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Generalized Factor Models: A Bayesian Approach

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GENERALIZED FACTOR MODELS: A BAYESIAN APPROACH

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Abstract

There is recent interest in the generalization of classical factor models in which the idiosyncratic factors are assumed to be orthogonal and there are identification restrictions on cross-sectional and time dimensions. In this study, we describe and implement a Bayesian approach to generalized factor models. A flexible framework is developed to determine the variations attributed to common and idiosyncratic factors. We also propose a unique methodology to select the (generalized) factor model that best fits a given set of data. Applying the proposed methodology to the simulated data and the foreign exchange rate data, we provide a comparative analysis between the classical and generalized factor models. We find that when there is a shift from classical to generalized, there are significant changes in the estimates of the structures of the covariance and correlation matrices while there are less dramatic changes in the estimates of the factor loadings and the variation attributed to common factors.

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1 Introduction

Financial and economic activities are characterized by some common movements with booms and depressions as the extreme examples. Cyclical behaviors or business cycles express the same common belief that there is a common movement between the variables. One can also refer to this common behavior as the state of the economy or the state of the market. If this common behavior is strong, it makes sense to represent the state of the economy or the market by an index characterizing the comovement of such economic or financial variables. A method that has been widely used to derive the common components of the variables is factor analysis. A factor model has two significant parts: common factors and idiosyncratic factors. Factor models have many applications in finance and economics: 1. *Asset pricing and risk measurement* (Ross (1976), Geweke and Zhou (1996)); 2. *Monitoring the economy and the market* (Forni, Hallin, Lippi and Reichlin (FHLR 2000), Stock and Watson (1998) and Kose, Otrok and Whiteman (2003)); 3. *Economic indicators and forecasting* (Stock and Watson (2002)); 4. *Trading strategies* (Tong (2006)).

Classical factor models and the need for new factor models. Classical factor models have been widely used in financial and economic applications since mid-1970s; however, these models have some limitations and use some restrictive assumptions (see Bai (2003) and FHLR (2000, 2001)): 1) Classical factor analysis assumes that cross-sectional dimension N is small and fixed. However, there are many large sample problems available these days; 2) Idiosyncratic factors are assumed to be orthogonal; however, this is an unrealistic and restrictive assumption. To relax these assumptions and extend the range of applications of factor models, there has been recent interest in finding more flexible and "generalized" factor models. Generalized factor models are proposed recently by Stock and Watson (1998, 2002), FHLR (2000, 2001), Bai and Ng (2002), Bai (2003) and Boivin and Ng (2006). However, these recently developed "generalized" factor models also have some limitations: 1) They assume weak idiosyncratic cross-sectional correlation; 2) The infinite cross-sectional dimension is the key assumption to identify the common components, idiosyncratic components and factor loadings. As discussed in FHLR (2000), for fixed number of cross-sections reasonable assumptions, without orthogonality, for identification of the factor structure can hardly be found.

In order to overcome these limitations, we develop a *Bayesian generalized factor model* allowing for non-orthogonality of the idiosyncratic factors and the flexibility of cross-sectional dimension. This proposed generalization is mainly for small samples but it can be applied to large samples as well. In this paper, we also evaluate the assumption of orthogonality of idiosyncratic factors and give a comparative analysis between the classical and generalized models to find out if there is a gain from a generalized factor

model. Moreover, we propose a unique methodology to choose the generalized factor model that best fits the given data set.

The assumptions, orthogonality of idiosyncratic factors and restrictions on the cross-sectional dimension, are necessary for identification of factors and the parameters of the models proposed in the current literature; however, their applications are limited to the extend that the data series satisfy these assumptions. Allowing the idiosyncratic factors to be correlated and working with small samples can make the factor analysis framework suited for a wider range of economic applications.

Recent developments and the need for improved models. Among the early studies that introduce factor models as a tool for the study of time series in the economics literature are Geweke (1977) and Sargent and Sims (1977). However, Chamberlain and Rothschild (1983) is the first study to allow for idiosyncratic cross-sectional correlation. They define an *approximate factor structure* and show that a weaker assumption that allows for some correlation in the idiosyncratic factors is sufficient for the result in Ross (1976) that says if there is a factor structure, then the mean asset returns are approximately linear functions of factor loadings. Stock and Watson (1998, 2002) extends this idea and applies in an index model which they call an *approximate dynamic factor model*. They work with asymptotic cross-sections and asymptotic time dimensions while allowing for weak cross-sectional correlation. In three other recent papers, FHLR (2000), FHLR (2004) and Forni and Lippi (2001) propose a factor model with non-orthogonal idiosyncratic components which they call the *generalized dynamic factor model*. Their model combines the characteristics of Geweke (1977) and Chamberlain and Rothschild (1983). Using the asymptotic analysis in both cross-sections and time series, they utilize the principal components method to estimate the common components and idiosyncratic components.

A very recent study on the generalization of the classical factor structure is Bai and Ng (2002) which develops a theory on estimating consistently the number of factors in a factor model within the framework of infinite cross sections N and infinite time dimension T. In another paper, Bai (2003) derives the limiting distributions and the rate of convergence of the estimated factors and factor loadings as N and T go to infinity. In both of these papers, the method of asymptotic principal components is used to estimate the factors and factor loadings.

These studies provide a tractable methodology to replace the classical orthogonality assumption with a more flexible one. However, these papers also have restrictive assumptions to achieve identification of the common and the idiosyncratic components. First, they assume weak idiosyncratic cross-sectional correlation. Second, they have to work with infinite cross-sectional dimensions. A methodology to generalize the classical factor structure for finite samples (either N finite, or both N and T finite) is still miss-

ing. As pointed out in FHLR (2000), reasonable assumptions, without orthogonality, for identification of the idiosyncratic and the common component is hard to find for fixed N. Hence the infinite cross-section is crucial for identification in the generalized models discussed so far.

In addition to these, the recent work by Boivin and Ng also supports the need for an alternative model that allows cross-sectional correlations and heteroscedastic variances for idiosyncratic factors within the framework of both small and large samples. Boivin and Ng (2006) shows that we do not need large data sets for better forecasting; idiosyncratic cross-sectional correlations and heteroscedastic variances of idiosyncratic factors are important assumptions to achieve precise estimation and have better forecasting results; and underestimating the number of factors has large efficiency loss in the factor estimates and forecasts. However, the idiosyncratic factors are correlated when the number of factors are underestimated. Therefore it is also important to know the patterns of covariations left unexplained by the common factors.

Major contributions and the main results of this paper. The main problem with the generalization of the classical factor structure, obviously, is the identification issue. If we can find proper priors on the model parameters, the Bayesian analysis of a non-identified model is always possible (Poirier (1998)). Thus, in this paper we propose a Bayesian approach for *generalized factor models* allowing for non-orthogonality of the idiosyncratic factors and the flexibility of cross-sectional dimension. To the best of our knowledge, this paper is the first Bayesian approach for generalized factor models. Moreover, the generalized factor methodology for small data is still missing; thus, this is the first study for generalized factor analysis of small data. While the proposed model is applied for the small cross-sectional data, it can also be used for the factor analysis of large data sets. Another nice feature of the model is that it can easily be transformed into a classical factor model because of the flexible setup of the model.

This study also provides a unique methodology for model selection. Using Bayes factors, we show how to select the generalized factor model that best fits the given data and characterizes the structure of the correlation and covariance matrices. The proposed model setup provides a flexible approach to determine the variations attributed to common and idiosyncratic factors.

Applying the proposed model to both the simulated data and the foreign exchange rates data, we provide a comparative analysis between the classical and generalized models, and evaluate the assumption of no cross-sectional correlation between the idiosyncratic factors. We find that when there is a shift from classical to generalized, there are significant changes in the estimates of the structure of the correlation matrix while we find less dramatic changes in the estimates of the factor loadings and the variation attributed to common factors.

The paper is organized as follows. Section 2 introduces the proposed generalized factor model. Section 3 describes how to make inference using the Bayesian framework and gives a simulation analysis of the model. Section 4 presents the data and comparative analysis with the foreign exchange rate data. Section 5 provides the model selection. Finally, section 6 summarizes the conclusions.

2 The Generalized Factor Model

In this section, we introduce and examine the generalized factor model and the prior beliefs associated with the model. This section is split into three parts. We describe the model in the first part. Then we introduce the prior beliefs that accompany the model to complete the generalization structure. The mechanics and advantages of the proposed methodology are provided in the last part of this section.

