

## **Man-Bites-Dog Business Cycle**

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# MAN-BITES-DOG BUSINESS CYCLES

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**ABSTRACT.** The newsworthiness of an event is partly determined by how unusual it is and this paper investigates the business cycle implications of this fact. In particular, we analyze the consequences of information structures in which some types of signals are more likely to be observed after unusual events. Such signals may increase both uncertainty and disagreement among agents and when embedded in a simple business cycle model, can help us understand why we observe (i) occasional large changes in macro economic aggregate variables without a correspondingly large change in underlying fundamentals (ii) persistent periods of high macroeconomic volatility and (iii) a positive correlation between absolute changes in macro variables and the cross-sectional dispersion of expectations as measured by survey data. These results are consequences of optimal updating by agents when the availability of some signals is positively correlated with tail-events. The model is estimated by likelihood based methods using individual survey responses and a quarterly time series of total factor productivity along with standard aggregate time series. The estimated model suggests that there have been episodes in recent US history when the impact on output of innovations to productivity of a given magnitude was more than eight times as large compared to other times.

## 1. INTRODUCTION

A well-known journalistic dictum states that “*dog-bites-man* is not news, but *man-bites-dog* is news”. That is, unusual events are more likely to be considered newsworthy than events that are commonplace. This paper investigates the business cycle implications of this aspect of news reporting. Particularly, we will demonstrate that a single and relatively simple mechanism can help us understand three features of business cycles. First, there can

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be large changes in aggregate variables like CPI inflation and GDP growth, but without an easily identifiable change in fundamentals of comparable magnitude. Second, there appear to be persistent episodes of increased macroeconomic volatility in the data. Third, measures of uncertainty as well as measures of cross-sectional dispersion of expectations are positively correlated with absolute magnitudes of changes in macro economic aggregates. These features can be explained by Bayesian agents optimally updating to signals that are more likely to be available about unusual events. The model can also help us understand a type of “crisis mentality” in which an intense media focus on the economy causes an increase in both agents’ uncertainty and sensitivity to new information.

Conceptually, information about the current state of the world can be divided into at least three categories. What we may call *local information* is information that agents observe directly through their interactions in markets, e.g. through buying and selling goods or through participating in the labor market. A second type of information is what we may call *statistics*. Statistics are collected and summarized by (often government) organizations and made available to a broader public through web sites and printed media. Statistics are normally reported regardless of the realized values of the variable that they refer to and often according to a pre-specified schedule. A third type is information provided by the news media, such as newspapers and television programs. News media may be the main source of information for a large section of the general population (e.g. Blinder and Krueger 2004). One service that the news media provides is to select what events to report. This editorial function of the news media is necessary since it is simply not possible for a newspaper or a television news program to report all events that have occurred on a given day or during a given week. The man-bites-dog dictum referred to above suggests that more unusual events are more likely to be selected for dissemination and more unusual events are thus more likely to become *news*. This makes news different from statistics since whether an outcome of an event is available as news depends on the realized outcome of the event. In order to have a terminology that is distinct from the one used by the literature studying how information about future productivity affect the economy today (e.g. Beaudry and Portier 2006 and Jaimovich and Rebelo 2009) “news” in the sense meant here will be referred to as man-bites-dog signals.

A prime example of man-bites-dog news reporting is the *Movers* segment on Bloomberg Television. In a typical segment, the price movements of a few stocks are reported along with short statements on the probable causes of these movements. The stocks in question are a small sub-sample of all stocks traded and are selected on the basis of having had the largest price movements during the day. Unusually large price movements are thus more likely to be reported than more common price movements. Because of the way that stock prices are selected for inclusion in the *Movers* segment, the variance of a stock’s price conditional on it being mentioned in the *Movers* segment is thus clearly larger than its unconditional variance.

The example of the *Movers* segment illustrates a more general point. When the availability of a signal depends on the realized value of the variable of interest, the *availability* of the signal is in itself informative. Below, we will prove this more formally in a general setting where agents want to form an estimate of a latent variable. There, it will be shown that man-bites-dog news selection introduces a form of conditional heteroscedasticity. From the agents’ perspective, it is *as if* the variable is drawn from a distribution with relatively more

probability mass in the tails when the man-bites-dog signal is available, compared to the variable's unconditional distribution. Since whether the man-bites-dog signal is available or not is a discrete event, this mechanism effectively splits the unconditional distribution of the latent variable into two distinct conditional distributions.

When signals are noisy, the information contained in the availability of a man-bites-dog signal will in general affect both the mean and variance of agents' posterior beliefs. Using a simple static setting, it can be shown that agents' conditional expectations about a latent variable respond more strongly to a man-bites-dog signal relative to a standard signal of the same precision. The intuition is straightforward: The availability of a man-bites-dog signal suggests that tail realizations are relatively more likely. This makes agents more willing to move their expectations about the latent variable further from their prior mean. When agents' optimal actions depend on these expectations, actions will also respond more strongly to a man-bites-dog signal than to a standard signal of the same precision.

Perhaps more surprising, observing a man-bites-dog signal can potentially make agents more uncertain about the latent variable. To understand this result, note that a man-bites-dog signal affects agents' posterior uncertainty through two different channels that work in opposite directions. The fact that the signal is available means that agents should redistribute probability mass towards the tails of the distribution since unusually large realizations are conditionally more likely when the man-bites-dog signal is available. This effect increases agents' uncertainty. But the signal also contains information about the realized value of the variable which decreases uncertainty. When the signal is sufficiently noisy, the increase in uncertainty from the first effect dominates and agents' posterior beliefs have a higher variance when the man-bites-dog signal is available compared to when the signal is not available.

In a dynamic setting, a larger posterior uncertainty in period  $t$  translates into a larger prior uncertainty in period  $t+1$ . By embedding a man-bites-dog information structure in a simple business cycle model similar to that of Lorenzoni (2009), we show that the propagation of uncertainty through time endogenously generates periods of persistently higher volatility in output and inflation. The mechanism is the following. Agents in the model need to solve a dynamic filtering problem in order to make optimal consumption and price setting decisions. In a given period, the weight agents put on new information is inversely related to the precision of their priors. Since a man-bites-dog signal in period  $t$  can increase the prior uncertainty in period  $t+1$ , agents may put more weight on all signals in period  $t+1$  relative to the case when there was no man-bites-dog signal available in period  $t$ . The increased sensitivity to new information can persist for several periods and implies that the impact of exogenous disturbances of a given magnitude can also be larger than usual for several periods. The mechanism can thus generate periods of higher volatility of macro economic aggregates as observed in the data and documented by Engle (1982), Stock and Watson (2003), Primiceri (2005) and Fernandez-Villaverde and Rubio-Ramirez (2010) among others. In related work, Gali and Gambetti (2009) document that the response of hours worked to a productivity shock is time-varying. This evidence is consistent with the mechanism of the model presented here. With labor as the sole input into production, the impact on output of a productivity shock can only be time-varying if the response of hours worked to a productivity shock is also time-varying.

Bloom (2009) and Bloom, Floetotto and Jaimovich (2011) analyze models in which firms respond to increased uncertainty by adopting a wait-and-see approach to capital investment and recruiting so that an increase in the second moment of the exogenous shocks generates a fall in output. These papers thus provides a story for how firms respond to increased uncertainty and where uncertainty has a direct effect on the level of output. By contrast, the man-bites-dog mechanism provides a story for how economic agents come to understand that conditional uncertainty has increased. In the model presented here, a large shock in levels is more likely to generate a man-bites-dog signal and thus more likely to lead to an increase in the conditional uncertainty. In related work, Bachmann and Moscarini (2011) propose an alternative mechanism that also implies that causality runs from realizations of first moment shocks to conditional uncertainty. In their paper, imperfectly informed firms tend to be more likely to experiment with prices after a large negative shock. This behavior allows firms to get a better estimate of the demand elasticity of the good that they are producing, but increases the conditional variance of profits in the short run.

As mentioned above, observing a man-bites-dog signal can increase the posterior uncertainty relative to the case when no man-bites-dog signal is observed. The same parameter restrictions that ensure that the posterior variance increases after a man-bites-dog signal also imply that the cross-sectional dispersion of expectations increases. This holds even if the signal is public (in the strong common knowledge sense of the word). In the data, we observe a positive correlation between the cross-sectional dispersion of forecasts (as measured by the *Survey of Professional Forecasters*) and the absolute magnitudes of changes in macro aggregates. Interpreted through the lens of the model, this suggest that the empirically relevant specification of the model may be one where the increase in uncertainty from conditioning on the availability of a man-bites-dog signal is dominating the increased precision due to the content of the signal.

In order to quantify the importance of the man-bites-dog aspect of news reporting I estimate the model on US data. In addition to standard macro variables like GDP, CPI inflation and the Federal Funds rate, I also use the quarterly time series of Total Factor Productivity constructed by John Fernald (2010) as well as individual survey responses from the *Survey of Professional Forecasters*. Using individual survey responses, i.e. the entire cross-section of individual survey responses rather than a mean or median response, has at least two advantages. First, and as documented by Mankiw, Reis and Wolfers (2004), there is significant time-variation in the dispersion of forecasts reported by survey respondents. Since the model can potentially fit this fact, individual survey responses can be exploited when estimating the model, allowing for a sharper inference about the precision of signals observed by agents and the timing of man-bites-dog events in the sample period. The second advantage of using individual survey responses rather than a median or mean expectation stems from the fact that the number of survey respondents varies over time. For instance, the number of respondents forecasting nominal GDP growth and CPI inflation varies between 9 (1990:Q2) and 50 (2005:Q4) in a sample that covers the period 1981:Q3 to 2010:Q4. Using individual survey responses and likelihood based estimation methods naturally incorporates that we have a (presumably) more representative sample of the population with 50 observations than with 9.

Most existing macro models imply that the cross-sectional dispersion of expectations is either zero, as in the full information rational expectations models, or non-zero but constant as in models with private but time-invariant information structures, e.g. Lorenzoni (2009), Mackowiak and Wiederholt (2009), Graham and Wright (2009), Nimark (2008, 2011) Angeletos and La'O (2009, 2010) or Melosi (2011). One exception is the sticky information models of Mankiw and Reis (2002) and Reis (2006a, 2006b). In sticky information models, only a fraction of agents update their information in each period and those who update, all observe the state perfectly. Because of this feature, the cross-sectional distribution of expectations implied by sticky information models is a mixture of degenerate distributions, with relative weights decreasing with the vintage of information that the forecasts are based on. It is because the implied cross-sectional dispersion of expectations in the model presented here is time-varying but continuous that it is possible to estimate the structural parameters of the model using likelihood based methods and individual survey responses. One methodological contribution of the paper is to demonstrate how dynamic models with time-varying information structures can be solved and estimated. This may be of independent interest to some readers.

In addition to the benchmark specification, I also estimate several closely related models. A Bayesian model comparison reveals that both time-variation in volatility and the number of periods in which the public signal about productivity is available generally improve the fit of the model. The benchmark man-bites-dog model thus fits better than a more restricted model with no public signals. However, the best fitting specification is a model with time-varying exogenous volatility in which the public signal about productivity is available in every period. This result may cast some doubt on the empirical relevance of the man-bites-dog mechanism. However, a closer analysis suggests that allowing for a public signal in periods of high volatility is more important than in periods of low volatility, which lends some qualified support for the mechanism presented here. Of course, a less favorable interpretation is that the time-varying volatility and cross-sectional dispersion observed in the data is explained by a mechanism unrelated to man-bites-dog news reporting.

The paper is structured as follows. The next section first defines the concept of a man-bites-dog signal formally and derives some general results. It then describes how a tractable man-bites-dog information structure can be constructed and analyzes how agents' beliefs are affected by a man-bites-dog information structure using a specific example where agents solve a filtering problem but make no economic decisions. In Section 3, beliefs are linked with decisions by the embedding of a man-bites-dog information structure in the static beauty contest game of Morris and Shin (2002). There, it is demonstrated that average actions respond more strongly to both fundamentals and noise shocks when there is a man-bites-dog signal available compared to the responses to a standard signal of the same precision. Section 4 presents a simple dynamic business cycle model similar to that of Lorenzoni (2009), but with a man-bites-dog information structure. This model is the main vehicle for the empirical part of the paper. Section 5 discusses how the business cycle model is solved and how the parameters are estimated. Section 6 contains the main empirical results. Section 7 briefly discusses three closely related alternative specifications and their in-sample fit relative to the benchmark model. Section 8 concludes.

## 2. SIGNALS AND UNUSUAL EVENTS

This section introduces the concept of man-bites-dog signals formally and contains the main theoretical results of the paper. We will start by being intentionally nonspecific about distributions and derive some general properties of man-bites-dog information structures that hold under minimal assumptions. A signal  $y$  about a latent variable  $x$  will be called a man-bites-dog signal if it is more likely to be available when the realization of  $x$  is more unusual in the sense of having a lower unconditional probability of occurring. When the availability of the signal  $y$  depends on the realized value of  $x$ , the availability of  $y$  by itself carries information about  $x$  independently of the particular realized value of the signal  $y$ . Bayes' rule then implies that it is *as if* the latent variable  $x$  is drawn from a different distribution when the signal  $y$  is available compared to when it is not. In particular, conditional on the signal  $y$  being available, the relative probability of unusual events increases. This section draws out the implications of this fact for how Bayesian agents update their beliefs in response to man-bites-dog signals.

**2.1. Signal availability and conditional distributions.** Denote the unconditional probability density function of the latent variable of interest  $x$  as  $p(x)$ . An unusual realization of  $x$  is thus a realization for which  $p(x)$  is small. We are interested in information structures in which the probability of observing the signal  $y$  about  $x$  is larger for relatively unusual realizations of  $x$ . To help distinguish between a particular realization of the signal  $y$  and the event that the signal  $y$  is available, the indicator variable  $S$  is defined to take the value 1 when the signal  $y$  about  $x$  is available and 0 otherwise. We can then define a man-bites-dog signal as follows.

**Definition 1.** *The signal  $y$  is said to be a man-bites-dog signal if for any two realizations of  $x$  denoted  $x'$  and  $x''$  such that*

$$p(x') < p(x'') \tag{2.1}$$

*the inequality*

$$p(S = 1 \mid x') > p(S = 1 \mid x'') \tag{2.2}$$

*holds.*

The first inequality in the definition simply establishes that  $x'$  is a more unusual realization than  $x''$ . The second inequality formalizes the notion that a more unusual realization of  $x$  is considered more newsworthy than a more common realization. Because the signal  $y$  is more likely to be available for some realized values of  $x$  than for others, the availability of the signal  $y$  is in itself informative about the distribution of  $x$ . More specifically, using Bayes' rule, the next proposition shows that conditional on the event that the signal  $y$  is available, the probability of more unusual realizations of  $x$  increases.

