	1322	168	127	77.63	2040.84	-979.46	-100.51	185.74	292.35	1253.21	465.60
60	1319	211	158	92.23	2361.97	-977.58	-126.13	187.59	140.77	1399.03	426.40
70	1289	158	229	128.98	3237.85	-957.88	-94.74	178.27	-240.06	1945.17	486.35
80	1280	242	154	86.07	2146.25	-952.28	-144.78	179.86	126.62	1175.81	241.17
90	1246	129	263	170.57	4652.40	-931.04	-77.55	157.07	-701.18	2942.63	495.13
100	1260	129	104	73.04	2090.97	-939.38	-77.26	161.24	210.53	1284.86	177.35
Discounted sum over 200 years					11019.94	-5042.14	-661.09		349.68		5666.39

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