Banking Competition and Stability: The Role of Leverage

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Abstract

This paper re-examines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting different types of risk and the role of leverage. We show that competition can affect portfolio risk, insolvency risk, liquidity risk, and systemic risk differently. The effect depends crucially on a bank's type of funding (retail deposits vs. wholesale debts) and whether leverage is exogenous or endogenous. In particular, we argue that while competition might increase financial stability in a classical originate-to-hold banking industry, the opposite can be true for an originate-to-distribute banking industry with a large fraction of market short-term funding.

Keywords: Banking Competition, Financial Stability, Leverage

JEL Classification: G21, G28

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1 Introduction

This paper re-examines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting different types of risk and the role of endogenous leverage. By means of a simple model we show how competition affects bank portfolio risk, insolvency risk, liquidity risk, and systemic risk in different ways. And we also emphasize that as banks optimally set their leverage in a competitive environments, any impact that competition may have on the riskiness of banks' assets can be amplified or mitigated by the change of banks' liability structures. The relationships between competition and various bank risks depend on a combination of banking characteristics. A search for a unified answer to the question of how bank competition affects financial stability could lead to incomplete, simplistic results.

Understanding the link between banking competition and financial stability is essential to the design of an efficient banking industry and its appropriate regulation. Because of the relevance of this topic, a large body of literature devoted to the issue has developed, with important contributions from both theoretical and empirical perspectives. Yet, in spite of the critical importance of the subject and notwithstanding today's improved understanding of its complexity, there is no clear-cut consensus on the impact of competition on banks' risk taking and on the resulting overall financial stability.[1]

Two main theoretical modelling approaches contend to explain the impact of banking competition on financial stability: the charter value view and the risk shifting view. The charter value theory, first put forward by Keeley (1990), assumes that banks choose their level of risk and argues that less competition makes banks more cautious when making their investment decisions, as in case of bankruptcy they will lose the present value of the future rents generated by their market power. Instead, proponents of the risk shifting hypothesis, which originated with Boyd and De Nicolo (2005), postulate that bank risks result from the borrowing firms' decisions, and point out that higher interest rates will lead firms to take more risks and therefore will increase the riskiness of banks' loan portfolios. Yet, both papers solely focus on the impact of competition on the riskiness of banks' asset, and the analyses are restricted to banks' insolvency risk. In order to establish an all-around view, we also need to understand how competition affects bank risks via the liability side, and to analyze extra layers of bank risks beyond insolvency. This has been the main motivation of the current paper.

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[1]See Beck, Jonghe, and Schepens (2011) for example. The authors show that the relationship between competition and financial stability is ambiguous and displays considerable cross-country variation.

Analyzing multi-layer bank risks and endogenous leverage should help to clarify the ambiguous and oftentimes contradictory results in the empirical literature, which reveals a great diversity in the concept of "financial stability"—with measurements ranging from non-performing loans to systemic crises.[2] Therefore, we believe that a first requirement for the analysis of the link between banking competition and financial stability is to build a model that encompasses different types of bank risks: portfolio risk, insolvency risk, liquidity risk, and systemic risk. In fact, we argue that part of the empirical ambiguity may stem from the fact that competition can affect different risks differently, and that the relationship can depend on a combination of banking characteristics.

A second requirement is to consider the endogeneity of bank leverage, a point that the literature has largely ignored.[3] It is important to recognize that any impact that competition may have on the riskiness of banks' assets can be amplified or mitigated by the simultaneous change of banks' leverage. For instance, if competition reduces the risk-shifting problem of banks' borrowers and results in lower portfolio risks, the safer loan portfolios can lead banks to take on more debt.[4] As a result, the insolvency risk of banks will not necessarily decrease, while their liquidity risk and systemic risk are likely to increase. Through affecting banks' insolvency, funding liquidity, and their vulnerability in systemic crises, leverage constitutes a central hub that connects all types of bank risk, and therefore plays a key role in the analysis of the impact of bank competition on financial stability. In our model, competition directly affects the riskiness of banks' assets, and as banks react by altering their leverage, all other types of risks are affected. In other words, in response to a change of competitive environment and that of the riskiness of assets, bank leverage, insolvency risk, and liquidity risk are jointly determined by the optimizing behaviors of banks.

Our approach builds on a large body of literature on banking competition that starts with the seminal paper of Keeley (1990) and Boyd and De Nicolo (2005). Martinez-Miera and Repullo (2010) refined Boyd and De Nicolo's argument by showing that the low profit resulting from

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[2]Table 3 offers a synthetic survey of the different choices in the measures of competition and bank risk in the empirical contributions to the analysis of the competition-financial stability link.

[3]For example, Boyd and De Nicolo (2005) consider banks solely financed by debt. Martinez-Miera and Repullo (2010) assume the cost of equity to be independent of banks' risk. An exception is Allen, Carletti, and Marquez (2009), where the authors show that as competition decreases charter values, banks' incentives to monitor borrowers are reduced. To provide banks with the proper incentives to monitor, one way is to require them to hold more capital.

[4]This is clearly illustrated in the extreme case where a bank's strategy is to maintain a given insolvency risk, which is in line with the idea of "economic capital" . In this case, any changes in portfolio risk are exactly offset by the bank's leverage adjustment.

competition reduces banks' buffer against loan losses and can, therefore, jeopardize financial stability. Wagner (2009) considers both banks' and entrepreneurs' incentives to invest in risky projects: Once entrepreneurs and banks move sequentially, the overall effect coincides with the charter value hypothesis. As mentioned, the fact that all these contributions focus solely on insolvency risk and ignore the endogeneity of leverage is the main motivation for our paper.

Because our objective is to explore the impact of competition on the different types of risk, our starting point has to be the micro-foundations of borrowing firms' risk taking. Following Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010), we assume that firms' investment decisions are subject to moral hazard, so that a higher interest rate leads them to take riskier investment projects. Consequently, greater banking competition decreases portfolio risk but reduces the banks' buffer provided by market power. We build a tractable model into which we embed the main mechanisms of Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010). The model is then extended to analyze banks' funding liquidity risk as well as endogenous leverage. We introduce bank liquidity risk and systemic liquidity crises using global games, and model bank leverage choice as a trade-off between cost of funding and bankruptcy cost. Using this framework, we examine the impact of banking competition on financial stability, both when leverage is set exogenous and when it is endogenous. When leverage is exogenous, which can be interpreted as capital regulations being binding, competition will always increase liquidity risk. If instead, leverage is endogenous, the change of liquidity risk will balance that of insolvency risk, with the two types of risk moving in opposite directions. Nevertheless, our model shows that the change of banks' total credit risk, defined as the sum of insolvency and liquidity risk, is dominated by the change of bank insolvency. The results are robust to the extension of systemic risk and contagion.

In sum, we show that competition can affect different bank risks differently, and that the relationships depend crucially on banks' liability structures. This means that any analysis of the impact of bank competition on financial stabilty should take into account banks' whether banks can adjust their leverage freely, and whether banks are financed by insured retail deposits or by uninsured wholesale funding. In particular, we show that the risk shifting hypothesis is satisfied for a low level of insured deposits and high levels of market power, while the charter value is correct in the opposite case. So competition may increase financial stability in a classical originate-to-hold banking industry funded by retail deposits, the opposite might hold true for an originate-to-distribute banking structure with a larger fraction of market short-term funding.

This result is helpful in understanding the apparent contradictions in the empirical results; it is also relevant to formulating new testable hypotheses as it predicts that the relationship should vary depending on the characteristics of the banking industry.

The paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the benchmark case, exploring how insolvency and liquidity risks are affected by banking competition under the assumption of exogenous leverage. In section 4, we determine endogenous bank leverage and analyze its impact on banks' insolvency and illiquidity. The results contrast with those under exogenous leverage. Section 5 extends the baseline model to study systemic risk and financial contagion. We devote section 6 to the empirical literature, reinterpreting the empirical findings with the refined definition of "financial stability" and forming new testable hypotheses. Relevant policy implications are discussed in section 7. Section 8 concludes.

2   Model Setup

2.1   Portfolio risk and competition

We consider a one-good, three-date ($t = 0, 1, 2$) economy where all agents are assumed to be risk neutral. There are three types of active agents: entrepreneurs, banks, and banks' wholesale financiers; and one type of purely passive agents: retail depositors. There are a continuum of entrepreneurs. They are penniless but have access to long-term, productive, but potentially risky projects. Each project requires one unit of initial investment and takes two periods to bear fruit. It yields a gross return of $x > 1$ if it succeeds, and 0 if it fails. Projects are subject to moral hazard: Each entrepreneur chooses privately a probability of success $P \in [0, 1]$ in order to maximize his expected utility.

$$E(U) = P(x - r) - \frac{P^2}{2b} \tag{1}$$

Here $r \in (1, x)$ is the gross loan rate charged by banks, and $P^2/2b$ denotes the disutility for exerting effort. The parameter $b \in (0, B]$ represents an entrepreneur's type, with a higher b implying a lower marginal cost of effort. The entrepreneurs' types are drawn from a uniform distribution $U(0, B)$ and are private information. Entrepreneurs' reservation utility is normalized to zero.

Because idiosyncratic risk diminishes in a bank's diversified portfolio of loans, we dispense with modeling this type of risk, and focus instead on a non-diversifiable bank-level risk that affects the whole portfolio. We assume that whether a project succeeds or not is jointly affected by an entrepreneur's choice P and a bank-level risk factor z that follows a standard normal distribution. The realization of z is identical for all loans in a bank's portfolio, but can vary across different banks. Following Vasicek (2002) and Martinez-Miera and Repullo (2010), we assume that the failure of a project is represented by a latent random variable y. When $y < 0$, a project fails. The latent variable y takes the following form

$$y = -\Phi^{-1}(1 - P) + z, \tag{2}$$

where Φ denotes the cumulative densitfy function of standard normal distribution. Thus a project defaults because of a combination of entrepreneur's moral hazard (a low P) and an unfortunate risk realization (a low z). For the sake of consistency, note that the probability of success P is given by:

$$Prob(y \geq 0) = 1 - Prob(y < 0) = 1 - Prob(z < \Phi^{-1}(1 - P)) = 1 - \Phi(\Phi^{-1}(1 - P)) = P.$$

The modeling complexity is necessary to analyze banks' leverage. For leverage to play any role, there must be imperfect correlation for loan defaults.[5] In the current setup, the bank-level risk factor z generates correlated loan defaults, and the entrepreneur heterogeneity makes the correlation imperfect.

Banks are well diversified and invest in a continuum of projects. We further assume the loan market is fully covered and all types of entrepreneurs are financed. When the projects mature, the loan portfolio generates a random cash flow that we denote by θ.

In order to focus on bank leverage and risk, we dispense with the specific modeling of loan market competition and consider the loan rate r as a sufficient statistic for the degree of competition. Since a lower r is associated with greater competition, our setup captures the driving force for risk reduction in Boyd and De Nicolo (2005) and is consistent with mainstream

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[5]Under the simplifying assumptions of either zero or perfect correlation, a bank's capital level cannot affect its insolvency risk. In the first case, the law of large numbers leads to zero risk for a well diversified portfolio, and banks do not need to hold any capital. In the second case, once the correlated defaults happen, any capital level lower than 100% is insufficient to prevent a bank failure.

competition models that predict competition will lead to lower spreads for bank interest rates.[6] The assumption is also in line with the empirical literature, which finds low interest margins associated with lower market concentration. (See Degryse, Kim, and Ongena (2009).)

