Static versus Dynamic Deferred Acceptance in School Choice: Theory and Experiment

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Abstract

In the context of school choice, we experimentally study how behavior and outcomes are affected when, instead of submitting rankings in the student proposing or receiving deferred acceptance (DA) mechanism, participants make decisions dynamically, going through the steps of the underlying algorithms. Our main results show that, contrary to theory, (a) in the dynamic student proposing DA mechanism, participants propose to schools respecting the order of their true preferences slightly more often than in its static version while, (b) in the dynamic student receiving DA mechanism, participants react to proposals by always respecting the order and not accepting schools in the tail of their true preferences more often than in the corresponding static version. As a consequence, for most problems we test, no significant differences exist between the two versions of the student proposing DA mechanisms in what stability and average payoffs are concerned, but the dynamic version of the student receiving DA mechanism delivers a clear improvement over its static counterpart in both dimensions. In fact, in the aggregate, the dynamic school proposing DA mechanism is the best performing mechanism.

Keywords: dynamic school choice, deferred acceptance, stability, efficiency.

JEL–Numbers: C78, C91, C92, D78, I20.

1 Introduction

Motivation

In school choice programs, parents can express preferences regarding the assignment of their children. Abdulkadiroğlu and Sönmez [1] show that prominent assignment procedures in the US had serious shortcomings (e.g., inefficiencies, manipulability) and employed matching theory to find a remedy. Under the student proposing deferred acceptance (DA) mechanism suggested in Abdulkadiroğlu and Sönmez [1], students submit an ordered list of schools to a central authority that then employs a matching algorithm to determine the final allocation of students to schools. This mechanism induces therefore a "static" game in which the students' strategies (or rankings, i.e., ordered lists) may or may not correspond to their true preferences. Their seminal paper has triggered a considerable theoretical and a growing experimental literature.

Much of the experimental literature has concentrated on showing that under the static student proposing deferred acceptance (DA) mechanism subjects do not realize that it is in their best interest to report the ranking that coincides with their true preferences (truth–telling). The reported truth–telling rates are between 47% and 75% in Chen and Kesten [4], 64% in Chen and Sönmez [6], between 44% and 65% in Klijn, Pais, and Vorsatz [19], and between 67% and 82% in Pais and Pintér [22]. Similarly, it is well known for the static school proposing DA mechanism, that is, when schools are making proposals and students are receiving offers, that in general students have incentives to misrepresent their true preferences. Featherstone and Mayefsky [10] however find in a laboratory experiment that subjects reveal their true preferences too often (manipulate not often enough).

The objective of our paper is to analyze by means of a laboratory experiment whether behavior is affected and better outcomes are obtained if, instead of submitting rankings to a central authority, subjects make decisions dynamically. Dynamic implementations of matching algorithms are used in college admissions in Brazil (Aygün and Bó [2]) and in Inner Mongolia (Chen and Kesten [5]), and in school choice in the Wake County Public School System (Dur *et al.* [8]). In the dynamic mechanisms we test in this paper, subjects go through the steps of an algorithm, either the student proposing or the school proposing DA. In the former case, subjects are prompted to send a new application upon receiving a rejection and, in the latter case, to immediately react to each offer received. This step by step approach may help subjects in an experiment to understand better how the underlying algorithm works. More importantly, by receiving feedback about their current status (direct experience), subjects have the possibility to use the information received, adapt behavior, and revise their strategies while the assignment process proceeds.

Theory

We first provide a theoretical analysis of the extensive–form games induced by the dynamic student proposing and the dynamic school proposing DA mechanisms when each school has exactly one seat. These results help to understand the incentives in the dynamic games and serve as a null

hypothesis for our experiment (when compared with the incentives in the static implementations).

For the student proposing DA mechanism we find that every Nash equilibrium outcome of the static game is also a Nash equilibrium outcome of the dynamic game. Since the set of stable matchings is a subset of the set of Nash equilibrium outcomes of the static game (see, e.g., Haeringer and Klijn [15]), every stable matching can also be sustained as an equilibrium outcome of the dynamic game. However, while proposers have incentives to submit lists that coincide with their true preferences in the static game, we show by means of an example that the strategy of "going down the list with respect to the true preferences," i.e., always applying to the best school that a student has not applied to yet, is not a weakly dominant strategy in the dynamic game, even though it remains a best reply when the other players also act according to a list (not necessarily corresponding to their true preferences).