**Proposition 1.** *The more unusual realization  $x'$  is relatively more likely when the signal  $y$  is available, i.e.*

$$\frac{p(x' \mid S = 1)}{p(x'' \mid S = 1)} > \frac{p(x')}{p(x'')} \tag{2.3}$$

*Proof.* Dividing Bayes' rule for conditional probabilities

$$p(x | S = 1) p(S = 1) = p(S = 1 | x) p(x) \quad (2.4)$$

for  $x'$  by the same expression for  $x''$  gives

$$\frac{p(x' | S = 1)}{p(x'' | S = 1)} = \frac{p(S = 1 | x') p(x')}{p(S = 1 | x'') p(x'')} \quad (2.5)$$

The proof then follows directly from the fact that the inequality (2.2) in Definition 1 implies that

$$\frac{p(S = 1 | x')}{p(S = 1 | x'')} > 1 \quad (2.6)$$

□

Proposition 1 states that the relative probability of the more unusual realization  $x'$  compared to the more common realization  $x''$  is larger conditional on the signal  $y$  being available. The availability of a man-bites-dog signal thus implies that probability mass should be re-distributed away from unconditionally more likely outcomes towards relatively less likely outcomes. By a completely symmetric argument we have

$$\frac{p(x' | S = 0)}{p(x'' | S = 0)} < \frac{p(x')}{p(x'')} \quad (2.7)$$

so that the absence of a man-bites-dog signal implies that probability mass should be re-distributed towards relatively more likely outcomes. Because the availability of the signal  $y$  is a discrete event, whether  $y$  is available or not thus effectively splits the unconditional distribution  $p(x)$  into the two distinct conditional distributions  $p(x | S = 1)$  and  $p(x | S = 0)$ .

**2.2. Unimodal symmetric distributions.** While the implications of a man-bites-dog information structure derived above hold for any distribution  $p(x)$ , from here on we will restrict our attention to unimodal symmetric distributions centered around a zero mean. The probability density function  $p(x)$  then takes a small value when the absolute value of  $x$  is large since realizations of  $x$  further out in the tails of  $p(x)$  are more unusual than realizations closer to the mean.

Figure 1 illustrates a man-bites-dog information structure for a unimodal and symmetric distribution  $p(x)$  (solid line). The dashed line illustrates the probability of observing the signal  $y$  conditional on different realizations of  $x$ . At the mean, there is approximately a 40% chance of observing the signal  $y$ . As realizations of  $x$  further from the mean are considered, the conditional probability of observing a signal increases towards 1 so that a signal  $y$  is available almost surely when the realization of  $x$  is far enough away from the mean. Graphically, that the conditional probability of observing  $y$  satisfies the definition of a man-bites-dog signal is implied by the fact that the slope of the dashed line and the solid line are of opposite signs (or both zero) for all values of  $x$ .

The distribution of  $x$  conditional on  $y$  being available can be backed out from  $p(x)$  and the conditional probability of observing  $y$  by using Bayes' rule

$$p(x | S = 1) = \frac{p(S = 1 | x) p(x)}{p(S = 1)}. \quad (2.8)$$



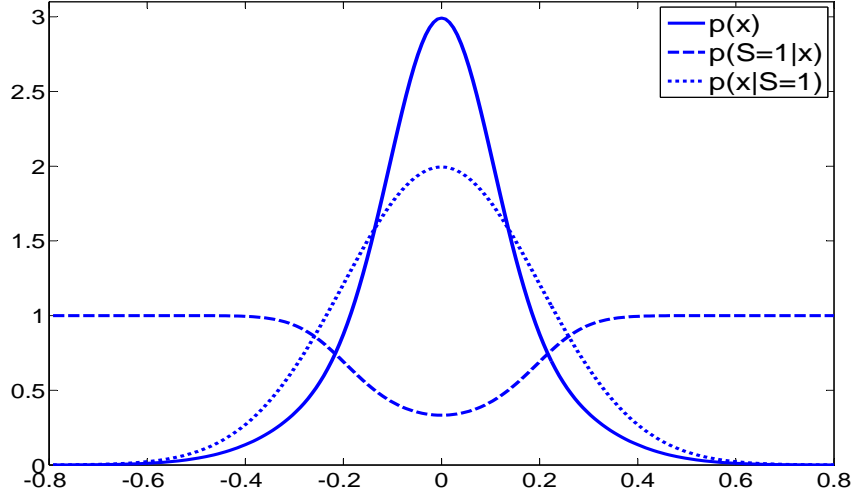


FIGURE 1. Unconditional distribution of  $x$ , conditional probability of observing the signal  $y$  and the implied conditional distribution of  $x$ .

if the unconditional probability of observing  $y$  is known. By construction, the conditional probability of observing  $y$  is increasing in the absolute value of  $x$ . Since the conditional distribution on the left hand side of (2.8) is proportional to the product of the unconditional distribution  $p(x)$  and the conditional probability  $p(S = 1 | x)$ , the conditional distribution  $p(x | S = 1)$  must have more probability mass in tails of the distribution relative to the unconditional distribution  $p(x)$ . This is illustrated by the dotted line in Figure 1.

Bayes' rule can also be used to plot the conditional distribution of  $x$  when  $y$  is not available. From the inequality (2.7) we know that it must have less probability mass in the tails than  $p(x)$  (not shown). A man-bites-dog information structure thus introduces a form of conditional heteroscedasticity in the distribution of  $x$ . From the agents' perspective, it is *as if* the latent variable  $x$  is drawn from a more dispersed distribution when the signal  $y$  is available compared to when it is not. It is important to note that this is true even when the signal content of  $y$  is not specifically about the variance, or second moment of,  $x$ . It is also important to keep in mind that the indicator variable  $S$  is a modeling device that we use to describe the event that the signal  $y$  is available and not a separate signal that agents can observe directly and independently of  $y$ .

**2.3. Reverse engineering a tractable man-bites-dog information structure.** So far, little has been assumed about the signal  $y$  apart from how the probability of observing it depends on the realized value of  $x$ . In order to describe a complete filtering problem of an economic agent, we need to be more specific not only about the exact nature of the signal  $y$  but also about other potential sources of information.

Throughout the rest of this section as well as in the next, agents indexed by  $j \in (0, 1)$  want to form an estimate of  $x$  conditional on all available information. There are two types

of signals. Agent  $j$  can always observe the private signal  $x_j$  which is the sum of the true  $x$  plus an idiosyncratic noise term

$$x_j = x + \varepsilon_j : \varepsilon_j \sim N(0, \sigma_\varepsilon^2) \quad \forall j. \quad (2.9)$$

where the variance of the idiosyncratic noise term  $\varepsilon_j$  is common across agents. There also exists a public signal  $y$

$$y = x + \eta : \eta \sim N(0, \sigma_\eta^2). \quad (2.10)$$

that may not always be available. As before, the indicator variable  $S$  takes the value 1 when  $y$  is available and 0 otherwise. The fact that all agents observe  $y$  when it is available is common knowledge (though this does not really matter until later).

To make  $y$  a man-bites-dog signal, we need to specify how the availability of  $y$  depends on the realized value of  $x$ . One approach would be to directly write down a parameterized functional form for the conditional probability  $p(S = 1 | x)$  that conforms to the definition of a man-bites-dog signal. That approach is feasible, but will in general not be sufficient to deliver tractable expressions for agents' posterior beliefs. Instead, we will use that the agents in the model always know whether the signal  $y$  is available or not. The agents thus never need to evaluate the unconditional distribution  $p(x)$ . When agents observe the signals  $x_j$  and  $y$  they instead update their beliefs from the conditional, "prior" distribution  $p(x | S)$ . We ensure tractability by specifying these conditional distributions to be conjugate to the distributions of the signals. By applying the results of Proposition 1 "in reverse", these conditional distributions can be parameterized to ensure that the inequalities in Definition 1 are satisfied so that we indeed have a man-bites-dog information structure.

**2.3.1. A tractable class of conditional distributions.** Updating normally distributed priors to normally distributed signals results in normally distributed posteriors, i.e. normal distributions are self-conjugate. A good choice given the signal structure (2.9) - (2.10) is thus to make the distributions  $p(x | S)$  conditionally normal. To this end, specify  $x$  conditional on  $S$  as

$$p(x | S = 0) = N(0, \sigma^2) \quad (2.11)$$

$$p(x | S = 1) = N(0, \gamma\sigma^2) \quad (2.12)$$

so that the unconditional distribution  $p(x)$  is a mixture normal

$$x \sim (1 - \omega) N(0, \sigma^2) + \omega N(0, \gamma\sigma^2) \quad (2.13)$$

The parameter  $\omega$  then determines how often the signal  $y$  is observed in the unconditional sense, i.e.  $\omega \equiv p(S = 1)$ .

**2.3.2. Verifying Definition 1.** The distributional assumptions (2.11) - (2.12) ensures tractability of agents' posterior beliefs. To make  $y$  a man-bites-dog signal, we need to parameterize these distributions so that the probability of observing  $y$  is larger for values of  $x$  that are relatively less likely under the unconditional distribution (2.13). The probability density function of a mixture of two normals both centered at zero is decreasing in the absolute value of  $x$ . Larger realizations of  $x$  are thus more unusual, so for  $y$  to be a man-bites-dog signal we need the probability  $p(S = 1 | x)$  to be increasing in the absolute value of  $x$ .

The distributional assumptions above together with Bayes' rule imply that the conditional distribution  $p(S = 1 | x)$  satisfies the equation

$$\frac{p(S = 1 | x)}{1 - p(S = 1 | x)} = \frac{\omega}{(1 - \omega)} \frac{1}{\sqrt{\gamma}} e^{(1 - \gamma^{-1}) \frac{x^2}{2\sigma^2}}. \quad (2.14)$$

The term  $(1 - \gamma^{-1})$  in the exponent on the right hand side of (2.14) is positive if  $\gamma > 1$ . Imposing this restriction on  $\gamma$  thus ensures that the conditional probability  $p(S = 1 | x)$  of observing the signal  $y$  is increasing in the absolute value of  $x$ . It is also clear from the expression that setting  $\gamma = 1$  makes the probability of observing  $y$  independent of  $x$ . (Expression (2.14) is derived in the Appendix.)

**2.3.3. Manipulating the probability of observing  $y$ .** Choosing  $\omega$ ,  $\gamma$  and  $\sigma^2$  let us manipulate the shape of the conditional probability of observing the signal  $y$ . For instance, a large value for  $\gamma$  implies that the probability of observing the signal  $y$  is low for values of  $x$  close to its mean. Similarly, observing  $y$  can be made an unconditionally rare event by setting  $\omega$  close to zero. While manipulating the parameters  $\omega$  and  $\gamma$  we may want to treat the unconditional variance of  $x$  as a primitive that we want to keep fixed. For all values of  $\omega$  and  $\gamma$  this is always possible since the variance of the mixture normal distribution (2.13) is given by

$$\sigma_x^2 = \omega\gamma\sigma^2 + (1 - \omega)\sigma^2 \quad (2.15)$$

and thus provides sufficient flexibility to hold  $\sigma_x^2$  fixed by scaling  $\sigma^2$ .

**2.4. Posterior beliefs.** Given the definitions (2.9) and (2.10) of the signals  $x_j$  and  $y$  and the distributional assumptions (2.11) - (2.12), agent  $j$ 's conditional expectations of  $x$  are then given by standard formulas for multiple signals with independent Gaussian noise processes. Denoting agent  $j$ 's information set  $\Omega_j^0$  when  $S = 0$  and  $\Omega_j^1$  when  $S = 1$ , the respective conditional expectations are then given by

$$E(x | \Omega_j^0) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}} x_j \quad (2.16)$$

and

$$E(x | \Omega_j^1) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} y \quad (2.17)$$

The weights on the signals are determined by the relative precision of the individual signals and the respective conditional distribution. The posterior variances are also given by the standard formulas

$$E[x - E(x | \Omega_j^0)]^2 = (\sigma_\varepsilon^{-2} + \sigma^{-2})^{-1} \quad (2.18)$$

or

$$E[x - E(x | \Omega_j^1)]^2 = (\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2})^{-1}. \quad (2.19)$$

depending on whether the signal  $y$  is available or not.

**2.5. Properties of conditional expectations with *man-bites-dog* signals.** The expressions for agents' posterior beliefs allow us to prove three results.

**Proposition 2.** *The (cross-sectional) average expectation of  $x$  responds more strongly to  $x$  when  $y$  is available.*

*Proof.* In the Appendix. □

The proof of Proposition 2 simply entails verifying that the sum of the coefficients on the two signals is larger when  $S = 1$  than the coefficient on the single private signal when  $S = 0$ . The weights agents put on  $y$  and  $x_j$  are inversely related to the precision of the conditional distributions  $p(x | S)$  and the sum of these weights thus go up unambiguously when  $y$  is available. Intuitively, agents are willing to update their expectations further when they know that a tail realization is more likely to have occurred, but part of the stronger response of expectations is simply due to the fact that agents have more information when the man-bites-dog signal is available. However, in the next section we show in simple static game that both expectations and aggregate actions also respond more strongly to a man-bites-dog signal than to a standard public signal of the same precision.

Proposition 2 above holds regardless of parameter values. The next two results derive conditions on the parameters of the model for when the stronger response of expectations to a man-bites-dog signal will be associated with an increase in uncertainty and cross-sectional dispersion of expectations.

**Proposition 3.** *The posterior uncertainty about  $x$  is larger when the signal  $y$  is observed relative to when it is not if the inequality*

$$\sigma_\eta^2 > \frac{\sigma^2}{(1 - \gamma^{-1})} \quad (2.20)$$

*holds.*

*Proof.* From (2.18) and (2.19) it is clear that the posterior variance is larger when  $y$  is available if the inequality

$$\sigma_\epsilon^{-2} + \sigma^{-2} > \sigma_\epsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2} \quad (2.21)$$

holds. The proof follows by rearranging (2.21) into the inequality in the proposition. □

That posterior uncertainty may increase when an additional signal is observed may at first appear counterintuitive but is in fact a natural consequence of Proposition 1. The event that the signal  $y$  is *available* make agents redistribute probability mass towards the tails of the distribution. This increases uncertainty. But observing the contents of the signal  $y$  is informative about the location of  $x$  which decreases uncertainty. Clearly, this second effect will be weaker when the signal is very noisy. When  $y$  is sufficiently noisy for the inequality (2.20) to hold, the former effect dominates and the posterior variance is larger than it would be if  $y$  was not available.

For a fixed variance of the noise in the signal  $y$ , the inequality in the proposition will also hold if  $\gamma$  is sufficiently large. A large  $\gamma$  implies that only very unusual realizations of  $x$  are significantly more likely to generate the signal  $y$ . The distribution  $p(x | S = 1)$  then has lot more probability mass in the tails relative to the unconditional distribution  $p(x)$  and the

signal  $y$  does not have to be very noisy for this effect to dominate. However, since the right hand side of (2.20) has a minimum of  $\sigma^2$  when  $\gamma \rightarrow \infty$ , that  $\sigma_\eta^2 > \sigma^2$  is a necessary condition for the uncertainty to be larger when  $y$  is available.

Since uncertainty may increase after observing  $y$ , one may think that risk-averse agents would be better off if the signal  $y$  was never available. This is not the case. It can be shown that even when the private signal  $x_j$  is uninformative, the unconditional expectation of the posterior variance of agents' beliefs is strictly smaller than the unconditional variance  $\sigma_x^2$  as long as the man-bites-dog signal is not infinitely noisy. This is related to a result from information theory stating that, in general, it is possible that some realizations of signals may increase entropy, though on average entropy must decrease when conditioning on more information (see Theorem 2.6.5 of Cover and Thomas 2006).<sup>1</sup>

**Corollary 1.** *When the inequality*

$$\sigma_\eta^2 > \frac{\sigma^2}{(1 - \gamma^{-1})} \quad (2.22)$$

*holds, the cross sectional dispersion of expectations about  $x$  is larger when  $y$  is observed compared to when it is not.*

The proof follows directly from the fact that the denominator in the weight agents put on the private signal in (2.16) and (2.17) is the same as the denominator in the posterior variances (2.18) and (2.19). The cross sectional dispersion is increasing in the weight on the private signal (holding the variance of the idiosyncratic noise constant). The same conditions that deliver a higher posterior variance thus also deliver more weight on the private signal and the intuition is also similar. If the public noise variance is high and the conditional probability of a tail event is high, agents will put more weight on other (e.g. private) sources of information.