2.1 The Model

The basic structure of the *generalized factor model* proposed in this paper is represented by,

$$y_{it} = \mathbf{\Lambda}_i \mathbf{f}_t + c_{ii}\varepsilon_{it} \tag{1}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. For the i^{th} cross-section, y_{it} is the observed variable at time t, $\mathbf{\Lambda}_i$ is the $1 \times r$ vector of factor loadings, ε_{it} is the idiosyncratic factor at time t and $\mathbf{f}_t$ denotes $r \times 1$ unobservable random factors at time t. In the model, c_{ii} is part of the decomposition of the variance-covariance matrix, which is one of the major components of the generalized factor model in this study. The mechanics of the generalization methodology is provided in section 2.3. A more convenient way to work with the model is the vector form which is expressed in an N dimensional dynamic model:

$$\mathbf{y}_t = \mathbf{\Lambda}\mathbf{f}_t + \mathbf{C}\boldsymbol{\varepsilon}_t \tag{2}$$

in which $\mathbf{y}_t$ is $N \times 1$ observable random variables, $\mathbf{\Lambda}$ is $N \times r$ factor loadings that is $\mathbf{\Lambda} = (\mathbf{\Lambda}_1', \ldots, \mathbf{\Lambda}_N')'$, $\boldsymbol{\varepsilon_t}$ is $N \times 1$ idiosyncratic factors, and $\mathbf{C} = diag(c_{11}, \ldots, c_{NN})$. Further assumptions on the model are:

$$\mathbf{f}_t \overset{i.i.d.}{\sim} N(\mathbf{0}, \mathbf{I}_r) \tag{3}$$

$$\boldsymbol{\varepsilon}_t \overset{i.i.d.}{\sim} N(\mathbf{0}, \mathbf{\Sigma}) \tag{4}$$

where $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})'$ and $\mathbf{\Sigma}$ is $N \times N$ positive definite matrix. Also, we assume that $Cov(\boldsymbol{\varepsilon}_{it}, \mathbf{f}_{jt}) = 0$ for all i,j and t. Orthogonality of the factors is a common

assumption to exploit the information from the data. We also assume $N > r$, that is, factor structure of the data is not more complicated than that of the observed data. Otherwise, there would not be much gain from the factor analysis. One should notice that variables on the left hand side of the equation (2) are observable whereas none of the variables on the right hand side are observable.

A significant difference in this setup is the allowance for cross-correlations across idiosyncratic components which is represented by the positive definite variance-covariance matrix $\mathbf{\Sigma}$. Thus, the cross sections of observables vector y_t can be contemporaneously correlated through both common factors $\mathbf{f}_t$ and idiosyncratic errors $\boldsymbol{\varepsilon}_t$. The variance-covariance matrix $\mathbf{\Sigma}$ and the diagonal matrix $\mathbf{C}$ are the two components of the model that allows the flexibility and the generalization of the classical factor models.

An additional assumption is on the factor loadings to avoid identification problems arising because of location shifts of the factor loading matrix (see Geweke and Zhou (1996)). The factor loading matrix is assumed to be of the following form:

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_{11} & 0 & 0 & \dots & 0 \\ \lambda_{21} & \lambda_{22} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{r1} & \lambda_{r2} & \lambda_{r3} & \dots & \lambda_{rr} \\ \lambda_{r+1,1} & \lambda_{r+1,2} & \lambda_{r+1,3} & \dots & \lambda_{r+1,r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \lambda_{N3} & \dots & \lambda_{Nr} \end{pmatrix} \tag{5}$$

where $\lambda_{ii} > 0$, $i = 1, \dots, r$.

One can observe that the specification of the model in equation (2) implies that the unconditional distribution of the observable variable is

$$\mathbf{y}_t \overset{i.i.d.}{\sim} N(\mathbf{0}, \mathbf{\Lambda\Lambda'} + \mathbf{C\Sigma C}) \tag{6}$$

We denote the covariance matrix of the observables by $\mathbf{\Omega}$ which is linked to the parameters by $\mathbf{\Omega} = \mathbf{\Lambda\Lambda'} + \mathbf{C\Sigma C}$.

2.2 Prior Distributions

The Bayesian method assigns uncertainty to the model parameters. Therefore parameters are assumed to be random variables in the Bayesian framework. Hence, prior distributions for the parameters, in addition to the data distribution, are defined in this subsection.

We consider a set of informative priors on the parameters of the model. Let $\mathbf{\Lambda_i^*} = (\boldsymbol{\lambda_{i1}}, \ldots, \boldsymbol{\lambda_{ii}})$ for $i = 1, \ldots, r$ and $\mathbf{\Lambda_i^*} = \mathbf{\Lambda_i}$ for $i = r+1, \ldots, N$. Let $\boldsymbol{\lambda} = (\mathbf{\Lambda_1^*}, \ldots, \mathbf{\Lambda_N^*})'$ be the $[(2N+1-r)r/2] \times 1$ vector of nonnegative elements of $\mathbf{\Lambda}$. We adopt the following class of priors on $\boldsymbol{\lambda}$ (or, equivalently $\mathbf{\Lambda}$), $\mathbf{C}$ and $\mathbf{\Sigma}$:

$$c_{ii} \overset{i.i.d.}{\sim} HN(\mathbf{0}, \underline{h}_c^{-1}), i = 1, \ldots, N \tag{7}$$

$$\boldsymbol{\lambda} \sim N(\underline{\boldsymbol{\lambda}}, \underline{\mathbf{H}}_{\lambda}^{-1}), \lambda_{ii} > 0, i = 1, \ldots, r \tag{8}$$

$$\underline{\nu}\mathbf{\Sigma}^{-1} \sim W(\mathbf{I}_N, \underline{\nu} + N + 1), \underline{\nu} > 2 \tag{9}$$

In the prior specification for $\mathbf{C}$, HN denotes a normal distribution truncated below at 0 and $\underline{h}_c$ is the precision hyperparameter. Prior for $\boldsymbol{\lambda}$ is normal with mean $\underline{\boldsymbol{\lambda}}$ and positive definite precision matrix $\underline{\mathbf{H}}_{\lambda}$. In this prior setup, factor loadings are allowed to be correlated in the prior, which gives more flexibility and interaction between loadings. The constraint $\lambda_{ii} > 0$ is for identification purposes due to the structural form in (5). The covariance matrix $\mathbf{\Sigma}$ is assumed to have a Wishart prior with the scale parameter $\underline{\nu}$, the identity scale matrix $\mathbf{I}_N$ and the degrees of freedom equal to the sum $\underline{\nu}+N+1$. The condition $\underline{\nu} > 2$ is derived from the restrictions of the Wishart distribution explained in Appendix A. How these prior specifications give us the flexibility to generalize the factor model is discussed in the next section.

2.3 Advantages of the Proposed Model Structure

In this section, we provide several advantages of the proposed model structure given by (2)-(4) and (7)-(9), and show how this structure generalizes the classical factor model. Lawley and Maxwell (1971) and Timm (2002) can be referred to for detailed description of the classical factor models. The main problem with the generalization of the classical factor structure is the identification issue. Therefore we first clarify the concept of identification and the related problems in the generalized factor framework in order to understand how the proposed approach in this study uniquely identifies factors, factor loadings and other parameters of interest. Then we explain further advantages of the model to show the usefulness of the flexible structure of the model in generalization, model selection and empirical applications.

Identification. Kadane (1974) provides that identification concept is a property of the likelihood function $L(\theta; Y)$ where, for example, $\theta = (\mathbf{\Lambda}, \mathbf{C}, \mathbf{\Sigma})$ in our case and, hence, is the same in both Bayesian and non-Bayesian analysis. Therefore the solution to the identification problem requires more information; however, this is usually not an additional data information but a non-data information. Although there is a consensus that the concept of identification and the solution requirement are the same whether a Bayesian or a non-Bayesian framework is considered, approaches to introduce additional

information to achieve the identification differ from one another. As Poirier (1998) indicated, if a proper prior is specified on all the parameters, a Bayesian analysis of a nonidentified model is always possible. The reader can refer to Poirier (1998) for more demonstrations and further discussions of the identification issue. On the other hand, a non-Bayesian approach requires additional restrictions and assumptions on the unknown parameters until they are identified.

Now we can specifically analyze the identification problem in the generalization of the factor structure. Let $\mathbf{\Phi}$ represent the idiosyncratic variance-covariance matrix in a generalized factor model –for example, $\mathbf{\Phi}$ is equal to $\mathbf{C\Sigma C}$ in our model– in order that $\mathbf{\Phi}$ is some $N \times N$ positive definite symmetric matrix and the variance-covariance matrix of the observables density is $\mathbf{\Omega} = \mathbf{\Lambda\Lambda'} + \mathbf{\Phi}$. Then the likelihood is $L(\mathbf{\Omega}; Y)$ or equivalently $L((\mathbf{\Lambda}, \mathbf{\Phi}); Y)$. The data would determine only the covariance matrix $\mathbf{\Omega}$; however, $\mathbf{\Lambda}$ and $\mathbf{\Phi}$ would not be identified because the right hand side of $\mathbf{\Omega} = \mathbf{\Lambda\Lambda'} + \mathbf{\Phi}$ has $(N^2+3N+2r)/2$ unknowns, whereas the left hand side has $(N^2+N)/2$ determined distinct elements. So there is clearly an identification problem. Proposing a Bayesian approach, we solve the identification problem by introducing the information through prior densities. We introduce proper priors for the model parameters $(\mathbf{\Lambda}, \mathbf{C}, \mathbf{\Sigma})$. This leads to proper posteriors and ensures the existence of posterior moments such as posterior means and posterior medians for all the parameters.

In addition, we need the assumption that $\mathbf{\Lambda}$ be of full column rank r to avoid identification problems arising through invariance of the model under location shifts of the factor loading matrix. Factors and factor loadings should be invariant under linear transformations. On this issue, following the setup used in the literature such as Aguilar and West (2000) and Geweke and Zhou (1996), we adopt the structural constraint of the form given in expression (5). This guarantees that $\mathbf{\Lambda}$ is of full column rank r. This structural form will ensure the invariance of the component $\mathbf{\Lambda\Lambda'}$ and uniquely identify the factors and loadings. This is just a technical model identification. It has no impact on the estimates of the observable variances, covariances, or their decompositions.

