On the Accumulation of Wealth under Aspirations

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Abstract

We analyze the effects of the introduction of aspirations both on the pattern of wealth accumulation along the life cycle of individuals displaying a "joy-of-giving" motive for bequests and on the evolution of wealth within a dynasty. We will show that the introduction of aspirations at different ages display different effects on the amount of saving of workers. However, both adult and old aspirations dampen the positive effect on wealth accumulation brought about by warm-glow altruism. Therefore, under aspirations, both bequest motivated and non-bequest motivated individuals will behave more similarly than when they do not exhibit aspirational concerns. We also show that the introduction of aspirations raises the speed of convergence to the dynastic steady state when there exist a bequest motive. However, when this motive is absent, aspirations lower the speed of convergence.

JEL classification codes: E21, D91.
Keywords: Aspirations, Saving, Bequests.

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1. Introduction

In this paper, we analyze the effects of the introduction of aspirations both on the pattern of wealth accumulation along the life cycle of individuals displaying a "joy-of-giving" motive for bequests and on the evolution of wealth within a dynasty. The presence of aspirations implies that the utility of individuals depends on the consumption experience of their parents. We will consider a general framework where adult individuals compare their level of consumption with that of their parents when they were also adult and, similarly, old individuals take into account the consumption of their parents when they were also old.

There is a strand of the literature that provides evidence in favour of time non-separable preferences and, in particular, of habit and aspiration formation. On the one hand, a large number of empirical studies about habit formation show that individuals' past consumption decisions affect the satisfaction derived from their current consumption (de la Croix and Urbain, 1998; Carrasco et al., 2005; Fuhrer and Klein, 2006; Alessie and Teppa, 2010). On the other hand, empirical evidence about the existence of aspirations associated with the involuntary transmission of tastes from one generation to the next is more scarce. For instance, Cox et al. (2004) estimate that parental preferences explain between 5 to 10 percent of the preferences of their children after controlling for their respective incomes. More recently, Charles et al. (2014) in the same line find that the intergenerational correlation in consumption ranges from 7 to 9 percent. Moreover, Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) provide surveys about the evidence on intergenerational transmission of tastes. Among the theoretical studies on the macroeconomic implications of aspirations, we could mention those of de la Croix (1996, 2001), de la Croix and Michel (1999, 2001), Alonso-Carrera et al. (2007), and Caballé and Moro-Egido (2014).

Our analysis will be conducted in the framework of an overlapping generations (OLG) economy where preferences of individuals display "joy of giving" or "warm-glow" altruism. This means that individuals' utility will be an increasing function of the amount of bequest left to their children, like in Yaari (1965) and Abel (1986). Several alternative motives leading to intergenerational transfers have been proposed in the literature. Among them, and in addition to joy of giving, we could mention strategic behavior (Bernheim et al., 1985), existence of incomplete annuity markets (Abel, 1985), and pure intergenerational altruism (Barro, 1974). However, the empirical evidence is not conclusive about the reasons why individuals make intergenerational transfers and probably the mechanism of intergenerational transmission of wealth is driven by a combination of all those motives. Moreover, Hurd (1987) showed through a simple model that the amount of wealth accumulation in all ages should be higher for altruistic individuals since these individuals wish to leave a bequest to their heirs and this desire provides them an additional motivation for saving. However, in the same paper, Hurd conducted an empirical analysis showing that the patterns of saving and consumption of altruistic individuals do not differ substantially from those who are not altruistic.[1] This empirical finding is thus at odds with the aforementioned theoretical implication of his model.

[1] In order to detect non-altruistic individuals, Hurd classifies as non-altruistic those individuals having no living children.

In this paper we find that aspirations affect the equilibrium levels of both bequest and saving in a direction similar to the one obtained by de la Croix and Michel (2001), Jellal and Wolff (2002) and Alonso-Carrera et al. (2007), who conducted the analysis under the assumption of altruistic preferences à la Barro (1974). Moreover, these authors focus their analysis on the effect of aspirations on the operativeness of the bequest motive and on the implications of the interaction between altruism and aspirations to induce self-restraint in parental consumption. We will focus instead on the implications for the pattern of wealth accumulation along the life cycle brought about by aspirations.

We will show analytically that, when a bequest motive is introduced in our otherwise standard OLG economy, the implications for accumulation of wealth are attenuated by the presence of aspirations. Therefore, the presence of aspirations makes the profile of consumption and saving of bequest motivated and non-bequest motivated individuals more similar than when no aspirational concerns are taken into account, which is consistent with the aforementioned empirical evidence obtained by Hurd.

When parents wish to leave a state to their children they raise the amount of saving in all ages of their lives. We will see that this increase in the accumulation of wealth is dampened if individuals form aspirations either when they are adult or when they are old. This dampening effect occurs even if the introduction of aspirations at different ages has not the same effect on wealth accumulation. In particular, the presence of adult aspirations results in a smaller amount of saving for workers as they want to mimic the amount of consumption of their parent when they were also workers. However, the introduction of old aspirations raises the amount of saving of workers as they want to shift consumption to the age were they are retired to imitate the standard of living of their parents when they were also old.

Another interesting result of our analysis is that the introduction of aspirations raises the speed of convergence to the dynastic steady state if individuals entertain a bequest motive. This is so because aspirations introduce a sluggish response in bequests, which constitute the main factor giving rise to history dependence within a dynasty. Since individuals want to mimic the level of consumption of their parents, bequests cannot adjust with the same degree of freedom as when aspirations were absent. However, if there is no bequest motive, the speed of convergence decreases when aspirations are introduced since now the profile of consumption becomes more history dependent due to the aspirational concerns. Finally, we show that, if the intensity of aspirations is sufficiently high (but no too high) and the bequest motive is present, the amounts of consumption and bequest display cycles around their steady state.

In our analysis we also perform some steady-state comparative statics concerning the interaction of aspirations with warm-glow altruism for families that have reached their stationary level of consumption, saving, and bequest. We check mainly trough a numerical analysis that the dampening effect of aspirations also hold when we compare two stationary equilibria with different configurations of parameter values.

The paper is organized as follows. Section 2 presents the general model with warm-glow altruism and aspirations. Section 3 analyzes the effect of bequest motive and aspirations on saving and inheritance. In Section 4, we analyze the effect of aspirations on the speed of convergence. In Section 5 and 6, we conduct the comparative statics analysis to characterize the effects of changes in the intensities of aspirations and bequest motives and the result arising from the interaction of these two phenomena for a given individual with exogenously given initial values of both aspirations and received inheritance. In Section 7 we make the numerical comparison between stationary equilibria. We conclude the paper in Section 8. The proofs appear in the appendix.

2. The Model

Let us consider a small open OLG economy, where individuals live for three periods and a new generation is born in each period. Each individual has offspring in the second period of his life and the exogenous number of children per parent is $n > 0$. We assume that agents make economic decisions only during the last two periods of their lives. During his first period of life an individual neither works nor consumes but just observes the amount of consumption of his parent. Each agent inelastically supplies one time unit of labor in the second period of his life and is retired in the third period. We index each generation by the period in which its members work, i.e., when they are adult.

There is a single commodity, which can be devoted to either consumption or saving. An adult individual of generation t distributes his net labor income and inheritance between consumption and saving. The budget constraint faced by this worker in period t is

$$w_t + b_t = c_t + s_t, \tag{2.1}$$

where w_t is the wage compensation received by this worker, c_t is his amount of consumption (hereinafter, adult consumption), b_t is the amount of inheritance he has received from his parent and s_t is the amount of saving.

When individuals are old, they receive a return on their savings, which is distributed between own consumption and bequests for their children. Therefore, the budget constraint of an old individual belonging to generation t will be

$$R_{t+1}s_t = x_{t+1} + nb_{t+1}, \tag{2.2}$$

where R_{t+1} is the gross rate of return on saving, b_{t+1} is the amount of bequest the individual leaves to each of his descendants (who where born in period t) and x_{t+1} is the amount of consumption of an old individual in period $t+1$ (hereinafter, old consumption). Thus, we are implicitly making an equal-treatment assumption as all the direct descendants of the same individual receive the same amount of inheritance.

We will assume that in each period individuals derive utility from the comparison of their own consumption with a consumption reference. As in de la Croix (1996), a generic member of the generation born in period $t-1$ inherits a certain level of aspirations a_t in period t. These aspirations are based on the standard of living achieved by his parent. More precisely, we assume that the inherited aspiration a_t^c of an adult individual of generation t is

$$a_t^c = c_{t-1}, \tag{2.3}$$

where c_{t-1} is his parent's amount of consumption when the parent was adult (second period of life). We posit the following additive specification for the aspiration adjusted consumption $\hat{c}_t$ of an adult individual belonging to generation t:

$$\hat{c}_t = c_t - \delta_c a_t^c, \quad \text{with } \delta_c \in [0, 1), \tag{2.4}$$

where δ_c is a parameter that represents the intensity of aspirations when the individual is adult. Thus, adult individuals who have acquired higher aspirations due to their parents' experience of consumption will require a larger amount of consumption in order to achieve the same level of utility. These aspirations arising when an individual is adult/worker will be dubbed adult aspirations.

Similarly, old individuals derive utility from the comparison between their own consumption when old and the consumption of their parents' old consumption. Therefore, the aspiration adjusted consumption $\hat{x}_{t+1}$ of an old individual in period $t+1$ is given by the following additive function:

$$\hat{x}_{t+1} = x_{t+1} - \delta_x a_{t+1}^x, \quad \text{with } \delta_x \in [0, 1) \,. \tag{2.5}$$

where a_{t+1}^x is the inherited aspiration of an old individual of generation t. This aspiration satisfies

$$a_{t+1}^x = x_t, \tag{2.6}$$

where x_t is his parent's amount of consumption when the parent was old (third period of life) and the value of the parameter δ_x measures the intensity of aspirations when an individual is old. The aspirations occurring when individuals are old/retired will be dubbed old aspirations.

The individual belonging to generation t derives utility from aspiration adjusted adult consumption, aspiration adjusted old consumption, and the amount left for inheritance. We posit the following separable utility function representing the preferences of an individual belonging to generation t:

$$U(\hat{c}_t, \hat{x}_{t+1}, b_{t+1}) = u_c(\hat{c}_t) + \beta u_x(\hat{x}_{t+1}) + \rho v(b_{t+1}), \tag{2.7}$$

where both β and ρ are positive and the functions u_c, u_x, and v are twice-differentiable, strictly increasing, strictly concave, and satisfy the typical Inada conditions at zero and at infinity. Note that we are generating positive bequests through a "joy-of-giving" motivation (as in Yaari, 1965; or Abel, 1986) so that the amount of bequests enters directly as an argument in the utility function. There are other motives for intergenerational transfer, such as altruistic preferences à la Barro (1974) and Becker (1981) where individuals derive utility from their children's indirect utility function, or through paternalistic preferences where individuals care about their offspring's level of consumption (Pollak, 1988). Under altruistic preferences, the last term in the utility (2.7) would be replaced by the indirect utility function of one's children, which is an increasing function of the amount of inheritance received by descendants. If preferences were paternalistic, the last term in the utility (2.7) would be replaced by the offspring's adult consumption, which in turn would be an increasing function of the amount b_{t+1} of inheritance. In both cases, the results would be qualitatively similar to those obtained under a joy-of-giving specification. However, a problem posed by these two alternative types of preferences is the potential existence of corner solutions when the bequest motive is not operative, i.e. when the amount of bequest in equilibrium is equal to zero. We will avoid this problem by assuming joy-of-giving preferences displaying an Inada condition when the amount b_{t+1} of bequest tends to zero.