This result can be contrasted to that of Kondor (2012). He shows that in a setting where two classes of agents are constrained in what type of private information they can acquire, a public signal may increase the dispersion between first and second order expectations. In the model presented here, a man-bites-dog signal decreases dispersion between different orders of expectation (not shown) but increases the cross-sectional dispersion of first order expectations.

### 3. MAN-BITES-DOG SIGNALS IN A BEAUTY CONTEST GAME

Above, the implications of a man-bites-dog information structure for agents' beliefs about the latent variable  $x$  were analyzed in some detail, but there were no economic decisions made by the agents. Here we introduce the information structure presented above into the beauty contest model of Morris and Shin (2002). This simple model will help us build intuition for the how a man-bites-dog information structure affects economic decisions that carries over to the dynamic business cycle model of the next section. Below, the main components of Morris and Shin's model are presented, though some derivations are relegated to the Online Appendix.

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<sup>1</sup>I am indebted to Mirko Wiederholt for pointing out this link to me.

**3.1. A beauty contest model.** The model of Morris and Shin (2002) consists of a utility function  $U_j$  for agent  $j$

$$U_j = -(1-r)(a_j - x)^2 - r(L_j - \bar{L}) \quad (3.1)$$

where  $a_j$  is the action taken by agent  $j$ . The terms  $L_j$  and  $\bar{L}$  are defined as

$$L_j \equiv \int (a_i - a_j)^2 di, \quad \bar{L} \equiv \int L_i di \quad (3.2)$$

Maximizing the expected value of the utility function (3.1) results in a first order condition for agent  $j$  given by

$$a_j = (1-r)E[x | \Omega_j] + rE[\bar{a} | \Omega_j] \quad (3.3)$$

where  $\bar{a}$  is the cross-sectional average action

$$\bar{a} \equiv \int a_i di \quad (3.4)$$

For a positive value of  $r$ , agent  $j$  thus wants to take an action that is close to the true value of  $x$  as well as close to the average action taken by other agents. The relative weights of these two objectives are determined by the parameter  $r$ . This basic structure is identical to that of the model in Morris and Shin (2002). As shown by Angeletos, Iovino and La'O (2011), agent  $j$ 's first order condition (3.3) is isomorphic to that of a firm in a simple business cycle model with monopolistic competition and dispersed information. In that setting, the action  $a_j$  corresponds to the optimal level of firm  $j$ 's output and the parameter  $r$  is a composite function of the parameters governing the curvature of the utility function and the elasticity of substitution between differentiated goods.

**3.2. The average action as a function of higher order expectations.** The expectation about the average action of other agents can be eliminated from agent  $j$ 's first order condition by repeated substitution. Taking averages of the resulting expression allows us to rewrite the average action  $\bar{a}$  as a weighted average of higher order expectations about  $x$

$$\bar{a} = (1-r) \sum_{k=1}^{\infty} r^{k-1} x^{(k)} \quad (3.5)$$

where the average  $k$  order expectation  $x^{(k)}$  is defined recursively as

$$x^{(k)} \equiv \int E[x^{(k-1)} | \Omega_j] dj \quad (3.6)$$

starting from the convention that  $x^{(0)} \equiv x$ . The expression for the average action (3.5) holds regardless of the assumed information structure. A man-bites-dog information structure will thus imply a different average action  $\bar{a}$  compared to the original model only to the extent that such an information structure will imply that higher order expectations in the two models differ.

**3.3. Higher order expectations and signals.** We can now embed the information structure defined in Section 2.3 into the model described above. Given the signal structure (2.9) and (2.10) and the distributional assumptions (2.11) - (2.12) the average  $k$  order expectation about  $x$  is given by

$$x^{(k)} = g_0^k x \quad (3.7)$$

when agents only observe  $x_j$  and by

$$x^{(k)} = g_y y + g_x^k (x - g_y y) \quad (3.8)$$

when the signal  $y$  is also available. The coefficients  $g_0$ ,  $g_x$  and  $g_y$  are given by

$$g_0 \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}, \quad g_x \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}, \quad g_y \equiv \frac{\sigma_\eta^{-2}}{\sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} \quad (3.9)$$

From the expression for the  $k$  order expectation (3.8) we can see that

$$\lim_{k \rightarrow \infty} x^{(k)} = g_y y \quad (3.10)$$

since  $0 < g_x < 1$ . Thus, just as in Morris and Shin's model, higher order expectations tend to be dominated by the public signal  $y$  (when available) as the order of expectation increases.<sup>2</sup>

**3.4. Aggregate responses to shocks and man-bites-dog signals.** In order to determine how the average action  $\bar{a}$  is affected by the man-bites-dog structure, substitute the expressions for the higher order expectations (3.7) and (3.8) into the average action expression (3.5). After simplifying, the average action is given by

$$\bar{a} = \frac{(1-r)g_0}{1-rg_0} x \quad (3.11)$$

when the signal  $y$  is not available and

$$\bar{a} = \frac{(1-r)g_x}{1-rg_x} x + \left(1 - \frac{(1-r)g_x}{1-rg_x}\right) g_y y \quad (3.12)$$

when it is. We can then prove the following.

**Proposition 4.** *The response of the average action  $\bar{a}$  to a given value of  $x$  is stronger when the signal  $y$  is available.*

*Proof.* In the Appendix. □

As in Proposition 2 above, proving the proposition entails verifying that the coefficient on  $x$  in (3.11) is always smaller than the sum of the coefficients on  $x$  and  $y$  in (3.12). The proposition holds for all parameter values, including when  $\gamma = 1$  and is thus true partly because expectations (of all orders) simply respond more strongly when there are more signals available. Perhaps more interestingly, the next proposition establishes that conditional on the public signal being available, the average action responds more strongly to a man-bites-dog signal than to a standard public signal of the same precision.

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<sup>2</sup>In the original model of Morris and Shin (2002), the unconditional variance of  $x$  (or  $\theta$  in their notation) is left undefined. Yet, the original model is a special case of the set-up here. The equilibrium is identical for the two models if we impose the parameter restrictions  $\gamma = \omega = 1$  and by letting  $\sigma_x^2$  (which then equals  $\sigma^2$ ) tend to infinity.

**Proposition 5.** *The average action responds more strongly to a change in  $x$  when a man-bites-dog signal is available, compared to the response of the average action when the public signal about  $x$  is always available (holding the precision of the signal and the unconditional variance of  $x$  fixed).*

*Proof.* In the Appendix. □

The proof of Proposition 5 uses the following two features of the model. First, the average action in (3.5) is an increasing function of all orders of expectations. Second, all orders of expectations respond more strongly to a given  $x$  when  $y$  is man-bites-dog signal compared to when it is not. To understand the second part, note that when  $\gamma > 1$  so that  $y$  is a man-bites-dog signal, by (2.15) the conditional variance  $\text{var}(x | S = 1)$  is larger than the unconditional variance, while if  $y$  is always available the unconditional and the conditional variance are by construction the same. Since agents are willing to update their first order expectations further when the conditional variance is large, first order expectations will respond more strongly to a change in  $x$  when  $y$  is man-bites-dog signal. This in turn implies that the conditional variance of first order expectations is higher when  $y$  is a man-bites-dog signal. By the same argument, agents are then willing to update their expectations about other agents' expectations further as well. Second order expectations thus also respond more to a given change in  $x$ , and so on.

A man-bites-dog information structure also changes how noise in the signal  $y$  affects the average action relative to the case when  $y$  is always available. Since  $y = x + \eta$  the coefficient on  $y$  in (3.12) determines how big an impact the noise shock  $\eta$  has on the average action  $\bar{a}$ . For the same reason that the average action responds more strongly to the fundamental  $x$  when a man-bites-dog signal is available, the impact of a noise shock in a man-bites-dog signal is also stronger than the impact of a noise shock in a standard signal. Of course, with a man-bites-dog information structure, the noise shock can only affect the average action when the signal  $y$  is available. This limits how much of the unconditional variance that can be explained by a noise shock.

This ends the theoretical part of the paper. Before turning to the estimated business cycle model we can sum up what has been demonstrated up to this point. First, conditional expectations and average actions respond more strongly when a man-bites-dog signal is available. Second, the stronger responses of expectations and actions when a man-bites-dog signal is available may be accompanied by either more or less posterior uncertainty and dispersion of expectations. We showed that uncertainty and expectation dispersion increase when the precision of the signal is low or when the parameter  $\gamma$  is large.

#### 4. A BUSINESS CYCLE MODEL

This section presents a simple business cycle model, following closely that of Lorenzoni (2009), but with a man-bites-dog information structure. As in the model by Lorenzoni, there are informationally separated islands that are subject to both common and island specific productivity shocks that cannot be distinguished by direct observation. Instead, island inhabitants need to form an estimate of the common component of productivity in order to make optimal consumption and price setting decisions. In the model, there is both island specific and public information and the man-bites-dog signal is specified as a public



signal about aggregate productivity. This means that information about large changes to productivity will be more correlated across agents than information about small changes.

**4.1. Preferences and technology.** The set up is a stationary version of the island economy described in Lorenzoni (2009). There is a continuum of islands indexed by  $j \in (0, 1)$  and on each island there is a continuum of firms indexed by  $i \in (0, 1)$  producing differentiated goods. On each island there is a representative household that consumes goods and supplies labor  $N_{j,t}$  to the firms on the island. Households are thus also indexed by  $j \in (0, 1)$  and a household on island  $j$  maximizes

$$E \sum_{s=0}^{\infty} \beta^s \left[ \exp(d_{j,t}) \ln C_{j,t} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi} \mid \Omega_{j,t} \right] \quad (4.1)$$

where  $\Omega_{j,t}$  is the information set of inhabitants of island  $j$  and  $C_{j,t}$  is the consumption bundle consumed by island  $j$  households defined as

$$C_{j,t} = \left( \int_{\mathcal{B}_{j,m}} \int_0^1 C_{i,m,j,t}^{(\delta-1)/\delta} di dm \right)^{\delta/(1-\delta)} \quad (4.2)$$

As in Lorenzoni (2009), households only consume goods from a subset  $\mathcal{B}_{j,m} \subseteq (0, 1)$  of islands. The set  $\mathcal{B}_{j,m}$  is drawn by nature in each period. The shock  $d_{j,t}$

$$d_{j,t} = d_t + \zeta_{j,t} : \zeta_{j,t} \sim N(0, \sigma_\zeta^2) \quad (4.3)$$

is a demand disturbance that is correlated across islands and the common component  $d_t$  follows an AR(1) process

$$d_t = \rho_d d_{t-1} + u_t^d : u_t^d \sim N(0, \sigma_d^2) \quad (4.4)$$

The demand disturbance  $d_t$  is not present in the original model by Lorenzoni (2009) but is needed here in order to avoid stochastic singularity when the model is estimated. Firm  $i$  on island  $j$  produce good  $i, j$  using the technology

$$Y_{i,j,t} = \exp(a_{j,t}) N_{i,j,t} \quad (4.5)$$

(The log of) productivity  $a_{j,t}$  is the sum of a common component  $a_t$  and the island specific component  $\varepsilon_{j,t}$

$$a_{j,t} = a_t + \varepsilon_{j,t} : \varepsilon_{j,t} \sim N(0, \sigma_\varepsilon^2) \quad \forall j, t. \quad (4.6)$$

The common productivity component  $a_t$  follows an AR(1) process

$$a_t = \rho_a a_{t-1} + u_t^a \quad (4.7)$$

The innovation to common productivity  $u_t^a$  is distributed as a mixture normal and is specified in detail below. Firms on island  $j$  are owned by island  $j$  households and set prices  $P_{j,t}$  to maximize discounted expected profits  $\Pi_{j,t}$

$$E \left[ \sum_{s=t}^{\infty} \theta^s \beta^s \frac{C_{j,t}}{C_{j,t+s}} \Pi_{j,t+s} \mid \Omega_{j,t} \right] = E \left[ \sum_{s=t}^{\infty} \theta^s \beta^s \frac{C_{j,t}}{C_{j,t+s}} (P_{j,t+s} Y_{j,t+s} - W_{j,t+s} N_{j,t+s}) \mid \Omega_{j,t} \right] \quad (4.8)$$

where  $\theta$  is the probability of not changing the price of a given good in a given period. The intertemporal budget constraint of households on island  $j$  equates expenditure with income

$$\frac{B_{j,t+1}}{R_t} + \int_{\mathcal{B}_{j,m}} \int P_{i,j,t} C_{j,m} di dm = B_{j,t} + W_{j,t} N_{j,t} + \int \Pi_{i,j,t} di. \quad (4.9)$$

$R_t$  is the nominal one period interest rate which (in logs) follows a Taylor rule

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_r r_{t-1} + u_t^r : u_t^r \sim N(0, \sigma_r^2) \quad (4.10)$$

and  $B_{j,t}$  are households on island  $j$ 's holdings of nominal bonds that pay one dollar in period  $t$ .

**4.2. Linearized equilibrium conditions.** The model presented above can be log linearized around a non-stochastic steady state, yielding the following equilibrium conditions. (i) An Euler equation determining the optimal intertemporal allocation of consumption

$$c_{j,t} = E[c_{j,t+1} | \Omega_{j,t}] - r_t + E[\bar{\pi}_{\mathcal{B}_{j,t+1}} | \Omega_{j,t}] + d_{j,t} \quad (4.11)$$

where  $\bar{\pi}_{\mathcal{B}_{j,t+1}}$  is the inflation of the goods basket consumed on island  $j$  in period  $t+1$ . (ii) A labor supply condition equating marginal disutility of labor supply with the marginal utility of consumption multiplied by the real wage

$$w_{j,t} - \bar{p}_{\mathcal{B}_{j,t}} = c_{j,t} + \varphi n_{j,t}. \quad (4.12)$$

(iii) A demand schedule for good  $j$  depending on the relevant relative price of good  $j$

$$y_{j,t} = \int_{\mathcal{C}_{j,t}} c_{m,t} dm - \delta \left( p_{j,t} - \int_{\mathcal{C}_{j,t}} \bar{p}_{m,t} dm \right) \quad (4.13)$$

where  $\int_{\mathcal{C}_{j,t}} \bar{p}_{m,t} dl$  is the log of the relevant price sub index for consumers from other islands buying goods from island  $j$ . (iv) An island  $j$  Phillips curve relating inflation on island  $j$  to the nominal marginal cost on island  $j$  and expected future island  $j$  inflation

$$\begin{aligned} p_{j,t} - p_{j,t-1} &= \lambda (\bar{p}_{\mathcal{B}_{j,t}} + c_{j,t} - p_{j,t} - a_{j,t}) + \lambda \varphi (y_{j,t} - a_{j,t}) \\ &+ \beta E(p_{j,t+1} - p_{j,t} | \Omega_{j,t}) \end{aligned} \quad (4.14)$$

where  $\bar{p}_{\mathcal{B}_{j,t}}$  is the relevant price subindex for consumers on island  $j$  and  $\lambda = (1-\theta)(1-\theta\beta)/\beta$ . The steps required to arrive at the linearized equilibrium conditions (4.11) - (4.14) are identical to those described in Lorenzoni (2009).