2.2 Bank funding and liquidity risk

Each bank holds a unit portfolio of loans,[7] and finances it with a mixture of debt and equity. At $t = 0$, a bank raises V_F from insured retail depositors, V_D from short-term wholesale creditors, and the rest, $V_E = 1 - V_D - V_F$, from equity holders. Because insured retail depositors are insensitive to banks' risk and play a purely passive role, we assume that their supply of funds is inelastic and fixed, $V_F = F$. We also assume that the financial safety net of deposit insurance is offered to banks at no cost.[8]

The wholesale debt is assumed to be raised in a competitive market where investors are risk neutral and require a market interest rate that is normalized to zero. Banks' debts are risky, demandable, jointly financed by a continuum of creditors, and promise a face value D at $t = 2$. Their short-term nature allows wholesale creditors to withdraw early at $t = 1$, before the banks' risky investment matures. Provided that a bank does not fail at $t = 1$, a creditor receives qD by running on the bank, with $1 - q \in (0, 1)$ representing an early withdrawal penalty.[9] Alternatively, the debt contract can be viewed as promising an interest rate qD/V_D at time $t = 1$ and D/V_D at time $t = 2$.

To simplify the analysis, we assume that banks can stay solvent at $t = 1$,[10] because in the absence of retail debt runs, a bank can sell its entire portfolio to repay those wholesale financiers who withdraw early. Yet, it is important to note that no bank failure at $t = 1$ does not

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[6]The opposite relationship may be obtained in models based on Broecker (1990) where an increase in the number of banks raises the probability for a bad borrower to get funded in equilibrium, which implies an increase in the equilibrium interest rate.

[7]The assumption that banks hold only loans but no cash obviously simplifies the model. Yet, to allow for endogenous cash holding would not change our results. In this model, the optimal cash holding always equals zero, because the low returns earned on cash reduces banks' overall profitability and generates thinner buffers against fire-sale losses. Despite the fact that cash helps to avoid loan fire-sales in the first place, funding liquidity will not be reduced by greater cash holding. Malherbe (2014) also provides another possible reason why cash holding may not reduce bank funding liquidity risk, arguing that cash hoarding can aggravate adverse selection and therefore amplifies liquidity problem in interbank markets.

[8]Assuming a flat deposit insurance premium that is based on the expected equilibrium debt ratio will not qualitatively change our results.

[9]Here q is exogenous and assumed to be sufficiently close to 1. This short-cut assumption is meant to capture the demandability of bank debts, which serves multiple economic purposes as suggested by papers like Diamond and Dybvig (1983), Calomiris and Kahn (1991), and Diamond and Rajan (2000).

[10]Alternative assumption that banks can fail at $t = 1$ entails rather extreme parameters, and does not hold once leverage is endogenous.

mean banks do not fail because of runs. A bank failure at $t = 2$ can happen if a run at $t = 1$ has forced the bank to liquidate a substantial fraction of its loan portfolio so that the remaining assets generate insufficient cash flows to repay $t = 2$ liabilities. For simplicity, we assume that the bankruptcy cost is sufficiently high so that, if bankruptcy happens, the wholesale creditors get zero payoff and only the deposit insurance company gets the residual cash flow.

We follow the literature of global games and model bank runs as a non-cooperative game of incomplete information. At $t = 1$ each wholesale creditor is assumed to privately observe a noisy signal $s_i = \theta + \epsilon_i$, where ϵ_i is pure noise that follows a uniform distribution on $[-\epsilon, \epsilon]$. Based on the signal, the wholesale creditors play a bank-run game. Each of them has two actions: to wait until maturity or to withdraw early. A creditor who chooses to withdraw early will receive qD as the bank does not fail on the intermediate date. The creditors who choose to wait will receive nothing if the bank is only able to pay early withdrawals at $t = 1$ but goes bankrupt at $t = 2$. If the bank does not fail at $t = 2$, the creditors who choose to wait will receive in full the promised repayment D.

A bank's loan portfolio takes two periods to mature. When facing early withdrawals, a bank has to sell a part of its long-term assets in a secondary market at a fire-sale discount.[11] The liquidity mismatch results in the risk of bank runs. To model fire sales, we assume that by selling one unit of asset that generates cash flow θ, the bank obtains only a fraction of it.

$$\frac{\theta}{1 + \lambda}$$

Here $\lambda > 0$ reflects the illiquidity of banks' long-term assets that can be attributed to moral hazard, e.g., bankers' inalienable human capital in monitoring entrepreneurs, or adverse selection, e.g., buyers concerned with banks selling their 'lemon' loans.[12] We focus on the natural case where a bank run makes it more difficult for a bank to meet its debt liabilities, which occurs when the discount on the value of assets is greater than that on liabilities.[13]

$$\frac{1}{1 + \lambda} < q \tag{3}$$

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[11]We assume that banks sell fractions of diversified portfolios instead of individual loans, because this would lead to less adverse selection and helps to make bank assets more liquid.

[12]The alternative assumption of banks using collateralized borrowing generates similar results. See Morris and Shin (2009). In that case, $1/(1 + \lambda)$ reflects the hair-cut in the collateralized borrowing.

[13]Note also that condition (3) is always true as q approaches 1.

Inequality (3) captures the liquidity mismatch between banks' asset and liabilities, stating that it is costly to pay for short-term deposit withdrawals with the sale of long-term assets. If the condition is not satisfied, we will have a paradoxical scenario where banks can more easily meet their debt obligations in fire sales, and an insolvent bank with $\theta < D + F$ may be saved by a bank run, provided $(1 + \lambda)qD + F < \theta$.

2.3 Endogenous leverage

Banks set their leverage to maximize the equity value of existing shareholders. Since retail deposits are fixed in supply F, each bank optimizes the issuance of wholesale debt D.

We assume that the cost of capital is higher than the cost of debt, and denote by k the equity premium. So the expected return on equity is $1 + k$. The classical justification of the equity premium would be the tax benefits of debt.[14] Alternative justifications include the dilution costs à la Myers and Majluf (1984) and the renegotiation costs à la Diamond and Rajan (2000).

In the presence of bankruptcy costs, the optimal leverage will trade off the cost of equity with the expected costs of bankruptcy. The existence of liquidity risk makes the choice of leverage slightly more complex, because when evaluating the chance of bankruptcy, banks take into consideration both insolvency and illiquidity risk.

2.4 Time line

The timing of the model is summarized in the figure below.

t = 0	t = 1	t = 2
1. Banks choose capital structure (D). 2. Entrepreneurs choose P for a given r.	1. Wholesale creditors decide to run or not after observing private signals. 2. Banks that face runs sell their assets at a discount.	1. Returns realize. 2. Banks fail or survive. 3. Wholesale creditors who have waited get paid provided that their bank does not fail.

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[14]When a corporate tax is levied at a constant rate τ and debt repayments are exempted, k reflects the cost of losing tax shields. With $1 + k = 1/(1 - \tau)$, the model will provide the familiar expression that firms trade off between tax shields and bankruptcy costs.

3   Banking risks with exogenous leverage

We start with the case where bank leverage is exogenous and examine how competition affects different bank risks. We move through the spectrum of types of risk: from loan and portfolio risk, to insolvency risk, and to the risk of bank runs. In section 5.1, we will extend the model to incorporate also systemic risk and contagion.

3.1   Loan portfolio risk

In the spirit of Boyd and De Nicolo (2005), we show that bank competition reduces loan default risk by reducing entrepreneurs' moral hazard. Note that entrepreneurs' utility maximization yields the following probability of success.[15]

$$
P_b^* = \begin{cases} 1 & \text{if } b \in [1/(x-r), B] \\ b(x-r) & \text{if } b \in (0, 1/(x-r)) \end{cases}
$$

While an entrepreneur of type $b \geq 1/(x-r)$ will not default for any finite realization of z, loans issued to entrepreneurs with lower types can default. This makes a natural partition between risk-free and risky loans. Given the uniform distribution of b, a fraction α corresponds to risk-free loans

$$
\alpha \equiv 1 - \frac{1}{B(x-r)}, \tag{4}
$$

and the complementary fraction $(1-\alpha)$ to risky ones. The riskiness of a bank's loan portfolio is therefore reflected by $1-\alpha$, with a smaller α associated with higher risk.

As in Boyd and De Nicolo (2005), the riskiness of loan portfolio decreases with banking competition. When banks charge lower loan rates under fierce competition, entrepreneurs have more 'skin in the game' and therefore take less risk. As a result, the pool of safe loans grows.

$$
\partial \alpha / \partial r = \frac{-1}{B(x-r)^2} < 0 \tag{5}
$$

We further characterize banks' loan and portfolio risk by deriving the distribution of loan losses and that of cash flows. Denote the fraction of non-performing loans *in the risky pool* by γ. We show that γ follows a uniform distribution on $[0, 1]$, so that the expected loan loss in

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[15]Entrepreneurs' participation constraints are always satisfied, because their expected utility is non-negative for optimal P_b^*.

Figure 1: Cash flow distribution under two different levels of competition
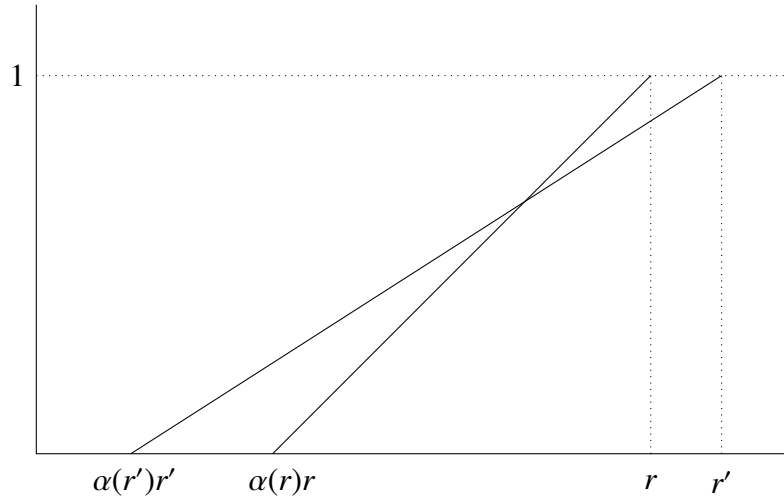
the risky pool is always 1/2. The riskiness of a loan portfolio depends solely on the size of the risky pool. When the risky pool shrinks under competition, the bank's portfolio risk decreases.

Lemma 1. *The loan loss γ, defined as the fraction of loan defaults in the risky pool, follows a uniform distribution on $[0, 1]$.*

Proof. See Appendix B.1. □

For a given α, a loan portfolio generates the following cash flow θ.

$$\theta \equiv \alpha r + (1 - \alpha)[0 \cdot \gamma + r \cdot (1 - \gamma)] = r - (1 - \alpha)r\gamma$$

The stochastics of the cash flow is driven by the random loan loss γ. Since γ enters the expression linearly, the cash flow θ also follows a uniform distribution, on support $[\alpha r, r]$. Figure 1 depicts two cumulative distribution functions of cash flows, associated with different levels of competition. When competition intensifies (loan rate drops from r' to r), the distribution function becomes steeper, implying a less volatile cash flow. Analytically, one can show that $\sigma(\theta) = (1 - \alpha)^2 r^2 \sigma(\gamma)$ monotonically increases in r.

Lemma 2. *A bank's loan portfolio generates a random cash flow $\theta \sim U(\alpha r, r)$. When competition reduces loan rate r, the volatility of cash flow decreases.*

11

3.2 Insolvency risk

Less risk-shifting and lower portfolio risk, however, do not necessarily imply a lower bank insolvency risk, because competition also reduces bank profits which can be used as a buffer against loan losses. In this subsection, we study how competition affects a bank's insolvency risk for a given level of debt.