Let us assume that the good of this economy is produced by means of a production function displaying constant returns to scale in capital and labor. In our small open economy, capital is fully mobile and labor is not mobile. Under competitive input markets this implies that the rental price of a unit of capital is constant and equal to its international level r. Therefore, the gross rate of return on savings satisfies $R_{t+1} = 1 + r \equiv R$ for all t. We will assume throughout the paper that the interest rate r is strictly positive, i.e., $R > 1$. Moreover, the equilibrium capital to labor ratio becomes constant and, thus, the marginal productivity of a unit of labor (which is equal to the competitive real wage per unit of labor) is also constant, $w_t = w$ for all t.

3. Aspirations, saving, and bequests

Individuals maximize (2.7) with respect to $\{c_t, x_{t+1}, b_{t+1}, s_t\}$ subject to (2.1), (2.2), (2.4) and (2.5), taking as given a_t^c, a_{t+1}^x, b_t, w_t and R_{t+1}. If we plug the aspiration formation equations (2.3) and (2.6), and the competitive rental prices of inputs $w_t = w$ and $R_{t+1} = R$ into the solution of this individual problem, we obtain the following first order conditions for the individual belonging to generation t:

$$u_c'(\hat{c}_t) = \beta R_{t+1} u_x'(\hat{x}_{t+1}) \tag{3.1}$$

and

$$n\beta u_x'(\hat{x}_{t+1}) = \rho v'(b_{t+1}), \tag{3.2}$$

where equation (3.1) give us the optimal allocation of consumption along the life cycle and equation (3.2) give us the optimal allocation of resources of an old individual between his own old consumption and the amount of bequest left to each of his direct descendants. Taking into account that in equilibrium

$$\hat{c}_t = w + b_t - s_t - \delta_c c_{t-1} \tag{3.3}$$

and

$$\hat{x}_{t+1} = Rs_t - nb_{t+1} - \delta_x x_t, \tag{3.4}$$

we can compute the effect on the values s_t and b_{t+1} of saving and bequest of changes in the bequest intensity ρ and in the aspirations intensities δ_c and δ_x when adult and old, respectively. Making use of (3.3) and (3.4) and implicitly differentiating the system of equations (3.1) and (3.2) with respect to the parameters δ_c , δ_x, and ρ we get the following partial derivatives, where we have suppressed the arguments of the functions to ease the notation:

$$\frac{ds_t}{d\rho} = -\frac{n\beta R u_x'' v'}{M} > 0, \tag{3.5}$$

$$\frac{ds_t}{d\delta_c} = -\frac{u_c''\left(n^2\beta u_x'' + \rho v''\right)c_{t-1}}{M} < 0, \tag{3.6}$$

$$\frac{ds_t}{d\delta_x} = \frac{\beta\rho R u_x'' v'' x_t}{M} > 0, \tag{3.7}$$

$$\frac{db_{t+1}}{d\rho} = -\frac{\left(u_c'' + \beta R^2 u_x''\right) v'}{M} > 0, \tag{3.8}$$

$$\frac{db_{t+1}}{d\delta_c} = -\frac{n\beta R u_c'' u_x'' c_{t-1}}{M} < 0,, \tag{3.9}$$

$$\frac{db_{t+1}}{d\delta_x} = -\frac{n\beta u_c'' u_x'' x_t}{M} < 0 \tag{3.10}$$

where $M = n^2\beta u_c'' u_x'' + \rho u_c'' v'' + \beta\rho R^2 u_x'' v'' > 0$.

Clearly, an increase in the intensity of warm-glow altruism, parameterized by the value of ρ, raises the amounts of both saving and bequest as a result of the shift of resources from adult consumption to the next generation. This effect of altruism is similar to the one obtained by Hurd (1987) and Bossmann et al. (2007) under pure altruism and joy-of giving, respectively. Moreover, when the intensity δ_c of adult aspirations increases, the utility associated with adult consumption diminishes while its marginal utility rises. Therefore, the optimal reaction of the individual is to increase his adult consumption c_t and reduce the values of the other arguments of his utility function, namely, old consumption x_{t+1} and bequests b_{t+1}. Finally , the shift from old to adult consumption results in a lower amount of saving. However, when the intensity δ_x of old aspirations rises, the utility associated with old consumption diminishes while its marginal utility rises so that individuals optimally react by augmenting their old consumption, which implies in turn an increase in saving and a decrease in the amount of bequest.

From now on, for simplicity we will assume that the function $U(\hat{c}_t, \hat{x}_{t+1}, b_{t+1})$ is not only additive separable but homothetic as in Abel (1986). Then, according to Katzner (1970, Theorem 2.4-4), the utility functions u_c, u_x and v must be isoelastic, i.e.,

$$u_c(z) = u_x(z) = v(z) = \begin{cases} \dfrac{z^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln z & \text{if } \sigma = 1, \end{cases}$$

with $\sigma > 0$. Under this parametric assumption we can obtain the explicit equilibrium values of adult and old consumption, saving and bequest:

$$c_t = \frac{1}{H}\left\{R\left(w + b_t\right) + \left[(R\beta)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c c_{t-1} - \delta_x x_t\right\}, \tag{3.11}$$

$$x_{t+1} = \frac{1}{H}\left\{R\left(R\beta\right)^{\frac{1}{\sigma}}\left(w + b_t\right) - R\left(R\beta\right)^{\frac{1}{\sigma}}\delta_c c_{t-1} + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x x_t\right\}, \tag{3.12}$$

$$b_{t+1} = \frac{1}{H}\left\{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(w + b_t\right) - R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_c c_{t-1} - \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_x x_t\right\}, \tag{3.13}$$

$$s_t = \frac{1}{H}\left\{\left[(R\beta)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\left(w + b_t\right) - \left[(R\beta)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c c_{t-1} + \delta_x x_t\right\},$$

where

$$H = R + (R\beta)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}. \tag{3.14}$$

Note that the linearity of the previous functions with respect to the state variables b_t, c_{t-1}, and x_t faced by an individual belonging to generation t is a direct consequence of the homotheticity of the utility function U and the assumed linearity of aspiration formation given in (2.4) and (2.5).

4. Transitional Dynamics

The evolution of consumption, saving and intergenerational transfers of the dynasty under consideration is entirely governed by the system of difference equations composed of (3.11), (3.12), and (3.13). The steady-state (or stationary) values of adult consumption, old consumption, and bequest can be found easily by just making $c_t = c_{t-1} = c$, $x_{t+1} = x_t = x$, and $b_{t+1} = b_t = b$ in that dynamic system and then solving for the steady-state value of adult consumption c, old consumption x, and bequest b. Those steady-state values are the following:

$$c = \frac{R\left(1-\delta_x\right)w}{J} \tag{4.1}$$

$$x = \frac{\left(1-\delta_c\right)R\left(R\beta\right)^{\frac{1}{\sigma}}w}{J} \tag{4.2}$$

$$b = \frac{R\left(R\beta\right)^{\frac{1}{\sigma}}\left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}w}{J}, \tag{4.3}$$

where

$$J = R\left(1-\delta_x\right) + \left(\beta R\right)^{\frac{1}{\sigma}}\left(1-\delta_c\right) + \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(n-R\right)\left(1-\delta_c\right)\left(1-\delta_x\right). \tag{4.4}$$

The following lemma provides a sufficient condition for the monotonic stability of the steady-state:

Lemma 4.1. *If*

$$\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{R + \left(\beta R\right)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} < 1 \tag{4.5}$$

and the aspirations intensities δ_c and δ_x are sufficiently small, then the dynamic system formed by equations (3.11), (3.12), and (3.13) converges monotonically to the steady state for adult consumption, old consumption and bequest given by (4.1), (4.2) and (4.3), respectively.

Note that under the assumption of the previous lemma the steady-state values c, x, and b are all strictly positive. To see this just observe that the numerators of (4.1), (4.2) and (4.3) are all strictly positive for sufficiently small values of δ_c and δ_x, whereas the denominator J defined in (4.4) tends to

$$R + (\beta R)^{\frac{1}{\sigma}} + \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} (n - R)$$

when δ_c and δ_x tend to zero. The previous expression is positive under the condition (4.5). If the utility functions u_c, u_x and v are logarithmic ($\sigma = 1$) this stability condition becomes simply

$$\frac{\rho R}{n\,(1 + \beta + \rho)} < 1.$$

In this case, the stability condition has a direct interpretation since the return to capital R and the bequest motive ρ should not be very high in order to prevent the dynasty from accumulating wealth per capita unboundedly. Obviously, a high rate of population growth will also eliminate the possibility of excessive accumulation of wealth as the initial wealth of individuals will be small when the family state has to be divided among many children.

Another natural issue related with the transition towards the steady state of the endogenous variables of the model refers to the speed at which the steady-state values are approached. We will consider the speed of convergence around the steady state. If we write the system formed by the linear difference equations (3.11), (3.12), and (3.13) in vector form (see the proof of Lemma 4.1 in the appendix), we see that the coefficient matrix $\mathbb{P}$ defined in (A.1) has three-eigenvalues, λ_1, λ_2, and λ_3. Therefore, the solution of the linear dynamic system will have the form

$$z_t = A_{z,1}\lambda_1^t + A_{z,2}\lambda_2^t + A_{z\,,3}\lambda_3^t + z, \quad t = 0, 1, ...$$

for $z_t = c_t, x_{t+1}, b_{t+1}$, and where z is the stationary value of z_t and the transpose vector $(A_{c,j}, A_{x,j}, A_{b,j})'$, for $j = 1, 2, 3$, is equal to $\kappa_j{\cdot}(m_{c,j}, m_{x,j}, m_{b,j})'$, where $(m_{c,j}, m_{x,j}, m_{b,j})'$ is an eigenvector associated with the eigenvalue λ_j of the matrix $\mathbb{P}$. The constants κ_j, $j = 1, 2, 3$, are pinned down by the initial values of c_{-1}, x_0 and b_0. Then, the speed of convergence of the economy could be measured by the fraction of the distance between the value of the generic variable z_t in period t and the stationary value z that the system travels in one period,

$$\frac{z_{t+1} - z_t}{z - z_t}.$$

Let us assume that $\lambda_{\max}$ is the largest eigenvalue. It is straightforward to see that

$$\lim_{t\to\infty} \frac{z_{t+1} - z_t}{z - z_t} = 1 - \lambda_{\max},$$

so that the value of the largest eigenvalue of the matrix $\mathbb{P}$ is inversely related with the speed of convergence around the steady-state.

The following Proposition characterizes the effect of the introduction of aspiration on the speed of convergence.

Proposition 4.2. *(a) If there exists a bequest motive ($\rho > 0$) then the speed of convergence around the steady-state increases when aspirations at any age are introduced;*

(b) If there is no bequest motive ($\rho = 0$) then the speed of convergence around the steady-state decreases when aspirations at any age are introduced.

In order to gain some intuition about the previous proposition, let us consider an economy where the agents' preferences exhibit a bequest motive but no aspirations ($\rho > 0, \quad \delta_c = 0, \quad \delta_x = 0$). In this case, the three eigenvalues of the dynamic system formed by the difference equations (3.11), (3.12), and (3.13) are

$$\lambda_1 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} \in (0,1), \;\; \lambda_2 = 0, \;\; \lambda_3 = 0.$$

Thus, the largest eigenvalue λ_1 determines the speed of convergence around the steady state and this eigenvalue remains the largest when aspirations are marginally introduced. As can be seen in the proof of Proposition 4.2 in the appendix, this eigenvalue decreases when aspirations are introduced (see (A.3) and (A.4)). Therefore, when there exists a bequest motive, the introduction of either adult or old aspirations results in faster local convergence.