**4.3. Local information.** The inhabitants of each island observe their own productivity and demand disturbances  $a_{j,t}$  and  $d_{j,t}$ . Since these contain a component that is common across islands, these local variables are informative about the aggregate state. In addition to these exogenous local signals, firms and households can also extract information about the aggregate state from observing the demand for their own good  $y_{j,t}$  and the price of the goods basket that they purchase. Lorenzoni (2009) assumes that the islands visited by the inhabitant of island  $j$  while shopping are drawn so that the price index of the goods basket consumed by island  $j$  inhabitants is equal to the aggregate price level plus a normally distributed island  $j$  specific shock. We will make a similar assumption, with an adjustment

to the mean of the normally distributed shock such that the signal is conditionally stationary. That is, the set  $\mathcal{B}_{j,t}$  is drawn such that

$$\bar{p}_{\mathcal{B}_{j,t}} = p_t + \xi_{j,t}^p : \xi_{j,t}^p \sim N(p_{j,t-1} - p_{t-1}, \sigma_{\xi^p}^2) \quad (4.15)$$

Since  $p_{j,t-1}$  is observable by the inhabitants of island  $j$  the price index of the good purchased by island  $j$  inhabitant is thus a noisy measure of aggregate inflation  $p_t - p_{t-1}$ , rather than of the aggregate price level as in Lorenzoni (2009). Reformulating the signal structure this way does not change anything substantial in the model but simplifies the representation of agents' filtering problems since all other variables except for the price level are stationary. Similarly, we assume that the set of islands  $\mathcal{C}_{j,t}$  is drawn such that (4.13) takes the form

$$y_{j,t} = y_t - \delta(p_{j,t} - p_t) + \xi_{j,t}^y : \xi_{j,t}^y \sim N(\delta[p_{j,t-1} - p_{t-1}], \sigma_{\xi^y}^2) \quad (4.16)$$

Again, the adjustment of the mean of the disturbance relative to Lorenzoni (2009) is made in order to keep signals stationary. As in Lorenzoni's original model, the shocks  $\xi_{j,t}^p$  and  $\xi_{j,t}^y$  are introduced in order to prevent local interactions from perfectly revealing the aggregate state.

**4.4. The joint distribution of signals and shocks.** The man-bites-dog signal structure is embedded in the business cycle model in a similar way as in the static setting discussed in Section 2 and 3. The unobservable variable of interest is the common component of productivity  $a_t$ . The joint distribution of the indicator variable  $s_t$  and the innovations  $u_t^a$  in (4.7) are specified such that a man-bites-dog signal is more likely to be generated when there has been a large (in absolute terms) innovation to the common productivity process  $a_t$ . The indicator variable  $s_t$  takes the value 1 when a man-bites-dog signal is generated in period  $t$  which occurs with unconditional probability  $\omega$ . Similar to the static setting of Section 2, a mixture normal density for  $u_t^a$  will be used to keep the filtering problem tractable

$$u_t^a \sim (1 - \omega) N(0, \sigma_a^2) + \omega N(0, \gamma \sigma_a^2). \quad (4.17)$$

with  $\gamma > 1$ . The unconditional variance of the productivity innovations  $u_t^a$  is then given by

$$E(u_t^a)^2 = (1 - \omega) \sigma_a^2 + \omega \gamma \sigma_a^2 \quad (4.18)$$

To complete the description of the joint distribution of innovations and signals, it is further assumed that when  $s_t = 1$  all households observe an additional public signal  $z_{a,t}$  given by

$$z_{a,t} = a_t + \eta_t : \eta_t \sim N(0, \sigma_\eta^2) \quad (4.19)$$

The signal  $z_t^a$  is thus a man-bites-dog signal and the vector of observables  $\mathbf{z}_{j,t}$  available to households and firms on island  $j$  in period  $t$  then is

$$\mathbf{z}_{j,t} = [a_{j,t} \ d_{j,t} \ y_{j,t} \ \bar{p}_{j,t} \ r_t \ s_t] \quad (4.20)$$

if  $s_t = 0$  and

$$\mathbf{z}_{j,t} = [a_{j,t} \ d_{j,t} \ y_{j,t} \ \bar{p}_{j,t} \ r_t \ s_t \ z_t^a] \quad (4.21)$$

if  $s_t = 1$ . The information set  $\Omega_{j,t}$  of firms and households on island  $j$  evolves as

$$\Omega_{j,t} = \{\mathbf{z}_{j,t}, \Omega_{j,t-1}\} \quad (4.22)$$

This completes the description of the model. It is perhaps worth noting that the original model of Lorenzoni (2009) is nested in the model presented here by setting  $\rho_a = \omega = \gamma = 1$  and  $\sigma_d^2 = 0$ .

## 5. SOLVING AND ESTIMATING THE MODEL

There are two features of the model presented above that make standard solution methods for linear rational expectations models inapplicable. First, there is island specific information about variables that are of common interest to all islands. Natural state representations then tend to become infinite, due to the well-known problem of the infinite regress of expectations that arises when agents need to “forecast the forecasts of others” (see Townsend 1983 and Sargent 1991). Second, the precision of agents’ information is a function of the realized history of  $s_t$  and thus varies over time. This section briefly outlines how the solution method proposed in Nimark (2011) can be modified to solve a model with a time-varying information structure. Focus is on aspects of the solution method that help intuition for how time-varying information sets translate into time-varying equilibrium dynamics but a complete description of the solution algorithm is provided in the Online Appendix. The section ends with a description of how a posterior estimate of the parameters of the model and the history of  $s_t$  can be constructed using the Multiple-Block Metropolis-Hastings algorithm described in Chib (2001).

**5.1. Rationality and the dynamics of higher order expectations.** Nimark (2011) proposes an approximation method to solve linear rational expectations models with privately informed agents. Conceptually, the solution method has two components. The first is to put structure on higher order expectations, i.e. agents’ expectations about other agents’ expectations, by assuming that it is common knowledge that all agents form model consistent expectations. The second part is to use that the impact of higher order expectations on inflation and output is decreasing in the order of expectation. This second part is somewhat involved, and interested readers are referred to the original reference for more details. Here we briefly describe how common knowledge of model consistent expectations helps put structure on the dynamics of higher order expectations.

Let  $\mathbf{x}_t$  denote a vector containing the exogenous state variables  $a_t$  and  $d_t$  so that

$$\mathbf{x}_t \equiv \begin{bmatrix} a_t \\ d_t \end{bmatrix}. \quad (5.1)$$

With island specific information, the state of the model needs to be expanded relative to the full information case to also include average higher order expectations of current productivity  $a_t$  and the common demand shock  $d_t$ . The state can then be represented by the vector  $X_t$  defined as

$$X_t \equiv \left[ \mathbf{x}'_t \quad \mathbf{x}'^{(1)}_t \quad \dots \quad \mathbf{x}'^{(\bar{k})}_t \right]' \quad (5.2)$$

where

$$\mathbf{x}_t^{(k+1)} \equiv \int E \left[ \mathbf{x}_t^{(k)} \mid \Omega_t(j) \right] dj. \quad (5.3)$$

The constant  $\bar{k}$  is the maximum order of expectation considered.

To solve the model we need to find the law of motion for the vector  $X_t$ . The law of motion of  $\mathbf{x}_t$ , i.e. the first component of  $X_t$ , is given by (4.4) and (4.7). As usual in rational expectations models, first order expectations  $\mathbf{x}_t^{(1)}$  are optimal, i.e. model consistent estimates of the actual exogenous state vector  $\mathbf{x}_t$ . The knowledge that other traders form model consistent estimates allow traders to treat average first order expectations as a stochastic process with known properties when they form second order expectations. Imposing this structure on all orders of expectations allows us to find the law of motion for the hierarchy of expectations  $X_t$  as a function of the structural parameters of the model.

The agents inhabiting the islands of the model use the Kalman filter to form an estimate of all the average higher order expectations in the state vector  $X_t$ . Island  $j$ 's estimate of  $X_t$  follows the update equation

$$\underbrace{E[X_t | \Omega_{j,t}]}_{\text{posterior}} = (I - K_t D_t) \underbrace{E[X_t | \Omega_{j,t-1}]}_{\text{prior}} + K_t \underbrace{\mathbf{z}_{j,t}}_{\text{signal}} \quad (5.4)$$

where  $K_t$  is the Kalman gain and  $D_t$  is a matrix that maps the state into the observable vector  $\mathbf{z}_{j,t}$ . The Kalman filter thus plays a dual role. It is used by individual agents to form an estimate of the state vector. But since the aggregate state vector  $X_t$  is made up of the cross-sectional average of individual state estimates determined by (5.4), the Kalman update equation above also determines the law of motion of the aggregate state  $X_t$ .

**5.2. Time-varying state dynamics.** The Kalman update equation (5.4) describes how agents combine prior beliefs with period  $t$  signals. When signals are very precise or when the prior is very uncertain, agents will put more weight on the signals and less weight on the prior. For the same reasons that expectations respond more strongly in the static example of Section 2, agents' expectations about productivity will respond more strongly to a productivity innovation of a given size when a man-bites-dog signal is available.

A man-bites-dog event may also have persistent effects on the relative weight on the prior and the signals in agents' update equation. The reason is that if, for example, there is a man-bites-dog episode in period  $t$  that increases the posterior uncertainty in period  $t$ , then this will translate into an increase in prior uncertainty in period  $t + 1$ . A larger posterior uncertainty in period  $t$  thus translates into more weight being put on signals observed in period  $t + 1$ . Through this channel, the economy may become more responsive to shocks for several periods after a man-bites-dog event.

Since a period  $t$  realization of the indicator variable  $s_t$  may have persistent effects on the dynamics of the state  $X_t$  we need to keep track of the history of  $s_t$  (which we denote  $s^t$ ) to determine period  $t$  equilibrium dynamics. However, as long as the filtering errors of agents follow a stable process, i.e. do not accumulate over time, we do not need to keep track of the entire history of  $s_t$  but only its most recent realizations. How far back in time the realizations of  $s_t$  matter depends on the eigenvalues of the process that propagates the variance of agents' filtering errors through time. If the underlying processes are not very persistent, or if information is very accurate, filtering errors do not tend to be long lived and only a few of the past realizations of  $s_t$  influence current dynamics. In general, how many lags of  $s_t$  that are relevant for period  $t$  dynamics depends on the parameters of the model and needs to be checked on a case-by-case basis.

The strategy we follow is to specify a maximum lag of  $s_t$ , say  $s_{t-\mathcal{T}}$ , where  $\mathcal{T}$  should be large enough so that the Kalman gain  $K_t$  in agents' update equation (5.4) is invariant to changes in the histories of  $s_t$  up to period  $t - \mathcal{T}$ . That is, setting  $\mathcal{T} = 4$  is an appropriate choice if the changes in the Kalman gain  $K_t$  depending on whether  $s_{t-\mathcal{T}-1}, s_{t-\mathcal{T}-2}, \dots$  etc equals 1 or 0 are so small that it does not justify the increased computational burden of including one more lag of  $s_t$ . With 2 different exogenous regimes (i.e.  $s_t \in \{0, 1\}$ ) there will be  $2^{\mathcal{T}}$  relevant different histories  $s^t$  and  $2^{\mathcal{T}}$  different Kalman gains  $K_t$  in the update equation (5.4).<sup>3</sup>

The equilibrium law of motion of  $X_t$  is a vector autoregression of the form

$$X_t = M(s^t)X_{t-1} + N(s^t)\mathbf{u}_t : \mathbf{u}_t \sim N(0, I) \quad (5.5)$$

where the dependence of the matrices  $M$  and  $N$  on  $s^t$  is a consequence of the dependence of the law of motion of the state on the Kalman gain  $K_t$  in (5.4). There are thus also  $2^{\mathcal{T}}$  different laws of motion, or endogenous regimes, for  $X_t$ .

**5.3. Aggregate inflation and output.** Taking averages of the consumption Euler equation (4.11) and the Phillips curve (4.14) and collecting the resulting expressions in vector form gives the vector Euler equation

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = A \int E \left( \begin{bmatrix} \pi_{t+1} \\ y_{t+1} \end{bmatrix} \mid \Omega_{j,t} \right) + (B + C) X_t + G_r r_{t-1} \quad (5.6)$$

linking current inflation and output to the average expectation of the same variables in the next period. To solve out the expectations term we conjecture (and verify) that inflation and output can be expressed as linear functions of the state  $X_t$  with period  $t$  parameters depending on the history of man-bites-dog events  $s^t$

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = G(s^t)X_t + G_u \mathbf{u}_t + G_r r_{t-1} \quad (5.7)$$

The matrices  $G_u$  and  $G_r$  capture the direct effect of the monetary policy shock and lagged interest rate respectively. Since the interest rate is observable these matrices do not depend on  $s^t$ .<sup>4</sup> When forming expectations about period  $t + 1$  inflation and output agents will need to take into account the probability that there will be a man-bites-dog signal available in the next period. Expectations of output and inflation thus depend on the probability  $\omega$  that  $s_{t+1}$  will take the value 1. The time-varying matrix  $G(s^t)$  can be computed by noting that for a given conjectured law of motion (5.5) and the linear function (5.7), we can express current inflation and output as

$$\begin{aligned} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} &= \omega AG(s_1^{t+1}) M(s_1^{t+1}) H X_t \\ &+ (1 - \omega) AG(s_0^{t+1}) M(s_0^{t+1}) H X_t \\ &+ (B + C) X_t + G_u \mathbf{u}_t + G_r r_{t-1} \end{aligned} \quad (5.8)$$

<sup>3</sup>It is perhaps worth pointing out here that while we truncate the history of  $s_t$  used to compute the Kalman gain  $K_t$ , agents still condition on the entire history of observables  $\mathbf{z}_{j,t}$ .

<sup>4</sup>There is also an indirect affect of a monetary policy shock on inflation and output since such a shock will affect the higher order expectations in  $X_t$ . This effect is time-varying and works through the matrices  $N(s^t)$  and  $G(s^t)$ .

where  $s_n^{t+1}$  denotes the history  $s^{t+1}$  with  $s_{t+1} = n$ . The matrix  $H$  is defined so that

$$\int E[X_t | \Omega_{j,t}] dj = HX_t \quad (5.9)$$

That is,  $H$  moves a vector of average higher order expectations one step “up” in orders of expectations and is used in (5.8) to compute the average expectation in (5.6). Equating coefficients in (5.7) and (5.8) implies

$$\begin{aligned} G(s^t) &= \omega AG(s_1^{t+1}) M(s_1^{t+1}) H \\ &\quad + (1 - \omega) AG(s_0^{t+1}) M(s_0^{t+1}) H \\ &\quad + (B + C) \end{aligned} \quad (5.10)$$

The matrices  $G(s^t)$  depend on the law of motion (5.5) through  $M(s^{t+1})$  in (5.10). At the same time, the matrices  $M(s^t)$  and  $N(s^t)$  in the law of motion for the state  $X_t$  depend on  $G(s^t)$ . This dependence arises because how informative the signal vector  $\mathbf{z}_{t,j}$  is about the state  $X_t$  is partly a function of  $G(s^t)$ . The Online Appendix describes a fixed point algorithm that can be used to find the equilibrium dynamics of the model for each of the  $2^T$  relevant histories of  $s_t$ .