A bank is solvent if its cash flow meets its liability, $\theta = r - (1 - \alpha)r\gamma \geq F + D$. The inequality gives a critical level of loan loss, $\hat{\gamma}$.

$$\hat{\gamma} \equiv \frac{r - (F + D)}{(1 - \alpha)r}$$

A bank with a realized loan loss greater than $\hat{\gamma}$ will become insolvent. For $\gamma \sim U(0, 1)$, this implies that the solvency probability is equal to $\hat{\gamma}$. A bank's pure insolvency risk ρ_{SR}, i.e., the risk of failure in the absence of bank runs, takes the following form.

$$\rho_{SR} \equiv 1 - \hat{\gamma} = \frac{(F + D) - \alpha r}{(1 - \alpha)r} \tag{6}$$

Note that insolvency risk is not monotonic in r. The reason is the same as in Martinez-Miera and Repullo (2010). Banking competition has two countervailing effects on insolvency: On one hand, lower loan rates reduce entrepreneurs risk-taking so that loan losses decrease (reduced risk-shifting). On the other hand, competition also makes banks' interest margin thinner and banks less profitable, reducing the buffer available to absorb loan losses (reduced buffer). The overall effect depends on parameters and is characterized in the following proposition.

Proposition 1. *For a given capital structure, a bank's insolvency risk is reduced by competition if and only if $r^2 > x(F + D)$.*

Proof. See Appendix B.2. □

The intuition behind condition $r^2 > x(F + D)$ is as follows. When project returns x are sufficiently high, entrepreneurs will have enough stake in their projects, and the change of loan rate will not substantially affect their incentive to take risk (analytically, note that $\partial^2(1 - \alpha)/\partial x\partial r < 0$). In this case, the reduced loan rate under competition mainly translates into a thinner capital buffer against loan losses. Therefore, for $x > r^2/(F + D)$, the buffer reduction effect dominates and competition increases insolvency. Graphically, $r^2 > x(F + D)$ is equivalent to two conditions: (1) $\partial(\alpha r)/\partial r > 0$ so that the distribution function satisfies a single crossing

Figure 2: Change of insolvency risk under exogenous leverage
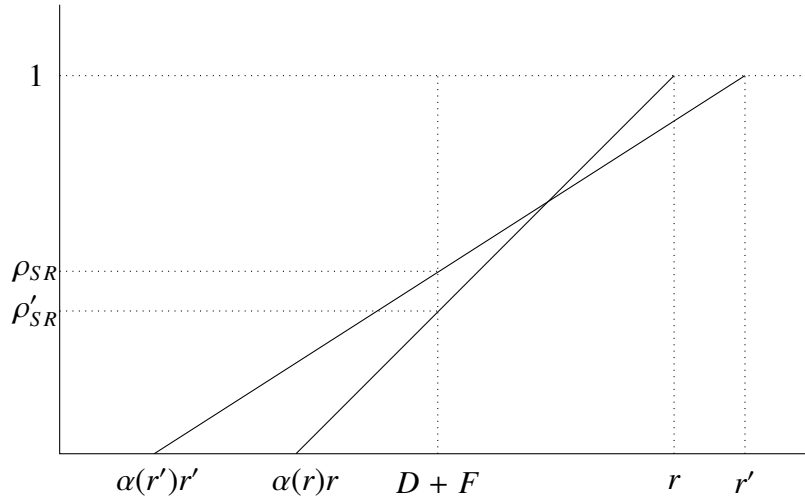
condition, and (2) the face value of debt should be to the left of the crossing point. Figure 2 illustrates such a scenario: As banking competition weakens and the loan rate rises from r to r', solvency probability drops from ρ_{SR} to ρ'_{SR}.

3.3 Funding liquidity risk and bank run

In this section we use the global games approach of Carlsson and Van Damme (1993) to examine banks' funding liquidity risk. We derive a critical level of cash flow below which a bank fails because of funding illiquidity. That is, the bank is able to repay its $t = 2$ liability in full if no run occurs at $t = 1$, but is going to default if sufficiently many wholesale creditors withdraw early. Such a panic, or speculative bank run, can be found in papers like Morris and Shin (2000), Goldstein and Pauzner (2005), and Rochet and Vives (2004).

We present in this paper a setup whose simplicity and tractability allows us to define funding liquidity risk and to study how competition affects it. In particular, we are able to show that, there exists a critical cash flow level $\theta^* = F + [1 - q(1 - (1 + \lambda)q)]D$, below which a wholesale bank run will happen and a bank will fail because of illiquidity. The critical value is greater than the bank's liability $D + F$. Intuitively, in order to maintain the confidence of financiers, a bank has to be more than barely solvent; it needs to be able to absorb potential fire-sale losses in order to convince the financiers that it is in their own interest not to run. Also note that θ^*, as intuition would suggest, increases in λ and D, so that greater fire-sale losses and more exposure to unstable short-term funding lead to a higher chance of illiquidity. The result is summarized in proposition 2.

13

Proposition 2. *There exists a unique equilibrium for the bank run game at the interim date. One can derive a unique critical cash flow $\theta^* = F + \mu D$, where $\mu = 1 - q[1 - (1 + \lambda)q] > 1$. If $\theta < \theta^*$, a wholesale debt run will occur and the bank will fail. Otherwise, no run occurs and the bank stays solvent.*

Proof. See Appendix A. □

We give here an outline of the proof, and refer readers to the proof in Appendix A. First, we show that there exist upper and lower dominance regions: when a bank's cash flow exceeds a critical level θ^U, it is a dominant strategy for wholesale financiers to wait; and when a bank's cash flow falls below a critical level θ^L, it is a dominant strategy for the wholesale financiers to run. For intermediate cash flows between θ^L and θ^U, multiple equilibria exist under complete information, and refinement can be achieved with incomplete information. In particular, if each wholesale financier privately observes a noisy signal $s_i = \theta + \epsilon_i$, and each of them follows a switching strategy—to run on the bank if and only if his signal s_i is lower than a critical level s^*, we show that marginal financiers who observe $s_i = s^*$ hold a posterior belief that the fraction of financiers who will follows a uniform distribution on $[0, 1]$. And financiers who observe a higher signal $s_i > s^*$ (a lower signal $s_i < s^*$) hold more opportunistic (pessimistic) beliefs. Finally, in a limiting case where the noise diminishes and the noisy signals are arbitrarily close to θ, we can derive the critical θ^* from the indifferent condition of marginal financiers.

For $\theta \sim U[\alpha r, r]$, the probability for a bank to fail because of a bank run can be calculated as follows.[16]

$$\rho_{IL} \equiv \frac{(\mu - 1)D}{(1 - \alpha)r} = \frac{q[(1 + \lambda)q - 1]D}{(1 - \alpha)r} \tag{7}$$

Note that the liquidity risk increases in q and λ, because a greater λ implies a bigger fire-sale loss, and a greater q allows wholesale financiers to suffer less from their early withdrawals. Both make bank runs more likely.

When a bank's wholesale debt D is exogenous, the pure liquidity risk will increase with competition. The result follows directly from the comparative statics.

$$\frac{\partial \rho_{IL}}{\partial r} = (\mu - 1)\frac{-D}{(1 - \alpha)^2}\frac{\partial(1 - \alpha)}{\partial r} < 0$$

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[16]Contrary to the pure insolvency risk, the amount of stable funds provided by insured deposits, F, is absent from the above measure of risk, because retail depositors do not have any incentive to run on the bank. The same would hold true for long-term debt, as by definition it is impossible for long-term claim holders to run on the bank.

Intuitively, competition contributes to illiquidity by reducing the expected cash flows. For a given level of fire-sale losses (λ) and a given level of wholesale debt (D), the lower cash flow due to intensified competition leads to a thinner buffer against fire-sale losses. Creditors who withdraw early will then cause a greater loss to those who wait. As the negative externalities aggravate, the coordination failure will happen more frequently among the wholesale creditors, and therefore a bank run becomes more likely.

In practice, it is difficult to distinguish between bank failures due to insolvency and those due to illiquidity. Given the observational equivalence, it is useful to examine a bank's total credit risk (ρ_{TCR}), i.e., the probability of bankruptcy for either solvency or liquidity reasons. Since pure insolvency and pure illiquidity are disjoint events, we have the total credit risk $\rho_{TCR} \equiv \Pr(\theta < \theta^*) = \Pr(\theta \leq D + F) + \Pr(D + F < \theta \leq \theta^*) = \rho_{SR} + \rho_{IL}$.

$$\rho_{TCR} = \frac{(F + \mu D) - \alpha r}{(1 - \alpha)r} \tag{8}$$

Examining the first order derivative with respect to r, one can verify that banking competition reduces total credit risk ($\partial \rho_{TCR}/\partial r > 0$) if and only if

$$r^2 > x(F + \mu D). \tag{9}$$

Note that condition (9) is more stringent than the condition in Proposition 1. Once funding liquidity risk is taken into account, for a parameter constellation satisfying $x(F + \mu D) > r^2 > x(F + D)$, banking competition would decrease pure insolvency risk but increase total credit risk. In other words, when illiquidity risk is considered, the set of parameters where the result of Boyd and De Nicolo (2005) applies will shrink.

Proposition 3. *For a given level of debt obligation, the probability that a bank fails because of a bank run monotonically increases with competition. The total credit risk, defined as the risk of bank failures due to either insolvency or illiquidity, decreases with competition if and only if $r^2 > x(F + \mu D)$.*

Proposition 1 and 3 suggest that even if banks do not adjust their leverage according to changing competitive environment, competition can affect different risks differently. Solely focusing on one dimension of risk can lead to a biased judgment of the overall effect.

4 The impact of endogenous leverage

Although banks' leverage decisions are restricted by regulation, banks still have the ability to choose the buffer above and beyond regulatory capital requirements as well as the maturity structure of their debt. Leverage plays a crucial role in the determination of insolvency risk: A low-risk portfolio financed with high leverage can end up generating a high insolvency risk. Consequently, the bank's optimal leverage choice may offset any reduction in portfolio risk due to competition. It is therefore crucial to study how the previous results change when leverage is endogenous.

4.1 Endogenous leverage

While a higher debt level saves on costly capital, it also entails a greater chance of bankruptcy (caused by either insolvency or illiquidity). Banks rationally set their leverage to equalize the marginal cost and the marginal benefit.

Banks choose their capital structure to maximize the leveraged firm value to existing shareholders. If ω is the fraction of the bank sold to new shareholders, existing shareholders obtain

$$(1 - \omega) \int_{F+\mu D}^{\bar{\theta}} \Big[\theta - F - D\Big] h(\theta, r) d\theta,$$

where bank cash flow θ has a density function $h(\theta, r)$ on support $[\underline{\theta}, \bar{\theta}]$. The bank will raise V_E from new shareholders, V_D from wholesale short term creditors, and F from insured depositors. And the three sources of funding should provide the required amount of investment, $V_E + V_D + F = 1$. The optimal leverage is the solution to the maximization program below.

$$\max_{V_E, V_D} (1 - \omega) \int_{F+\mu D}^{\bar{\theta}} \Big[\theta - F - D\Big] h(\theta, r) d\theta$$

$$s.t. \quad V_E = \frac{\omega}{1 + k} \int_{F+\mu D}^{\bar{\theta}} \Big[\theta - F - D\Big] h(\theta, r) d\theta$$

$$V_D = \int_{F+\mu D}^{\bar{\theta}} D h(\theta, r) d\theta$$

$$V_E + V_D + F = 1$$

Adding the three constraints to the objective function we obtain the unconstrained optimization, with $\int_{\underline{\theta}}^{F+\mu D} Fh(\theta, r)d\theta$ reflecting the subsidy from deposit insurance.

$$\max_{D} \int_{F+\mu D}^{\bar{\theta}} [\theta + k(F + D)]h(\theta, r)d\theta + (1 + k) \int_{\underline{\theta}}^{F+\mu D} Fh(\theta, r)d\theta - (1 + k) \tag{10}$$

The corresponding first order condition can be written compactly as

$$-\mu(\mu + k)D^*h(F + \mu D^*, r) + \int_{F+\mu D^*}^{\bar{\theta}} kh(\theta, r)d\theta = 0,$$

or with H denoting the c.d.f. of θ,

$$D^* = \frac{k[1 - H(F + \mu D^*, r)]}{\mu(\mu + k)h(F + \mu D^*, r)}. \tag{11}$$