If we consider instead an economy with no bequest motive where agents only display either adult aspirations ($\rho = 0, \quad \delta_c > 0, \quad \delta_x = 0$) or old aspirations ($\rho = 0, \; \delta_c = 0, \; \delta_x > 0$), the three eigenvalues of the dynamic system become

$$\lambda_1 = 0, \;\; \lambda_2 = \frac{(R\beta)^{\frac{1}{\sigma}}\,\delta_c}{R + (R\beta)^{\frac{1}{\sigma}}} \in (0,1), \;\; \lambda_3 = 0,$$

and

$$\lambda_1 = 0, \;\; \lambda_2 = 0, \;\; \lambda_3 = \frac{R\delta_x}{R + (R\beta)^{\frac{1}{\sigma}}} \in (0,1),$$

respectively. We clearly see that when, there is no bequest motive and there are adult (resp. old) aspirations only, the largest eigenvalue is λ_2 (resp. λ_3), which is increasing in the intensity δ_c (resp. δ_x) of aspirations. Therefore, the speed of convergency decreases with the aspiration intensities when bequests are absent.

When both bequests and aspirations are removed from our economy, convergence to the steady state is achieved instantaneously given our assumption of fixed wages and interest rates. The introduction of either a bequest motive or an aspirational concern, introduce some inertia in the dynamic system so that the convergence is not instantaneous any longer. Both bequests and aspirations make individual decisions dependent of their parents decisions. Even if these two preference features result in a lower speed of convergence when they are introduced separately, their interaction ends up displaying an offsetting effect. If aspirations are marginally introduced when individuals exhibit a bequest motive ($\rho > 0$), bequest are still the main driving force for local convergence. However, the strength of this bequest motive in governing the intergenerational linkage becomes weaker as now individuals condition the bequest they leave to the achievement of a level of consumption adjusted to their aspirations. This stickiness in the bequest decisions brought about by the introduction of aspirations results thus in less inertia and thus faster convergence to the steady state. Similarly, when individuals exhibit aspirations, the introduction of the joy-of-giving motive for bequests reduces the dominating inertia associated with aspirations since now individuals will also choose their consumption taking into account the utility arising from the amount of bequests they leave.

As discussed in de la Croix (1996), de la Croix and Michel (1999), and Caballé and Moro-Egido (2014), a high intensity of aspirations might be a source of endogenous fluctuations around the steady state. In particular, if there are only adult aspirations (i.e., $\delta_x = 0$) in our model, it can be proved that the eigenvalues of the matrix $\mathbb{P}$ are real and positive if

$$\delta_c \in \left[0, \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[2R+(R\beta)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}-2R^{\frac{1}{2}}\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+(R\beta)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left((R\beta)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2}\right)$$

Similarly, when only old aspirations are present (i.e., $\delta_c = 0$), the eigenvalues of the matrix $\mathbb{P}$ are real and positive if

$$\delta_x \in \left[0, \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[R+2(R\beta)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}-2(R\beta)^{\frac{1}{2\sigma}}\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+(R\beta)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2}\right).$$

Moreover, the upper values of the previous two intervals are bifurcations where the eigenvalues become complex. Therefore, oscillations could arise in the economy under strong aspirations. The models of de la Croix (1996) and de la Croix and Michel (1999), do not display transmission of wealth through bequests but endogenous rental prices of labor and capital. Moreover, they only consider adult aspirations. In this case, individuals subject to strong adult aspirations will consume a lot when they are adult and thus they will save very little. The next generation will receive a small labor income due the small stock of capital installed in the economy, which will result in a low level of consumption so that consumption oscillations will appear. In our setup with bequests and exogenous rental prices, the mechanism for oscillations under strong aspirations is even more straightforward. Consider a generation that consumes a lot for an aspirational motive (i.e., to achieve the same standard of living of their parents either when adult or when old). This will result in a reduction in the amount left as bequests, which in turn will reduce the life-time income, and thus the consumption of the next generation. Therefore, intergenerational oscillations of consumption will arise naturally.

Finally, for even larger values of the aspirations intensities, the eigenvalues become real again but the dominating eigenvalue (either λ_1 or λ_2) turns out to be decreasing in the aspirations intensity so that the speed of convergence decreases with aspirations. This is due to the fact that, when aspirations are sufficiently large, they become the dominating force driving the speed of convergence. In this case, a larger intensity of aspirations results in a larger inertia from the past and, hence, in a lower speed of convergence. In particular, these new bifurcations associated with large values of aspiration intensities occur when

$$\delta_c = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[2R+(R\beta)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+2\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+(R\beta)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}R^{\frac{1}{2}}\right]}{\left(n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+(R\beta)^{\frac{1}{\sigma}}\right)^2} \quad \text{if } \delta_x = 0$$

and

$$\delta_x = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[\left(R+2(R\beta)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)+2(R\beta)^{\frac{1}{2\sigma}}\left(R+n\left(\frac{R}{n}\rho\right)^{\frac{1}{\sigma}}+(R\beta)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2} \quad \text{if } \delta_c = 0.$$

Note also that if the bequest motive is absent ($\rho = 0$), then the previous bifurcations do not appear.

Figure 1 depicts the values of the eigenvalues of the dynamic system for small values of both the aspiration intensities and the bequest motive. We see in this figure that, when there is a bequest motive, the introduction of aspirations raises the speed of convergence and the transition remains monotonic. However, when the intensity of aspirations becomes larger, then oscillations associated with the presence of complex eigenvalues appear. Finally, for even larger values of the aspiration intensities, oscillations disappear and the convergence becomes monotonic at a local speed that is decreasing in the aspiration intensities.

[Insert Figure 1]

5. Aspirations and wealth accumulation

In this section, we are going to study how the presence of aspiration and bequest motives modifies the pattern of wealth accumulation along the life cycle of an individual belonging to generation t who has received a given amount of inheritance b_t and who is exposed to the given levels of aspirations c_{t-1} and x_t. We will consider two stages in the accumulation process: first, the individual's accumulation of wealth until the end of the adult period, i.e., until the end of his participation in the labor market, which is summarized by the difference $s_t - b_t$ (adult accumulation) and, second, the wealth accumulation that takes place while the individual is retired, which is collected by the difference $nb_{t+1} - s_t$ (old accumulation). Finally, we can consider the total accumulation of wealth along the life cycle, which will be given by the difference between asset holdings at the final and the beginning of the individual's life $nb_{t+1} - b_{t.}$

As follows from the derivatives (3.6)-(3.8) computed under general utility functions, saving increases with the intensity ρ of the warm-glow altruism, decreases with the intensity δ_c of adult aspirations, and increases with the intensity δ_x of old aspirations. Hence, these comparative statics effects are automatically translated into the pattern of adult accumulation $s_t - b_t$ as the initial inheritance b_t of the individual is given. Similarly, since the amount of bequest left to each descendant b_{t+1} is decreasing in the intensities of both types of aspirations, δ_c and δ_x, and increasing in the bequest motive ρ, the same comparative statics result applies to the pattern of total accumulation $nb_{t+1} - b_t$ for a given initial inheritance b_t received by the individual.

The analysis of the effect of aspirations and bequest motives on wealth accumulation by an old individual is more complex since the two terms of the difference $nb_{t+1} - s_t$ are endogenous. Concerning the effect of the intensity ρ of warm-glow altruism, we can compute the following partial derivatives:

$$\frac{\partial \left(nb_{t+1} - s_t\right)}{\partial \rho} = \frac{n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[R-1+(R\beta)^{\frac{1}{\sigma}}\right]}{\rho\sigma H^2}\left(R\left(w+b_t\right) - \delta_x x_t - R\delta_c c_{t-1}\right) > 0.$$

where H is defined in (3.14). Therefore, even if the bequest motive raises both the total amount of bequests nb_{t+1} and the amount s_t saved, the effect on nb_{t+1} is a direct effect that dominates the side effect on saving. Note, however, that the sign of the previous derivative relies crucially on the existence of a positive net return on saving, $R - 1 > 0$, as we have assumed. If the net return on saving were negative, larger bequests could require a larger amount of saving as old consumption does not decrease enough in response to an increase in the joy-of-giving motive parametrized by the value of ρ.

The effects of adult aspirations on the accumulation of wealth by a generic old individual is summarized by the following derivative:

$$\frac{\partial \left(nb_{t+1} - s_t\right)}{\partial \delta_c} = \frac{(R\beta)^{\frac{1}{\sigma}}\left[1 + (1-R)\,n\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}\right]}{H} c_{t-1} \gtrless 0. \tag{5.1}$$

Note that the intensity δ_c of aspirations on adult consumption pushes down both the amount of saving and the amount of bequests. The net effect is ambiguous in general. If the net return on saving were negative $R \leq 1$, then the effect of δ_c on adult accumulation will be clearly negative. However, if the natural condition $R > 1$ holds, then we have the following:

$$\frac{\partial \left(nb_{t+1} - s_t\right)}{\partial \delta_c} \gtrless 0 \text{ if and only if } \frac{\beta}{\rho} \gtrless \frac{(R-1)^{\sigma}}{n^{1-\sigma}}.$$

In particular, if we consider the ratio β/ρ as a measure of the importance of old consumption relative to bequests in individual preferences, we see that the higher (lower) the value of this ratio, the higher (lower) will be the impact on old accumulation. If old consumption is very important relative to bequests then the amount of saving will be much larger than the amount of bequest. Therefore, the negative impact of δ_c on saving will be of larger size than the negative impact on the already small amount of bequests. This results in a positive net effect on the accumulation of wealth $nb_{t+1} - s_t$ by an old individual. The converse result will hold when the ratio β/ρ is small, i.e., when bequest motives are more important than the appetite for consumption when old. In particular, note that

$$\left.\frac{\partial \left(nb_{t+1} - s_t\right)}{\partial \delta_c}\right|_{\rho=0} > 0,$$

so that old accumulation increases with the intensity of old aspiration when there is no bequest motive. Clearly, in this case $b_{t+1} = 0$ and saving decreases with the intensity δ_c of adult aspirations.

Finally, the intensity δ_x of old aspirations lower the level of wealth accumulation of an old individual. This is a direct consequence of the induced increase in the amount of saving and the decrease in the amount of bequests given in (3.7) and (3.10). Obviously, as the level of old aspirations rises, old individuals want to consume more and this results in a lower value of $nb_{t-1} - s_{t.}$

Table 1 summarizes all the effects of warm-glow altruism and aspirations on the accumulation of capital along the life cycle.

[Insert Table 1]

6. Bequest motive under aspirations

In this section, we consider the introduction of a bequest motive when individuals exhibit aspirations. We have already seen in the previous section that warm-glow altruism tends to increase the accumulation (or to decrease the disaccumulation) of wealth for both adult and old agents. According to the empirical work of Hurd (1987), there is no significative difference between the individuals with living children (who are assumed to display some bequest motive) and individuals with no living children (who cannot have any desire to leave bequest). We will see that aspirations weaken the effect of altruism so that individuals exhibiting aspirations are less sensitive to the presence of altruism. Therefore, the existence of aspirations makes the pattern of accumulation of bequest motivated individuals and non-bequest-motivated individuals more similar. Thus, aspirations may partially explain the result of Hurd concerning the insensitivity of wealth accumulation with respect to altruism.