Flexibility of the Model and the Generalization Issue. After studying the identification problem, our next issue is to give a full analysis of the prior beliefs that have significant role on the generalization of the classical factor model. Normal prior on $\mathbf{\Lambda}$ is a standard prior assumption for the coefficients. The critical part of the model setup is the construction of the covariation attributed to idiosyncratic factors; that is, the covariance matrix of the idiosyncratic components $\mathbf{C\Sigma C}$ and the prior beliefs on the parameters of this covariance structure. This part requires particular attention since it characterizes the generalization of the classical factor analysis. What are the implications of the structural form of the idiosyncratic covariance matrix $\mathbf{C\Sigma C}$ for the generalization? What do the priors associated with this covariance structure, specifically priors on $\mathbf{C}$ and $\mathbf{\Sigma}$, imply for this task? Now we will discuss this important

feature of the model.

First of all, a nice feature of the generalized model is that if $\mathbf{\Sigma} = \mathbf{I}_N$, then the factor model defined in (2), (3) and (4) fall into the classical factor modeling where idiosyncratic components are cross-sectionally uncorrelated. In this special case, the variance-covariance matrix of the observables is $\mathbf{\Omega} = \mathbf{\Lambda}\mathbf{\Lambda}' + \mathbf{C}\mathbf{C}$. In the proposed model setup, $\underline{\nu}$ is the key parameter to determine the amount of idiosyncratic cross-sectional correlation. In addition to the structural form $\mathbf{C}\mathbf{\Sigma}\mathbf{C}$, having $\underline{\nu}$ in front of $\mathbf{\Sigma}$, setting the degrees of freedom equal to $\underline{\nu} + N + 1$ and also setting the scale matrix equal to $\mathbf{I}_N$ in the prior provide three advantages. First, the size of the scale parameter $\underline{\nu}$ gives us the flexibility to set the level of idiosyncratic correlations. As $\underline{\nu}$ increases [decreases], $\mathbf{\Sigma}$ gets tightened [loosened] up at around the scale matrix $\mathbf{I}_N$. At the extreme case, $\mathbf{\Sigma} \xrightarrow{p} \mathbf{I}_N$ as $\underline{\nu} \to \infty$; this leads us to the classical factor model. The algebraic derivation of this idea is available in Appendix A. Second, as $\underline{\nu}$ changes, only variances and covariances of the elements of $\mathbf{\Sigma}$ change whereas the mean of $\mathbf{\Sigma}$ is kept the same. This is what we desired because we just intend to investigate how the parameter estimates change with different levels of idiosyncratic correlations. We just do not want mean shifts; therefore $\mathbf{\Sigma}$ is centered at $\mathbf{I}_N$ for any value of $\underline{\nu}$. This is also demonstrated in Appendix A. Third, as N changes, neither variances and covariances nor the mean changes in the prior belief, technical details of which are provided in Appendix A. So the prior beliefs are the same whether we include additional observable variables in the model or not.

On the other hand, the diagonal matrix $\mathbf{C}$ in the covariance structure is a technical part to handle the estimation of idiosyncratic variance-covariance matrix $\mathbf{C}\mathbf{\Sigma}\mathbf{C}$ as the estimate of $\mathbf{\Sigma}$ changes due to a change in the size of $\underline{\nu}$. For example, when $\underline{\nu}$ is sufficiently large, $\mathbf{\Sigma}$ is almost equal to the identity matrix. In this case, idiosyncratic standard deviations are estimated by diagonal elements of $\mathbf{C}$. Hence, $\mathbf{C}$ displays as a scaling depending on whether $\mathbf{\Sigma}$ has a loose or tight prior.

The major technical contribution of this paper is the specification characterized by the covariance structure $\mathbf{C}\mathbf{\Sigma}\mathbf{C}$ and the priors in (7) and (9). This is a unique specification that can be used to determine the covariance and correlation structures between idiosyncratic components in factor analysis or between error terms in linear models.

Model Selection. Another advantage of the proposed model structure is about the model selection. We use Bayes factors to choose the model that best explain the correlation structure of the data, which is a model comparison between a "more" classical model and a "more" generalized model. The calculation of the Bayes factor for model selection in this study is quite straightforward because different factor models are characterized by the priors, (7)-(9), rather than the body of the model, (2). This idea is explained in detail in section 5.

Small Data or Large Data. One final comment about the proposed model is its usefulness in the applied analysis. For the convenience of many repeated experiments and simulations for this study, we take a small number of cross-sections; however, the model works for the estimation and inference in both small sample and large sample analysis. We applied the model for large cross-sections of up to sixty with simulated data but we do not want to discuss the large data analysis in this study because this is beyond the scope of this paper.

3 Bayesian Inference for the Factors and Parameters

Bayesian inference requires us to compute the posterior distributions and sample from these distributions. Therefore we start this section with posterior densities and sampling algorithms. Then we discuss the verification of the accuracy of the analytic derivations, posterior simulators and the computer coding.

3.1 Posterior Distributions and Sampling

Explicit forms of the posterior densities for the model parameters are not available and the closed forms of the conditional posterior densities can not be derived for some parameters. Therefore, we need to employ hybrid Markov Chain Monte Carlo (MCMC) sampling algorithms in order to get the draws from each posterior density. Let $\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_T)$. The kernel of the joint posterior p.d.f. $p(\mathbf{C}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}, \mathbf{F}|\mathbf{Y})$ is proportional to the multiplication of the prior densities and the observables density:

$$p(\mathbf{C}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}, \mathbf{F}|\mathbf{Y}) \propto p(\mathbf{C})p(\boldsymbol{\lambda})p(\boldsymbol{\Sigma})p(\mathbf{F})p(\mathbf{Y}|\mathbf{C}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}, \mathbf{F}) \tag{10}$$

We solve out the following conditional posteriors from this kernel of joint posterior of parameters for simulation purposes in MCMC sampling.

We do not have a known distributional form for the conditional posterior of $\mathbf{C}$. The kernel of the conditional posterior distribution of $\mathbf{C}$, $p(\mathbf{C}|\mathbf{Y}, \boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \mathbf{F})$, is given as in the following function:

$$exp\{-\frac{1}{2}(\underline{h}_c \sum_{i=1}^{N} c_{ii}^2 + \sum_{t=1}^{T} \mathbf{z}_t)\}(\prod_{i=1}^{N} c_{ii})^{-T} \prod_{i=1}^{N} I_{[0,\infty)}(c_{ii}) \tag{11}$$

where $\mathbf{z}_t = (\mathbf{y}_t - \boldsymbol{\Lambda}\mathbf{f}_t)'(\mathbf{C}\boldsymbol{\Sigma}\mathbf{C})^{-1}(\mathbf{y}_t - \boldsymbol{\Lambda}\mathbf{f}_t)$. We apply the Metropolis within Gibbs sampling algorithm to draw from the conditional posterior of $\mathbf{C}$. Metropolis sampling is applied to draw from each c_{ii} conditional on other c_{jj}'s ($j \neq i$) and other parameters of

the model. Gibbs sampling part of this algorithm enters when this drawing is recursively applied for each c_{ii}.

We need to introduce an additional notation before we continue the characterization of the conditional posterior of $\boldsymbol{\lambda}$. Let $\mathbf{F}_i$ be the matrix consisting of the first i columns of $\mathbf{F}$ for $i = 1, \ldots, r$ and $\mathbf{F}_i = \mathbf{F}$ for $i = r+1, \ldots, N$. Then let $\mathbf{F}^* = diag(\mathbf{F}_1, \ldots, \mathbf{F}_N)'$. The conditional posterior of $\boldsymbol{\lambda}$ (or, equivalently $\boldsymbol{\Lambda}$) is a multivariate normal distribution

$$\boldsymbol{\lambda}|(\mathbf{Y}, \mathbf{C}, \boldsymbol{\Sigma}, \mathbf{F}) \sim N(\overline{\boldsymbol{\lambda}}, \overline{\mathbf{H}}_\lambda^{-1}) \tag{12}$$

where

$$\overline{\mathbf{H}}_\lambda = \underline{\mathbf{H}}_\lambda + \mathbf{F}^{*'}(\mathbf{C}\boldsymbol{\Sigma}\mathbf{C} \otimes \mathbf{I}_T)^{-1}\mathbf{F}^*$$
$$\overline{\boldsymbol{\lambda}} = \overline{\mathbf{H}}_\lambda^{-1}[\underline{\mathbf{H}}_\lambda.\underline{\boldsymbol{\lambda}} + \mathbf{F}^{*'}(\mathbf{C}\boldsymbol{\Sigma}\mathbf{C} \otimes \mathbf{I}_T)^{-1}\mathbf{Y}]$$

with a truncation of the elements λ_{ii}'s, $i = 1 \ldots r$, below at 0.