**5.4. Estimating the model.** The solved model is a state space system and standard likelihood based methods are applicable to estimate its parameters. In addition to the 19 structural parameters, which we denote

$$\Theta = \{\rho_a, \rho_d, \sigma_a, \sigma_d, \sigma_r, \sigma_\varepsilon, \sigma_\zeta, \sigma_{\xi 1}, \sigma_{\xi 2}, \sigma_\eta, \delta, \varphi, \phi_\pi, \phi_y, \phi_r, \theta, \beta, \omega, \gamma\} \quad (5.11)$$

we also want to construct a posterior estimate of the indicator variable  $s_t$  that keeps track of whether there was a man-bites-dog signal available in period  $t$  or not. Below we describe how this can be done by sampling from the two conditional distributions  $p(\Theta | s^T, Z^T)$  and  $p(s^T | \Theta, Z^T)$ . Dividing the sampling from the joint posterior distribution of  $\Theta$  and  $s^T$  into two conditional blocks lets us get around the problem that unlike the agents inside the model, as econometricians we do not observe the regimes  $s_t$  directly. Conditional on a given history  $s^T$ , the model is linear-Gaussian and it is straightforward to evaluate the likelihood.

**5.5. The data.** The time series used to estimate the model are US CPI inflation, the Federal Funds rate, US real GDP, the quarterly time series of total factor productivity (TFP) constructed by John Fernald (2010) and individual survey response data from the *Survey of Professional Forecasters* (SPF). The data is quarterly and the sample ranges from 1981:Q3 to 2010:Q4. The start date is chosen based on the availability of survey data for inflation forecasts and the end date is the date of the most recent data on real GDP and total factor productivity. CPI inflation is de-trended using a linear trend and the same trend is taken out of the Federal Funds rate. Real GDP is de-trended using the HP-filter with a smoothing coefficient of 1600. The survey data used are the individual survey responses of one quarter ahead forecasts of CPI inflation and nominal GDP growth taken from the *Survey of Professional Forecasters* available from the web site of the Federal Reserve Bank of Philadelphia. Inflation forecasts are de-trended using the CPI inflation trend. Nominal GDP growth forecasts are de-trended by subtracting the inflation trend and the growth rate of the real GDP (HP-filter) trend.

We denote the vector of observables in period  $t$  as  $Z_t$ . All elements in  $Z_t$  have natural counterparts in the model. Specifically, TFP will be taken as a noisy measure of the common productivity component and the vector of survey forecasts  $\mathbf{f}'_t$  are taken to be representative of the expectations of the inhabitants of the islands in the model.

In the benchmark specification, the vector of observables are thus given by

$$Z_t = [ a_t \quad \pi_t \quad y_t \quad r_t \quad \mathbf{f}'_t ]' \quad (5.12)$$

and linked to the state of the model by the measurement equation

$$Z_t = \bar{D}(s^t)X_t + D_r r_{t-1} + \bar{R}(s^t)\mathbf{u}_t \quad (5.13)$$

Due to the fact that the number of survey respondents is not constant in the sample, the dimensions of both  $\bar{D}$  and  $\bar{R}$  are time-varying. The elements of  $\bar{D}$  and  $\bar{R}$  also vary over time. The elements of the matrix  $\bar{D}$  is time-varying since the function mapping the state into endogenous variables is time-varying. The elements of the matrix  $\bar{R}$  is time-varying since the cross-sectional dispersion of forecasts is time-varying in the model.

Individual respondents can be tracked in the SPF. So while in theory, it is possible to exploit a (limited) panel dimension in the SPF responses, it is not feasible to do so in practise. This would require that in order to evaluate the likelihood, we as econometricians would need to carry along an individual state for each respondent in the SPF, thereby increasing the state dimension by a multiplier of 50. Instead, we treat individual survey responses of inflation  $f_{t,\pi}^j$  and nominal GDP growth forecasts  $f_{t,\pi+\Delta y}^j$  as independent draws from the distributions

$$f_{t,\pi}^j \sim N \left( \int E [\pi_{t+1} | \Omega_{j,t}] dj, \sigma_{f_\pi}^2(s^t) \right) \quad (5.14)$$

$$f_{t,\pi+\Delta y}^j \sim N \left( \int E [\Delta y_{t+1} + \pi_{t+1} | \Omega_{j,t}] dj, \sigma_{f_{\pi+\Delta y}}^2(s^t) \right) \quad (5.15)$$

The variances  $\sigma_{f_\pi}^2(s^t)$  and  $\sigma_{f_{\pi+\Delta y}}^2(s^t)$  are the model implied cross-sectional variances of inflation and nominal GDP growth expectations. These are functions of the structural parameters  $\Theta$  as well as the history  $s^t$  and thus vary over time.

**5.6. Priors.** We will use uniform priors on all parameters governing the variances in the model and on the parameters  $\omega, \gamma$  and  $\sigma_\eta^2$ , i.e the parameters that are directly linked to the man-bites-dog information structure. We impose informative, but relatively diffuse, priors on the remaining parameters in  $\Theta$ . The priors are reported in the first column of Table 1. The history of the indicator variable  $s_t$  is assigned a uniform prior, reflecting that we have no prior information about the timing or frequency of man-bites-dog events in the sample.

**5.7. Estimation Algorithm.** The posterior distribution of  $\Theta$  and  $s^T$  are estimated using a Multiple-Block Metropolis algorithm (see Chib 2001). It exploits the fact that, conditional on a history of man-bites-dog regimes  $s^T$ , the model is in linear-Gaussian state space form. An outline of the algorithm is as follows:

- (1) Specify initial values  $\Theta_0$  and  $s_0^T$ .
- (2) Repeat for  $j = 1, 2, \dots, J$ 
  - (a) Block 1: Draw  $\Theta_j$  from  $p(\Theta | s_{j-1}^T, Z^T)$



- (b) Block 2: Draw  $s_j^T$  from  $p(s^T | \Theta_j, Z^T)$   
 (3) Return values  $\{\Theta_0, \Theta_1, \dots, \Theta_J\}$  and  $\{s_0^T, s_1^T, \dots, s_J^T\}$

The Multiple-block Metropolis algorithm is similar to (and nests as a special case) the Gibbs sampling algorithm. In both algorithms, the parameters are divided into blocks to exploit that it is simpler to sample from the conditional distributions  $p(\Theta | s_{j-1}^T, Z^T)$  and  $p(s^T | \Theta_j, Z^T)$  than from the joint posterior  $p(s^T, \Theta | Z^T)$ . However, here, both Step 2(a) and 2(b) are executed using a Metropolis step rather than by drawing directly from the full conditional distribution. The Online Appendix describes the algorithm in detail.

**5.8. Posterior parameter estimates.** The posterior mode and the 95 per cent probability intervals for the parameters in  $\Theta$  are reported in the column labeled *Benchmark* in Table 1. The results are based on 2 600 000 draws from the Multiple-block Metropolis algorithm and the Online Appendix contains diagnostic checks for convergence and plots of the posteriors distributions. The mode refers to the parameter vector in the Markov chain that achieved the highest posterior likelihood.

Most parameters appear to be well-identified, including those with uniform priors. The two exceptions are the standard deviations of the island specific consumption demand shock  $\zeta_{j,t}$  and the shock to the demand for goods produced on an individual island  $\xi_{j,t}^y$ . In the absence of data on consumption and output on individual islands, these parameters will affect the likelihood function only via the island inhabitants beliefs about common productivity and demand. In the model, there are four island specific signals but only two unobserved aggregate fundamentals. There is thus a redundancy of signals in the sense that there are many different combinations of signal precisions that will result in the same posterior beliefs about common productivity and demand shocks. This explains why the posterior estimates of the standard deviations  $\sigma_{\xi_y}$  and  $\sigma_\zeta$  are clearly bounded away from zero, but otherwise not sharply identified. However, because of the redundancy of signals, this lack of identification is also without consequences for the results presented in the next section.

The posterior estimates of the parameters governing the properties of the man-bites-dog information structure suggest that the conditional variance when a man-bites-dog signal is available is substantially larger compared to when the signal is not available. That is, the posterior estimate of  $\gamma$  is large, with a posterior mode at 5.01. It is also relatively precisely estimated, and clearly bounded away from 1. The posterior mode of  $\omega$  suggest that the unconditional probability of a man-bites-dog event is around 30 per cent. The man-bites-dog signal is estimated to be rather noisy, but the standard deviation of the noise  $\sigma_\eta$  is relatively precisely estimated. How these parameter estimates interact to determine the dynamics implied by the man-bites-dog information structure is analyzed in the next section.

Table 1: Posterior Parameter Estimates

$\Theta$	Prior Dist	Benchmark			Mean surveys			Alt (i)			Alt (ii)			Alt (iii)		
		Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode	2.5%-97.5%	Mode
Preferences etc																
$\varphi$	N(1,0.2)	0.94	0.74-1.14	1.06	0.79-1.21	1.00	0.99-1.02	1.32	1.31-1.35	0.94	0.76-1.08					
$\delta$	N(1,0.2)	1.28	1.17-1.45	0.75	0.47-1.17	0.84	0.83-0.96	0.73	0.61-0.90	1.43	0.68-1.68					
$\beta$	Be(0.99,0.01)	0.99	0.99-0.99	0.99	0.99-0.99	0.99	0.99-0.99	0.97	0.97-0.98	0.99	0.99-0.99					
$\theta$	Be(0.75,0.1)	0.86	0.84-0.87	0.88	0.85-0.92	0.82	0.82-0.85	0.75	0.74-0.76	0.90	0.87-0.91					
Exogenous aggregate processes																
$\rho_a$	Be(0.9,0.05)	0.96	0.96-0.97	0.93	0.89-0.97	0.95	0.95-0.97	0.90	0.90-0.91	0.98	0.97-0.98					
$\rho_d$	Be(0.7,0.1)	0.51	0.474-0.55	0.85	0.78-0.89	0.67	0.65-0.67	0.68	0.64-0.69	0.66	0.62-0.69					
$\sigma_a$	U(0,1)	0.006	0.005-0.007	0.003	0.002-0.006	0.01	0.01-0.01	0.01	0.01 - 0.01	0.01	0.01-0.01					
$\sigma_d$	U(0,1)	0.032	0.030-0.034	0.004	0.003-0.008	0.02	0.016-0.018	0.01	0.01-0.01	0.02	0.02-0.02					
$\sigma_r$	U(0,1)	0.006	0.005-0.008	0.003	0.002-0.004	0.01	0.01-0.01	0.03	0.03-0.03	0.01	0.01-0.01					
Island specific processes																
$\sigma_\varepsilon$	U(0,5)	0.20	0.16-0.26	0.06	0.034-0.75	0.03	0.02-0.03	0.08	0.07-0.08	0.07	0.06-0.08					
$\sigma_{\xi^p}$	U(0,5)	0.008	0.008-0.008	0.10	0.03-1.58	0.01	0.01-0.07	0.08	0.07-0.11	0.01	0.01-0.01					
$\sigma_{\xi^y}$	U(0,5)	1.49	0.54-4.50	0.23	0.08-2.00	0.18	0.13-0.18	0.14	0.09-0.17	1.50	1.16-2.08					
$\sigma_\zeta$	U(0,5)	3.07	0.87-4.90	0.01	0.00-0.02	0.12	0.08-0.12	0.16	0.12-0.18	1.51	1.14-1.85					
Man-bites-dog parameters																
$\omega$	U(0,1)	0.31	0.27-0.36	0.33	0.20-0.44	1*	-	0.38	0.31-0.44	0.38	0.33-0.43					
$\gamma$	U(1,10)	5.01	4.44-5.77	4.24	1.16-5.11	1*	-	4.05	3.91-4.44	5.67	5.06-6.31					
$\sigma_\eta$	U(0,1)	0.45	0.36-0.57	0.06	0.03-0.088	0.05	0.03-0.05	-	-	0.32	0.28-0.37					
Taylor rule parameters																
$\phi_r$	Be(0.5,0.1)	0.10	0.04-0.16	0.48	0.40-0.71	0.11	0.09-0.11	0.28	0.26-0.29	0.30	0.25-0.34					
$\phi_\pi$	N(1.5,0.1)	1.48	1.41-1.57	1.43	1.27-1.54	1.36	1.36-1.41	1.74	1.62-1.77	1.47	1.40-1.55					
$\phi_y$	N(0.5,0.1)	0.31	0.29-0.34	0.00	0.00-0.08	0.28	0.27-0.28	0.00	0.00-0.00	0.38	0.35-0.41					
Measurement error variance in survey means																
$\sigma_{f_\pi}$	U(0,1)	-	-	0.001	0.001-0.002	-	-	-	-	-	-					
$\sigma_{f_{\pi+\Delta y}}$	U(0,1)	-	-	0.007	0.001-0.008	-	-	-	-	-	-					

Notes:  $N(\mu, \sigma)$  is the normal distribution with mean  $\mu$  and s.d.  $\sigma$ .  $Be(\mu, \sigma)$  is the beta distribution with mean  $\mu$  and s.d.  $\sigma$ .  $U(a,b)$  is the uniform distribution over the interval (a,b). \* indicates a calibrated parameter.

## 6. ESTIMATED MAN-BITES-DOG DYNAMICS

The estimated model can be used to quantify the contribution of the man-bites-dog mechanism to business cycle dynamics. This section describes how the propagation of shocks changes when there is a man-bites-dog signal available and presents estimates of how the probability of observing a man-bites-dog signal depends on the absolute size of the innovation to productivity. We also present posterior estimates of the historical probabilities of man-bites-dog events in the sample period and relate these estimates to a *News Heard Index* from the Michigan Survey. The section ends with a quantitative assessment of the persistent effects a man-bites-dog event has on the sensitivity of output to productivity shocks.

**6.1. Impulse propagation with and without man-bites-dog signals.** Figure 2 illustrates the impulse responses of inflation (left column) and output (right column) to 1 s.d. innovation to productivity (top row), demand (middle row) and noise in the man-bites-dog signal (bottom row). The dashed black lines describe the responses when there is no man-bites-dog signal available in any period and the solid blue lines describe the responses when there is a man-bites-dog signal available in the impact period but none before or after. The dotted lines are the 95 per cent credible intervals.

**6.1.1. Productivity shocks.** Inflation falls and output increases in a gradual, hump-shaped pattern after a positive innovation to productivity whether a man-bites-dog signal is available or not. The responses of inflation and output to a productivity shock of a given magnitude are substantially larger if the shock coincides with the availability of a man-bites-dog signal. The difference is particularly large for the response of output. At the posterior median, the response of output in the impact period is about twice as large when there is a man-bites-dog signal available compared to when there is no such signal. The peak response, at about 0.35 of a percentage point compared to 0.2 of a percentage point, is also substantially larger when there is a man-bites-dog signal available. The fall in inflation is about 20 per cent larger on impact when a man-bites-dog signal is available. Since the exogenous shock is the same whether a man-bites-dog signal is available or not, the stronger responses must be caused by expectations responding more strongly when a man-bites-dog signal is available.