In order to obtain how aspirations contribute to the effect of the introduction of altruism we need to compute the second cross derivative of the endogenous variable under consideration with respect to the intensity ρ of bequest motive and the intensity of aspirations parameterized by the values of the parameters δ_c and δ_x. Note that in our analysis we take as given the values of the initial wealth b_t and of aspirations, c_{t-1} and x_t, of a generic member of the generation t. Let us consider first the effects of adult aspirations, which are summarized in the following cross derivatives:

$$\left.\frac{\partial^2 (s_t - b_t)}{\partial\rho\partial\delta_c}\right|_{\delta_c=0,\delta_x=0} = -\frac{Rn\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} c_{t-1}}{\sigma\rho H^2} < 0, \tag{6.1}$$

$$\left.\frac{\partial^2 (nb_{t+1} - s_t)}{\partial\rho\partial\delta_c}\right|_{\delta_c=0,\delta_x=0} = -\frac{Rn\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R - 1 + (R\beta)^{\frac{1}{\sigma}}\right] c_{t-1}}{\sigma\rho H^2} < 0, \tag{6.2}$$

$$\left.\frac{\partial^2 (nb_{t+1} - b_t)}{\partial\rho\partial\delta_c}\right|_{\delta_c=0,\delta_x=0} = -\frac{Rn\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R + (R\beta)^{\frac{1}{\sigma}}\right] c_{t-1}}{\sigma\rho H^2} < 0. \tag{6.3}$$

We have evaluated the previous derivatives at $\delta_c = 0$ and $\delta_x = 0$, that is, the previous signs hold for low values of the intensity of aspirations. For higher values, the resulting expressions for the derivatives are extremely messy and cannot be sign explicitly. However, we will conduct a numerical simulation to check the robustness of the signs of the corresponding comparative statics. We see from the previous derivatives that the positive effect of warm-glow altruism on wealth accumulation at all stages of the lifetime is dampened under aspirations associated with adult consumption. If this type of aspirations are introduced in the utility function, then the value $c_t - \delta_c c_{t-1}$ of adjusted adult consumption becomes smaller. As the isoelastic utility function displays decreasing absolute risk aversion, the degree of concavity of the utility u becomes higher ceteris paribus. In our non-stochastic environment this translates into a lower willingness to change the level of adult consumption, and this results in turn in a lower impact of the introduction of bequest motive on saving and bequest. Thus, the positive effect of bequest motive on adult and total accumulation becomes smaller when a positive value of δ_c is introduced (see (6.1) and (6.3)). Moreover, the introduction of adult aspirations dampens the positive effect of warm-glow altruism on the amount of wealth accumulation by old individuals even if this accumulation might be increasing in the intensity of adult aspirations (see (5.1) and (6.2)).

The impact of the intensity δ_x of old aspirations associated with old consumption on the effects of the introduction of bequest motive are also summarized in the following partial cross-derivatives evaluated at $\delta_c = 0$ and $\delta_x = 0$:

$$\left.\frac{\partial^2 (s_t - b_t)}{\partial \rho \partial \delta_x}\right|_{\delta_c=0,\delta_x=0} = -\frac{n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} x_t}{\sigma \rho H^2} < 0, \tag{6.4}$$

$$\left.\frac{\partial^2 (nb_{t+1} - s_t)}{\partial \rho \partial \delta_x}\right|_{\delta_c=0,\delta_x=0} = -\frac{n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R - 1 + (R\beta)^{\frac{1}{\sigma}}\right] x_t}{\sigma \rho H^2} < 0,$$

$$\left.\frac{\partial^2 (nb_{t+1} - b_t)}{\partial \rho \partial \delta_x}\right|_{\delta_c=0,\delta_x=0} = -\frac{n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R + (R\beta)^{\frac{1}{\sigma}}\right] x_t}{\sigma \rho H^2} < 0.$$

Here, a higher strength of old aspirations lowers the positive impact of the introduction of warm-glow altruism on saving, bequests, and wealth accumulation of adult and old individuals. As it happens under adult aspirations, parental consumption induces some inertia in the behavior of individuals, which results in a weaker response to the introduction of bequest motive. Note also that the introduction of old aspirations dampens the positive effect of warm-glow altruism on the amount of wealth accumulation by adult individuals even if old aspirations raise the wealth accumulation of those old individuals (see (3.7) and (6.4)). Table 2 summarizes the sign of the previous cross partial derivatives.

[Insert Table 2]

In order to analyze the robustness of the dampening effect of both types of aspirations when bequest motivations are introduced, we perform the following numerical analysis. We choose the value of the preference parameters $\beta = 1/2$ and, following Iacoviello (2008), the value $w = 2/3$ for the wage. We assume an annual rate of population growth of 1% and an annual interest rate of 2% and consider that each period lasts for 30 years.[2] We maintain these parameter values for the remaining numerical exercises. As initial values of bequest b_t, adult aspirations c_{t-1}, and old aspirations x_t, we choose the steady-state values corresponding to a benchmark with no warm-glow altruism, $\rho = 0$, which implies, $b_t = 0$, and no aspirations, $\delta_c = \delta_x = 0$.[3] Here, we keep this initial values fixed since we are going to analyze how the optimal decisions of individuals are affected by bequest motive and aspirations and to do so we should also take as given the exogenous variables of the maximization problem faced by individuals. In the next section we will perform instead the comparative statics on the steady-state, that is, we will compare the pattern of wealth accumulation of individuals in stationary equilibria associated with different values of the parameters of the model. In this latter case, the initial values of bequest and aspirations should also be adjusted as a response to the change in parameter values.

Figures 2 and 3 depict the effect of altruism on adult accumulation of wealth, $s_t - b_t$, for different strictly positive values of the intensities δ_c and δ_x of aspirations and of the parameter σ referred to the inverse of elasticity of the utility functions. In particular, we compare the benchmark case where $\delta_c = \delta_x = 0$, with values of the aspirations intensities equal to 1/2. Figure 2 considers the values $\sigma = 1$ and $\sigma = 2$ while Figure 3 refers to the value $\sigma = 1/2$. We see that the slopes of all the wealth accumulations as functions of the intensity of warm-glow altruism are smaller for positive values of aspiration intensities. The same conclusion arises from Figures 4 and 5 concerning the accumulation of wealth for old individuals, $nb_{t+1} - s_t$. Finally Figures 6 and 7 refer to the accumulation of wealth along the whole life of individuals, $nb_{t+1} - b_t$. Therefore, the dampening effect of aspirations presented above extends beyond the simple introduction of bequest motive and aspirations.

[Insert Figures 2 to 7]

7. Steady-state effects

In this section we will analyze how the introduction of aspirations affects the change in the pattern of wealth accumulation brought about by bequest motivations in the steady state. This means that we are going to compare the individual optimal decision concerning saving and bequest of a generation in the steady state, where $c_t = c$, $x_{t+1} = x$, $s_t = s$, and $b_{t+1} = b$ for all t, with the optimal decision under different values of the parameters characterizing warm-glow altruism and aspirations once the dynasty has reached the new steady-state associated with these new parameter values. The steady-state values of consumption and bequest are given in (4.1), (4.2), and (4.3) while the steady-state value of saving is $s = w + b - c$. We start with the effects on the stationary value of saving and bequests of the intensity ρ of bequest motive and aspirations δ_c and δ_x. The following derivatives summarize the results:

$$\frac{\partial s}{\partial \rho} = \frac{(1-\delta_c)(1-\delta_x) R \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[(1-\delta_x) n + (1-\delta_c)(R\beta)^{\frac{1}{\sigma}}\right] w}{\sigma \rho J^2} > 0,$$

[2] In terms of the parameters of our madel we get $n = (1.01)^{30} = 1.348$ and $R = (1.02)^{30} = 1.811$.

[3] The initial values for parental consumption are $c_{t-1} = 0.444$ and $x_t = 0.403$ for $\sigma = 1$, $c_{t-1} = 0.437$ and $x_t = 0.416$ for $\sigma = 2$, and $c_{t-1} = 0.459$ and $x_t = 0.376$ for $\sigma = 1/2$.

$$\frac{\partial b}{\partial \rho} = \frac{(1-\delta_c)(1-\delta_x) R \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[(1-\delta_x) R + (1-\delta_c)(R\beta)^{\frac{1}{\sigma}}\right] w}{\sigma \rho J^2} > 0,$$

$$\frac{\partial s}{\partial \delta_c} = -\frac{(1-\delta_x) R (R\beta)^{\frac{1}{\sigma}} \left[1 + (1-\delta_x) n \left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}\right] w}{J^2} < 0,$$

$$\frac{\partial b}{\partial \delta_c} = -\frac{R^2 (1-\delta_x)^2 \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} w}{J^2} < 0,$$

$$\frac{\partial s}{\partial \delta_x} = \frac{R (R\beta)^{\frac{1}{\sigma}} (1-\delta_c) \left[1 - (1-\delta_c) \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right] w}{J^2} \gtrless 0, \tag{7.1}$$

$$\left.\frac{\partial s}{\partial \delta_x}\right|_{\rho=0} = \frac{R (R\beta)^{\frac{1}{\sigma}} (1-\delta_c) w}{\left[(1-\delta_x) R + (1-\delta_c)(R\beta)^{\frac{1}{\sigma}}\right]^2} > 0, \tag{7.2}$$

$$\frac{\partial b}{\partial \delta_x} = -\frac{R (R\beta)^{\frac{2}{\sigma}} (1-\delta_c)^2 \left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}} w}{J^2} < 0.$$

where J is defined in (4.4). We see that the sign of the partial derivatives are the same as the ones obtained in Section 3 for the non-stationary values of bequest and saving. Note however that here in order to sign the effect of the intensity δ_x of old aspirations on saving we need to evaluate the partial derivative (7.1) at $\rho = 0$ (see (7.2)) so that the sign applies to low levels of warm-glow altruism. In general, the partial derivative (7.1) has an ambiguous sign.

Concerning the effects on the accumulation of wealth by adult individuals, $s - b$, by old individuals, $nb - s$, and total accumulation $(n-1)b$ of changes in the intensity of bequest motive, we obtain the following partial derivatives:

$$\frac{\partial (s-b)}{\partial \rho} = -\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} (1-\delta_x)^2 (1-\delta_c) w}{\sigma \rho J^2} (R - n) \gtrless 0, \tag{7.3}$$

$$\frac{\partial (nb-s)}{\partial \rho} = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} (1-\delta_c)(1-\delta_x) \left[n(1-\delta_x)(R-1) + (R\beta)^{\frac{1}{\sigma}} (1-\delta_c)(n-1)\right] w}{\sigma \rho J^2} \gtrless 0, \tag{7.4}$$

$$\frac{\partial \left[(n-1) b\right]}{\partial \rho} = (n-1) \frac{\partial b}{\partial \rho} \gtrless 0 \text{ if and only if } n \gtrless 1. \tag{7.5}$$

We see in (7.3) that adult accumulation $s - b$ increases (decreases) with the degree of warm-glow altruism in a steady state if $R < (>) n$. If the gross return R from saving is small then saving must increase a lot relative to bequest per children to generate an increase in the transfer to the next generation. Similarly, if the number n of children is high, saving should also increase a lot so as to raise the amount of bequest per child. Concerning the the accumulation of wealth $nb - s$ for old individuals individuals, we see that the sign of the partial derivative (7.4) is strictly positive whenever the gross return R from saving and the gross rate n of population growth are both larger or equal than 1, and at least one of them is strictly larger than 1. Obviously, on the one hand, the higher is the return on saving the less is the need for increasing the amount of savings for the bequest motive. On the other hand, the higher is the number of children per parent, the larger should be the total amount nb bequeathed to the heirs. These two effects explain the positive sign of the partial derivative (7.4) for this empirically relevant case. Finally, the sign of the effect on total accumulation along the life cycle, $(n-1)b$, depends on whether the net rate of population growth is positive ($n > 1$) or negative ($n < 1$). Clearly, the higher is the rate of population growth the larger the amount of wealth individuals must accumulate to endow their children with the stationary amount of inheritance.