The conditional posterior of $\boldsymbol{\Sigma}$ is inverted Wishart in the following form

$$\underline{\nu}\boldsymbol{\Sigma}^{-1}|(\mathbf{Y}, \mathbf{C}, \boldsymbol{\Lambda}, \mathbf{F}) \sim W(\overline{\mathbf{G}}^{-1}, \underline{\nu} + T + N + 1) \tag{13}$$

where the posterior scale matrix is defined by $\overline{\mathbf{G}} = \mathbf{I}_N + \mathbf{A}\mathbf{A}'/\underline{\nu}$ in which the $N \times T$ matrix $\mathbf{A} = (\mathbf{A}_1, \ldots, \mathbf{A}_T)$ and the $N \times 1$ matrix $\mathbf{A}_t = \mathbf{C}^{-1}(\mathbf{y}_t - \boldsymbol{\Lambda}\mathbf{f}_t)$.

The conditional posterior of $\mathbf{F}$ is a multivariate normal distribution. For $t = 1, ..., T$, the conditional posterior of $\mathbf{f}_t$ is of the form

$$\mathbf{f}_t|(\mathbf{Y}, \mathbf{C}, \boldsymbol{\Lambda}, \boldsymbol{\Sigma}) \sim N(\overline{\boldsymbol{\mu}}_t, \overline{\mathbf{H}}^{-1}) \tag{14}$$

where $\overline{\boldsymbol{\mu}}_t = \boldsymbol{\Lambda}'(\boldsymbol{\Lambda}\boldsymbol{\Lambda}' + \mathbf{C}\boldsymbol{\Sigma}\mathbf{C})^{-1}\mathbf{y}_t$ and $\overline{\mathbf{H}} = [\mathbf{I}_r - \boldsymbol{\Lambda}'(\boldsymbol{\Lambda}\boldsymbol{\Lambda}' + \mathbf{C}\boldsymbol{\Sigma}\mathbf{C})^{-1}\boldsymbol{\Lambda}]^{-1}$. Furthermore, $Cov(\mathbf{f}_t, \mathbf{f}_s|\mathbf{Y}, \mathbf{C}, \boldsymbol{\Sigma}, \boldsymbol{\Lambda}) = \mathbf{0}$ for $t \neq s$.

In all the posterior sampling studies of the paper, we use the following MCMC algorithm. We adopt the Gibbs sampling algorithm as our MCMC procedure to sample from the posterior densities. However the Metropolis within Gibbs sampling is applied to draw from the joint kernel for the diagonal elements of $\mathbf{C}$. Here is how the full MCMC is run:

• First, in order to draw from the joint kernel for the diagonal elements of $\mathbf{C}$, we draw each diagonal element c_{ii} once at a time conditional on the others. Hence, for $i = 1, \ldots, N$, we draw from the kernel of c_{ii} conditional on c_{jj}'s, $j \neq i$.

• Second, we draw $\boldsymbol{\lambda}$'s from the multivariate Normal posterior in (12).

• Third, we draw $\boldsymbol{\Sigma}$'s from the Wishart posterior in (13).

• Last, we draw $\mathbf{F}$'s equation by equation from the multivariate Normal posterior in (14).

Let the output from the full MCMC run be denoted by $\{\mathbf{C}^{(m)}, \boldsymbol{\Lambda}^{(m)}, \boldsymbol{\Sigma}^{(m)}, \mathbf{F}^{(m)}\}_{m=1}^M$ for future reference.

3.2 Simulation Analysis

We have to verify the accuracy of the analytic derivations, posterior simulators explained in the previous section and computer coding incorporating the ideas before we proceed to the applications of the proposed model. It is essential that all of these be error-free in order to have reliable simulated and empirical results. In all of the data generations below to run the tests, we set $N = 5$, $T = 20$ and $r = 2$ and hyperparameters in the priors are simply taken as $\underline{h}_c = 1$, $\underline{\boldsymbol{\lambda}} = 0$, $\underline{\mathbf{H}}_{\lambda}$ equal to identity matrix and $\underline{\nu} = 120$. Accuracy of simulation results are also verified with various other values of model parameters. Furthermore, number of iterations is 10000, 500 of which are used in the burn-in period.

We first conduct the joint distribution test proposed in Geweke (2004) which is designed to detect the errors in posterior simulators. These errors could be in the analytic derivations or in the computer coding. In other words, this is a test for the consistency of the posterior simulator explained in the previous subsection with the data density (6) and prior densities (7), (8) and (9). Some of the test functions and their chi-square probability values are reported in Table 1. These joint distribution tests suggest that our posterior simulators seem error-free.

In order to further check for the accuracy of our procedure, we test the convergence of the algorithm using artificially created data. So we run an experimental study to highlight the convergence of the posterior simulator. First, we create observable data and latent factors using the expressions (2), (3) and (4). We call these artificially created factors the true factors. Parameters of the model are constructed using the priors specified in section 2.2 with the hyperparameters listed above. We use the MCMC algorithm described in section 3.1 to estimate the unobservable factors and unobservable model parameters created artificially. We confirm the convergence of the algorithm using different segments of the entire simulation. Chi-square probability values for some selected parameters are presented in Table 2.

4 Data and the Comparative Analysis

We implement the methodology described in section 2 and give a comparative analysis between different specifications of the generalized factor model and the classical factor model. We apply the proposed methodology to the foreign exchange rates (forex). The forex data are the seven major foreign currencies that span the EMU (European Monetary Union) era from 1999:1 to 2003:12. The results derived in this study present some evidence of how the Euro is strongly linked to world currencies in such a short

period of time.

4.1 Data

We use the foreign exchange rate (forex) data publicly available on the New York Federal Reserve Bank's website. The data are monthly foreign exchange rates dated from January 1999 to December 2003 and transformed to monthly percentage returns by setting $y_{it} = 100(p_{it} - p_{i(t-1)})/p_{i(t-1)}$ where p_{it} is the dollar price of foreign currency at month t. We use every month's last trading day in these calculations. The data are demeaned because of the purposes of this study. The seven major international currencies to be analyzed are the Australian Dollar (AUD), Canadian Dollar (CAD), European Euro (EUR), Japanese Yen (JPY), Swedish Krona (SEK), Swiss Franc (CHF) and the British Pound (GBP). There are several reasons why we select these currencies. These are the seven major currencies that are internationally traded in deep and relatively liquid financial markets. Short-term and long-term interest rates are readily available for them. Moreover, these seven currencies serve as a gauge of financial pressure on the value of the dollar.

Summary statistics of the data are reported in Table 3, Table 4, and Table 5. Table 3 reports sample means and sample standard deviations. Standard deviations vary between 1.85 and 3.12. Table 4 presents sample autocorrelations. Clearly, exchange rate returns have low autocorrelations as most financial asset returns do. Highest autocorrelation appears in the Euro, yet that dies out soon after second lag. If the percentage growth rates for currencies were a Gaussian white noise, the approximate lower and upper confidence bounds would be (-0.2582, 0.2582) and all of the currencies would be inside the interval. That is, all the autocorrelations are insignificant. Hence, in most of the factor analysis papers the returns are assumed to be independent and identically distributed.

Correlation coefficients between currency returns are reported in Table 5. First, all coefficients are positive and most of them are significantly high. These indicate that correlations are clearly nonzero and there may be a strong common behavior between the currencies. Obviously, European currencies are more correlated with each other than the rest of them. For example, the Swede Krona, Swiss Franc and Euro are highly correlated while the UK Pound is a little less correlated with other European currencies. The Canadian Dollar has a large correlation coefficient with the Australian Dollar while its correlation coefficients with the rest of the world is smaller. On average, the Japanese Yen and Canadian Dollar have the lowest correlation numbers whereas the Euro has the largest value.

4.2 Comparative Analysis

This section presents the first empirical results with the generalized factor model developed in this study. We run a comparative analysis to understand the impact of a move from a classical factor model to a generalized factor model on the estimates of functions of model parameters and the latent factors. In the model, we have a set of informative priors which requires specifying the hyperparameters that determine the prior densities. Priors on the factor loadings are centered at zero and the prior covariance matrix is set to $\underline{\mathbf{H}}_\lambda = (1/5) \times \mathbf{I}_N$. The hyperparameter on the prior for $\mathbf{C}$ is set to be $\underline{h}_c = 1/5$. The prior specification of $\mathbf{\Sigma}$ has a significant role on the generalization of the classical framework as discussed in section 2.3. There is one parameter left to move freely, the scale parameter $\underline{\nu}$ for $\mathbf{\Sigma}$. This is the key parameter in terms of the generalization setup. Its determination is the key point because determining the scale parameter is equivalent to determining the correlation structure between idiosyncratic factors and the correlation structure of the observed variables. Discussions in the following three paragraphs are based on a one-factor model. Although we present the results from a one-factor model in the rest of this section, all the discussions and conclusions are valid for two- and three-factor models as well.

We start the comparative analysis with the investigation of changes in the estimates of the variation explained by common factors, $\frac{\mathbf{\Lambda_i'\Lambda_i}}{\mathbf{\Lambda_i'\Lambda_i}+c_{ii}^2\sigma_{ii}}$, the variation attributed to idiosyncratic factors, $\frac{c_{ii}^2\sigma_{ii}}{\mathbf{\Lambda_i'\Lambda_i}+c_{ii}^2\sigma_{ii}}$ and factor loadings Λ_i, which are often of research interest in the literature. However we only discuss the variation due to common factors since the second expression is the complement of the first one and the factor loadings can easily be derived from the first one. The estimates of these expressions are widely used in finance and economics. For example, the Arbitrage Pricing Models (see Geweke and Zhou (1996)) and portfolio management problems (see Aguilar and West (2000)). Therefore which model we use to estimate is quite important in financial decision problems.