In the special case of uniform distribution, equation (11) takes the simple form of

$$D^* = \frac{r - F}{\mu^2/k + 2\mu}. \tag{12}$$

As intuition would suggest, D^* increases in the cost of capital k, and decreases in the associated liquidity risk μ. And the risky debt that a bank issues is proportional to its *maximum* residual cash flow after paying insured deposits F.[17]

Proposition 4. *A bank that maximizes its value by trading off the benefits of debt versus its bankruptcy cost sets its debt to $D^* = (r - F)/[\mu^2/k + 2\mu]$.*

Proof. See Appendix B.3. □

Being junior, risky and demandable, the wholesale funding D is the most relevant debt in the model. Denote a bank's leverage ratio by l. The leverage ratio can be measured by the face value of its risky debt over the available expected cash flow:

$$l \equiv \frac{D^*}{E(\theta - F)} = \frac{1}{\mu^2/k + 2\mu} \cdot \frac{2(r - F)}{(1 + \alpha)r - 2F}. \tag{13}$$

The leverage ratio is not monotonic in banking competition. Its comparative statics with respect to r depends on the relative strength of two countervailing effects: (1) when r increases,

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[17]Note that $\lim_{\mu \searrow 1} 1/[\mu^2/k + 2\mu] = 1/(1/k + 2) < 1$. Since $1/[\mu^2/k + 2\mu]$ monotonically decreases in μ, it holds that $1/[\mu^2/k + 2\mu] < 1/(1/k + 2) < 1$. A bank cannot issue more risky debt claims than its maximum cash flow after paying the risk-free F and is unwilling to issue risky debt more than a fraction of $(r - F)$.

a bank generates a higher cash flow and can issue more claims, including risky debts, which we call "cash flow effects"; (2) a higher r implies stronger risk-shifting by entrepreneurs, leading to higher portfolio risk and curbs leverage via bankruptcy costs, which we call "risk effects". The overall effect depends on the relative magnitude of the two forces. For a low level of F, the cash flow effect dominates. As r rises, banks have more cash flow available to its wholesale financiers, and the leverage ratio increases.

Corollary 1. *The leverage ratio $D^*/E(\theta - F)$ decreases with competition (decreases with loan rate r) if and only if $xF < r^2$. Otherwise, the result reverses.*

Proof. See Appendix B.3. □

4.2 Risk under endogenous leverage

With exogenous leverage D replaced by endogenous D^*, the different bank risks under endogenous leverage are defined analogously to equations (6) and (7) - (8). Denoting the risks accordingly with a superscript star, one can write insolvency risk, liquidity risk, and total credit risk as follows.

$$\rho_{SR}^* \equiv 1 - \frac{r - F - D^*}{(1 - \alpha)r} = 1 - \left[1 - \frac{1}{\mu^2/k + 2\mu}\right]\frac{r - F}{(1 - \alpha)r} \tag{14}$$

$$\rho_{IL}^* \equiv (\mu - 1)\frac{D^*}{(1 - \alpha)r} = \frac{\mu - 1}{\mu^2/k + 2\mu}\frac{r - F}{(1 - \alpha)r} \tag{15}$$

$$\rho_{TCR}^* \equiv 1 - \frac{r - F - \mu D^*}{(1 - \alpha)r} = 1 - \frac{\mu + k}{\mu + 2k}\frac{r - F}{(1 - \alpha)r} \tag{16}$$

How competition affects bank risks under endogenous leverage follows directly from the comparative statics.

Proposition 5. *When banks set their leverage endogenously, pure insolvency and total credit risk will decrease with banking competition and funding liquidity risk will increase with banking competition, if and only if $r^2 > xF$.*

Proof. See Appendix B.4. □

Proposition 5 states that under endogenous leverage, pure insolvency and liquidity risk always move in the opposite direction, with the latter dominant in determining total credit risk. Compared to proposition 4, it should be clear that endogenous leverage has a crucial impact on

the various risks already identified. To emphasize this, we now visualize the comparative statics for insolvency, illiquidity, and total credit risk. For parameters $x = 1.25$, $B = 50$, $F = 0.90$, $D = 0.055$, $k = 1.1$, and $\mu = 1.05$, one can depict the following relationships between various bank risks and loan rate, under exogenous and endogenous leverage respectively.

Banking Competition and Total Credit Risk: Exogenous vs Endogenous Leverage
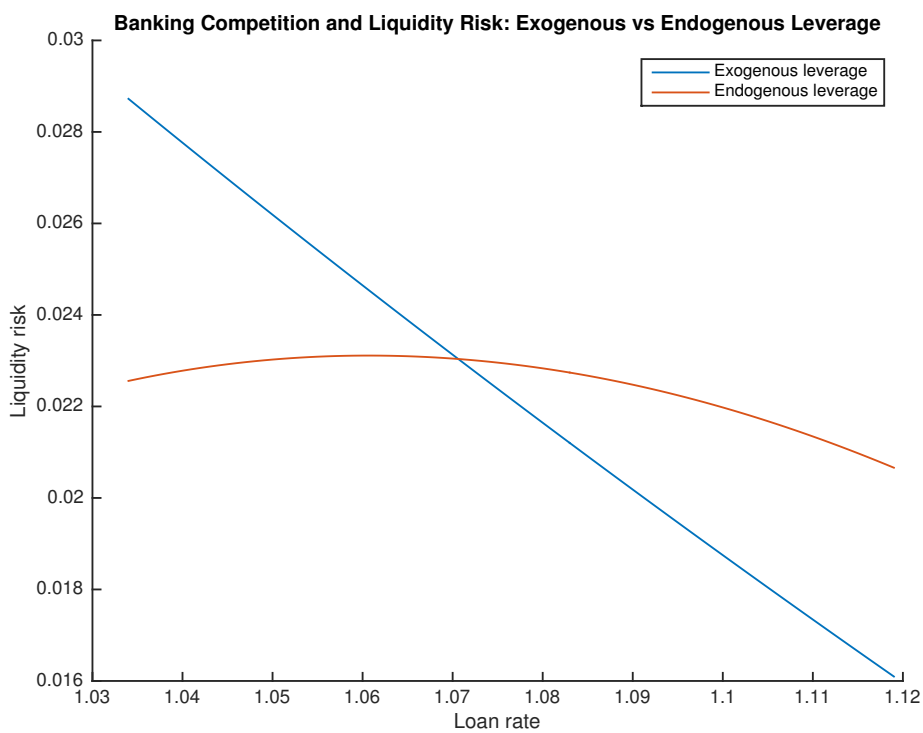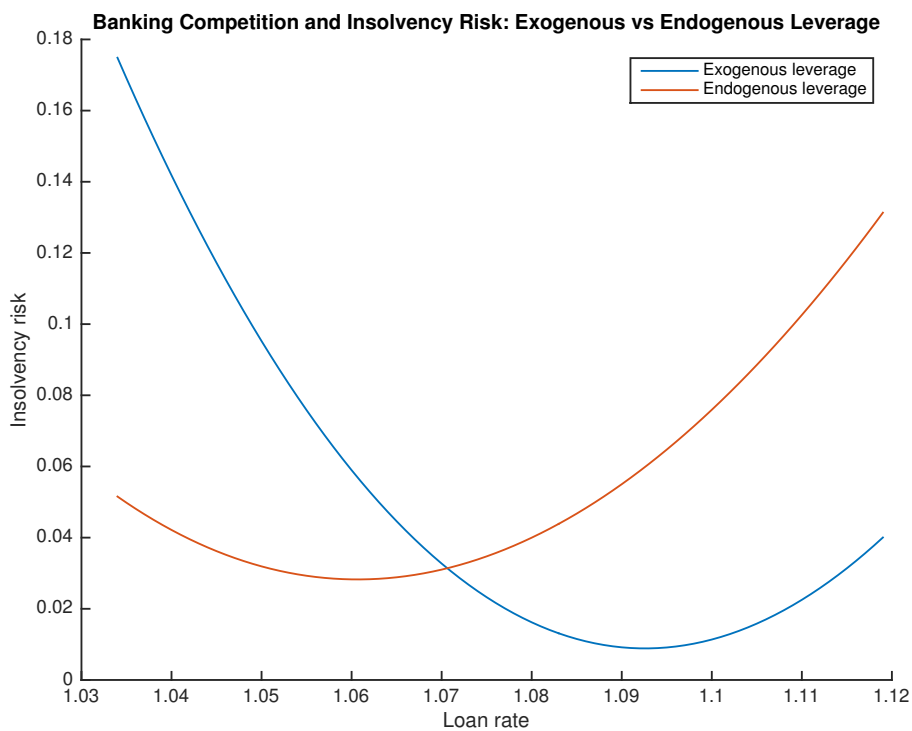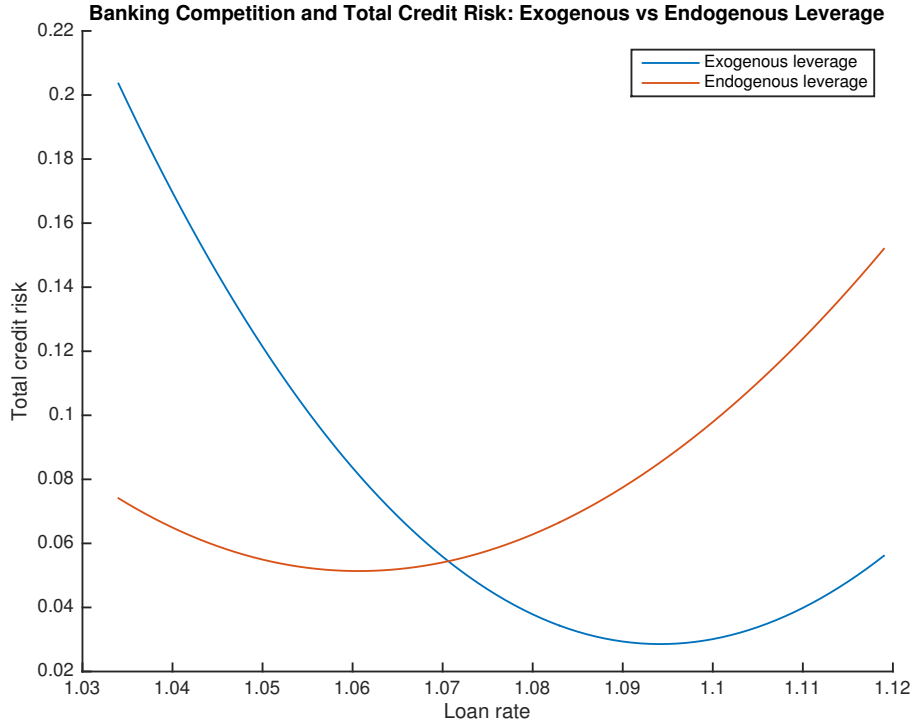
4.3 Interpretation

Overall, our results state that the impact of competition on financial stability critically depend on the type of banking industry that is considered. Two possible cases emerge. The case $r^2 > xF$, corresponds to less productive firms facing high borrowing costs, while banks obtain high margins and raise funding in the market (low level of insured deposits). When this is the case, total credit risk is reduced with competition. As a limit case, $F = 0$ can be interpreted as investment banking. More competition means safer investment banking. Alternatively, the case $r^2 < xF$, corresponds to highly productive firms facing low borrowing costs, with banks mainly financed through deposits. In such environment, the opposite result holds: banking competition reduces financial stability. This correspond to classic retail banking with low margins and prudent funding through insured deposits.

Although our model does not pretend to provide robust results that hold true in every environment, it is worth noticing the key ingredients that determine here the impact of bank competition on the different types of financial stability. Banks' liability structure, and in particular the amount of short term wholesale funding, is central to the relationship between competition and bank risk. Our model's conclusions provide a much richer view of the link between banking competition and financial risk than is usually considered.