We next show the partial derivatives that characterize the effect of the intensity δ_c of adult aspirations on the stationary pattern of wealth accumulation:

$$\frac{\partial (s-b)}{\partial \delta_c} = -\frac{\left[R\left(R\beta\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)\right]\left[1-\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)\left(R-n\right)\right]w}{J^2} \gtrless 0,$$

$$\left.\frac{\partial (s-b)}{\partial \delta_c}\right|_{\rho=0} = -\frac{R\left(R\beta\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)w}{\left[\left(1-\delta_x\right)R+\left(1-\delta_c\right)\left(R\beta\right)^{\frac{1}{\sigma}}\right]^2} < 0, \tag{7.6}$$

$$\frac{\partial (nb-s)}{\partial \delta_c} = \frac{Rw\left(R\beta\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)\left[1-n\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)\left(R-1\right)\right]}{J^2} \gtrless 0,$$

$$\left.\frac{\partial (nb-s)}{\partial \delta_c}\right|_{\rho=0} = \frac{R\left(R\beta\right)^{\frac{1}{\sigma}}\left(1-\delta_x\right)w}{\left[\left(1-\delta_x\right)R+\left(1-\delta_c\right)\left(R\beta\right)^{\frac{1}{\sigma}}\right]^2} > 0, \tag{7.7}$$

$$\frac{\partial\left[\left(n-1\right)b\right]}{\partial \delta_c} = \left(n-1\right)\frac{\partial b}{\partial \delta_c} \gtrless 0 \text{ if and only if } n \lessgtr 1.$$

Finally, the effect of the intensity δ_x of old aspirations on the stationary pattern of wealth accumulation is given by the following partial derivatives:

$$\frac{\partial (s-b)}{\partial \delta_x} = \frac{R(R\beta)^{\frac{1}{\sigma}}(1-\delta_c)w}{J^2} > 0,$$

$$\frac{\partial (nb-s)}{\partial \delta_x} = -\frac{Rw(R\beta)^{\frac{1}{\sigma}}(1-\delta_c)\left[1+\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}(R\beta)^{\frac{1}{\sigma}}(1-\delta_c)(n-1)\right]}{J^2} \gtrless 0,$$

$$\left.\frac{\partial (nb-s)}{\partial \delta_x}\right|_{\rho=0} = \frac{R(R\beta)^{\frac{1}{\sigma}}(\delta_c-1)w}{\left[R(1-\delta_x)+(1-\delta_c)(R\beta)^{\frac{1}{\sigma}}\right]^2} < 0, \tag{7.8}$$

$$\frac{\partial\left[(n-1)b\right]}{\partial \delta_x} = (n-1)\frac{\partial b}{\partial \delta_x} \gtrless 0 \text{ if and only if } n \lessgtr 1.$$

Note that the signs of the comparative static exercises for adult and old accumulation of wealth are the same as the ones obtained in Section 3 when the dynasty was not in its steady state We need however to assume in some cases that the intensity of bequest motive is close to zero (see (7.6), (7.7), and (7.8)). Concerning the total wealth accumulation $(n-1)b$ along the lifetime, the sign depends in an obvious way on the rate of population growth. Table 3 summarizes the signs of the comparative statics exercises on the stationary equilibrium.

[Insert Table 3]

To conclude our analysis we can evaluate whether the aspirations dampen or exacerbate the effect of warm-glow altruism when we make the comparison between to steady-sates. As before, we need to compute the partial cross derivative of the steady state values s, b, $s-b$, $nb-s$, and $(n-1)b$ with respect to the bequest motive intensity ρ and the aspirations intensities δ_c and δ_x when adult and old, respectively. The exact expressions of these cross derivatives are extremely messy and are available from the authors under request but their sings are shown in Table 4 under some parametric restrictions.

[Insert Table 4]

To assess the effect of aspirations when they are introduced non-marginally, we have the numerical exercises contained in Figures 8 to 13. These figures are the counterparts of Figures 2 to 7 but evaluated at the steady state values, i.e., we do not take the initial inheritance and aspirations as fixed since we use instead the stationary values b of inheritance and c and x of aspirations, which vary endogenously with the parameter values ρ, δ_c and δ_x. The main conclusion of these figures is that the presence of either adult or old aspirations dampens the effect of warm-glow altruism on the pattern of wealth accumulation. We see that the slopes of the functions displayed exhibit a lower slope in absolute value. In particular, observe in Figures 8 and 9 how the stationary adult accumulation of wealth $s-b$ is decreasing in the intensity of bequest motive ρ.

However, this negative effect becomes less strong when aspirations exist. This numerical analysis allows to extend the conclusion at which we arrived when we considered the individual decision with exogenous initial values of bequest and aspirations to the stationary patterns of capital accumulation. In all these new graphs we have used the same parameter values as in the previous figures.

[Insert Figures 8 to 13]

8. Conclusion

We have developed a simple OLG model that enable us to study the effect of the introduction of warm-glow altruism and aspirations on the individual pattern of wealth accumulation. Our results show that the introduction of aspirations at different ages display different effects on the amount of adult saving for given initial values of bequest and aspirations. However, both adult and old aspirations make the introduction of the bequest motive less effective in changing the pattern of accumulation. Therefore, under aspirations both bequest motivated and non-bequest motivated individuals will behave more similarly than when they do not exhibit aspirational concerns. Moreover, the dampening effect of aspirations prevails when we compare stationary allocations.

We have also shown that, if the bequest motive is present, the marginal introduction of aspirations at any age makes a dynasty converge faster towards its steady state since intergenerational transfers play a less relevant role, which makes individual decisions less dependent of their ancestors' decisions. However, the converse result holds when there is no bequest motive. In this latter case, the introduction of aspirations gives raise to some inertia as individual decisions concerning consumption will depend on parental consumption, which results in turn in a lower speed of convergence.

References

Abel, A. (1985), Precautionary Saving and Accidental Bequests, *American Economic Review* 75, 777-791.

Abel, A. (1986), Capital Accumulation and Uncertain Lifetimes with Adverse Selection, *Econometrica* 54, 1079-1097.

Alessie, R. and Teppa, F. (2010), Saving and Habit Formation: Evidence from Dutch Panel Data, *Empirical Economics* 38, 385-407.

Alonso-Carrera, J., Caballé, J. and Raurich X. (2007), Aspirations, Habit Formation, and Bequest Motive, *Economic Journal* 117, 813-836.

Barro, R.J. (1974), Are Government Bonds Net Wealth?, *Journal of Political Economy* 82, 1095-1117.

Boyd, R. and Richerson, P. (1985), Culture and the Evolutionary Process, University of Chicago Press, Chicago, IL.

Becker, G. (1981), A Treatise on the Family, Harvard University Press, Cambridge, MA.

Bernheim, B., Schleifer, A. and Summers, L. (1985), The Strategic Bequest Motive, *Journal of Political Economy* 93 1045-1075.

Bossmann, M., Kleiber, C. and Waelde, K. (2007), Bequests, Taxation and the Distribution of Wealth in a General Equilibrium Model, *Journal of Public Economics* 91, 1247-1271.

Carrasco, R., Labeaga, J.M. and Lopez-Salido, J.D. (2005), Consumption and Habits: Evidence from Panel Data, *Economic Journal*, 115, 144-65.

Caballé, J. and Moro-Egido, A.I. (2014), Effects of Aspirations and Habits on the Distribution of Wealth, *Scandinavian Journal of Economics* 116, 1012-1043.

Cavalli-Sforza, L. and Feldman, M. (1981), Cultural Transmission and Evolution: A Quantitative Approach, Princeton University Press, Princeton, NJ.

Charles, K.K., Danziger, S., Li, G. and Schoeni, R. (2014), The Intergenerational Correlation of Consumption Expenditures", *American Economics Review: Papers and Proceedings,* 104, 136-140.

Cox, D., Ng, S. and Waldkirch, A. (2004), Intergenerational Linkages in Consumption Behavior, *Journal of Human Resources* 32, 355-381.

de la Croix, D. (1996), The Dynamics of Bequeathed Tastes, *Economics Letters* 53, 89-96.

de la Croix, D. (2001), Growth Dynamics and Education Spending: The Role of Inherited Tastes and Abilities, *European Economic Review* 45, 1415-1438.

de la Croix, D. and Michel, P. (1999), Optimal Growth when Tastes Are Inherited, *Journal of Economic Dynamics and Control* 23, 519-37.

de la Croix, D. and Michel, P. (2001), Altruism and Self-Restraint, *Annales d'Economie at de Statistique* 63-64, 233-260.

de la Croix, D. and Urbain, J.P. (1998), Intertemporal Substitution in Import Demand and Habit Formation, *Journal of Applied Econometrics* 13, 589–612.

Fuhrer J.C. and Klein, M.W. (2006), Risky Habits: on Risk Sharing, Habit Formation, and the Interpretation of International Consumption Correlations. *Review of International Economics*, 14, 722–740.

Hurd, M. D. (1987), Savings of the Elderly and Desired Bequests, *American Economic Review* 77, 298-312.

Iacoviello, M. (2008), Household Debt and Income Inequality, 1963-2003, *Journal of Money, Credit and Banking* 40, 930-965.

Jellal, M. and Wolff, F.C. (2002), Altruistic Bequests with Inherited Tastes, *International Journal of Business and Economic* 1, 95-113.

Katzner, D.W. (1970), Static Demand Theory, The Macmillan Company, New York, NY.

Pollak, R. A. (1988), Tied Transfers and Paternalistic Preferences, *American Economic Review* 78, 240-244.

Yaari, M. (1965), Uncertain Lifetime, Life Insurance, and the Theory of the Consumer, *Review of Economic Studies* 32, 137-150.

A. Appendix

Proof of Lemma 4.1. We can rewrite the system composed of the difference equations (3.11), (3.12), and (3.13) in matrix form as

$$\begin{bmatrix} c_t \\ x_{t+1} \\ b_{t+1} \end{bmatrix} = \mathbb{P} \times \begin{bmatrix} c_{t-1} \\ x_t \\ b_t \end{bmatrix} + \frac{1}{H} \begin{bmatrix} R \\ R\left(R\beta\right)^{\frac{1}{\sigma}} \\ R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \end{bmatrix} w,$$

where H is given in (3.14) and the coefficient matrix $\mathbb{P}$ is

$$\mathbb{P} = \frac{1}{H} \begin{bmatrix} \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c & -\delta_x & R \\ -R\left(\beta R\right)^{\frac{1}{\sigma}}\delta_c & \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x & (R\beta)^{\frac{1}{\sigma}} R \\ -R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_c & -\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_x & R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \end{bmatrix}. \tag{A.1}$$

The characteristic polynomial of the matrix $\mathbb{P}$ is

$$P\left(\lambda\right) = \lambda^3 - \mu_1\lambda^2 + \mu_2\lambda - \mu_3, \tag{A.2}$$

where

$$\mu_1 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x}{H},$$

$$\mu_2 = \frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R\delta_c + R\delta_x + n\delta_c\delta_x\right)}{H},$$

$$\mu_3 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_c\delta_x}{H}.$$

Moreover, if $\delta_c = 0$ and $\delta_x = 0$ the characteristic polynomial becomes

$$P\left(\lambda\right) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{H}\right]\lambda^2$$

so that the eigenvalues become

$$\lambda_1 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} > 0,$$

$\lambda_2 = 0$ and $\lambda_3 = 0$. Finally, since the eigenvalues are continuous functions of the parameters δ_c and δ_x, the three eigenvalues will lie in the interior of the unit circle for a sufficiently small value of parameters measuring the intensity of aspirations if $\lambda_1 < 1$. *Q.E.D.*

Proof of Proposition 4.2.