The comparison of the estimates derived using different model specifications are displayed in Table 6. The reported numbers are the posterior means and numerical standard errors. As the scale parameter $\underline{\nu}$ increases [decreases], $\mathbf{\Sigma}$ gets tightened [loosened] up at around $\mathbf{I}_N$ and the model converges to a more classical factor model [more generalized model]. $\underline{\nu} = 5$ is quite a small number to represent a model with an extremely loose covariance matrix $\mathbf{\Sigma}$ and $\underline{\nu} = 500$ is quite a large number to show a model with an extremely tight covariance matrix $\mathbf{\Sigma}$. As the model converges to the classical factor model, variation attributed to the common factor rises on average. That is, there is an increase on average in the estimates as we move from a "loose" model to a "tight" model. This result is not a surprise because we are imposing the assumption that there is almost no cross-correlation between idiosyncratic factors and we are forc-

ing the common factors to explain all of the covariation between the variables when we use the classical factor model. Since the estimated variance is the same whether it is a "loose" model or a "tight" model, we can deduce from Table 6 that the estimates for factor loadings increase as we move from generalized to classical model.

We can summarize the empirical results from Table 6 as follows. The variation attributed to the common factor achieves the highest percentage values for the Euro, Krona and Franc while its smallest values are for the Canadian Dollar and the Japanese Yen. Interestingly, these latter currencies seem apart from the common movements of the other five major currencies. On the other hand, the Euro displays a very high variation, 97%, explained by the common factors. It seems that the common factor is almost a "Euro factor".

Next we compare the changes in the idiosyncratic correlation coefficients for the "loose" and "tight" models. Mean absolute idiosyncratic correlation coefficients for $\underline{\nu} = 5$ and $\underline{\nu} = 500$ are displayed in Table 7 and Table 8, respectively. The idiosyncratic correlations are quite big if we employ the "loose" model, that is, the generalized factor model, whereas they are quite small if we apply the "tight" model, that is, "almost" classical factor model. Considering the fact that the classical factor model apriori assumes the zero-correlation between idiosyncratic components, we can understand how big the amount of correlation that is not captured by the classical factor analysis is. For example, unexplained correlation between CAD and AUD is 0.6558, however this number is zero when classical factor model is employed.

Lastly, we examine the factor estimates in the same one-factor model for various model specifications discussed above. Figure 1 demonstrates the factor estimates over time for four values of the scale parameters, that is (a) $\underline{\nu} = 5$, (b) $\underline{\nu} = 10$, (c) $\underline{\nu} = 100$ and (d) $\underline{\nu} = 500$. Factor estimates do not change as we move from the classical model to the generalized model. One empirical result one can observe from Figure 1 is that the U.S. dollar depreciates over the time.

In this section, we show an implementation of the generalized factor model and run a comparative analysis between the estimates of the generalized and classical factor models. In addition, we evaluate the assumption that the idiosyncratic factors in a classical factor model are cross-sectionally orthogonal. From the outputs of various model specifications, we find that there are slight decreases in the estimates of factor loadings and variation due to common factors (hence, slight increases in the estimates of the variation due to idiosyncratic factors), and there is no change in the factor estimates as we switch from the classical to the generalized model setup. However, we find substantial changes in the structure of the correlation matrix. These evaluations are in terms of statistical estimates for some functions of parameters which are often of research interest, but whether these differences make significant changes in economic

and financial applications is future research interest.

5 Model Comparison

The purpose of this section is two-fold: First, we want to propose a methodology to select the generalized factor model that best explains the given data. We wish to do a formal model comparison between different specifications of the generalized factor structure, including the classical factor structure. We use Bayes factors to choose the model that best fits the data. The calculation of the Bayes factor for model selection in this study simplifies substantially because different factor models are characterized by different prior beliefs rather than the body of the model in (2). Second, we want to run a comparative analysis between models. We show that the data favor a generalized factor model over a classical factor model unless a significant amount of each covariation within the data is explained by the common factors. Moreover, we study the implications of the selected model setup for the structures of the covariance and correlation matrices.

Each specification of the model is characterized by the choice of the scale parameter $\underline{\nu}$. So selection of the model for the true characterization of the data is determined by selection of the scale parameter. This analysis also provides an answer to the problem of model comparison between the classical and generalized framework. The prior hyperparameters for the posterior simulation analysis are given in section 4.2.

Bayesian model selection proceeds by pairwise comparison of the models through their Bayes factors. A nice feature of the proposed model setup is that the computation of the Bayes factor simplifies substantially since any pair of models have the same conditional probability densities of observables but have different prior densities. The derivation of the Bayes factor is available in Appendix B. The resulting Bayes factor for a pair of models is

$$BF^o = E\{\frac{p(\mathbf{\Sigma}|\underline{\nu}_1)}{p(\mathbf{\Sigma}|\underline{\nu}_2)}|\mathbf{Y}, \underline{\nu}_2\} \tag{15}$$

where $p(\mathbf{\Sigma}|\underline{\nu}_1)$ and $p(\mathbf{\Sigma}|\underline{\nu}_1)$ are prior densities for the models with scale parameters $\underline{\nu}_1$ and $\underline{\nu}_2$, respectively. Let $\{\mathbf{\Sigma}^{(m)}\}_{m=1}^M$ be the output from the posterior simulator $p(\mathbf{\Sigma}|\mathbf{Y}, \underline{\nu}_2)$. Then, the Bayes factor can be approximated consistently by

$$BF = \frac{1}{M}\sum_{m=1}^{M}\frac{p(\mathbf{\Sigma}^{(m)}|\underline{\nu}_1)}{p(\mathbf{\Sigma}^{(m)}|\underline{\nu}_2)} \tag{16}$$

under the conditions that posterior moment in expression (15) be well defined and finite, and the ratios of prior densities $\frac{p(\mathbf{\Sigma}|\underline{\nu}_1)}{p(\mathbf{\Sigma}|\underline{\nu}_2)}$ be bounded in the parameter space

of $\underline{\nu}_2$ (Geweke (2005)). Data favors the model characterized by the parameter $\underline{\nu}_1$ if $BF > 1$ whereas it favors the model characterized by the parameter $\underline{\nu}_2$ if $BF < 1$. It is essential that the model characterized by the parameter $\underline{\nu}_2$ be more diffuse (less restrictive); hence we must have $\underline{\nu}_2 < \underline{\nu}_1$. Another important point, however, is that we are not able to compute the Bayes factor consistently if $\underline{\nu}_2$ is a lot smaller than $\underline{\nu}_1$ because we lose the numerical efficiency as the difference between these two numbers grows substantially. For example, we can hardly compare a model with $\underline{\nu}_2 = 10$ and a model with $\underline{\nu}_1 = 400$.

In section 5.1, we run an experiment using the simulated data to implement and illustrate the model selection method. In section 5.2, we explore the empirical implications of the model selection for the foreign exchange rate data and study the structures of the covariance and correlation matrices implied by the generalized factor model.

5.1 Results with Simulated Data

We build a pseudo-true model and study the implications of the model selection procedure in the light of the simulated data from this model. We construct the pseudo-true model with $N = 7$, $T = 60$ and $r = 2$ and $\mathbf{\Sigma} = \mathbf{I}_N$. To determine the model parameters $\mathbf{\Lambda}$ and $\mathbf{C}$, we draw them from their priors given the values of the hyperparameters in section 4.2. Under these specifications, the data are generated according to the expressions (2), (3) and (4). This two-factor model is the "true model" to be estimated by the generalized factor model. Observe that all the covariations are driven by two common factors in this simulation study.

We next apply the generalized factor model to the simulated data. Using the MCMC output $\{\mathbf{\Sigma}^m\}_{m=1}^M$ from the generalized factor model with $r = 1$, $r = 2$ and $r = 3$, we compute the Bayes factors. Bayes factors for various pairs of models conditional on one factor, two factors and three factors are presented in Tables 9, 10 and 11, respectively. The models compared using the Bayes factor are pairwise listed in the first two columns. The first column represents the model characterized by $\underline{\nu}_1$ and the second column displays the model characterized by $\underline{\nu}_2$ in equation (16). Using Bayes factors, we can select the best generalized factor model for a given number of factors. Assuming a one-factor model (Table 9), marginal likelihood rises sharply until $\underline{\nu}_1 = 7$. Then it drops slowly afterwards and become flatter as the model approaches a classical model. The shape of the log marginal likelihood function is depicted in Figure 2, which has the mode at $\underline{\nu}_1 = 7$. Hence the model characterized by the scale parameter $\underline{\nu}_1 = 7$ is chosen under the framework of a one-factor model. So idiosyncratic factors are not orthogonal and some covariations are not explained by the common factor. This is no surprise because our "true model" is a two-factor model with orthogonal idiosyncratic

components. Then we try the two-factor and three-factor model.

With the two-factor model whose results are reported in Table 10, the marginal likelihood increases until the model completely converges to the classical factor setup. This picture is depicted in Figure 3. The three-factor model (Table 11) has the same picture as the two-factor model. The only difference is that the log marginal likelihood function (Figure 4) is flatter for the three-factor model. So the log marginal likelihood favors the two-factor model which is the "true model". After this verification and illustration of the model selection methodology, now we can apply the model to the foreign exchange rates data.