20

1. To begin with, the impact of banking competition on financial stability depends upon the borrowing firms' project returns x. In highly productive economies, bank competition constitutes a threat to financial stability. The impact of Boyd and De Nicolo moral hazard effect is limited, and the key determinant of the link between bank competition and financial stability is the role of the buffer generated by banks' market power a la Martinez-Miera and Repullo. Comparing Proposition 1 and Proposition 5 we observe that the threshold for x that inverts the relationship from banking competition to financial stability is reached much earlier if we take into account the endogeneity of banks' leverage. This is the case because banks will be more conservative in their choice of leverage as competition increases, so that the strength of the Boyd and De Nicolo's argument is weakened and the charter value argument dominates. The argument can also be reinterpreted considering the business cycle. In a boom, banking competition jeopardizes financial stability, while, in a bust it reduces banks' risks.

2. The existing level of market power is also essential in our framework. For high market power competition reduces bank fragility, nevertheless a threshold may exist (provided that $xF > 1$) beyond which the result is reversed. This is interesting from a policy perspective as it provides a more nuanced prescription than the usual one: in order to sustain financial stability, it might be interesting to promote competition up to a certain threshold, but beyond that point, competition will lead to higher banking risks.

3. The role of stable funds is critical for our result. In a traditional banking industry funded through deposits and long term bonds (equivalent in our context to insured deposits) where $xF > r^2$, competition will be detrimental to financial stability. Instead in a banking industry where wholesale short term (possibly interbank) funding is prevalent, the Boyd De Nicolo argument will prevail.

4. More generally, two types of banks, corresponding to the two possible signs of $xF - r^2$, may coexist and will react in a different way to a generalized increase in competition. For banks that rely less on stable funding $xF < r^2$, and in particular for investment banks, an increase in competition will increase financial stability. Instead, for banks with high levels of deposits and lower market power, for which the inequality $xF > r^2$ is satisfied, the opposite occurs and the main effect of banking competition is to reduce the banks buffer and to encourage higher leverage.

5 Extension: contagion and systemic risk

A natural extension is to explore what happens once we add the risk of financial contagion. We illustrate this in a two-bank setup, with two bank failures at the same time considered as a systemic crisis.[18] We make a stylized assumption that when both banks need to sell, the fire-sale discount rises from λ to λ'. This assumption captures the observation that secondary market prices tend to fall further when more banks fail and are forced to sell their assets.[19] Therefore, asset fire sales provide a channel for financial contagion: When the first bank goes under, asset prices decline, which magnifies the coordination failure among the debt holders at the other bank, leading to a second bank run.[20]

Following the structure of section 3 and 4, we first study how competition affects contagion and systemic risk under the assumption of exogenous leverage, and then move on to the case of endogenous leverage.

5.1 Contagion and systemic risk under exogenous leverage

Following the same procedure of section 3.4, one can derive a critical cash flow level

$$\theta^{**} = F + \mu'^*$$

with $\mu' = 1 - q[1 - (1 + \lambda')q] > \mu$. A bank whose cash flow falls between $[\theta^*, \theta^{**}]$ will not fail if the other bank does not face a run, but will fail because of illiquidity if a run happen to the other bank. Therefore, a bank of $\theta \in [\theta^*, \theta^{**}]$ is exposed to contagion. We define the risk of contagion as follows.

$$\rho_{CTG} \equiv Prob(\theta^* < \theta < \theta^{**}) = \frac{(\mu' - \mu)D}{(1 - \alpha)r} \tag{17}$$

A systemic crisis happens when the two banks fail simultaneously, which occurs with the following probability.

$$\rho_{SYS} = Prob(\theta < \theta^{**})^2 = \left[\frac{\theta^{**} - \alpha r}{(1 - \alpha)r}\right]^2 \tag{18}$$

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[18]The extension to n banks is straightforward.

[19]The decline in asset prices can be caused either by cash-in-the-market pricing or by informational contagion. In the former case, market prices are driven down by the limited supply of cash. In the latter, a large number of bank failures leads investors to form more pessimistic beliefs of banks' common risk exposures and lowers their willingness to pay for bank assets.

[20]For a full-fledged model where fire sales and bank runs mutually reinforce each other, see Li and Ma (2012).

Figure 3: Illustration of Various Bank Risk
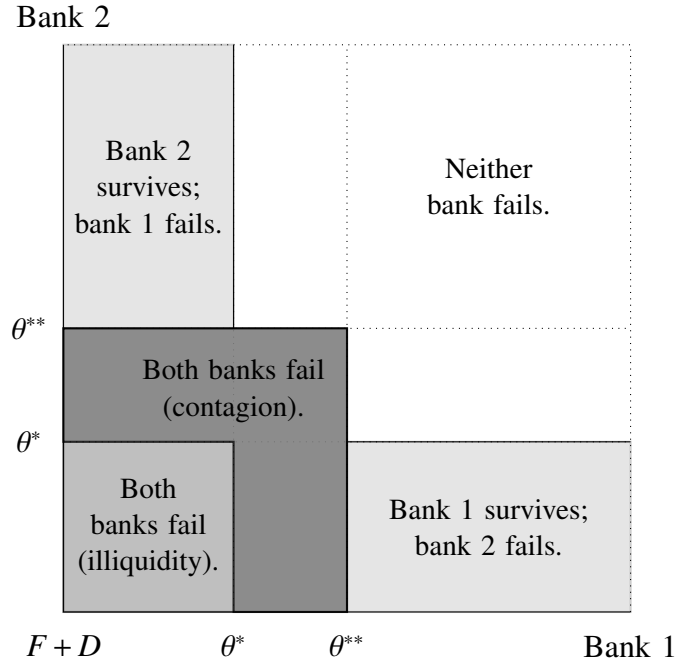
The liquidity risk, the exposure to contagion, and the risk of a systemic crisis are illustrated in Figure 3. As the competitive environment changes, the critical cash flow θ^* and θ^{**} shift, leading to the corresponding changes in various bank risks.

Competition affects systemic risk in two ways. First, it reduces banks' buffer against fire-sale losses, and therefore increases banks' exposure to contagion. On the other hand, it promotes banks' solvency if $r^2 > x(F + D)$, reducing the chance of the first firesale. Overall, competition reduces systemic risk if and only if $r^2 > x(F + \mu'D)$, which forms a counterpart to condition (9).

Proposition 6. *For a given level of debt obligation, banks' exposure to contagion increases with competition. Competition reduces the risk of systemic crises if and only if $r^2 > x(F + \mu'D)$.*

Proof. See Appendix B.5. □

5.2 Contagion and systemic risk under endogenous leverage

To simplify the analysis under endogenous leverage, we assume that regulators bail out both banks in a systemic crisis so that banks do not to take into account systemic risk when setting

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their leverage.[21] Consequently, the optimal wholesale debt remains the same as in equation
(B.23). Substituting that into (17) and (18), we obtain the probability of contagion and systemic
crisis under endogenous leverage.

$$\rho^*_{CTG} \equiv \frac{(\mu'^*}{(1-\alpha)r}$$

$$\rho^*_{SYS} \equiv Prob(\theta < \theta^{**})^2 = \left(1 - \frac{r - F - \mu'^*}{(1-\alpha)r}\right)^2$$

The impact of competition on banks' exposure to contagion (ρ^*_{CTG}) and the risk of a systemic
crisis (ρ^*_{SYS}) simply follow the comparative statics.

Proposition 7. *For $r^2 > xF$, while banks' exposure to financial contagion increases with com-
petition, the risk of a systemic crisis decreases, provided $\mu^2/k + 2\mu > \mu'$.*

Proof. See Appendix B.6. □

6   Reinterpreting the empirical literature

The difficulties in analyzing the link between competition and stability increase exponen-
tially when we turn to data, as empirical results are sometimes contradictory.[22]

Our model allows us to clarify and reconcile the empirical results because of the distinc-
tion we draw among different types of bank risks that competition affects differently.[23] In our
model, competition decreases banks' asset risk directly by reducing borrowers' risk-shifting.
But as banks react to the changes in asset risk by altering their leverage, all other types of
risks are indirectly affected. Therefore our reading of the empirical literature introduces drastic
differences depending on whether the evidence concerns the riskiness of banks' assets, or the

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[21]Relaxing this assumption would imply negative externalities of leverage: a bank that fails because of high
leverage is contagious to other banks. Since such cost is not taken into account in private decisions, banks are
likely to use greater leverage.

[22]In the end of this section, we summarize in Table 3 the empirical papers that we surveyed, highlighting the
whole variety of risk measurements studied and the diverse results obtained.

[23]Just like the multiple risk measures studied in the current paper, various competition measures abound in
the empirical literature, ranging from franchise value (Tobin's Q), to market concentration (e.g., HHI, C-n), to
structural measures (i.e., P-R H-stat., Lerner's index, Boone's indicator), and to institutions (e.g., contestability of
the market such as activity and entry restrictions). (See Degryse, Kim, and Ongena (2009) for a comprehensive
review on banking competition measures.) While some empirical studies, e.g., Claessens and Laeven (2004) and
Schaeck, Cihák, and Wolfe (2009), show that concentration is a poor proxy for bank competition, we are still left
with a wide range of possibilities, and the industrial organization literature does not provides us an unambiguous
answer which measure is most reliable.

riskiness of banks themselves. We suggest a progressive approach to understand the impact of competition on banks' risk-taking by refining the questions that are asked as successive layers.

1. Does competition increase the safety banks' portfolios? In other words, is Boyd & De Nicolo's basic result true?

2. Does competition increase the risk of bank insolvency?

3. Does competition increase the funding liquidity risk of banks?

4. Does competition increase banks' systemic risk?

Revisiting the empirical literature through this filter leads us to regroup the empirical results in a more complete and orderly way, taking into account explicitly endogenous leverage, and refusing to consider the different measures either equivalent or complementary in the assessment of the impact of competition on financial stability.

6.1   Portfolio risk: non-performing loans

The basic postulate of Boyd and De Nicolo (2005) is that competition will reduce the riskiness of banks' portfolio, an issue independent of the banks' leverage decision. In contrast, the charter value hypothesis posits that banks' overall investment strategy will be more risky when the opportunity cost of bankruptcy is lower. Therefore, knowing whether Boyd & De Nicolo's basic conjecture is in line with empirical evidence is a crucial step forward. In order to measure the riskiness of assets, measures like stock volatility, as in Demsetz, Saidenberg, and Strahan (1996) and Brewer and Saidenberg (1996), are contaminated by leverage; whereas non-performing loans (NPLs) appears as a natural candidate. This is confirmed by the fact that a bulk of the literature takes NPLs as one of the key dependent variables in their analysis.[24]

Restricting the measurement of asset risk to NPLs implies focusing on a very specific dimension of the broad link between competition and stability, where we might hope for some consensus. Unfortunately the evidence is still mixed even with this drastic reduction. The initial paper on charter value, Keeley (1990), did not consider NPLs but rather the overall risk of bank failure. The use of NPLs can be found in more recent works such as Salas and Saurina (2003) and Yeyati and Micco (2007). The authors found an increase in non-performing loans as bank

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[24]Some caveats are in order regarding the accuracy of this measurement. First, banks can manipulate NPLs by rolling over bad loans. Second, a risky loan granted today will only default in the future, e.g., after a two-year lag if we follow Salas and Saurina (2003), and the rate of default will depend on the business cycle (Shaffer 98).

competition increased in Spain and in eight Latin American countries respectively. Jiménez, Lopez, and Salas (2010) also find supporting evidence for charter value hypothesis, but only if market power is measured by Lerner indices. Support for risk-shifting hypothesis can be found in Boyd and Jalal (2009), who find competition negatively correlated with loan losses for banks both in the US and in 134 non-industrialized countries. The finding is further corroborated by Berger, Klapper, and Turk-Ariss (2009). Having studied a cross-section of banks in 23 developed countries, the authors conclude that banks with a higher degree of market power exhibit significantly more loan portfolio risk.