In the static model of Section 3 above, the average action responded more strongly when a man-bites-dog signal was available. This was so partly because expectations simply respond more when there is more information available. However, in Section 3 it was also shown that the aggregate response was stronger to a man-bites-dog signal compared to the response to a standard public signal of the same precision. To quantify the relative importance of the man-bites-dog and the *more information* effect, we can solve the model under the alternative assumption that the availability of a man-bites-dog signal is uncorrelated with the magnitude of the innovation to productivity. The result of this exercise is illustrated by the red dashed-dotted lines in Figure 2. There, we can see that almost the entire difference between the responses with and without a man-bites-dog signal is explained by the man-bites-dog effect.

**6.1.2. Demand shocks.** The middle row of Figure 2 shows that both inflation and output increase after a demand shock. Neither of the responses depend substantially on there being a man-bites-dog signal available or not. Both inflation and output respond with a geometric decay after impact, with approximately the same persistence and shape as that of

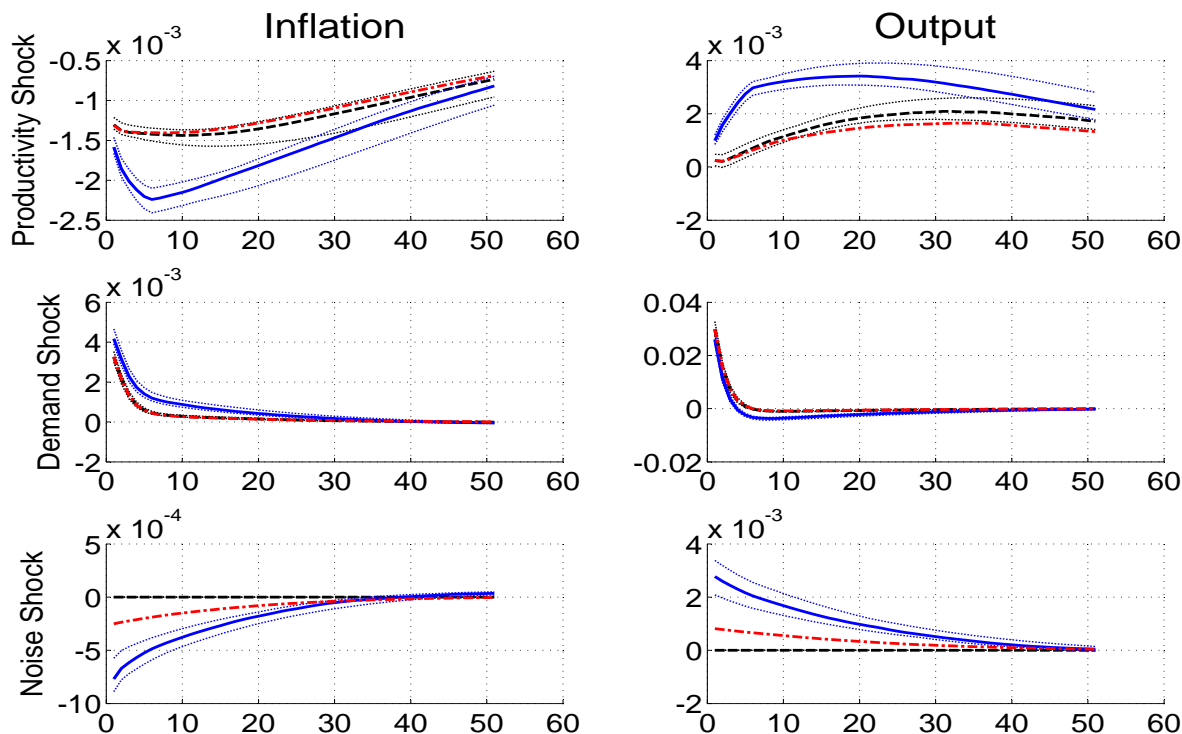


FIGURE 2. Posterior impulse response functions. Blue solid lines are for man-bites-dog signal available in impact period and black dashed lines for no man-bites-dog signal. Dotted lines show the 95 per cent posterior credible intervals. The dash-dot red lines are the responses of inflation and output when the availability of the signal  $z_t^a$  is uncorrelated with the innovation to productivity.

the exogenous shock  $d_t$ . This is so because relative to the variance of the innovations to  $d_t$ , agents have quite precise information about  $d_t$  so that their (higher order) expectations of the demand shock are close to the actual shock.

6.1.3. *Noise shocks.* The agents in the economy use all available information optimally. Nevertheless, since the man-bites-dog signal is noisy, agents will sometimes inadvertently respond to a pure noise shock. The effects such a response have on inflation and output are plotted in the bottom row of Figure 2. Qualitatively, the initial responses of inflation and output to a noise shock in the man-bites-dog signal are similar to the responses to a true innovation to productivity. That is, inflation falls and output increases. The initial response of both inflation and output is larger to a 1 s.d. pure noise shock than to a 1 s.d. actual productivity shock. This is so because the estimated standard deviation of the noise shock is much larger than the standard deviation of the true innovation to productivity. For shocks of the same magnitude, the response is weaker to a pure noise shock.

Comparing the response to a noise shock in a man-bites-dog signal to the response when the availability of the public productivity signal is uncorrelated with the underlying shock, about half of the response to a noise shock is due to the *more information* effect. Of course,

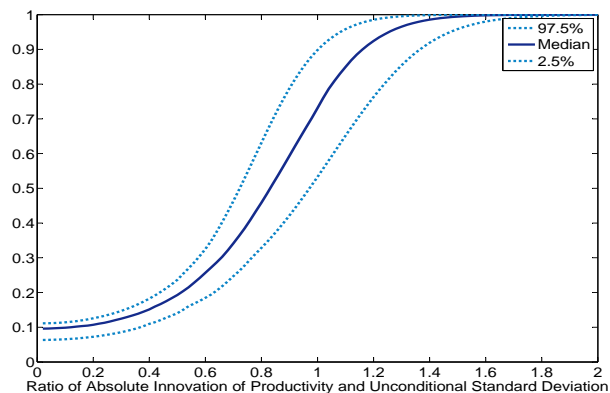


FIGURE 3. The probability of observing a man-bites-dog signal conditional on the absolute size of the innovation to productivity.

when no signal is available, a noise shock cannot affect neither inflation nor output. This explains why the black dashed lines in the bottom row of Figure 2 are flat at zero.

In the original model of Lorenzoni (2009), the responses to shocks in the public signal about common productivity look like the responses to demand shocks discussed above (hence the title of Lorenzoni’s paper). That is, both inflation and output increases in response to a noise shock. The different prediction of the current model is not driven directly by the man-bites-dog information structure. For instance, the model presented here also predicts that noise shocks are inflationary if the Taylor rule coefficient on output, i.e.  $\phi_y$ , is sufficiently low (holding the other parameters fixed at their estimated posterior modes).<sup>5</sup> That the posterior estimates do not suggest inflationary noise shocks may be due to the fact that the specification presented here includes “actual” demand shocks, i.e. the same type of shocks Lorenzoni (2009) seeks to replace with noise shocks. By construction, they will absorb much of the positive co-movement between inflation and output.

**6.2. The conditional probability of observing a man-bites-dog signal.** The probability of generating a man-bites-dog signal is increasing in the absolute size of an innovation to productivity when  $\gamma > 1$ . Above, we saw that when a productivity or a noise shock coincided with a man-bites-dog signal, the responses of inflation and output are much stronger compared to when no signal is available. This introduces a non-linearity in the model’s responses to productivity shocks. Since large innovations to productivity are more likely to generate the stronger responses, the man-bites-dog information structure makes both recessions and booms sharper than they otherwise would be.

Figure 3 illustrates just how large innovations to productivity have to be to significantly increase the probability of a man-bites-dog event. There, the posterior probability that  $s_t = 1$  (y-axis) is plotted as a function of the ratio of the absolute value of an innovation and the unconditional standard deviation of innovations (x-axis). For innovations close to the

<sup>5</sup>The role of the monetary policy in determining whether the response to noise shocks is more supply-like or demand-like is analyzed in more detail in Rousakis (2013).

mean, the probability of observing a man-bites-dog signal is about 10 per cent. Innovations larger than 1.5 unconditional standard deviation generate a man-bites-dog signal and the stronger responses of inflation and output almost surely.

**6.3. Historical man-bites-dog episodes.** The top panel of Figure 4 displays the posterior probabilities of man-bites-dog events for the benchmark specification. The shaded areas are the NBER dated recessions. In addition to the NBER dated recessions, there are several episodes, particularly in the 1980s and 2000s, that are assigned a high probability of being man-bites-dog episodes. In comparison, the 1990s have much fewer periods with a substantial probability of having been man-bites-dog events.

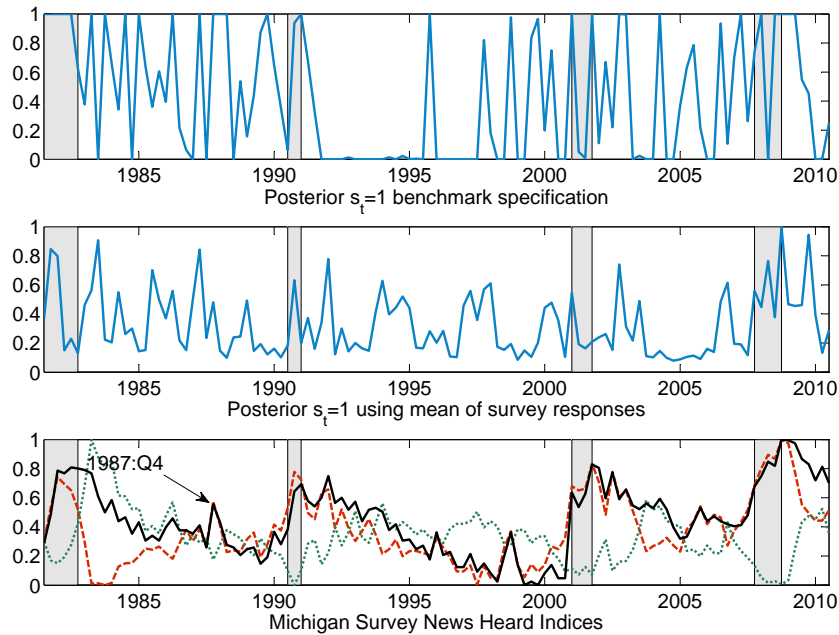


FIGURE 4. Posterior historical probability of  $s_t = 1$  and Michigan Survey News Heard indices.

As suggested by the theory, the quarters that are assigned a high probability of being a man-bites-dog episode are also quarters with larger than average (in absolute terms) productivity innovations. The standard deviation of the innovations in the quarters assigned a probability larger than 90 per cent is 0.0076 compared to 0.0064 for the full sample. To ensure that this result is not an artefact of the structure imposed on the data by the model, the innovations were computed from a separate univariate AR(1) model for the TFP series.<sup>6</sup>

<sup>6</sup>The numbers are very similar when the innovations from the full model are used, though the TFP process is then estimated to be more persistent compared to the estimates from the separate TFP model.

**6.4. News media and historical man-bites-dog episodes.** In the introduction, the *Movers* segment on Bloomberg Television was given as an example of man-bites-dog news reporting. It seems unlikely that individual events, perhaps with the exception of the 1987 stock market crash, directly cause what is identified as man-bites-dog episodes by the model. An alternative interpretation of these episodes at the macro level is that, at certain times, the economy for various reasons becomes one of the main sources of news stories, dominating network news and newspaper front pages. According to the man-bites-dog dictum, this should be more likely to happen when macro economic developments are in some sense unusual.

One way to check more directly whether what the model interprets as man-bites-dog events are indeed related to the intensity of news coverage is to compare the posterior probabilities that  $s_t = 1$  with the fraction of respondents in the Michigan Survey that have heard either unfavorable or favorable news “during the last months”. This data was not used in estimation and thus provides an independent check on how reasonable the estimates produced by the model are. The bottom panel of Figure 4 contains an index of the number of respondents that have heard any news about the economy (black solid line). The bottom panel also includes indices for the fraction of respondents that have heard any unfavorable (red dashed line) or any favorable (green dotted line) news about the economy.<sup>7</sup> It is clear that the unfavorable news heard index increases around recessions and around the stock market crash in 1987:Q4. The biggest spike in the good news index is in the early 1980s as the economy recovered from the Volcker disinflation recession.

In addition to these large and easily identifiable events there are many smaller, high-frequency movements in these indices. To analyze more formally how the *Any News Heard Index* relates to what the model interprets as man-bites-dog episodes, we can compute the posterior correlation between  $s_t$  and the index. For the benchmark specification this correlation is 0.24. While the positive correlation between the timing of man-bites-dog events and the *Any News Heard Index* does not provide a direct test of the causal link proposed in this paper, it does provide evidence that what the model identifies as man-bites-dog episodes are indeed periods associated with an increase in the news coverage on the economy.

**6.5. Man-bites-dog events and the cross-sectional dispersion of expectations.** Because of the man-bites-dog information structure, the precision of agents information sets varies over time. This means that the weight agents put on island specific signals and, as a consequence, the cross-sectional dispersion of expectations will also vary over time. Since the cross-sectional dispersion of expectations in period  $t$  is a function of the history  $s^t$ , the time-variation in the cross-sectional dispersion of survey responses is particularly useful for identifying man-bites-dog episodes in the sample. This is illustrated in Figure 5, where the cross-sectional standard deviation of survey responses is plotted together with the corresponding fitted dispersion from the benchmark specification.

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<sup>7</sup>The indices are constructed by computing the fraction of survey respondents that have heard either favorable or unfavorable news. The fractions are re-normalized to have a minimum of 0 and a maximum of 1 to make them more easily comparable in a single graph. This normalization is an affine transformation and does not affect the computed correlations.

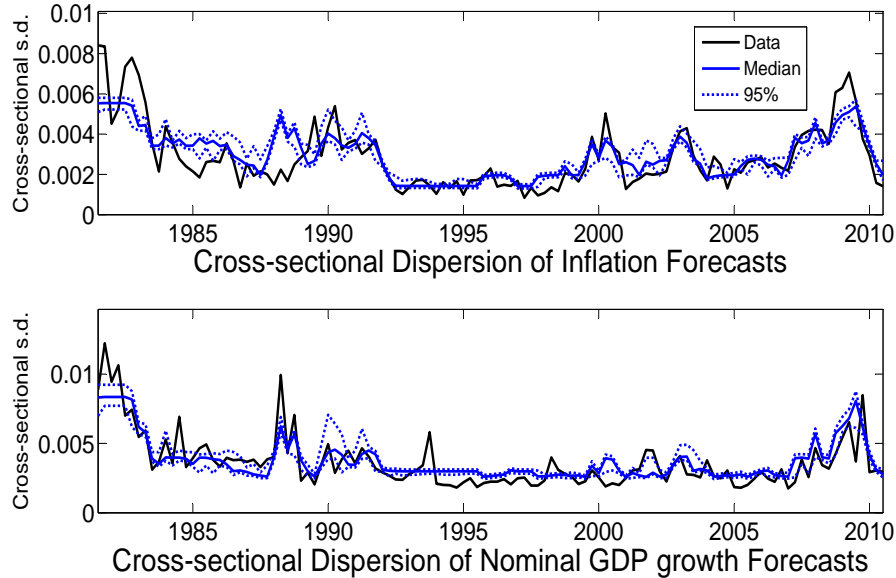


FIGURE 5. Historical and posterior model implied dispersion of survey responses.