We show for the school proposing DA mechanism that the set of Nash equilibrium outcomes is equal to the set of stable matchings. Then, since the set of Nash equilibrium outcomes in the static game is equal to set of stable matchings whenever each school has exactly one seat (Gale and Sotomayor [13], Roth [24]), we can conclude that the sets of Nash equilibrium outcomes coincide for both implementations in our setting. Moreover, it is well known that the static mechanism is manipulable and that the set of "truncation" strategies, whereby the student removes a tail of least preferred schools from her true preference list, is strategically exhaustive in the sense that for each strategy a student may use, the induced match can be replicated or improved upon by some truncation of her true preferences (see, e.g., Jaramillo *et al.* [18] and Roth and Vande Vate [25]). Nevertheless, in the dynamic game we can construct strategy profiles so that for all truncations of the true preferences, going down the list with respect to the truncation is not a best reply. So, a wider range of behavior can again be supported for the dynamic implementation.

Laboratory experiments

Our laboratory experiment is designed to analyze the effects of a dynamic implementation of the DA mechanism. We consider four problems with four students and four schools with one seat each that differ in the degree of heterogeneity of preferences of students and priorities of schools. In consistence with the theoretical analysis, subjects assume the role of students, schools are passive and simply follow their priorities. In order to account for the possibility of learning, each problem is played six times throughout an experimental session. Our treatment conditions correspond to the different matching mechanisms. The two benchmark treatments are the static student proposing DA mechanism (*treatment CI*) and the static school proposing DA mechanism (*treatment CS*). These treatments are compared with their dynamic counterparts, the dynamic student proposing DA mechanism (*treatment DI*) and the dynamic school proposing DA mechanism (*treatment DS*).

The experimental results show that subjects are often not truthful as proposers, but they respect the true order of their preferences slightly more often in treatment DI than in treatment CI. This difference is however not strong enough to find a clear overall improvement of the dynamic over the static implementation in terms of stability and efficiency (average payoffs). On the other hand, for the school proposing DA mechanism it turns out that subjects react to proposals by respecting their

true order and by not accepting schools in the tail of their true preferences significantly more often in the dynamic than in the static implementation. As a consequence, stable matchings are reached with a significantly higher frequency and average payoffs are substantially higher in treatment DS than in treatment CS. In fact, the dynamic school proposing DA mechanism outperforms even the two student proposing DA mechanisms in these two dimensions.

A couple of experimental papers look into dynamic implementations of deferred acceptance. Haruvy and Ünver [17] study repeated interactions between the two sides of the market. They do so under two information levels on others' preferences and, contrary to our experiment, allow proposers to repeat offers. Haruvy and Ünver [17] find that acting according to the true preferences is a good predictor of proposers' behavior, particularly under low information, and that the proposer optimal stable matching is reached in most cases. Our paper is more closely related to a recent study by Echenique, Wilson, and Yariv [9], who report an array of laboratory experiments where subjects go through the steps of the DA algorithm. One important difference with respect to our study is that in Echenique, Wilson, and Yariv [9] both sides of the market are active, whereas we take schools to be passive. Consequently, in our setting subjects do not have to form beliefs about the behavior of the other side of the market, a simplification that could help in making better decisions. This is true in what receivers' behavior is concerned, as receivers fail to truncate in Echenique, Wilson, and Yariv [9], while often reverting to this strategy in our treatment DS. In fact, and as mentioned above, receivers act according to a truncation significantly more in the dynamic school proposing DA than in its static version, a comparison that is absent in Echenique, Wilson, and Yariv [9]. Moreover, in Echenique, Wilson, and Yariv [9] a stable matching (mainly the receiver optimal one) is reached in about 48% of the cases. We obtain higher frequencies (between 68% in treatment CI and 90% in treatment DS), which may be explained by the fact that our experiment is simpler as markets are smaller.

Organization

We proceed as follows. In Section 2, we introduce the experimental design and procedures. Section 3 contains the theoretical analysis. In Section 4, we present and discuss our experimental results. In Section 5, we conclude. The detailed experimental instructions are relegated to the Appendix.

2   Laboratory experiment

Our experiment is designed to analyze the differences between a static and a dynamic implementation of the DA mechanism in school choice problems. The students i_1, i_2, i_3 and i_4 seek to obtain a seat at the schools s_1, s_2, s_3, and s_4. Each school offers exactly one seat. The preferences of the students and the priorities of the schools in the four problems subjects face during the experiment are depicted in Table 1. The information in Table 1 is common knowledge.