Since market structure and banks' loan losses can be jointly determined, researchers have also examined natural experiments, such as nationwide banking in the US, in order to establish a causal relationship between competition and stability. But again, contradictory results arise. While Jayaratne and Strahan (1998) report that "Loan losses decrease by about 29 basis points in the short run and about 48 basis points in the longer run after statewide branching is permitted", Dick (2006) finds out that "charged-off losses over loans (...) appears to increase by 0.4 percentage point following deregulation".[25]

6.2   Individual bank insolvency

Because of the endogeneity of banks leverage, a lower portfolio risk does not necessarily translate into a lower bank default risk. For example, Salas and Saurina (2003) show that a bank's capital ratio increases in its Tobin's Q, thus providing evidence on the (endogenous) reaction of leverage to charter value. This potential divergence between portfolio risk and insolvency risk is best illustrated in Berger, Klapper, and Turk-Ariss (2009): in spite of finding NPLs positively correlated with banks' market power, the authors show that banks of greater market power have lower insolvency risk because of their higher capital ratios.

Since Keeley (1990) the literature has been focusing on the risk of individual bank failure. In his classic paper, Keeley (1990) considers the market-value capital-to-asset ratio and the interest cost on large, uninsured CD's. Following his approach, Demsetz, Saidenberg, and Strahan (1996) use seven different measures of BHCs' risks and in each of them franchise

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[25]Banking deregulation is usually associated with a removal of entry barriers that will increase competition. Yet, deregulation may also affect the range of financial products banks are allowed to invest in and the structure of financial institutions. Therefore, banking liberalization does not only affect competition, but also a whole set of bank strategies. This implies that using deregulation to identify the impact of competition can end up with exploring the stability effect of a package of liberalization measures, where the reduced market power may be only an undistinguishable part of this joint product.

value is statistically significant, thus providing support to the charter value theory.[26] Brewer and Saidenberg (1996) found also corroborating evidence that the stock returns volatility was negatively related to S&L franchise values as measured by the market-to-book asset ratio.

In our judgement, a bank's z-score and distance to default can be natural measurements for its insolvency risk, and many empirical papers take these as their main risk measurement.[27] Still, there are important nuances in the empirical findings. Beck, Jonghe, and Schepens (2011) find on average a positive relationship between banks' market power and their z-scores. Nevertheless, they report large cross-country variation in this relationship, with it being negative in many cases. Boyd and Jalal (2009) further challenged the charter value hypothesis: using both US and cross-country samples the authors consistently find competition negatively correlated with bank insolvency risk, measured by lower z-scores and actual bank failures. Confirming this view, De Nicolo and Ariss (2010) show that large loan market rents predict lower bank capitalization and higher probabilities of bank failures.

6.3   Individual bank illiquidity

Funding liquidity risk has largely been overlooked by the empirical studies on bank competition, mainly because it is difficult to distinguish a bank failure that is due to illiquidity from one that is caused by insolvency, Goodhart (1987). Nevertheless, a number of recent papers have proposed indicators for bank funding liquidity risk. Based on accounting information, Brunnermeier, Gorton, and Krishnamurthy (2012) and Brunnermeier, Krishnamurthy, and Gorton (2013) suggest a liquidity mismatch index that captures funding liquidity risk from both asset and liability sides.[28] Based on market information, Morris and Shin (2004) identify extra yield due to illiquidity risk in their study of bond pricing, and Veronesi and Zingales (2010) construct a bank run index using CDS spreads. Such developments make it possible to study the relationship between bank competition and funding liquidity risk.

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[26]The risk measurements include annualized standard deviation of weekly stock returns, systematic risk, firm-specific risk, capital-to-assets Ratio, loans-to-assets ratio, commercial and industrial loans-to-assets ratio and loan portfolio concentration.

[27]A bank's z-score is calculated $(RoA + E/A)/\sigma(RoA)$, and is meant to capture a bank's distance from bankruptcy. Distance-to-default is defined similarly using stock market information. Note that even though theoretical models made no prediction concerning how competition affects bank leverage, empirical studies have indirectly taken into account leverage as leverage a component of the risk measures.

[28]See Bai, Krishnamurthy, and Weymuller (2013) for an empirical implementation of this measure.

6.4 Systemic risk

The analysis of systemic risk is, obviously, even more difficult since it often has to deal with cross-country analysis, and the precise definition of a banking crisis itself as well as its timing are also subject to different interpretations. Thus, while some authors define a systemic crisis by system-wide public interventions, or 10% of the banking industry being affected, others like Anginer, Demirgüç-Kunt, and Zhu (2012) and De Nicolo, Bartholomew, Zaman, and Zephirin (2004) prefer to measure the probability of systemic risk by pairwise correlation of banks' distance-to-default or by constructing an indicator of the probability of failure for the five largest banks.[29]

Beck, Demirgüç-Kunt, and Levine (2006) analyze a sample of 69 countries over a 20-year period, and find that more concentrated national banking systems are subject to a lower probability of systemic banking crisis. Still, they point out that concentration need not be related to market power, as already mentioned by Claessens and Laeven (2004), and that other measures of competition may lead to the opposite result. Contradicting that result, Schaeck, Cihák, and Wolfe (2009) show, using the Panzar and Rosse H-Statistic as a measure for competition in 45 countries during 1980-2005, that more competitive banking systems are less prone to systemic crises, and that the time elapsed between crises is longer in a competitive environment.

On this issue, our paper's empirical prediction states that an increase of competition can have different effects depending upon the amount of insured retail deposits, and the profitability of projects and banks' spreads, thus suggesting new lines for future empirical research based on distinguishing different types of banks. If we interpret our model literally, this would be to distinguish banks with low deposit to asset ratios from those with high deposit to asset ratios. More generally, one could divide banks according to their funding strategies, according to whether a bank is funded by wholesale or retail debts.

7   Discussion and policy implications

Because the aim of our paper is to clarify the multiple concepts of risk and the key role of leverage, we have made a number of drastic simplifying assumptions that lead to relatively simple propositions but cannot be easily generalized. Indeed, our framework considerably un-

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[29]It should be noted that with newly developed measurements of systemic risk such as marginal expected short-falls or *CoVaR* in Adrian and Brunnermeier (2010), one can also link a bank's market power to its contribution to the systemic risk.

Table 1: Does banking *competition* lead to *instability*? Various risk and competition measurements, and diverse results from the empirical literature.

Paper	Risk	Competition	Results	Data Source
Keeley (1990)	Interest Cost	Tobin's q	Yes	US
Demsetz, Saidenberg, and Strahan (1996)	Stock Volatility	Market-Book Value	Yes	US
Brewer and Saidenberg (1996)	Stock Volatility	Market-Book Value	Yes	US S&L banks
Jayaratne and Strahan (1996, 1998)	NPLs	Deregulation	No	US
Salas and Saurina (2003)	Loan Loss	Tobin's q	Yes	Spain
De Nicolo and Loukoianova (2005)	Z-Score	HHI	No	Non-industrialized
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Concentration	Yes	Cross-Country
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Contestability	No	Cross-Country
Dick (2006)	Loan Loss	Deregulation	Yes	US
Yeyati and Micco (2007)	Z-Score & NPLs	P-R H-Stat.	Yes	Latin America
Schaeck and Cihák (2010a)	Capitalization	P-R H-Stat.	No	Developed Countries
Dell'ariccia, Igan, and Laeven (2012)	Lending standard	Number of banks	Yes	US
Boyd and Jalal (2009)	Loan Loss	HHI	No	US/Cross-Country
Boyd and Jalal (2009)	Z-Score	HHI	No	US/Cross-Country
Boyd, De Nicolo, and Loukoianova (2009)	Crisis Dummy	HHI/C3	No	Cross-Country
Berger, Klapper, and Turk-Ariss (2009)	NPLs	Lerner Index/HHI	No	Developed Countries
Berger, Klapper, and Turk-Ariss (2009)	Z-Score	Lerner Index/HHI	Yes	Developed Countries
Schaeck, Cihák, and Wolfe (2009)	Crisis Dummy	P-R H-Stat.	No	Cross-Country
Schaeck, Cihák, and Wolfe (2009)	Duration until crisis	P-R H-Stat.	No	Cross-Country
Schaeck and Cihák (2010b)	Z-Score	Boone's Indicator	No	US/EU
Jiménez, Lopez, and Salas (2010)	NPLs	Lerner Index	Yes	Spain
Jiménez, Lopez, and Salas (2010)	NPLs	HHI/C5	No	Spain
Beck, Jonghe, and Schepens (2011)	Z-Score	Lerner Index	Yes	Cross-Country
De Nicolo and Ariss (2010)	Z-Score	Deposit market rent	Yes	Europe
De Nicolo and Ariss (2010)	Z-Score	Loan market rent	No	Europe
Dick and Lehnert (2010)	Personal bankruptcy	Deregulation	Yes	US
Dick and Lehnert (2010)	NPLs/risk management	Deregulation	No	US
Anginer, Demirgüç-Kunt, and Zhu (2012)	D-to-D Correlation	Lerner Index	No	Cross-Country

derstates the complexity of the issue, because competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, securitization, cash hoarding, and so on, which are all abstracted from in the current setup. Yet, the result still conveys the key messages of the paper: (1) banking competition affects different types of risk differently; and (2) endogenous leverage is a central hub that both reflects changes in the cash flow riskiness and affects all different aspects of banking risk.

Another reason why we consider our model as a special case is that we start with the risk-shifting hypothesis as a priori. The argument of endogenous leverage is however more general and also applies to models that take charter value hypothesis as a priori. In that case, when competition induces banks to choose riskier portfolios with lower charter values, banks will optimally adjust their leverage to balance the change in the portfolio risk. Again, leverage and risks will be jointly determined by banks' optimization behaviors. Yet as the priori flips, the overall results might reverse as compared to those in the current paper.

Also our model's main objective is not to address the design of overall banking regulation, and consequently, from that perspective, it suffers from two limitations: On the one hand,

it does not take into account the impact of competition on increasing productivity through the Shumpeterian creative destruction process, Dick (2006). On the other hand, it does not consider the supply of credit, which is exogenously set as all firms receiving financing. In spite of this, it is interesting to consider the implications that our results have for regulatory policies. Two main lessons can be drawn: the first regarding the impact of competition in general, and the second regarding capital and liquidity regulation.

The first lesson is that a one-size-fits-all approach to the analysis of the link between banking competition and financial stability is insufficiently rigorous. To be more precise, we conclude that the link depends, among other things, on the degree of market power of financial institutions. If financial institutions have high market power, then competition reduces total bankruptcy risk (the sum of insolvency and liquidity risk) in financial institutions, confirming the risk shifting hypothesis of Boyd and De Nicolo. Still, in this high market power case, we show that the impact is dampened by the increase in liquidity risk that stronger competition causes. On the other hand, once the banking industry is sufficiently competitive, the inequality is reversed and additional competition leads to financial instability, thus confirming the charter value assumption. From that perspective, the optimal policy depends upon whether market power is above or below a threshold that depends upon firms' productivity as well as upon banks' liability structure.

Second, a simple extension of our framework, consisting in distinguishing wholesale short-term market funding from long-term market funding, also has implications regarding liquidity regulation. Indeed, we show that a more competitive banking industry has a higher level of liquidity risk, proportional to the amount of short-term market funding, if leverage is exogenous. This is directly related to capital regulation, because if capital regulation is binding, leverage becomes exogenous. As a consequence, liquidity regulation, as suggested by Basel III, may reduce the liquidity risk that is implied by fiercer competition.

8   Concluding remarks

In this paper we model explicitly the credit risk created by borrowers' moral hazard and examine how banks optimally adjust their leverage in light of various risks. Our theoretical framework enables us to clarify the concept of financial stability, which has multiple dimensions ranging from portfolio risk to systemic risk. We show that the idea of finding an unified

relationship between banking competition and financial stability that would hold across different types of banks and risks has no theoretical foundation. This result can help to explain the diverse findings in the empirical literature. We further establish that banks' leverage and liability structure play a key role in determining the impact of banking competition on stability. As a consequence, this opens road for new empirical analysis on the competition-stability link that should depend upon the type of banks and the state of the economy.