5.2 Empirical Results

The generalized factor model is applied to the foreign exchange rates data in order to explore the empirical implications of the proposed model setup. As in the simulated data analysis, we show how to choose the generalized factor structure that best characterize the data and study the implications of the model for the structures of the covariance and correlation matrices.

Bayes factors for the models with one factor, two factors and three factors are reported in Tables 12, 13 and 14, respectively. Pairs of models compared using the Bayes factor in (16) are listed in the first two columns and their corresponding Bayes factors are given in the third column. NSEs are also reported in each table. From Table 12, one can clearly see that the marginal likelihood increases steeply until $\underline{\nu}_1 = 12$ and then falls gradually afterwards. Hence the log marginal likelihood function have a picture as graphed in Figure 5. The mode is at $\underline{\nu}_1 = 12$. Therefore we can conclude that the best model specification is at $\underline{\nu}_1 = 12$ for a one-factor model. Table 13 shows that, with two factors, the more restrictive the model is, the higher the log marginal likelihood is. The graph of the log marginal likelihood should be like the one depicted in Figure 6. Thus the data favors the restrictive model with a diagonal idiosyncratic covariance matrix. Results for a three-factor model in Table 14 are quite similar to those for the two-factor model. Bayes factors favor the restricted model again but now the log marginal likelihood function, displayed in Figure 7, is flatter than the one for the two-factor model. These results are almost the same as the ones derived using the simulated data in the previous section. Only the one-factor model requires a loose $\mathbf{\Sigma}$, hence favors a less restrictive model. Thus from the analysis of log marginal likelihoods we conclude that two factor model is chosen for the foreign exchange rates since the intuitive idea behind the factor analysis is to explain the covariation between observed variables with the minimum number of factors.

With two and three factors, the best model is characterized by a quite large hyper-parameter $\underline{\nu}$. Therefore the idiosyncratic factors for these models are orthogonal, which implies a generalized factor model equivalent to a classical factor model. However, with a one-factor model the idiosyncratic factors are not orthogonal but correlated. For this model, the proposed structure allows us to analyze the patterns of covariance and correlation that are not driven through the dynamics of the common factor. Tables 15 and 16 show that these patterns of covariation left unexplained by the common factor are significant amount. For example, much of the covariation between the Australian Dollar and the Canadian Dollar are determined by the idiosyncratic factor but not the common factor. This may not be surprising because the common factor is almost a "Euro factor".

We can summarize this section in three parts. As a first result, we show how to choose the best generalized factor model in financial and economic applications. Second, for any given number of factors, we are able to select the model that best characterize the idiosyncratic component as well as the common component of the covariance and correlation matrices. This allows us to study the patterns of covariations within a data set. Third, even in a factor model where the computation of the marginal likelihood to determine the number of factors is complicated, as in this model, we can still make inferences about the number factors that best characterize the data.

6 Concluding Remarks

In this study, we describe and implement a Bayesian approach for generalized factor models in which idiosyncratic factors can be correlated and identification does not require any restrictions neither on cross-sectional dimensions nor on time dimensions. We also propose a methodology to choose the generalized factor model that best describe the factor structure of a given data set. This model selection methodology is specific to factor analysis questions; however, it is more efficient –in terms of time cost, analytical derivations and computational effort– to employ this method than the marginal likelihood method. For the convenience of many repeated experiments and simulations for this study, the model is applied for the analysis of a small cross-sectional data set but it can be used for large data analysis as well.

Applying the model to the foreign exchange rate data, we discuss whether the generalization of the classical factor model makes any changes in the estimates for the functions of factor loadings and factors, and evaluate the controversial assumption of no cross-sectional correlation between idiosyncratic factors. We find that when there is a shift from classical to generalized model, there are significant changes in the estimates of correlation structure while we find less dramatic changes in the estimates of the

factor loadings and variation attributed to common factors.

This study can initiate possible further research projects that can be based on the idea of generalization. The model and the ideas in this paper can lead to several empirical and technical extensions. First, the current paper presents the statistical differences between the outputs of generalized and classical factor models. However, an interesting empirical question will be to study the differences these models will make in economic and financial applications. Second, the generalized factor models are relatively new ideas. Therefore their applications in empirical problems are just expanding recently. Pattern analysis of covariances and correlations –for example, patterns of covariations in bank default rates or international markets– are just some possible empirical applications. Third, the generalized factor model for serially correlated data can be a major extension of this project. Forth, the proposed model selection approach is relatively an efficient method; however, a development of a model with hierarchical structure may improve the efficiency, which is an on-going research project.

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Appendix

A Analysis of the Prior For Variance-Covariance Matrix

Let $\mathbf{U}$ be $N \times N$ symmetric and positive definite matrix. If $\mathbf{U}$ follows an inverted Wishart distribution with scale matrix $\mathbf{G}$ and degrees of freedom v, that is, $\mathbf{U} \sim IW(\mathbf{G}, v)$, then first and second moments are given by

$$E(\mathbf{U}) = \frac{\mathbf{G}}{v - 2N - 2}, v - 2N - 2 > 0$$

$$Var(u_{ij}) = \frac{g_{ii}g_{jj} + \frac{(v-2N)}{(v-2N-2)}g_{ij}^2}{(v-2N-1)(v-2N-2)(v-2N-4)}, v - 2N - 4 > 0$$

$$Cov(u_{ij}, u_{kl}) = \frac{\frac{2}{(v-2N-2)}g_{ij}g_{kl} + g_{ik}g_{jl} + g_{il}g_{kj}}{(v-2N-1)(v-2N-2)(v-2N-4)}, v - 2N - 4 > 0$$

for all i, j, k, l and $G = (g_{ij})$.

In the prior specification of $\mathbf{\Sigma}$, $\underline{\nu}\mathbf{\Sigma}^{-1} \sim W(\mathbf{I}_N, \underline{\nu} + N + 1)$ implies that $\frac{1}{\underline{\nu}}\mathbf{\Sigma} \sim IW(\mathbf{I}_N, \underline{\nu} + 2N + 2)$. Hence, with $v = \underline{\nu} + 2N + 2$ and $\mathbf{G} = \mathbf{I}_N$, we have $\underline{\nu} > 2$ and following prior moments

$$E(\mathbf{\Sigma}) = \mathbf{I}_N$$

$$Var(\sigma_{ii}) = \frac{2}{\underline{\nu} - 2}$$

$$Var(\sigma_{ij}) = \frac{\underline{\nu}}{(\underline{\nu} + 1)(\underline{\nu} - 2)}$$

for $i \neq j$ and $\mathbf{\Sigma} = (\sigma_{ij})$. Variances go to zero as $\underline{\nu}$ increases and converge to infinity as $\underline{\nu}$ approaches to 2. Hence variances are monotone decreasing functions of $\underline{\nu}$. Similarly one can straightforwardly show that these are true for covariances as well.

B Derivation of the Bayes Factor

Using Bayes factors, we compare two (generalized) factor models to choose the one that better characterizes the data. Each specification of the model is represented by the choice of the scale parameter $\underline{\nu}$. Let $\underline{\nu}_1$ and $\underline{\nu}_2$ represent the two competing models, say model 1 and model 2. In the model setup of this paper, these models share the

same conditional probability densities of observables but have different prior densities. Therefore the computation of the Bayes factor simplifies substantially.

Let $\boldsymbol{\delta} = (\mathbf{C}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}, \mathbf{F})$ be the set of model parameters and latent factors. Let also Δ_1 and Δ_2 be parameter spaces for models 1 and 2, respectively. Then the Bayes factor in favor of model 1 is

$$\begin{aligned} BF^o &= \frac{\int_{\Delta_1} p(\boldsymbol{\Sigma}|\underline{\nu}_1)p(\mathbf{C})p(\boldsymbol{\lambda})p(\mathbf{F})p(\mathbf{Y}|\mathbf{C},\boldsymbol{\lambda},\boldsymbol{\Sigma},\mathbf{F})d\boldsymbol{\delta}}{\int_{\Delta_2} p(\boldsymbol{\Sigma}|\underline{\nu}_2)p(\mathbf{C})p(\boldsymbol{\lambda})p(\mathbf{F})p(\mathbf{Y}|\mathbf{C},\boldsymbol{\lambda},\boldsymbol{\Sigma},\mathbf{F})d\boldsymbol{\delta}} \\ &= \frac{\int_{\Delta_1} [p(\boldsymbol{\Sigma}|\underline{\nu}_1)/p(\boldsymbol{\Sigma}|\underline{\nu}_2)]p(\boldsymbol{\Sigma}|\underline{\nu}_2)p(\mathbf{C})p(\boldsymbol{\lambda})p(\mathbf{F})p(\mathbf{Y}|\mathbf{C},\boldsymbol{\lambda},\boldsymbol{\Sigma},\mathbf{F})d\boldsymbol{\delta}}{\int_{\Delta_2} p(\boldsymbol{\Sigma}|\underline{\nu}_2)p(\mathbf{C})p(\boldsymbol{\lambda})p(\mathbf{F})p(\mathbf{Y}|\mathbf{C},\boldsymbol{\lambda},\boldsymbol{\Sigma},\mathbf{F})d\boldsymbol{\delta}} \\ &= \int_{\Delta_2} \frac{p(\boldsymbol{\Sigma}|\underline{\nu}_1)}{p(\boldsymbol{\Sigma}|\underline{\nu}_2)} \frac{p(\boldsymbol{\Sigma},\mathbf{C},\boldsymbol{\lambda},\mathbf{F},\mathbf{Y}|\underline{\nu}_2)}{p(\mathbf{Y}|\underline{\nu}_2)} d\boldsymbol{\delta} \\ &= E\{\frac{p(\boldsymbol{\Sigma}|\underline{\nu}_1)}{p(\boldsymbol{\Sigma}|\underline{\nu}_2)}|\mathbf{Y},\underline{\nu}_2\} \end{aligned}$$

The condition $\Delta_1 \subseteq \Delta_2$ should be satisfied for the validity of the derivation above.