Overall, the model does a good job at fitting both the average level and the time-variation of the dispersion in both inflation and nominal GDP growth survey responses. Two exceptions are the periods with very high dispersion in the early 1980s and the current crisis during which the model under-predicts the dispersion in the data. The model also somewhat over-predicts the inflation forecast dispersion and under-predicts the nominal GDP growth forecast dispersion following the stock market crash in 1987:Q4.

Comparing Figure 4 and 5, it is clear that most of the episodes to which the model assigns a high probability of being man-bites-dog episodes are associated with an increase in the dispersion of either the inflation forecasts or the nominal GDP growth forecasts. In particular, the dispersion of survey forecasts are relatively flat in the 1990s when only a few quarters were assigned a high probability of being man-bites-dog events.

*6.5.1. Using the mean of survey responses.* It is clear from the evidence presented above that the time-variation in the dispersion of survey forecasts is important for identifying historical man-bites-dog episodes. It may therefore be of interest to estimate the model without using the information in the cross-section of survey responses. The second panel in Figure 4 shows the posterior estimate for  $s_t$  when the mean of the inflation and nominal GDP growth survey responses are used to estimate the model. Relative to the other specifications, there are fewer quarters that are assigned a probability close to 1 of being a man-bites-dog event and there are practically none where a man-bites-dog event is completely ruled out. That the posterior is less precise is not surprising since less sample information was used in the estimation.



Without individual survey responses, the model has to identify man-bites-dog episodes from the movements of the aggregate variables and the mean of the surveys. Larger movements of these variables are then required to provide strong evidence of a man-bites-dog event. The standard deviation of the innovations in the quarters assigned a probability larger than 90 per cent is 0.0132 compared to 0.075 for the benchmark specification.

Relative to the specification using individual survey responses, the posterior correlation between  $s_t$  and the *Any News Heard Index* drops to 0.13. That the Index is more strongly correlated with the posterior estimate of  $s_t$  when the individual survey responses are used in estimation provides further support for the model's prediction that news media coverage increases dispersion. The correlation is also much less precisely estimated and the 95 per cent probability interval ranges from 0.05 to 0.29, compared to 0.19-0.34 for the benchmark specification. Also, when no individual survey responses are used for estimation, the model under predicts the dispersion in the survey responses by an order of magnitude.

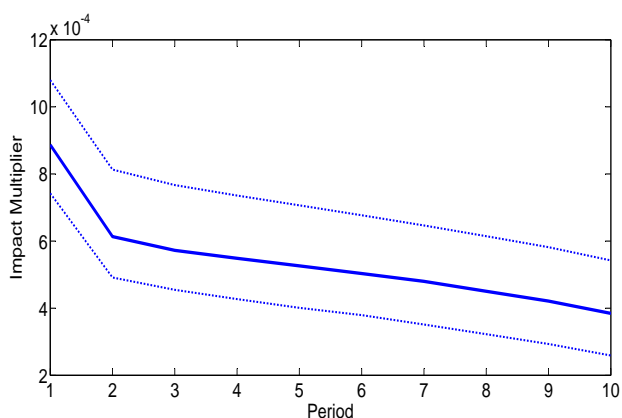


FIGURE 6. Impact multiplier persistence: Impact on output of 1 s.d. innovation to productivity after single man-bites-dog event.

**6.6. The cyclicalty of dispersion and uncertainty.** Bloom (2009) reports that the cross-sectional dispersion in the GDP forecasts in the *Survey of Professional Forecasters* is strongly correlated with standard measures of uncertainty, such as stock market volatility. In the model presented here, there is a strong positive correlation between conditional uncertainty and the dispersion of individual expectations. Bloom (2009) also documents that uncertainty and cross-sectional dispersion tend to increase around recessions. The correlation between growth rates of nominal GDP and the spread between the 25th and the 75th percentile of nominal GDP growth survey responses is -0.06 over the sample period used in this paper. The man-bites-dog model is symmetric and large positive and large negative shocks are equally likely to trigger a man-bites-dog event. There is some support in the data that the magnitude of shocks matters at least as much for dispersion as their sign. While nominal GDP forecast dispersion is negatively correlated with nominal GDP growth rates, the correlation between *absolute* growth rates of nominal GDP and dispersion is 0.31. Absolute changes of the CPI are also more strongly correlated with the dispersion than actual

changes. The correlation with the spread is 0.32 for actual changes in CPI and 0.37 for absolute changes.

**6.7. Endogenous persistence in volatility.** The impulse response functions in the top panel of Figure 2 traced out the responses of inflation and output to a single productivity shock, i.e. there were no further impulses to productivity after the impact period. However, a single man-bites-dog signal may affect how responsive the economy is to innovations to productivity for several periods after the signal was observed. As explained in Section 5.2 above, if the posterior uncertainty about the state increases in period  $t$ , then this translates into an increased prior uncertainty in period  $t + 1$ . Since how much weight agents put on new information depends inversely on the precision of their prior, changes in posterior uncertainty in period  $t$  will affect how responsive the economy is to shocks also in period  $t + 1$ . Figure 6 plots the output multiplier on a productivity shock the periods after a man-bites-dog signal is observed. That is, Figure 6 plots the posterior estimate of the relevant elements of  $G(s^t)N(s^t)$  from the model solution (5.5) - (5.7) for  $s_t = 1$  in the impact period and  $s_t = 0$  in all other periods.

The impact multiplier is largest in the period when the man-bites-dog signal is observed. A 1 s.d. innovation to productivity increases output by about 0.09 of a percentage point when accompanied by a man-bites-dog signal. The impact multiplier then slowly converges towards 0.004, which is the level associated with no man-bites-dog signals.

The persistence in the volatility of the endogenous variables is consistent with the evidence from less structural models such as the GARCH model estimated on US real GDP growth by Bhar and Hamori (2003). It is perhaps worth pointing out that the gradual and monotonic decay of the impact multiplier after a man-bites-dog event is an empirical result and not a necessary implication of the model. With a very precise man-bites-dog signal, posterior uncertainty in period  $t$  decreases when a man-bites-dog signal is available, making agents less sensitive to new information in period  $t + 1$ . The impact multiplier in period  $t + 1$  would then be lower than that associated with no man-bites-dog signals and we would have observed a negative “overshooting” of the impact multiplier in the periods after the man-bites-dog event occurred.

The effect on the sensitivity of inflation and output accumulates over time if there are several man-bites-episodes occurring in close succession. This is illustrated in Figure 7 where the posterior estimate of the impact multiplier on output of a 1 s.d. innovation to productivity is plotted. In the 1980s, to which the model assigns a high probability of several man-bites-dog episodes, the impact multiplier is persistently above the level associated with no man-bites-dog events. Only in the more tranquil 1990s does the impact multiplier decrease to the level associated with no recent man-bites-dog events. At the peaks in the early 1980s and during the recent financial crisis, the impact multiplier is about eight times as large as it was in the mid 1990s.

These results are also related to the findings of Coibion and Gorodnichenko (2011). Using survey data, they document that expectations are updated faster, in the sense that average expectations in surveys respond more strongly, during periods of higher macroeconomic volatility. Though Coibion and Gorodnichenko are looking at a partly different sample period and focus on lower frequency movements, this is exactly the qualitative prediction

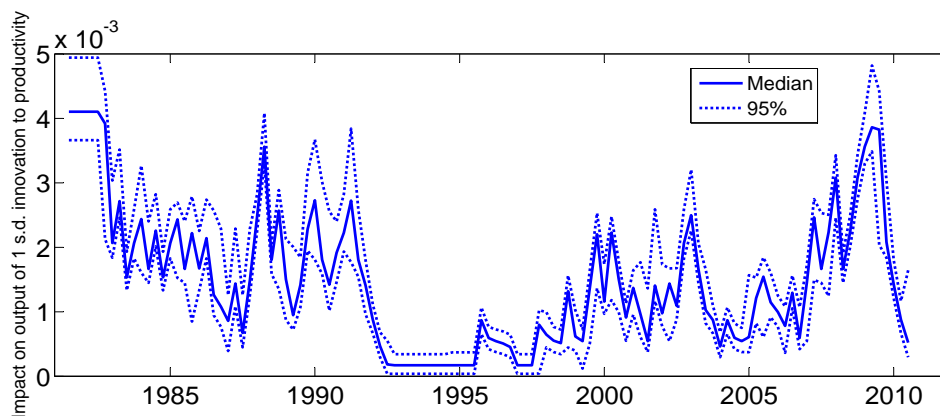


FIGURE 7. Posterior estimate of historical impact on output of a 1 s.d. innovation to productivity. Median and 2.5%-97.5% probability interval.

made by the model presented here: A man-bites-dog event increases the volatility of macro aggregates *because* expectations are updated faster.

The man-bites-dog model predicts that the magnitude of the response of output to a productivity shock of a given size varies over time. Given that the only variable input into production is labor, the time-varying responses of output to productivity shocks must be driven by time-varying responses of hours worked. Using a SVAR with time-varying parameters, Gambetti (2006) and Gali and Gambetti (2009) identify the impulse responses of hours worked to a permanent productivity shock using long-run restrictions. While the focus of these papers is on the decline in overall volatility in the post-1984 period, it is evident from their results that there are also higher frequency changes in the responsiveness of hours worked to technology shocks. This provides further evidence in support of the mechanism proposed here.

*6.7.1. Time-varying volatility and other modeling approaches.* The man-bites-dog information structure shares some features with both ARCH-type and stochastic volatility models. In ARCH (and related) models, the standard deviation of shocks is a function of the square of past innovations. Large level shocks thus deterministically increase the standard deviation of future shocks. With a man-bites-dog information structure, large shocks to the level of productivity are more likely to generate a man-bites-dog event and a persistent increase in the volatility of inflation and output. Unlike in ARCH-type models, here there is only a probabilistic relationship between the magnitude of level shocks and the future volatility of inflation and output.

In structural models with stochastic volatility, one can distinguish between shocks to the level of the exogenous variables and a shock to the standard deviation of the exogenous innovations. In non-linear models such as those in Bloom (2009), Bloom, Floetotto and Jaimovich (2011) and Fernández-Villaverde, Guerron-Quintana and Rubio-Ramírez (2012), a shock to the standard deviation has a direct first order effect on both the level and the dynamics of the endogenous variables. In linearized models with stochastic volatility such

as that of Justiniano and Primiceri (2008), a shock to the standard deviation affects neither the level nor the propagation of the endogenous variables. The model presented here lies somewhere in between these two model classes: A man-bites-dog signal that is not accompanied by a shock to the level of the exogenous variables or to the noise in the man-bites-dog signal does not perturb the economy from its steady state. However, a man-bites-dog signal does change the propagation of level shocks and the dynamics by which the model returns to that steady state.

In stochastic volatility models such as those of Justiniano and Primiceri (2008) and Fernández-Villaverde *et al* (2012), an increase in the standard deviation of shocks has to be inferred from the realized volatility of the macro economic aggregates. In the man-bites-dog model estimated using survey data, we can also identify “false alarms”, i.e. periods when the endogenous variables would have responded strongly to a shock to productivity if such a shock would have occurred. One such example is the sharp spike in the impact multiplier plotted in Figure 7 after the stock market crash in 1987:Q4. The estimated increase in the impact multiplier is driven entirely by the sharp increase in the dispersion of survey responses after the stock market crash, since neither productivity, nor inflation nor output had particularly large realizations in that quarter. This is also clear from inspecting the time series for  $s_t$  in the middle panel of Figure 5, which does not have a spike in 1987:Q4. The only thing that differs relative to the benchmark model is that  $s_t$  in the middle panel was estimated using the mean of the surveys rather than the individual responses. Similarly, other classes of models that infer the standard deviation of exogenous shocks from the realized volatility of macro aggregates generally have a smoothly declining standard deviation from 1984 onwards until around the early 1990s recession.

The man-bites-dog mechanism is more restrictive than many of the alternative approaches in that there is a single discrete event that drives the time variation in volatility in all endogenous variables. More flexible models such as those in Justiniano and Primiceri (2008) and Fernández-Villaverde, Guerron-Quintana and Rubio-Ramírez (2010) allow for a continuum of possible volatilities and that the volatilities of different shocks can vary independently of each other.

## 7. ALTERNATIVE SPECIFICATIONS

The man-bites-dog mechanism implies that a single process determines both how the information available to agents and how the conditional volatility of shocks varies over time. In this section I investigate the implications of relaxing some of the restrictions implied by this theory. In particular, I estimate the following three alternative specifications that have reduced form representations that are either nested or are close to the benchmark specification: (i) The public signal  $z_t^a$  about common productivity is always available and is thus not informative about the conditional variance of productivity, implying the restriction  $\gamma = \omega = 1$ . (ii) The signal  $z_t^a$  is never available but the innovations to productivity are drawn from the mixture normal distribution (4.17) with the regimes directly observable by the agents. This specification is equivalent to imposing the restriction on the benchmark model that the noise in the man-bites-dog signal has infinite variance, i.e.  $\sigma_\eta^2 = \infty$ . (iii) The signal  $z_t^a$  is always available and the innovations to productivity are drawn from the mixture

normal distribution (4.17). The log-likelihood evaluated at the posterior mode of the three specifications as well as for the benchmark specification are listed in Table 2 below.

A simple way to compare the relative fit of different models that takes into account the number of estimated parameters is to use the Schwarz approximation (see Schwarz 1978) to the posterior odds ratio. It can be computed as

$$PO \approx e^{\log L(\mathbf{y}^T | \hat{s}_B^T, \hat{\Theta}_B) - \log L(\mathbf{y}^T | \hat{s}^T, \hat{\Theta}) - \frac{1}{2}(\dim \Theta_B - \dim \Theta) \ln T} \quad (7.1)$$

where a  $B$  subscript denotes the benchmark specification and  $\dim \Theta$  denotes the number of freely estimated parameters. The resulting relative probability of the benchmark model compared to the alternatives are reported in the bottom row of Table 2.

**Table 2**

	Benchmark	(i) $\gamma = \omega = 1$	(ii) $\sigma_\eta^2 = \infty$	(iii) Pub. signal + regimes
$\log L \left( Z^T \mid \hat{s}^T, \hat{\Theta} \right)$	3091.1	2999.0	2982.7	3097.4
$PO$	1	$e^{96.9}$	$e^{110.8}$	$e^{-6.3}$

There appears to be overwhelming evidence against specifications (i), i.e. the model without time-varying volatility, which is assigned a near zero posterior odds ratio relative to the benchmark model. The time-variation in volatility and in the cross-sectional dispersion of forecasts thus help to substantially improve the fit of the man-bites-dog model relative to a model without these features.

To understand the relative fit of the three models that feature time-varying volatility, it is useful to think of them as differing only in terms of the availability of the public signal about productivity. In specification (ii) the public signal about productivity is never available. This results in a fit that is worse than specification (i) without time-varying volatility. The public signals is thus at least as important as time-varying volatility for explaining the data. In the benchmark man-bites-dog model, the public signal is available only when the volatility of the innovations to productivity is high. The benchmark model's fit is substantially better than specification (ii) which is assigned a near zero posterior odds ratio. In specification (iii) the public signal about productivity is always available. This is the best fitting model. The fit thus generally improves with the number of periods in which the public signal is available. One reason for this is that the noise component in the public signal acts as an aggregate shock, and the more periods in which this shock is active, the more flexibility the model has to fit the data. That the best fitting specification is the one in which the public signal is always available may cast some doubt on the empirical relevance of the man-bites-dog mechanism. However, a closer analysis suggest an alternative interpretation that is more favourable to the man-bites-dog mechanism.