Appendix A   Unique bank run equilibrium (Proposition 2)

To find equilibrium of the bank run game for all possible values of θ, we start by establishing the existence of upper and dominance regions. We derive critical cash flow levels θ^U and θ^L, such that when $\theta > \theta^U$, to wait is a dominant strategy, and when $\theta < \theta^L$, to run is a dominant strategy.

Note that for waiting to be a dominant strategy, a wholesale financier i should find it in his own interest not to run on the bank even if all other financiers do so. In other words, even if all other wholesale financiers withdraw early, the bank should be solvent and able to repay financier i in full. In this case, financier i receives qD by running, D by waiting, and will find to wait a dominant strategy.

To derive θ^U, note that when all other wholesale financiers withdraw early, the bank has to liquidate $f = qD(1 + \lambda)/\theta$ fraction of its asset, and will stay solvent if and only if

$$(1 - f)\theta = \left(1 - \frac{qD(1 + \lambda)}{\theta}\right)\theta = \theta - q(1 + \lambda)D \geq F.$$

Therefore we have the upper dominance region defined by

$$\theta^U = F + q(1 + \lambda)D.$$

If $\theta \in (\theta^U, \overline{\theta}]$, it is a dominant strategy for wholesale financiers to wait. Intuitively, the wholesale financiers will not run if the bank's cash flow is sufficient to cover fire-sale loss $[q(1 + \lambda) - 1]D$ in addition to its full liability $F + D$.

There exists also a critical cash flow θ^L such that when θ falls below θ^L, to run is a dominant strategy for all wholesale financiers. This happens when a bank is fundamentally insolvent, $\theta < F + D$. In this case, by waiting, a wholesale financier will receive nothing; but by running

on to the bank, he can receive qD. Therefore, we have found a lower dominance region defined by $\theta \in [\underline{\theta}, \theta^L]$, with

$$\theta^L = F + D.$$

By our assumption $q(1 + \lambda) > 1$, we have $\theta^L < \theta^U$. In the intermediate range between θ^L and θ^U, multiple equilibria exist under complete information. A wholesale financier finds it in his interest to run if the other financiers run, and will choose to wait if all other financiers stay.

Now we refine the multiple equilibria by introducing incomplete information, assuming that each wholesale financier i receives a private signal $s_i = \theta + \epsilon_i$, $\epsilon_i \sim U(-\epsilon, \epsilon)$. Therefore for a given realization of θ, the private signals that wholesale financiers receive are uniformly distributed on the interval $[\theta - \epsilon, \theta + \epsilon]$. We focus on the case where all wholesale financiers follow a threshold strategy: to run if they privately observe a signal $s_i > s^*$, and to wait if $s_i < s^*$, with s^* being the critical level of signal. Furthermore, we focus on a limiting case where ϵ approaches 0 so that s^* can be arbitrarily close to θ. In this case, wholesale financiers run if and only if a bank's cash flow is smaller than a critical $\theta^* = s^*$. It takes the following steps to derive the unique threshold.

First, for a given threshold, we derive the proportion of wholesale financiers who run as a function of the true fundamental. We denote the proportion of financiers who run by $L(\theta, s^*)$, because the decision to run depends both on the strength of fundamental (captured by the first element) and actions of the other players' (captured by the second element). For a realization of θ, we have three cases. (1) When $\theta + \epsilon < s^*$, even the highest possible private signal is below the threshold s^*. All investors will run, and $L(\theta, s^*) = 1$. (2) When $\theta - \epsilon > s^*$, even the lowest possible private signal is above the threshold s^*. Thus all players will wait, and $L(\theta, s^*) = 0$. (3) When θ falls into the intermediate range $[s^* - \epsilon, s^* + \epsilon]$, according to the threshold strategy, the fraction of wholesale financiers who will run can be written as follows.

$$L(\theta, s^*) = prob(s_i < s^* | \theta) = \frac{s^* - (\theta - \epsilon)}{2\epsilon} \tag{A.19}$$

Second, we derive financiers' posterior belief on $L(\theta, s^*)$. We start with a "marginal player" who observes $s_i = s^*$, and show that he will hold a posterior belief L following a uniform distribution on $[0, 1]$. To see this result, note that because each player observes only a noisy signal (probability distribution) about fundamental, θ in expression (A.19) is uncertain from the perspective of the players. Since the proportion of players who run is a function of θ, each

player forms a posterior belief (a conditional probability distribution) about the proportion based on his private signal. For the marginal player who observes $s_i = s^*$, we have[30]

$$Prob\left(L(\theta, s^*) \leq \hat{L}\Big|s_i = s^*\right) = Prob\left(\frac{s^* - (\theta - \epsilon)}{2\epsilon} \leq \hat{L}\Big|s_i = s^*\right) = Prob\left(\theta \geq s^* + \epsilon - 2\hat{L}\epsilon\Big|s_i = s^*\right)$$

On the other hand, we know that conditional on $s_i = s^*$, the marginal player has a posterior belief that fundamental θ is uniformly distributed on $[s^* - \epsilon, s^* + \epsilon]$. Therefore, the probability above can be written as

$$Prob\left(L(\theta, s^*) \leq \hat{L}\Big|s_i = s^*\right) = \frac{(s^* + \epsilon) - (s^* + \epsilon - 2\epsilon\hat{L})}{2\epsilon} = \hat{L}.$$

And we prove that the marginal player holds a posterior belief $L(\theta, s^*|s_i = s^*) \sim U(0, 1)$.

Now we move onto slightly more complicated cases where $s_i \neq s^*$. Without losing generality, we focus on a player who observes $s_i > s^*$. We show that such a player perceives a mixed distribution and believes $L(\theta, s^*|s_i > s^*)$ to be uniformly distributed on interval $\left[0, 1 - \frac{s_i - s^*}{2\epsilon}\right]$ with a positive probability mass at 0.

Note that an investor who receives a signal s_i holds a posterior belief that the fundamental follows a uniform distribution on $[s_i - \epsilon, s_i + \epsilon]$. For $s_i > s^*$, the upper bound of the support is greater than $s^* + \epsilon$. And we know that when $\theta > s^* + \epsilon$, all investors will wait and $L = 0$. In fact with $s_i > s^*$, we can divide the support of posterior belief about θ into two sections: $[s_i - \epsilon, s^* + \epsilon]$ and $[s^* + \epsilon, s_i + \epsilon]$, with the second section corresponding to a posterior belief $L(\theta, s^*|s_i) = 0$. Therefore, a player i who receives $s_i > s^*$ perceives a positive probability mass that $L = 0$. On the other hand, the posterior belief of θ continues to have a uniform distribution on $[s_i - \epsilon, s^* + \epsilon]$. Since this interval is covered by the intermediate range of $[s^* - \epsilon, s^* + \epsilon]$, $L(\theta, s^*)$ is given by expression (A.19), and we can derive the posterior belief on L as follows.

$$Prob\left(L(\theta, s^*) \leq \hat{L}\Big|s_i\right) = Prob\left(\frac{s^* - (\theta - \epsilon)}{2\epsilon} \leq \hat{L}\Big|s_i\right) = Prob(\theta \geq s^* + \epsilon + \hat{L}2\epsilon|s_i)$$

Because the player perceives a uniform distribution of θ on $[s_i - \epsilon, s^* + \epsilon]$, the probability above can be written as

$$\frac{(s^* + \epsilon) - (s^* + \epsilon + \hat{L}2\epsilon)}{(s^* + \epsilon) - (s_i - \epsilon)} = \frac{\hat{L}}{1 - (s_i - s^*)/2\epsilon},$$

and this is a uniform distribution on $[0, 1 - (s_i - s^*)/2\epsilon]$.

---

[30]Here $L(\theta, s^*)$ takes the form of equation (A.19) because $s^* \in [s^* - \epsilon, s^* + \epsilon]$.

Note that a player who observes $s_i > s^*$ holds a more optimistic belief that a smaller proportion of creditors will run (as reflected by the positive probability mass on $L = 1$). As the marginal creditor who observes $s_i = \theta^*$ is indifferent between running on the bank or not, the player who observes $s_i > \theta^*$ will strictly prefer to wait. The analysis of a player who observes $s_i < s^*$ follows exactly the same procedure. One can show that from the perspective of such a player, L again has a mixed distribution: It is uniformly distributed on $\left[\frac{s^*-s_i}{2\epsilon}, 1\right]$, and has with a positive probability mass at 1. Thus, a player who observes $s_i < s^*$ will be more pessimistic than the marginal player and strictly prefer to run on the bank.

Now we have established that financiers who observe $s_i < s^*$ will prefer to run, and those who observe $s_i > s^*$ will prefer to wait. We continue to derive explicitly the critical signal level s^*. By the definition of switching equilibrium, the marginal player who observes $s_i = s^*$ should be indifferent between running on the bank or not. And s^* is given by the indifference condition. Let $L^C(\theta)$ be the critical fraction of early withdrawals that makes a bank of cash flow θ fail, we have

$$L^C(\theta) = \frac{\theta - F - D}{[(1 + \lambda)q - 1]D}.$$

As a critical player holds a belief $L \sim U(0, 1)$, in expectation, he receives

$$V_{wait} \equiv \int_0^{L^C(\theta)} DdL + \int_{L^C(\theta)}^1 0dL \quad \text{by waiting, and}$$

$$V_{run} \equiv \int_0^{L^C(\theta)} qDdL + \int_{L^C(\theta)}^1 qDdL \quad \text{by running on the bank.}$$

His indifference condition therefore can be written as

$$\Delta \equiv V_{wait} - V_{run} = \int_0^{L^C(\theta)} (1 - q)DdL - \int_{L^C(\theta)}^1 qDdL = (1 - q)DL^C(\theta) - qD(1 - L^C(\theta)) = 0,$$

which further reduces to $q = L^C(\theta)$. We can then solve for a critical θ^*.

$$\theta^* = F + D + q[(1 + \lambda)q - 1]D$$

Graphically, with the payoff difference between 'wait' and 'run' denoted by Δ, we can draw Δ as a step function of L. Since a marginal creditor holds a belief that $L \sim U(0, 1)$, the critical θ^* is determined such that the area below $\Delta = 0$ is equal to that above $\Delta = 0$, as illustrated by Figure 4.

34

Figure 4: Payoff differences and the decision to run
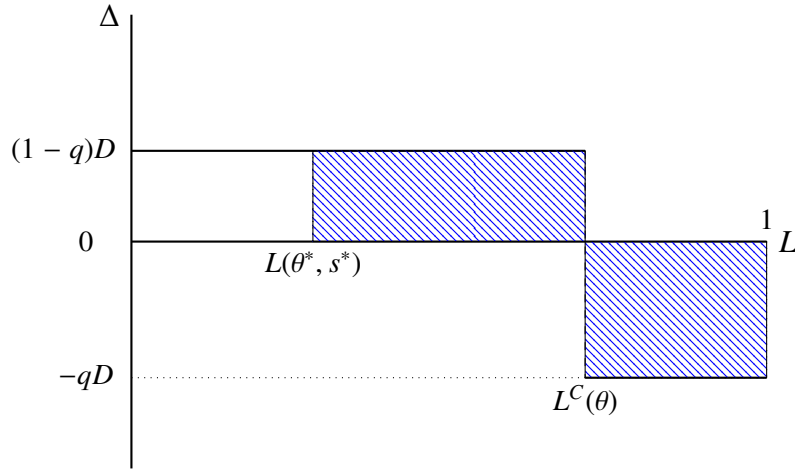
Defining $\mu \equiv 1 + q[(1 + \lambda)q - 1]$, one can show $\mu > 1$ because $(1 + \lambda)q > 1$. With $\theta^* = F + \mu D > F + D$, a bank whose cash flow is greater than its liability $F + D$ will fail because of illiquidity, when its cash flow falls below the critical θ^*.