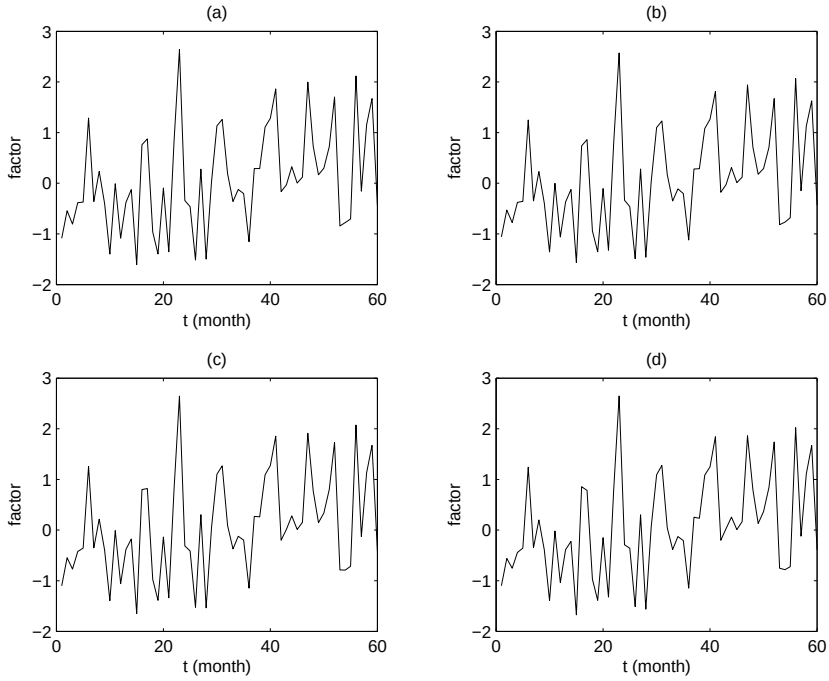

Figure 1: Factor estimates over time for four values of the scale parameter: (a) scale parameter=5, (b) scale parameter=10, (c) scale parameter=100 and (d) scale parameter=500.

Table 1: Testing for coding error

Test Functions	P-values for equality of means
trace(YY')	0.72
max. eigenvalue of (YY')	0.62
min. eigenvalue of (YY')	0.89
σ_{11}	0.37
F_{11}	0.94

The test proposed in Geweke (2004) is conducted to check for errors in the analytic derivations and the computer programming. Some selected samples of the test functions are presented. All results pass the test at the 10% level.

Table 2: Testing convergence

Variables of interest	P-values for equality of means
$\mathbf{\Omega}_{11}$	0.65
$\mathbf{\Omega}_{13}$	0.89
$\mathbf{\Lambda}_1\mathbf{\Lambda}_1'$	0.53
$\mathbf{\Lambda}_1\mathbf{\Lambda}_3'$	0.96

Convergence test results for Posterior Simulation of covariance matrix, $\mathbf{\Omega} = \mathbf{\Lambda}\mathbf{\Lambda}' + \mathbf{C}\mathbf{\Sigma}\mathbf{C}$. Some selected components are presented. Convergence is verified at the 10% level.

Table 3: Summary statistics

CURRENCY	Mean	Std. Dev.
AUD	0.3678	3.1289
CAD	0.2325	1.8539
EUR	0.1944	2.9681
JPY	0.1925	2.8398
SEK	0.1265	3.0114
CHF	0.2386	2.9419
GBP	0.1931	2.2033

Sample means and sample standard deviations for currency percentage returns.

Table 4: Autocorrelations

CURRENCY	lag 1	lag 2	lag 3	lag 4
AUD	0.0796	-0.0848	0.0192	0.0997
CAD	0.054	-0.0734	-0.1304	0.0758
EUR	0.1725	-0.0022	-0.0499	-0.0916
JPY	0.1103	-0.0523	0.2333	-0.1519
SEK	0.0754	-0.048	0.0905	0.0472
CHF	0.0957	0.0217	-0.0366	-0.2389
GBP	-0.0449	-0.1204	0.2452	-0.0219

Sample autocorrelations for currency percentage returns.

Table 5: Cross-correlations

CURRENCY	AUD	CAD	EUR	JPY	SEK	CHF	GBP
AUD	1	0.7523	0.5214	0.3034	0.5847	0.3665	0.3519
CAD	0.7523	1	0.3509	0.1867	0.4542	0.1811	0.1605
EUR	0.5214	0.3509	1	0.3347	0.8954	0.9492	0.6985
JPY	0.3034	0.1867	0.3347	1	0.3605	0.3388	0.2665
SEK	0.5847	0.4542	0.8954	0.3605	1	0.8102	0.6191
CHF	0.3665	0.1811	0.9492	0.3388	0.8102	1	0.6722
GBP	0.3519	0.1605	0.6985	0.2665	0.6191	0.6722	1

Sample correlation coefficients between currency percentage returns.

Table 6: Variation attributed to the common factor

Model	AUD	CAD	EUR	JPY	SEK	CHF	GBP
$\underline{\boldsymbol{\nu}} = \mathbf{5}$	0.2516	0.1199	0.9800	0.1123	0.7674	0.8927	0.4512
	(0.0006)	(0.0005)	(0.0002)	(0.0004)	(0.0004)	(0.0003)	(0.0006)
$\underline{\boldsymbol{\nu}} = \mathbf{10}$	0.2579	0.1232	0.9777	0.1139	0.7721	0.9007	0.4546
	(0.0006)	(0.0005)	(0.0001)	(0.0004)	(0.0004)	(0.0002)	(0.0006)
$\underline{\boldsymbol{\nu}} = \mathbf{20}$	0.2784	0.1341	0.9837	0.1200	0.7889	0.9084	0.4665
	(0.0006)	(0.0005)	(0.0001)	(0.0004)	(0.0003)	(0.0002)	(0.0006)
$\underline{\boldsymbol{\nu}} = \mathbf{100}$	0.2991	0.1481	0.9922	0.1200	0.7972	0.9072	0.4747
	(0.0005)	(0.0005)	(0.0000)	(0.0004)	(0.0003)	(0.0001)	(0.0005)
$\underline{\boldsymbol{\nu}} = \mathbf{500}$	0.2955	0.1482	0.9948	0.1283	0.8094	0.9079	0.4982
	(0.0006)	(0.0005)	(0.0000)	(0.0004)	(0.0003)	(0.0001)	(0.0005)

Variation attributed to the common factor, $\frac{\boldsymbol{\Lambda}_\mathbf{i}'\boldsymbol{\Lambda}_\mathbf{i}}{\boldsymbol{\Lambda}_\mathbf{i}'\boldsymbol{\Lambda}_\mathbf{i}+c_{ii}^2\sigma_{ii}}$ in a one-factor model.
Numerical standard errors are in the parenthesis. Number of iterations is 30000.

Table 7: Idiosyncratic correlations, scale parameter=5

CURRENCY	AUD	CAD	EUR	JPY	SEK	CHF	GBP
AUD	1.0000	0.6558	0.3394	0.1606	0.2959	0.4093	0.1169
	(0)	(0.0004)	(0.0012)	(0.0006)	(0.0009)	(0.0010)	(0.0005)
CAD	0.6558	1.0000	0.3398	0.1196	0.3147	0.4276	0.1373
	(0.0004)	(0)	(0.0012)	(0.0005)	(0.0009)	(0.0010)	(0.0006)
EUR	0.3394	0.3398	1.0000	0.2031	0.2932	0.3125	0.2044
	(0.0012)	(0.0012)	(0)	(0.0009)	(0.0011)	(0.0012)	(0.0009)
JPY	0.1606	0.1196	0.2031	1.0000	0.1464	0.1463	0.1078
	(0.0006)	(0.0005)	(0.0009)	(0)	(0.0006)	(0.0007)	(0.0005)
SEK	0.2959	0.3147	0.2932	0.1464	1.0000	0.2599	0.1213
	(0.0009)	(0.0009)	(0.0011)	(0.0006)	(0)	(0.0009)	(0.0005)
CHF	0.4093	0.4276	0.3125	0.1463	0.2599	1.0000	0.1449
	(0.0010)	(0.0010)	(0.0012)	(0.0007)	(0.0009)	(0)	(0.0007)
GBP	0.1169	0.1373	0.2044	0.1078	0.1213	0.1449	1.0000
	(0.0005)	(0.0006)	(0.0009)	(0.0005)	(0.0005)	(0.0007)	(0)

Mean absolute idiosyncratic correlation coefficients when $\underline{\nu} = 5$ in a one-factor model. Numerical standard errors are in the parenthesis. Number of iterations is 30000.