In the sample, there are 117 periods and at the posterior mode of the benchmark model 30 of those are man-bites-dog episodes. The posterior odds ratio of specification (ii) and the benchmark model implies that adding the public signals to these 30 high-volatility periods increases the likelihood by 113.2 log points (i.e.  $110.8 + \frac{1}{2} \ln T$ ). Adding the public signal also to the remaining 87 periods, i.e. going from the benchmark model to specification (iii), increases the likelihood only by an additional 6.3 log points. This suggest that in terms of fitting the data, the public signal is more important in periods with high volatility than

in periods with low volatility, which lends some qualified support for the man-bites-dog mechanism. Of course, the evidence presented here is also consistent with the interpretation that there is some other mechanism, unrelated to the one proposed here, that explains the time-variation in volatility and the cross-sectional dispersion of forecasts better than the man-bites-dog mechanism.

## 8. CONCLUSION

News media perform an editorial service to its audience by selecting which events to report. This paper has presented tools to analyze how beliefs and economic decisions are affected if more unusual events are considered more newsworthy and therefore are more likely to be reported. We defined a man-bites-dog signal to be a signal that is more likely to be available after unusual realizations of a latent variable. Under general conditions, the availability of a man-bites-dog signal then makes rational Bayesian agents redistribute probability mass towards unconditionally less likely realizations. The absence of such a signal makes agents redistribute probability mass towards unconditionally more likely realizations. Using an explicit and tractable example, we showed that posterior uncertainty and the cross-sectional dispersion of expectations can either increase or decrease after a man-bites-dog signal is observed, but that conditional expectations always respond more strongly. In the context of the beauty contest model of Morris and Shin (2002), we also demonstrated that the average action responds more strongly to a man-bites-dog signal than to a standard signal of the same precision.

Conceptually, the information structure proposed here differs from the ex ante perspective taken by most of the existing literature on rational inattention, e.g. Sims (1998, 2003) and Mackowiak and Wiederholt (2009). In that literature, agents pay more attention to those variables that are most useful *on average*. In contrast, here realizations of shocks matter for what type of signals that are available. There is nothing inherent in the rational inattention approach though that makes an ex ante perspective necessary. For instance, Matejka (2011) develops a model of rational inattention in which it is optimal for agents to let the precision of signals depend on the realization of shocks. The information structure in that paper thus also depend on the realizations of shocks. However, the availability of signals in Matejka's model is constant and thus do not carry any additional information about the distribution of the variables of interest.

In the second part of the paper, a simple business cycle model was presented and estimated in which large innovations to productivity are more likely to generate a public signal. The estimated model suggests that there have been episodes recent US history in which the impact of an innovation to productivity on aggregate output was more than eight times larger than at other times. The increased sensitivity of macro aggregates to productivity innovations were found to be persistent, lasting about 2 years after a single man-bites-dog event. The model thus captures aspects of a "crisis mentality" in which there is an intense media focus on the economy and yet, while there is more information produced and broadcast about the economy, uncertainty and sensitivity to new information appear to increase. We also presented corroborative and independent evidence from the *Michigan Survey* that the episodes identified by the model as man-bites-dog events were indeed associated with higher

than normal news coverage of the economy. This correlation is stronger when individual survey responses are used which provides further support for the mechanism in the model.

The model was estimated by likelihood based methods using both the quarterly total factor productivity time series constructed by Fernald (2010) and individual survey responses from the *Survey of Professional Forecasters* along with more standard macro indicators. Using a time series of TFP as an observable variable has obvious advantages in terms of disciplining the model, especially since one of the aims of the paper has been to quantify the extent of time-variation in the impact of TFP shocks on other variables. Using the cross-section of individual survey responses allowed us to incorporate the information in the time-variation in the dispersion of survey responses into the posterior estimates of the parameters of the model. Particularly, we showed that the cross-sectional dimension in the *Survey of Professional Forecasters* is informative about the timing of man-bites-dog events. In order to exploit the time variation in the cross-sectional dispersion of the survey data, it is necessary to have a model that can fit this fact. The paper makes a methodological contribution by demonstrating how a model with time-varying information sets can be solved and estimated. This may be of separate interest to some readers.

The model presented here features a restricted form of stochastic volatility. In less restricted models such as those of Justiniano and Primiceri (2008) and Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2012), persistence in the volatility of endogenous variables is caused by persistence in the volatility of the exogenous shocks. In the model presented here, the volatility of exogenous productivity is restricted to be an i.i.d. process but the filtering problem of the agents generates persistence in the volatility of the endogenous variables. To the extent that we can observe the exogenous shocks directly, this distinction is a testable difference between the two approaches. Even though the model is conditionally linear, changes in variances have first order effects on the propagation of shocks through the filtering problem of the agents. This aspect of the model does not depend on the man-bites-dog mechanism per se. The solution method proposed here could thus relatively easily be extended to more general stochastic volatility specifications including specifications that allow for persistence in the exogenous regimes.

This paper has argued that the man-bites-dog signals provide an intuitive and plausible mechanism that can explain several features of the business cycle. However, we have not presented a direct test of the causal link between the news media and the macro economy implied by the theory. Peress (forthcoming) uses newspaper strikes to identify the effect of news media on asset price volatility and stock market trading volume. While outside the scope of the present paper, it would be interesting to use a similarly direct strategy to test the predictions of the man-bites-dog mechanism.

Finally, one limitation of the framework presented here is that the availability of signals depends only on the realized value of exogenous shocks. In practise, unusual developments of endogenous variables are surely also considered newsworthy. However, modeling the availability of man-bites-dog signals as depending on the realized values of endogenous variables is at the moment computationally unfeasible.

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## APPENDIX A. PROOF OF PROPOSITION 2

**Proposition 2.** *The average expectation of  $x$  responds more strongly to  $x$  when  $S=1$  than when  $S=0$ .*

*Proof.* We need to show that the sum of the coefficients on the private signal  $x_j$  and the public signal  $y$  in the conditional expectation

$$E(x | \Omega_j^1) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}y \quad (\text{A.1})$$

when  $S = 1$  is larger than the coefficient on the private signal

$$E(x | \Omega_j^0) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}x_j \quad (\text{A.2})$$

when  $S = 0$ . Simply comparing the expected average expectation conditional on  $x$  for  $S = 0$

$$\int E[x | \Omega_j^0] dx = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}x_j dx \quad (\text{A.3})$$

$$= \left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right)x \quad (\text{A.4})$$

and  $S = 1$

$$E\left[\int E(x | \Omega_j^1) dx | x\right] = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}x_j dx + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}x \quad (\text{A.5})$$

$$= \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right)x \quad (\text{A.6})$$

means that the proposition is true if the inequality

$$\left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) < \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) \quad (\text{A.7})$$

holds. The last expression can with a little algebra be rearranged to

$$\gamma^{-1} < 1 + \sigma_\eta^{-2} \quad (\text{A.8})$$

which is always true since  $\gamma > 1$  and  $\sigma_\eta^{-2} > 0$ .  $\square$

## APPENDIX B. DERIVING EXPRESSION (2.14) IN SECTION 2.X

Start by dividing Bayes rule for conditional probabilities for  $p(S = 1 | x)$

$$p(S = 1 | x) = \frac{p(x | S = 1)p(S = 1)}{p(x)} \quad (\text{B.1})$$

with the corresponding expression for  $p(S = 0 | x)$

$$p(S = 0 | x) = \frac{p(x | S = 0)p(S = 0)}{p(x)} \quad (\text{B.2})$$

to get

$$\frac{p(S = 1 | x)}{p(S = 0 | x)} = \frac{p(x | S = 1)p(S = 1)}{p(x | S = 0)p(S = 0)} \quad (\text{B.3})$$

Substitute in the distributional assumptions for  $p(x | S = 1)$  and the unconditional probabilities of observing  $y$

$$\frac{p(S = 1 | x)}{p(S = 0 | x)} = \frac{\frac{1}{\sqrt{\gamma}\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\frac{x^2}{\gamma\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}} \frac{\omega}{1 - \omega} \quad (\text{B.4})$$

Rearrange and simplify to get (2.14) from the main text

$$\frac{p(S = 1 | x)}{1 - p(S = 1 | x)} = \frac{\omega}{1 - \omega} \frac{1}{\sqrt{\gamma}} e^{(1-\frac{1}{\gamma})\frac{x^2}{2\sigma^2}} \quad (\text{B.5})$$

where we also used that  $p(S = 0 | x) = 1 - p(S = 1 | x)$ .

#### APPENDIX C. PROOF OF PROPOSITION 4

**Proposition 4** *The response of the average action  $\bar{a}$  to a given value of  $x$  is stronger when the signal  $y$  is available.*

*Proof.* We need to prove that

$$\frac{(1-r)g_0}{1-rg_0} < \frac{(1-r)g_x}{1-rg_x} + \left(1 - \frac{(1-r)g_x}{1-rg_x}\right)g_y \quad (\text{C.1})$$

Divide everywhere by  $(1-r)$  to get

$$\frac{g_0}{1-rg_0} < \frac{g_x}{1-rg_x} + \frac{g_y}{1-r} - \frac{g_x g_y}{1-rg_x} \quad (\text{C.2})$$

and rearrange to get

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y}{1-rg_x} + \frac{g_y}{1-r} \quad (\text{C.3})$$

Since  $0 < g_x < 1$  it follows that

$$\frac{g_y}{1-rg_x} < \frac{g_y}{1-r} \quad (\text{C.4})$$

and it is thus sufficient to prove that

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y + g_y}{1-rg_x} \quad (\text{C.5})$$

for the proposition to hold. Now if  $g_x > g_0$  we have that

$$\frac{(1-r)g_0}{1-rg_0} < \frac{(1-r)g_x}{1-rg_x} \quad (\text{C.6})$$

and the proof follows immediately since the term

$$\left(1 - \frac{(1-r)g_x}{1-rg_x}\right)g_y \quad (\text{C.7})$$

on the right hand side of (C.1) is positive. On the other hand, if  $g_x < g_0$  it is sufficient to prove that

$$g_0 < g_x - g_x g_y + g_y \quad (\text{C.8})$$

We know that the two expressions (2.17) and (3.8) must result in the same average first order expectation implying that

$$g_x - g_x g_y + g_y = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} \quad (\text{C.9})$$

which can be rearranged to

$$g_x - g_x g_y + g_y = \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) \quad (\text{C.10})$$

From (3.9) we know that

$$g_0 = \left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) \quad (\text{C.11})$$

so that the inequality (C.8) is equivalent to

$$\left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) < \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) \quad (\text{C.12})$$

which was demonstrated to hold generally in the proof of Proposition 2.  $\square$

#### APPENDIX D. PROOF OF PROPOSITION 5

**Proposition 5** *The average action responds more strongly to a change in  $x$  when a man-bites-dog signal is available, compared to the response of the average action when the public signal about  $x$  is always available (holding the precision of the signal and the unconditional variance of  $x$  fixed).*

*Proof.* To prove the proposition, it is helpful to denote quantities associated with what we may call the standard signal model with an asterisk (\*). The standard public signal  $y^*$  is always available and defined as

$$y^* = x + \eta : \eta \sim N(0, \sigma_\eta^2) \quad (\text{D.1})$$

and thus have the same precision  $\sigma_\eta^{-2}$  as the man-bites-dog signal. The information set of agent  $j$  in the standard signal model is given by

$$\Omega_j^* = \{x_j, y^*\} \quad (\text{D.2})$$

We want to compare the response of the average action  $\bar{a}$  when a man-bites-dog signal is available with the response of the average action  $\bar{a}^*$  in the standard signal model while holding the unconditional variance of  $x$  fixed by imposing that  $\sigma_x^{*2} = \sigma_x^2$ .

The expression (3.5) for the average action  $\bar{a}$  as a function of higher order expectations shows that the average action is increasing in all orders of expectations about  $x$  and this expression is valid regardless of which information structure we consider. To prove the proposition, it is thus sufficient to show that all orders of expectation about  $x$  respond more strongly when  $y$  is a man-bites-dog signal compared to in the standard model.

The formula (3.8) for the  $k$  order expectation in the main text is convenient for deriving the average action as a function of  $x$  and  $y$ , but is more cumbersome to manipulate for our present purposes. We will therefore derive an alternative (but equivalent) expression for  $x^{(k)}$  here.

In the man-bites-dog model, the average first order expectation of  $x$  when  $y$  is available is of the form

$$x^{(1)} \equiv \int E(x | x_j, y) dj \quad (\text{D.3})$$

$$= c_x x + c_y y \quad (\text{D.4})$$

Leaving the coefficients undefined for now, note that substituting in the expression for  $x^{(1)}$  into the definition of the second order expectation we get

$$x^{(2)} \equiv \int E(x^{(1)} | x_j, y) dj \quad (\text{D.5})$$

$$= c_x (c_x x + c_y y) + c_y y \quad (\text{D.6})$$

and by repeated substitution we arrive at the general expression

$$x^{(3)} = c_x (c_x (c_x x + c_y y) + c_y y) + c_y y \quad (\text{D.7})$$

$$\vdots \quad (\text{D.8})$$

$$x^{(k)} = c_x^k x + \sum_1^k c_x^{k-1} c_y y \quad (\text{D.9})$$

Following the same steps, we can write the  $k$  order expectation in the standard model as

$$x^{*(k)} = c_x^{*k} x + \sum_1^k c_x^{*k-1} c_y^* y^* \quad (\text{D.10})$$

To prove the proposition, it is thus sufficient to show that the coefficients  $c_x, c_y, c_x^*$  and  $c_y^*$  in (D.9) and (D.10) satisfy

$$c_x > c_x^* \text{ and } c_y > c_y^*$$

Given the parameter restrictions on the standard signal model, the average first order expectation about  $x$  is given by

$$x^{*(1)} \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma_x^{*-2}} x + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma_x^{*-2}} y \quad (\text{D.11})$$

which corresponds to the expression (2.17) in the man-bites-dog model. Now, since  $\gamma > 1$  and  $0 \leq \omega < 1$  the variance  $\sigma^{*2}$  in the standard model is smaller than the conditional variance  $\gamma\sigma^2$  in the man-bites-dog model, i.e.

$$\begin{aligned} \sigma_x^{*2} &= \sigma_x^2 \\ &= \omega\gamma\sigma^2 + (1 - \omega)\sigma^2 \\ &< \gamma\sigma^2. \end{aligned}$$

The coefficients  $c_x$  and  $c_y$  in (D.9) are then larger than the corresponding coefficients  $c_x^*$  and  $c_y^*$  in (D.10) since  $\sigma_x^{*2} < \gamma\sigma^2$  implies that

$$c_x = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} > \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma_x^{*-2}} = c_x^* \quad (\text{D.12})$$

and

$$c_y = \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} > \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma_x^{*-2}} = c_y^* \quad (\text{D.13})$$

which completes the proof.  $\square$