Appendix B   Proof of propositions

Appendix B.1   Proof of lemma 1

To derive the uniform distribution of loan loss γ, take a risky type $\tilde{b} < 1/(x - r)$; and define the fraction of entrepreneurs below $\tilde{b}$ in the risky pool by $\tilde{\gamma}$. We have

$$\tilde{\gamma} = \frac{\tilde{b} - 0}{1/(x - r) - 0} = \tilde{b}(x - r). \tag{B.20}$$

Consider the critical realization $\tilde{z} = \Phi^{-1}(1 - P^*_{\tilde{b}})$ such that an entrepreneur of type $\tilde{b}$ will not default but all types $b < \tilde{b}$ will. So for $z = \tilde{z}$, one will have $\gamma = \tilde{\gamma}$. To derive the distribution of γ, notice that

$$
\begin{aligned}
F(\tilde{\gamma}) &\equiv Prob(\gamma < \tilde{\gamma}) = Prob(z > \tilde{z}) = 1 - Prob(z < \tilde{z}) \\
&= 1 - \Phi(\Phi^{-1}(1 - P^*_{\tilde{b}})) = P^*_{\tilde{b}} \\
&= \tilde{b}(x - r).
\end{aligned}
$$

By equation (B.20), we have $\tilde{b} = \tilde{\gamma}/(x - r)$. Substitution yields

$$F(\tilde{\gamma}) = \tilde{\gamma},$$

implying $\gamma \sim U(0, 1)$.

Appendix B.2   Proof of proposition 1

On the comparative statics of insolvency risk, computation is simplified if we consider its complementary probability, $1 - \rho_{SR} = [r - (F + D)]/(1 - \alpha)r$. Examining its first order derivative with respect to r, we obtain:

$$
\begin{aligned}
\frac{\partial(1 - \rho_{SR})}{\partial r} &= \frac{1}{(1 - \alpha)^2 r^2}\left[(1 - \alpha)r - \frac{\partial[(1 - \alpha)r]}{\partial r}[r - (F + D)]\right] \\
&= \frac{1}{(1 - \alpha)^2 r^2}\left[(1 - \alpha)r - [(1 - \alpha) - \frac{\partial \alpha}{\partial r}r][r - (F + D)]\right] \\
&= \frac{1}{(1 - \alpha)^2 r^2}\left[\frac{\partial \alpha}{\partial r}r^2 + [(1 - \alpha) - \frac{\partial \alpha}{\partial r}r](F + D)\right].
\end{aligned}
$$

Recall that $\partial \alpha/\partial r = -1/B(x - r)^2$ and $(1 - \alpha) = 1/B(x - r)$. Taking out the common factor, we will have

$$
\begin{aligned}
\frac{\partial(1 - \rho_{SR})}{\partial r} &= \frac{1}{(1 - \alpha)^2 r^2}\frac{\partial \alpha}{\partial r}\left[r^2 + [-(x - r) - r](F + D)\right] \\
&= \frac{1}{(1 - \alpha)^2 r^2}\frac{\partial \alpha}{\partial r}[r^2 - x(F + D)]
\end{aligned}
$$

Therefore,
$$\frac{\partial \rho_{SR}}{\partial r} = \frac{-1}{(1 - \alpha)^2 r^2}\frac{\partial \alpha}{\partial r}[r^2 - x(F + D)].$$

Pure insolvency risk is reduced by competition if and only if

$$r^2 > x(F + D). \tag{B.21}$$

Appendix B.3   Proof of proposition 4

It is especially convenient to work with the loan loss $\gamma \sim U(0, 1)$. To facilitate exposition, we denote
$$\hat{\gamma}_\mu \equiv \frac{r - (F + \mu D)}{(1 - \alpha)r}.$$

36

$\hat{\gamma}_\mu$ is a counterpart of $\hat{\gamma}$: It defines a critical loan loss the bank will survive once liquidity risk is taken into account. The general optimization program then takes the following specific form.

$$\max_{\omega,D} \left\{ (1 - \omega) \int_0^{\hat{\gamma}_\mu} \left[\theta - F - D \right] d\gamma \right\}$$

$$s.t. \quad V_E = \frac{\omega}{1 + k} \int_0^{\hat{\gamma}_\mu} \left[\theta - D - F \right] d\gamma$$

$$V_D = \int_0^{\hat{\gamma}_\mu} D d\gamma$$

$$V_F = F$$

$$V_E + V_D + V_F = 1$$

After substituting the constraints, the program simplifies to:

$$\max_D \left\{ \int_0^{\hat{\gamma}_\mu} \left[\theta + k(D + F) \right] d\gamma + (1 + k) \int_{\hat{\gamma}_\mu}^1 F d\gamma - (1 + k) \right\} \tag{B.22}$$

The maximization program has the following first order condition[31]

$$\left[r \frac{\partial \hat{\gamma}_\mu}{\partial D} - \frac{(1 - \alpha)r}{2} 2\hat{\gamma}_\mu \frac{\partial \hat{\gamma}_\mu}{\partial D} \right] + k\hat{\gamma}_\mu + k(F + D^*) \frac{\partial \hat{\gamma}_\mu}{\partial D} - (1 + k)F \frac{\partial \hat{\gamma}_\mu}{\partial D} = 0$$

and yields the optimal level of risky debt

$$D^* = \frac{r - F}{\mu^2/k + 2\mu}. \tag{B.23}$$

Appendix B.4   Proof of corollary 1

Denote $c \equiv 1/(\mu^2/k + 2\mu)$. The leverage ratio l can be rewritten as $l = \frac{2c(r-F)}{(1+\alpha)r-2F}$. The comparative statics follow directly from the first order condition.

$$\begin{aligned}
\frac{\partial l}{\partial r} &= \frac{2c}{[(1 + \alpha)r - 2F]^2} \left[[(1 + \alpha)r - 2F] - (r - F)[\frac{\partial \alpha}{\partial r}r + (1 + \alpha)] \right] \\
&= \frac{2c}{[(1 + \alpha)r - 2F]^2} \left[-(1 - \alpha)F - \frac{\partial \alpha}{\partial r}r(r - F) \right] \\
&= \frac{2c(1 - \alpha)}{[(1 + \alpha)r - 2F]^2} \frac{r^2 - xF}{x - r}
\end{aligned}$$

---

[31]It is straightforward to check that the second order condition is satisfied, $-(1 - \alpha)r\left(\frac{\partial \hat{\gamma}_\mu}{\partial D}\right)^2 + 2k\frac{\partial \hat{\gamma}_\mu}{\partial D} < 0$.

The comparative statics depends on the sign of

$$r^2 - xF. \tag{B.24}$$

When $r^2 > xF$, $\partial l/\partial r > 0$, banking competition leads to a lower leverage ratio.

Appendix B.5   Proof of proposition 5

With $c \equiv 1/(\mu^2/k + 2\mu)$. The pure insolvency risk, illiquidity risk, and total credit risk can be written as the following, with again total credit risk being the summation of pure insolvency risk and illiquidity risk. The comparative statics with respect to r follow from the definitions.

$$
\begin{aligned}
\rho_{SR}^* &\equiv 1 - \frac{r - F - D^*}{(1-\alpha)r} &&= 1 - \frac{(1-c)(r-F)}{(1-\alpha)r} \\
\rho_{IL}^* &\equiv (\mu - 1)\frac{D^*}{(1-\alpha)r} &&= \frac{c(\mu-1)(r-F)}{(1-\alpha)r} \\
\rho_{TCR}^* &\equiv 1 - \frac{r - F - \mu D^*}{(1-\alpha)r} &&= 1 - \frac{(1-\mu c)(r-F)}{(1-\alpha)r}
\end{aligned}
$$

(1) Comparative statics: Insolvency risk

$$\frac{\partial \rho_{SR}^*}{\partial r} = -(1-c)\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right)$$

We have shown $c < 1$. So the expression shares the same sign as

$$\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right). \tag{B.25}$$

(2) Comparative statics: Liquidity risk

$$\frac{\partial \rho_{IL}^*}{\partial r} = (\mu - 1)c\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right),$$

With $\mu > 1$, the sign will be opposite to that of expression (B.25).

(3) Comparative statics: Total credit risk

$$\frac{\partial \rho_{TCR}^*}{\partial r} = -(1-\mu c)\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right)$$

Note that $\mu c = 1/(\mu/k + 2) < 1$. The comparative statics of total credit risk is again determined by the sign of expression (B.25).

Therefore when competitive environment changes, pure insolvency risk moves in the opposite direction as pure liquidity risk. With the latter dominating, total credit risk changes in the same direction as that of pure insolvency. Now we characterize the condition that

$$
\begin{aligned}
\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right) &= \frac{1}{(1-\alpha)^2 r^2}\left[(1-\alpha)r - [-\frac{\partial\alpha}{\partial r}r + (1-\alpha)](r-F)\right] \\
&= \frac{1}{(1-\alpha)^2 r^2}\left[\frac{\partial\alpha}{\partial r}r^2 + [-\frac{\partial\alpha}{\partial r}r + (1-\alpha)]F\right] \\
&= \frac{1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}[r^2 - rF - (x-r)F] \\
&= \frac{1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}(r^2 - xF).
\end{aligned}
$$

With $\partial\alpha/\partial r < 0$, competition increases insolvency risk, decreases liquidity risk, and increases total credit risk if and only if

$$r^2 > xF. \tag{B.26}$$

Appendix B.6   Proof of proposition 6

A systemic crisis takes place if both banks' cash flow fall below θ^{**}, i.e., $\rho_{SYS} = Prob(\theta < \theta^{**})^2$. This allows us to obtain:

$$
\begin{aligned}
\frac{\partial\rho_{SYS}}{\partial r} &= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}Prob(\theta < \theta^{**}) \\
&= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}\left(1 - \frac{r-\theta^{**}}{(1-\alpha)r}\right) \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\left[(1-\alpha)r - [(1-\alpha) - \frac{\partial\alpha}{\partial r}r](r-\theta^{**})\right] \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}[r^2 - x\theta^{**}] \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}[r^2 - x(F + \mu'D)].
\end{aligned}
$$

As $\partial\alpha/\partial r < 0$, the sign of the comparative statics is determined by $r^2 - x(F + \mu'D)$: the risk of a systemic crisis decreases with competition if and only if $r^2 > x(F + \mu'D)$.

Appendix B.7   Proof of proposition 7

The proof resembles that of proposition 5, with comparative statics again determined by the sign of the following expression.

$$\frac{\partial}{\partial r}\Big(\frac{r - F}{(1 - \alpha)r}\Big)$$

Note that for the exposure to contagion, we have

$$\frac{\partial \rho^*_{CTG}}{\partial r} = (\mu' - \mu)\frac{\partial}{\partial r}\Big(\frac{D^*}{(1 - \alpha)r}\Big) = \frac{\mu' - \mu}{\mu^2/k + 2\mu}\frac{\partial}{\partial r}\Big(\frac{r - F}{(1 - \alpha)r}\Big),$$

and for the risk of a systemic crisis, we have

$$
\begin{aligned}
\frac{\partial \rho^*_{SYS}}{\partial r} &= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}Prob(\theta < \theta^{**}) \\
&= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}(1 - \frac{r - F - \mu'^*}{(1 - \alpha)r}) \\
&= -2Prob(\theta < \theta^{**})\Big(1 - \frac{\mu'}{\mu^2/k + 2\mu}\Big)\frac{\partial}{\partial r}\Big(\frac{r - F}{(1 - \alpha)r}\Big).
\end{aligned}
$$

Therefore, when $r^2 > xF$, loan competition leads to greater exposure to contagion but a smaller chance of systemic crisis, provided $\mu^2/k + 2\mu > \mu'$.

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