(a) Since we are only interested in the effect of the introduction of aspirations, we will consider the marginal introduction of adult aspirations and old aspirations separately. Let us first consider the introduction of adult aspirations from a starting situation with no aspirations when $\rho > 0$. To this end, let us take the characteristic polynomial (A.2) and make $\delta_x = 0$ to get

$$P(\lambda) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda^2 + \left[\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_c}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda.$$

Here one of the eigenvalues equals zero since the parental old consumption x_t is not a state variable for an individual of generation t and, hence the value of the initial condition x_0 is irrelevant. The other two eigenvalues λ_1 and λ_2 are equal to the conjugate pair

$$\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c \pm \left[\left(R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c\right)^2 - 4\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_c\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\right]^{\frac{1}{2}}}{2\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]}.$$

If we take the largest of this two eigenvalues, λ_1 say, and perform the derivative with respect to the aspiration intensity δ_c and then we evaluate the derivative when $\delta_c = 0$, we obtain

$$\left.\frac{d\lambda_1}{d\delta_c}\right|_{\delta_c=0,\delta_x=0} = -\frac{R}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} < 0. \tag{A.3}$$

Similarly, we can replicate the argument for the introduction of aspirations on old consumption. The characteristic polynomial in (A.2) with $\delta_c = 0$ becomes now

$$P(\lambda) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda^2 + \left[\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_x}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda.$$

In this case, one of the eigenvalues is again equal to zero since the parental adult consumption c_t is not a state variable for an individual of generation t and, hence the value of the initial condition c_{-1} does not affect his decision. The other two eigenvalues λ_1 and λ_2 are equal to the conjugate pair

$$\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x \pm \left[\left(R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x\right)^2 - 4\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_x\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\right]^{\frac{1}{2}}}{2\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]}.$$

If we take the largest of this two eigenvalues, λ_1 say, and perform the derivative with respect to the aspiration intensity δ_x and then we evaluate the derivative when $\delta_x = 0$, we obtain

$$\left.\frac{d\lambda_1}{d\delta_x}\right|_{\delta_c=0,\delta_x=0} = -\frac{(R\beta)^{\frac{1}{\sigma}}}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} < 0. \tag{A.4}$$

Therefore, we can conclude that the introduction of aspirations when $\rho > 0$ either on adult or in old consumption increases the speed of convergence around the steady state.

(b) When there is no bequest motive, $\rho = 0$, and $\delta_x = 0$, the characteristic polynomial (A.2) becomes

$$P(\lambda) = \lambda^3 - \left[\frac{(R\beta)^{\frac{1}{\sigma}}\,\delta_c}{R + (R\beta)^{\frac{1}{\sigma}}}\right]\lambda^2,$$

so that there is only an eigenvalue different for zero,

$$\lambda_2 = \frac{(R\beta)^{\frac{1}{\sigma}}\,\delta_c}{R + (R\beta)^{\frac{1}{\sigma}}},$$

which is clearly increasing in the intensity δ_c of adult aspirations. Similarly, when there is no bequest motive, $\rho = 0$, and $\delta_c = 0$, the characteristic polynomial (A.2) becomes

$$P(\lambda) = \lambda^3 - \left[\frac{R\delta_x}{R + (R\beta)^{\frac{1}{\sigma}}}\right]\lambda^2,$$

so that there is also only an eigenvalue different for zero,

$$\lambda_3 = \frac{R\delta_x}{R + (R\beta)^{\frac{1}{\sigma}}},$$

which is also increasing in the intensity δ_x of old aspirations. Therefore, the introduction of either adult or old aspirations when bequests are absent results in a lower speed of convergence. *Q.E.D.*

	s_t	b_{t+1}	$s_t - b_t$	$nb_{t+1} - s_t$	$nb_{t+1} - b_t$
$\frac{\partial}{\partial \rho}$	> 0	> 0	> 0	> 0	> 0
$\frac{\partial}{\partial \delta_c}$	< 0	< 0	< 0	$\gtreqless 0 \ \left(\text{if } \frac{\beta}{\rho} \gtreqless \frac{(R-1)^\sigma}{n^{1-\sigma}}\right)$ $> 0 \ (\text{if } \rho = 0)$	< 0
$\frac{\partial}{\partial \delta x}$	> 0	< 0	> 0	< 0	< 0

Table 1. Comparative statics of saving, bequest and wealth accumulation.

	s_t	b_{t+1}	$s_t - b_t$	$nb_{t+1} - s_t$	$nb_{t+1} - b_t$
$\left.\frac{\partial^2}{\partial \rho \partial \delta_c}\right\vert_{\delta_c=0,\delta_x=0}$	< 0	< 0	< 0	< 0	< 0
$\left.\frac{\partial^2}{\partial \rho \partial \delta_x}\right\vert_{\delta_c=0,\delta_x=0}$	< 0	< 0	< 0	< 0	< 0

Table 2. The dampening effect of aspirations on bequest motive

	s	b	$s-b$	$nb-s$	$(n-1)b$
$\frac{\partial}{\partial\rho}$	>0	>0	$\gtreqless 0$ (if $R \lesseqgtr n$)	>0 (if $n>1$)	$\gtreqless 0$ (if $n \gtreqless 1$)
$\frac{\partial}{\partial\delta_c}$	<0	<0	<0 (if $\rho=0$)	>0 (if $\rho=0$)	$\gtreqless 0$ (if $n \lesseqgtr 1$)
$\frac{\partial}{\partial\delta x}$	>0	<0	>0	<0 (if $\rho=0$)	$\gtreqless 0$ (if $n \lesseqgtr 1$)

Table 3. Comparative statics of stationary saving, bequest and wealth accumulation.

	s	b	$s-b$	$nb-s$	$(n-1)b$
$\frac{\partial^2}{\partial\rho\partial\delta_c}\Big\vert_{\delta_c=0,\delta_x=0}$	<0 (if $R \lesseqgtr n/2$)	<0	$\gtreqless 0$ (if $R \gtreqless n$)	<0	$\gtreqless 0$ (if $n \lesseqgtr 1$)
$\frac{\partial^2}{\partial\rho\partial\delta_x}\Big\vert_{\delta_c=0,\delta_x=0}$	$\gtreqless 0$	<0	$\gtreqless 0$ (if $R \gtreqless n$)	<0	$\gtreqless 0$ (if $n \lesseqgtr 1$)

Table 4. The dampening effect of aspirations on bequest motive.

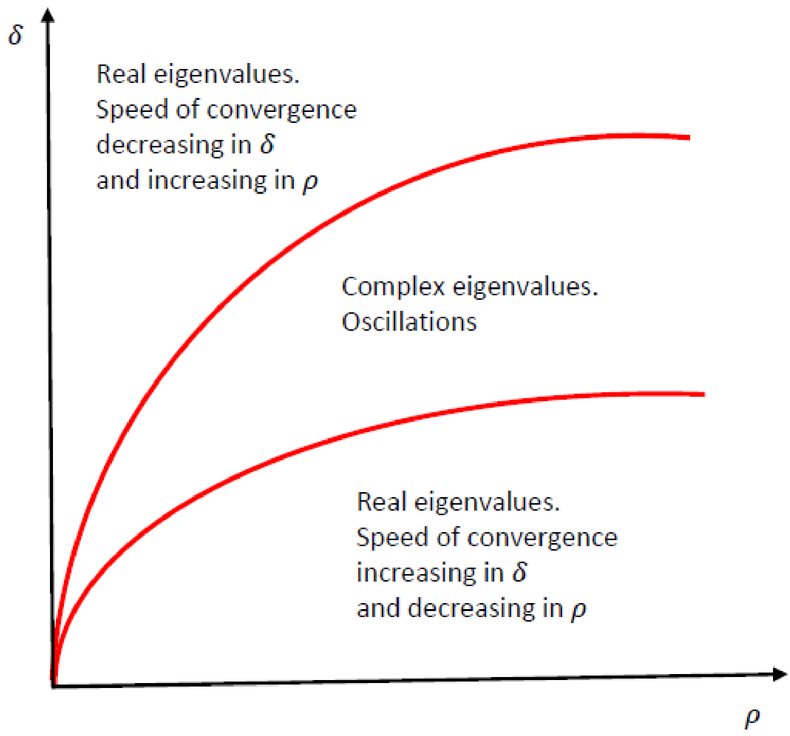

Figure 1. Eigenvalues for small values of the bequest motive and aspiration intensities.

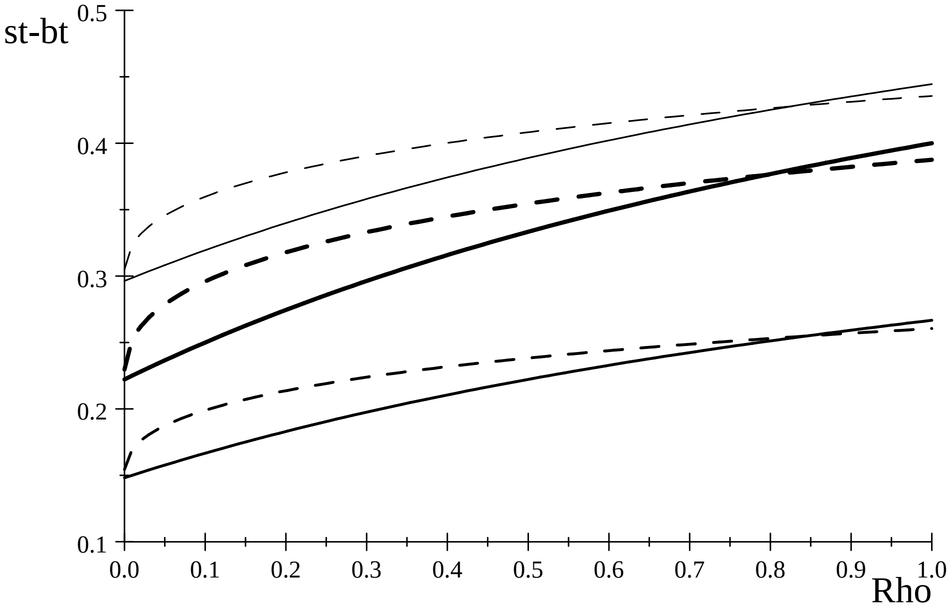

Figure 2. The effects of aspirations on adult accumulation of wealth $(s_t - b_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1$. Thick dash line: $\delta_c = 0, \delta_x = 0, \sigma = 2$.
Medium dash line: $\delta_c = 1/2, \delta_x = 0, \sigma = 2$. Thin dash line: $\delta_c = 0, \delta_x = 1/2, \sigma = 2$.

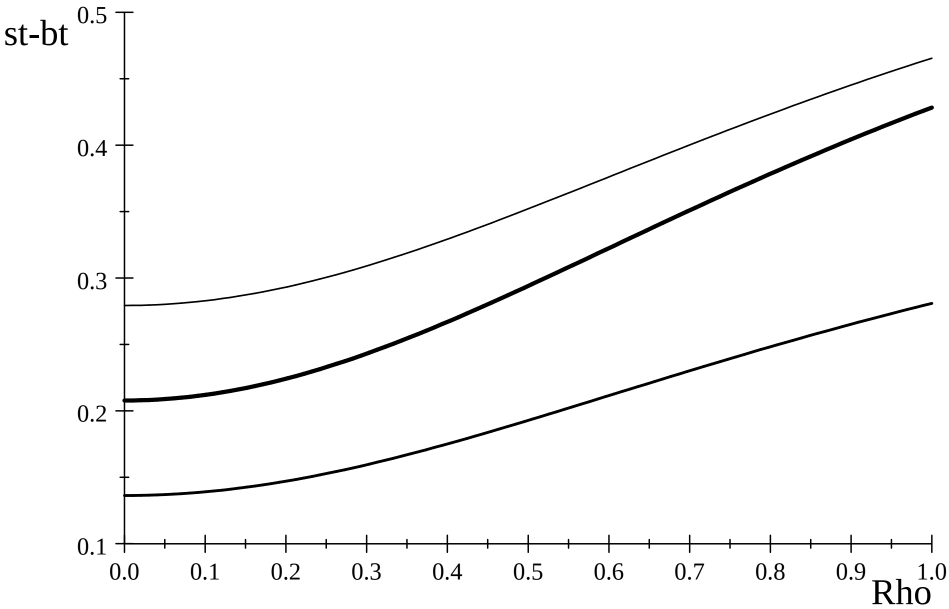

Figure 3. The effects of aspirations on adult accumulation of wealth $(s_t - b_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1/2$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1/2$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1/2$.