We selected the four problems in such a way that a wide variety of situations is covered. In Problem 1, each student is ranked last in her most preferred, third in her second most preferred,

Problem **1**	Preferences				Priorities			
	i_1	i_2	i_3	i_4	s_1	s_2	s_3	s_4
Best match	s_1	s_2	s_3	s_4	i_2	i_3	i_4	i_1
Second best match	s_2	s_3	s_4	s_1	i_3	i_4	i_1	i_2
Third best match	s_3	s_4	s_1	s_2	i_4	i_1	i_2	i_3
Fourth best match	s_4	s_1	s_2	s_3	i_1	i_2	i_3	i_4

Problem **2**	Preferences				Priorities			
	i_1	i_2	i_3	i_4	s_1	s_2	s_3	s_4
Best match	s_2	s_3	s_4	s_1	i_2	i_2	i_4	i_4
Second best match	s_3	s_4	s_1	s_2	i_1	i_3	i_3	i_1
Third best match	s_4	s_1	s_2	s_3	i_3	i_4	i_1	i_2
Fourth best match	s_1	s_2	s_3	s_4	i_4	i_1	i_2	i_3

Problem **3**	Preferences				Priorities			
	i_1	i_2	i_3	i_4	s_1	s_2	s_3	s_4
Best match	s_2	s_2	s_4	s_4	i_2	i_4	i_4	i_2
Second best match	s_1	s_3	s_3	s_1	i_3	i_3	i_1	i_1
Third best match	s_3	s_4	s_1	s_2	i_4	i_1	i_2	i_3
Fourth best match	s_4	s_1	s_2	s_3	i_1	i_2	i_3	i_4

Problem **4**	Preferences				Priorities			
	i_1	i_2	i_3	i_4	s_1	s_2	s_3	s_4
Best match	s_1	s_1	s_1	s_1	i_1	i_1	i_1	i_1
Second best match	s_2	s_2	s_2	s_2	i_4	i_2	i_2	i_2
Third best match	s_3	s_3	s_3	s_3	i_3	i_4	i_3	i_3
Fourth best match	s_4	s_4	s_4	s_4	i_2	i_3	i_4	i_4

Table 1: Preferences of students over schools (left) and priorities of schools over students (right).

second in her third most preferred, and first in her least preferred school. This problem thus resembles a situation where preferences (and priorities) are heterogeneous and the tension between the two sides of the market is maximal. The other extreme case, homogenous preferences (and priorities in the relevant parts) and no conflict between the two sides of the market, corresponds to Problem 4. Problems 2 and 3 are intermediate situations.

During the experiment, subjects assume the role of students. Schools are not strategic players. Given the information in Table 1, the subjects' task in the static mechanisms is to submit a ranking over schools (not necessarily the true preferences) to be used by a central clearinghouse to assign students to schools. Subjects are free to submit a list of less than four schools. So, for

example, a subject is allowed to submit the ranking s_3, s_1. We consider two static mechanisms, the "student proposing deferred acceptance" (treatment CI) and the "school proposing deferred acceptance" (treatment CS) mechanisms. For the particular school choice settings at hand, the two mechanisms are as follows:

STATIC MECHANISMS (CI AND CS)

Step 1.

(CI) Each student sends an application to the school she ranked first. If a student did not list any school, then she will not send any application.

Each school temporarily accepts the applicant with the highest priority and rejects all other applicants.

(CS) Each school offers a seat to its highest priority student.

Each student temporarily accepts the school she likes best according to her submitted ranking and rejects all other offers.

Step 2.

(CI) Each student who got rejected in the previous step sends an application to her next highest ranked school. If there is no such school, then the student will not send any application.

Among the previously retained (if any) and new applications, each school temporarily accepts the applicant with the highest priority and rejects all other applicants.

(CS) Each school that got rejected in the previous step makes an offer to its next highest priority student.

Among the previously retained (if any) and new offers, each student temporarily accepts the school she likes best according to her submitted ranking and rejects all other offers.

Steps 3, 4, . . .

(CI) Step 2 is repeated until no more applications are rejected. Each student is assigned to the school that holds her application. If a student got rejected by all schools she ranked, she is left unmatched.

(CS) Step 2 is repeated until no more offers are rejected. Each student is assigned to the school she holds the offer from. If a student does not hold an offer, she is left unmatched.

In our dynamic implementations students do not submit a ranking to the central clearinghouse. Instead, they go through each step of the corresponding static algorithm. So, while the static mechanisms induce a simultaneous move game, in the dynamic implementation subjects face a sequential move game. The descriptions of the dynamic versions of the student proposing DA mechanism (treatment DI) and the school proposing DA mechanism (treatment DS) are as follows:

Dynamic mechanisms (DI and DS)

Phase 1.