Table 8: Idiosyncratic correlations, scale parameter=500

CURRENCY	AUD	CAD	EUR	JPY	SEK	CHF	GBP
AUD	1.0000	0.0823	0.0361	0.0366	0.0438	0.0565	0.0339
	(0)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)
CAD	0.0823	1.0000	0.0362	0.0345	0.0456	0.0598	0.0349
	(0.0002)	(0)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
EUR	0.0361	0.0362	1.0000	0.0358	0.0358	0.0361	0.0356
	(0.0002)	(0.0002)	(0)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
JPY	0.0366	0.0345	0.0358	1.0000	0.0357	0.0345	0.0338
	(0.0002)	(0.0002)	(0.0002)	(0)	(0.0002)	(0.0002)	(0.0001)
SEK	0.0438	0.0456	0.0358	0.0357	1.0000	0.0431	0.0338
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0)	(0.0002)	(0.0001)
CHF	0.0565	0.0598	0.0361	0.0345	0.0431	1.0000	0.0339
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0)	(0.0001)
GBP	0.0339	0.0349	0.0356	0.0338	0.0338	0.0339	1.0000
	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0)

Mean absolute idiosyncratic correlation coefficients when $\underline{\nu} = 500$ in a one-factor model. Numerical standard errors are in the parenthesis. Number of iterations is 30000.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.2069	0.0635
7	6	1.1403	0.0470
8	7	0.9305	0.0276
11	10	0.7295	0.0136
21	20	0.6362	0.0054
101	100	0.8052	0.0023
501	500	0.9808	0.0002

Table 9: Estimates of the Bayes factor with $r = 1$ for simulated data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

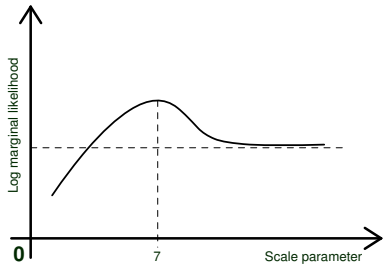

Figure 2: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the one-factor model applied to simulated data.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.7883	0.0896
11	10	1.4308	0.0259
21	20	1.1210	0.0073
101	100	1.0095	0.0007
501	500	1.0001	0.0001

Table 10: Estimates of the Bayes factor with $r = 2$ for simulated data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

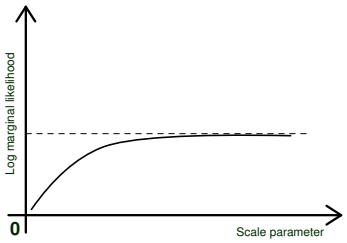

Figure 3: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the two-factor model applied to simulated data.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.5944	0.0735
11	10	1.2528	0.0216
21	20	1.0772	0.0077
101	100	1.0076	0.0007
501	500	1.0001	0.0001

Table 11: Estimates of the Bayes factor with $r = 3$ for simulated data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

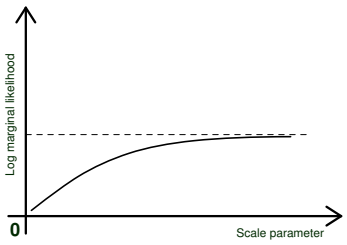

Figure 4: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the three-factor model applied to simulated data.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.4256	0.0675
11	10	1.1124	0.0263
12	11	1.0716	0.0180
13	12	0.9609	0.0126
21	20	0.9049	0.0081
101	100	0.9366	0.0010
501	500	0.9939	0.0002

Table 12: Estimates of the Bayes factor with $r = 1$ for forex data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

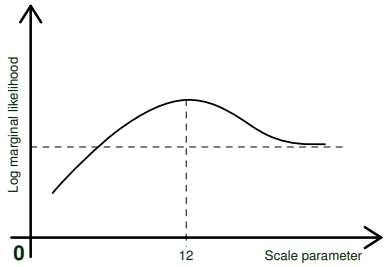

Figure 5: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the one-factor model applied to forex data.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.4564	0.0586
11	10	1.2221	0.0260
21	20	1.1226	0.0078
101	100	1.0087	0.0009
501	500	1.0003	0.0001

Table 13: Estimates of the Bayes factor with $r = 2$ for forex data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

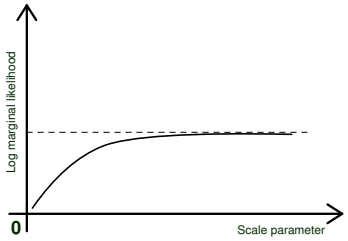

Figure 6: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the two-factor model applied to forex data.

$\underline{\nu}_1$	$\underline{\nu}_2$	Mean	NSE
6	5	1.2664	0.0359
11	10	1.1404	0.0240
21	20	1.0583	0.0068
101	100	1.0052	0.0008
501	500	1.0003	0.0001

Table 14: Estimates of the Bayes factor with $r = 3$ for simulated data. Posterior mean and numerical standard error are reported. Number of iterations is 10000.

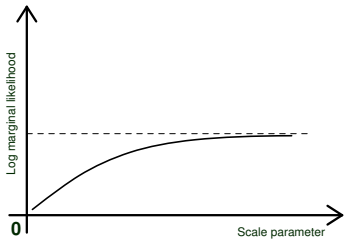

Figure 7: The log marginal likelihood as a function of the scale parameter $\underline{\nu}$ from the three-factor model applied to forex data.

Table 15: Unexplained covariance in one-factor model

CURRENCY	AUD	CAD	EUR	CAD	SEK	CHF	GBP
AUD	7.1261	2.8770	0.1371	0.9934	1.0261	-0.9069	-0.0018
	(0.0078)	(0.0040)	(0.0025)	(0.0053)	(0.0037)	(0.0026)	(0.0033)
CAD	2.8770	3.0090	0.0846	0.3401	0.7198	-0.6279	-0.2431
	(0.0040)	(0.0032)	(0.0016)	(0.0034)	(0.0023)	(0.0018)	(0.0022)
EUR	0.1371	0.0846	0.1910	-0.0363	0.1126	0.0446	0.0408
	(0.0025)	(0.0016)	(0.0012)	(0.0014)	(0.0014)	(0.0008)	(0.0009)
JPY	0.9934	0.3401	-0.0363	7.6221	0.3861	0.0869	0.1506
	(0.0053)	(0.0034)	(0.0014)	(0.0084)	(0.0030)	(0.0021)	(0.0033)
SEK	1.0261	0.7198	0.1126	0.3861	2.0067	-0.2859	-0.0170
	(0.0037)	(0.0023)	(0.0014)	(0.0030)	(0.0029)	(0.0011)	(0.0018)
CHF	-0.9069	-0.6279	0.0446	0.0869	-0.2859	0.8093	0.0386
	(0.0026)	(0.0018)	(0.0008)	(0.0021)	(0.0011)	(0.0018)	(0.0012)
GBP	-0.0018	-0.2431	0.0408	0.1506	-0.0170	0.0386	2.7448
	(0.0033)	(0.0022)	(0.0009)	(0.0033)	(0.0018)	(0.0012)	(0.0031)

Patterns of covariance left unexplained by the one-factor model denoted by $\mathbf{C\Sigma C}$. Numerical standard errors are in the parenthesis. Number of iterations is 30000.

Table 16: Idiosyncratic correlations in one-factor model

CURRENCY	AUD	CAD	EUR	JPY	SEK	CHF	GBP
AUD	1.0000	0.6190	0.2675	0.1494	0.2686	0.3852	0.1008
	(0)	(0.0004)	(0.0010)	(0.0006)	(0.0007)	(0.0008)	(0.0004)
CAD	0.6190	1.0000	0.2721	0.1112	0.2904	0.4063	0.1240
	(0.0004)	(0)	(0.0010)	(0.0005)	(0.0007)	(0.0008)	(0.0005)
EUR	0.2675	0.2721	1.0000	0.1757	0.2316	0.2325	0.1674
	(0.0010)	(0.0010)	(0)	(0.0008)	(0.0009)	(0.0010)	(0.0007)
JPY	0.1494	0.1112	0.1757	1.0000	0.1302	0.1193	0.0994
	(0.0006)	(0.0005)	(0.0008)	(0)	(0.0005)	(0.0005)	(0.0004)
SEK	0.2686	0.2904	0.2316	0.1302	1.0000	0.2490	0.1051
	(0.0007)	(0.0007)	(0.0009)	(0.0005)	(0)	(0.0008)	(0.0005)
CHF	0.3852	0.4063	0.2325	0.1193	0.2490	1.0000	0.1166
	(0.0008)	(0.0008)	(0.0010)	(0.0005)	(0.0008)	(0)	(0.0005)
GBP	0.1008	0.1240	0.1674	0.0994	0.1051	0.1166	1.0000
	(0.0004)	(0.0005)	(0.0007)	(0.0004)	(0.0005)	(0.0005)	(0)

Mean absolute idiosyncratic correlation coefficients for the one-factor model. Numerical standard errors are in the parenthesis. Number of iterations is 30000.