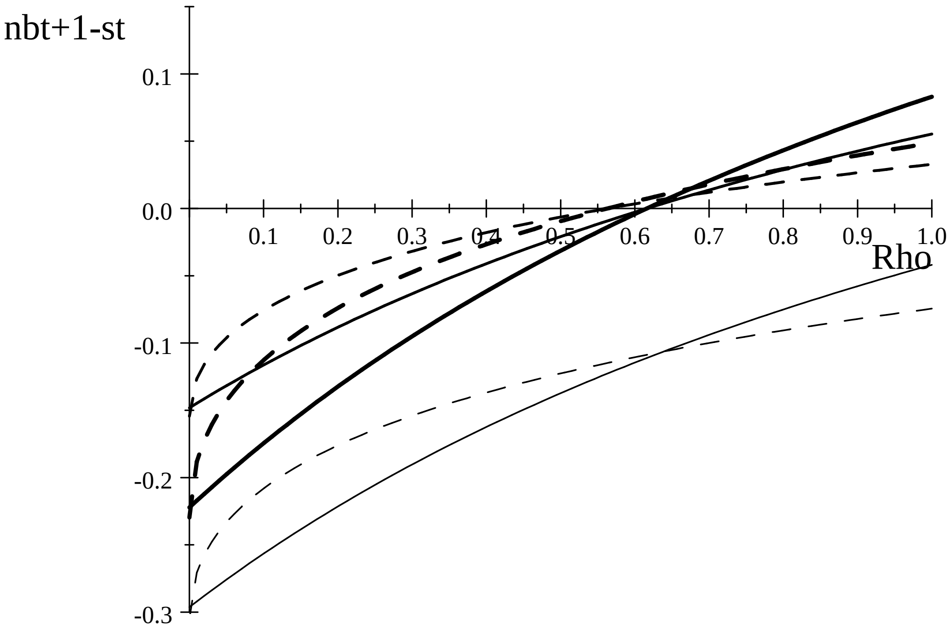

Figure 4. The effects of aspirations on old accumulation of wealth $(nb_{t+1} - s_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1$. Thick dash line: $\delta_c = 0, \delta_x = 0, \sigma = 2$.
Medium dash line: $\delta_c = 1/2, \delta_x = 0, \sigma = 2$. Thin dash line: $\delta_c = 0, \delta_x = 1/2, \sigma = 2$.

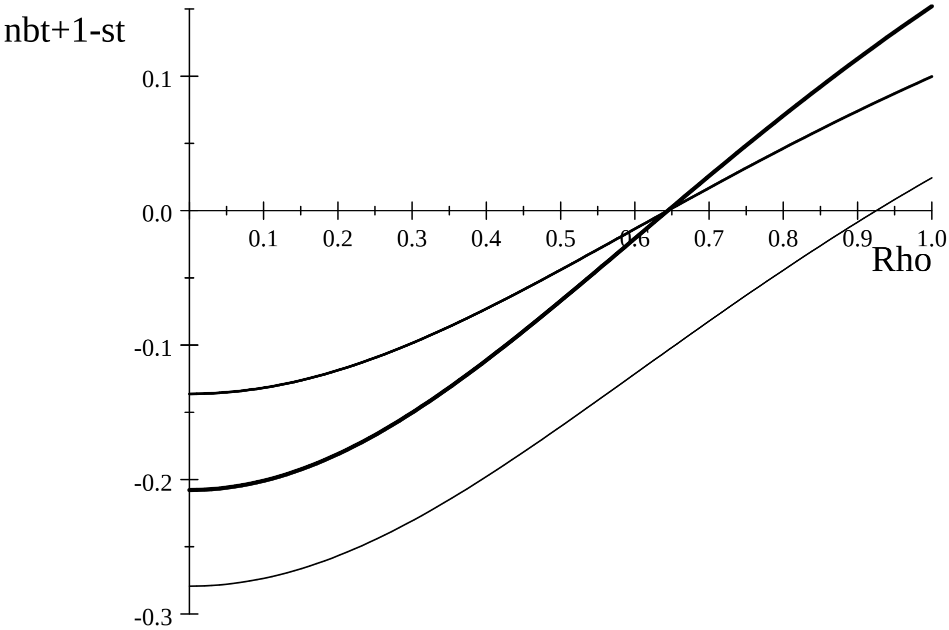

Figure 5. The effects of aspirations on old accumulation of wealth $(nb_{t+1} - s_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1/2$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1/2$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1/2$.

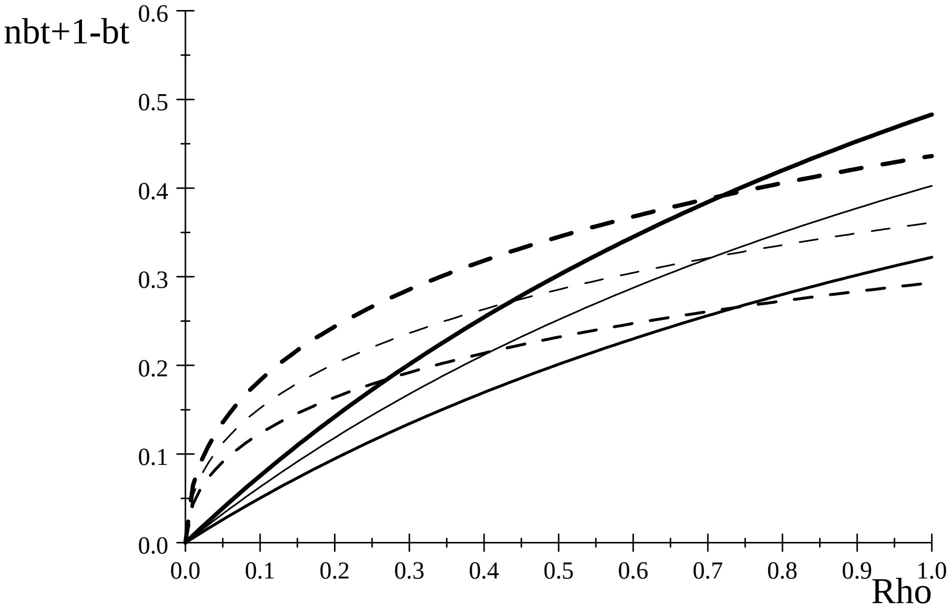

Figure 6. The effects of aspirations on lifetime wealth accumulation $(nb_{t+1} - b_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1$. Thick dash line: $\delta_c = 0, \delta_x = 0, \sigma = 2$.
Medium dash line: $\delta_c = 1/2, \delta_x = 0, \sigma = 2$. Thin dash line: $\delta_c = 0, \delta_x = 1/2, \sigma = 2$.

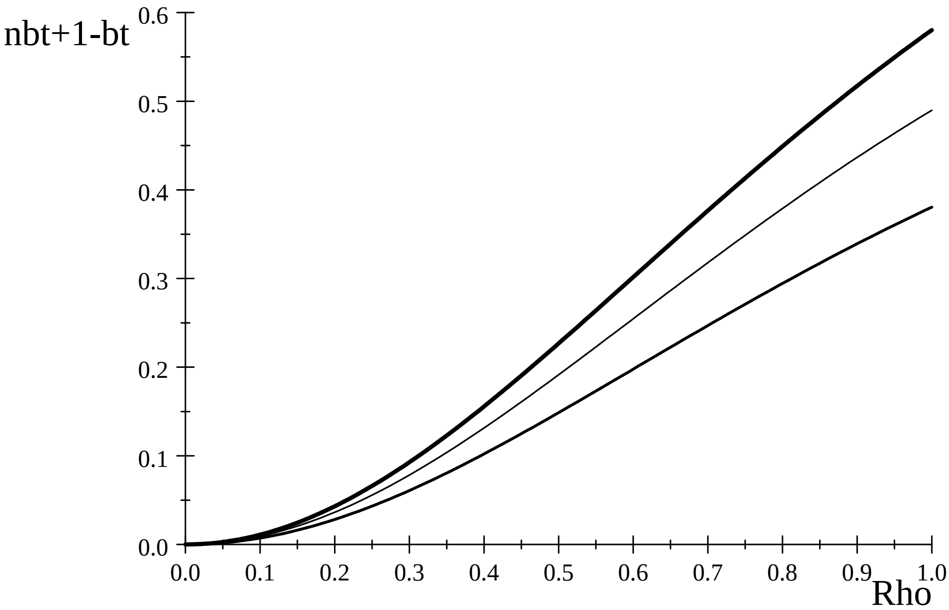

Figure 7. The effects of aspirations on lifetime wealth accumulation $(nb_{t+1} - b_t)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1/2$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1/2$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1/2$.

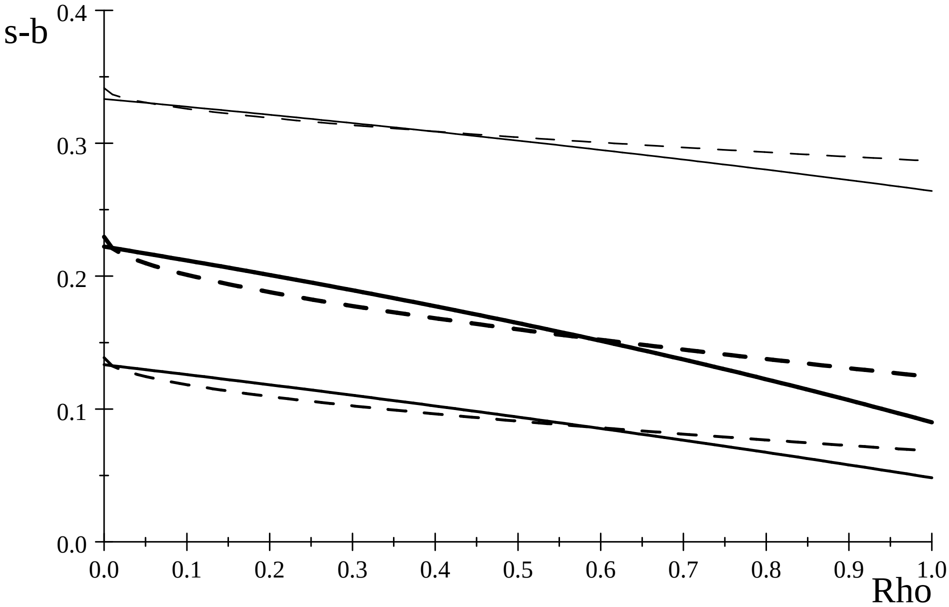

Figure 8. The effects of aspirations on stationary adult accumulation of wealth $(s-b)$.
Thick solid line: δ_c= $0, \delta_x = 0, \sigma = 1$. Medium solid line: δ_c=$1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: δ_c=$0, \delta_x = 1/2, \sigma = 1$. Thick dash line: δ_c= $0, \delta_x = 0, \sigma = 2$.
Medium dash line: δ_c= $1/2, \delta_x = 0, \sigma = 2$. Thin dash line: δ_c=$0, \delta_x = 1/2, \sigma = 2$.

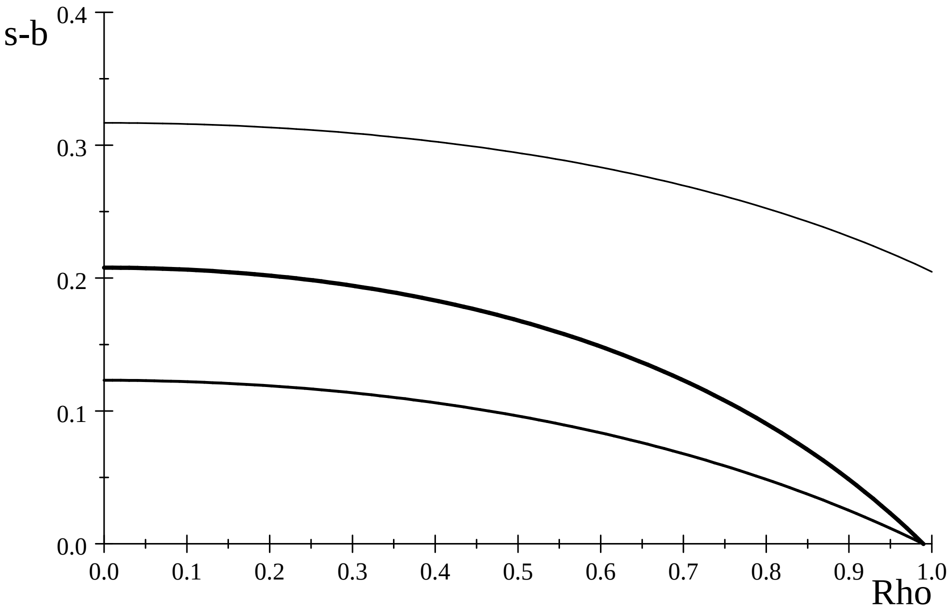

Figure 9. The effects of aspirations on stationary adult accumulation of wealth $(s-b)$.
Thick solid line: δ_c= $0, \delta_x = 0, \sigma$ =$1/2$. Medium solid line: δ_c=$1/2, \delta_x = 0, \sigma$ =$1/2$.
Thin solid line: δ_c=$0, \delta_x = 1/2, \sigma$ =$1/2$.

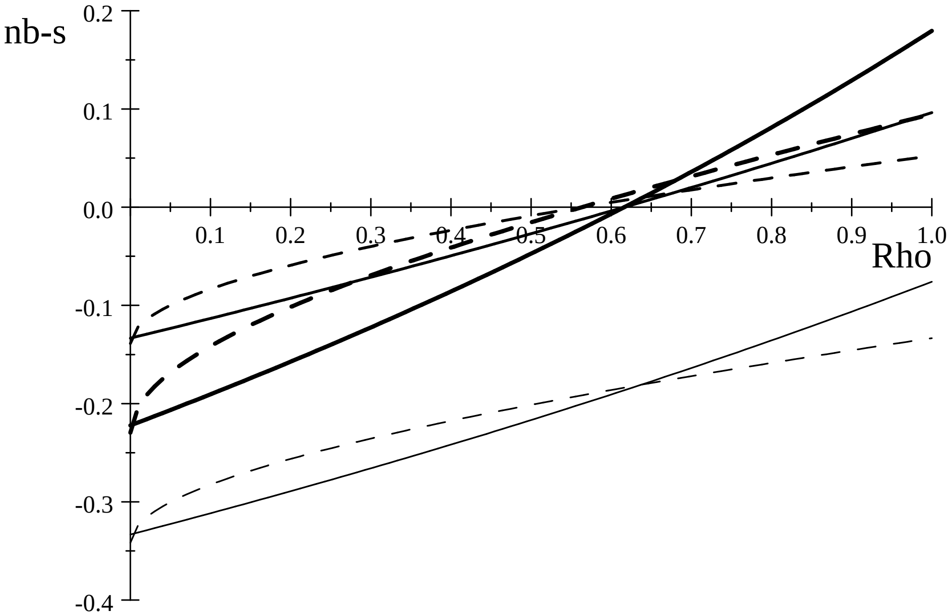

Figure 10. The effects of aspirations on stationary adult accumulation of wealth $(nb - s)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1$. Thick dash line: $\delta_c = 0, \delta_x = 0, \sigma = 2$.
Medium dash line: $\delta_c = 1/2, \delta_x = 0, \sigma = 2$. Thin dash line: $\delta_c = 0, \delta_x = 1/2, \sigma = 2$.

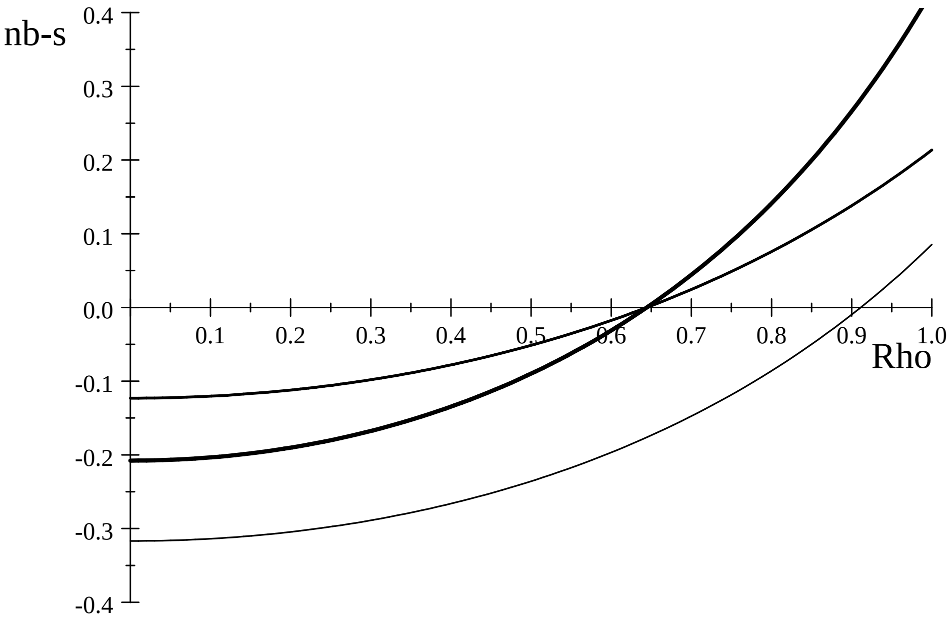

Figure 11. The effects of aspirations on stationary adult accumulation of wealth $(nb - s)$.
Thick solid line: $\delta_c = 0, \delta_x = 0, \sigma = 1/2$. Medium solid line: $\delta_c = 1/2, \delta_x = 0, \sigma = 1/2$.
Thin solid line: $\delta_c = 0, \delta_x = 1/2, \sigma = 1/2$.

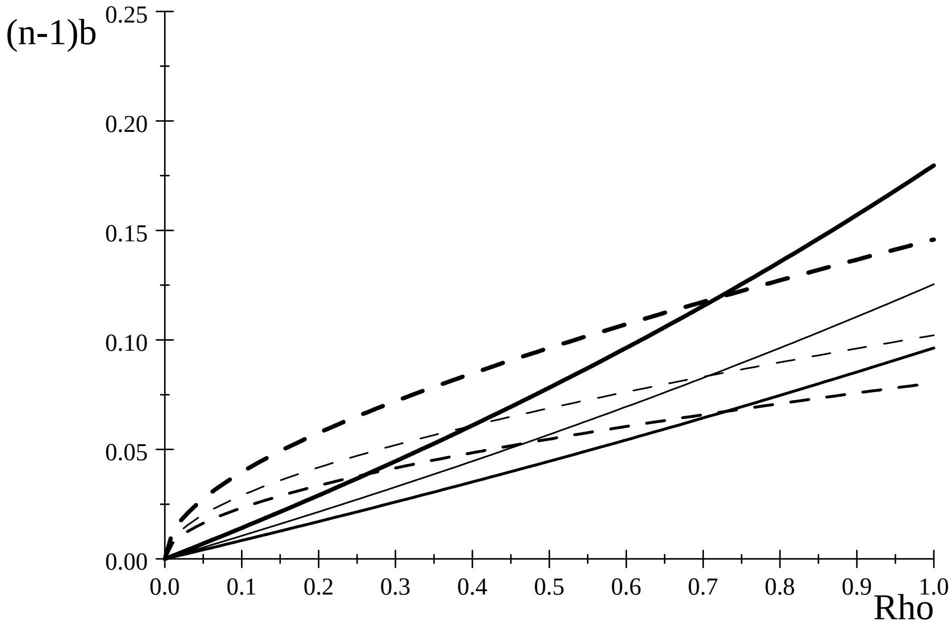

Figure 12. The effects of aspirations on stationary lifetime accumulation of wealth ($(n-1)b$)
Thick solid line: δ_c= $0, \delta_x = 0, \sigma = 1$. Medium solid line: δ_c=$1/2, \delta_x = 0, \sigma = 1$.
Thin solid line: δ_c=$0, \delta_x = 1/2, \sigma = 1$. Thick dash line: δ_c= $0, \delta_x = 0, \sigma = 2$.
Medium dash line: δ_c= $1/2, \delta_x = 0, \sigma = 2$. Thin dash line: δ_c=$0, \delta_x = 1/2, \sigma = 2$.

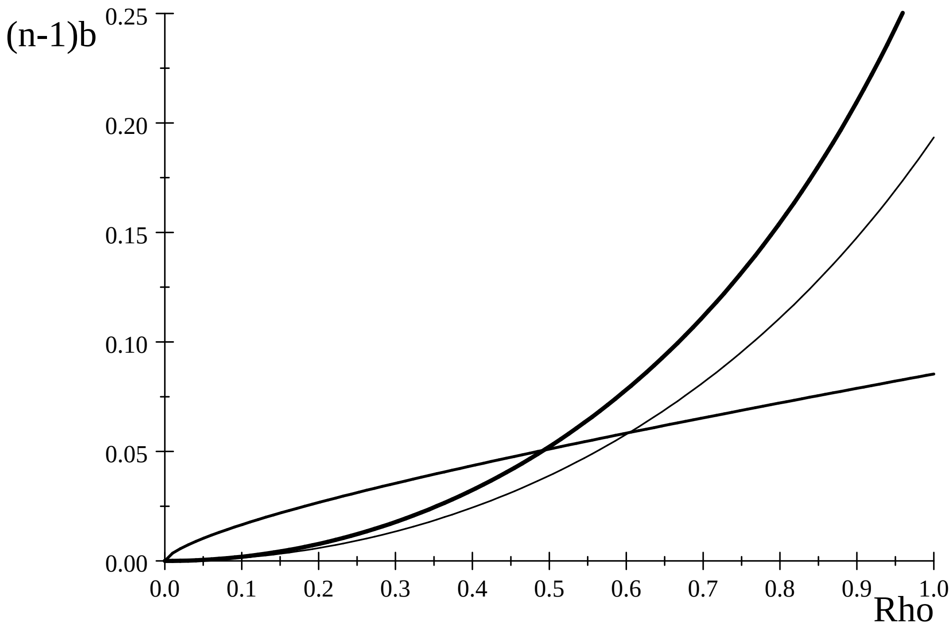

Figure 13 The effects of aspirations on stationary lifetime accumulation of wealth ($(n-1)b$)
Thick solid line: δ_c= $0, \delta_x = 0, \sigma$ =$1/2$. Medium solid line: δ_c=$1/2, \delta_x = 0, \sigma$ =$1/2$.
Thin solid line: δ_c=$0, \delta_x = 1/2, \sigma$ =$1/2$.