(DI) Each student is asked to send an application that either contains the name of a school or is left empty. If a student decides to send an empty application, she will not be able to send future applications.

Each school temporarily accepts the applicant with the highest priority and rejects all other applicants.

Each student is informed about the status of the application, that is, if it is retained or whether it got rejected. If a student sent an empty application, she is informed that she did not obtain a seat.

(DS) Each school offers a seat to its highest priority student.

Each student decides to retain at most one of the offers received.

Phase 2.

(DI) The students who sent an application to a school in the previous phase and got rejected are asked to send a new application (that consists of a school the student has not yet applied to or is left empty). If a student decides to send an empty application, she will not be able to send future applications.

Among the previously retained (if any) and new applications, each school temporarily accepts the applicant with the highest priority and rejects all other applicants.

Each student who sent a new application to a school and each student whose application was retained in the previous phase is informed about the status of the application, that is, if it is retained or whether it got rejected. Each student who sent an empty application is informed that she did not obtain a seat.

(DS) Whenever a school's offer is rejected by a student, the school makes an offer to its next highest priority student.

Each student who received at least one new offer can retain at most one offer (among the new offers and the retained one, if it exists).

Phases 3, 4, . . .

(DI) Phase 2 is repeated until each student decides not to send any further application, has been rejected by all schools, or has her application retained by a school. Each student is assigned to the school that holds her application. If a student got rejected by all schools or sent an empty application, she is left unmatched.

(DS) Phase 2 is repeated until no more offers are rejected. Each student is assigned to the school she holds the offer from. If a student does not hold an offer, she is left unmatched.

The experiment was programmed within the z–Tree toolbox provided by Fischbacher [12] and carried out at Lineex (www.lineex.es) hosted at the University of Valencia. In total, 192 undergraduates from various disciplines participated in the experiment. We run one session with 48 subjects per treatment. At the beginning of the session, subjects were anonymously matched into groups of four. Within each group, one subject was assigned the role of student i_1, another subject the role of i_2, and so forth. Groups and roles did not change over the course of the experiment. Participants were told that the experiment would take a total of 24 rounds and that preferences and priorities would change each six rounds. Before the first round, subjects went individually over an illustrative example in order to get used to the matching procedure. Afterwards, we implemented a trial period that was not taken into account for payment. This helped subjects to get familiar with the computer software. At the beginning of each round, the computer screen presented the preferences of all group members and the priorities of the four schools. Subjects took then their respective decisions. At the end of each round, subjects got to know the outcome of the matching process. In treatments DI and DS, due to the sequential nature of the game, subjects knew the status of their application at each point of the process. No explicit information about the behavior of the other group members was provided. At the end of the experiment, one round was randomly selected for payment. Subjects received 24, 20, 16, and 12 experimental currency units (ECU) if they ended up in their most, second most, third most, and least preferred school. They got 7 ECU if they remained unmatched. Each ECU was worth 1 Euro.[1]

3   Theory

In this section we start with the definition of a school choice problem and then provide a broad description of the extensive–form games induced by the two dynamic mechanisms, DI and DS. Since the two mechanisms differ in very important aspects, we then focus on each game in turn.

3.1   School choice problem

A *school choice problem* is a five-tuple $\langle I, S, q, P_S, P_I \rangle$ where

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[1]The experimental instructions and programs are available from the authors upon request.

- I is a finite set of students (individuals).

- S is a finite set of schools.

- $q_s = 1$ for each $s \in S$, where q_s is the number of seats at $s \in S$.

- $P_S = (P_s)_{s \in S}$ is a profile of strict priority relations for schools, where P_s is a complete, irreflexive, and transitive binary relation over $I \cup \{s\}$ for $s \in S$.

- $P_I = (P_i)_{i \in I}$ is a profile of strict preference relations for students, where P_i is a complete, irreflexive, and transitive binary relation over $S \cup \{i\}$ for $i \in I$.

We say that $s \in S$ finds $i \in I$ *acceptable* if and only if $iP_s s$ and that $i \in I$ finds $s \in S$ *acceptable* if and only if $sP_i i$. In the problems we consider, a *matching* of students to schools is a function $\mu : I \cup S \to I \cup S$ such that for all $i \in I$ and $s \in S$:

- $\mu(i) \in S \cup \{i\}$,

- $\mu(s) \in I \cup \{s\}$, and

- $s = \mu(i)$ if and only if $i = \mu(s)$,

where the notation $v = \mu(v)$ means that agent v is unmatched under μ and $v' = \mu(v)$ denotes that agent v' is agent v's partner under matching μ. We let $\mathcal{M}$ denote the set of matchings.

A matching μ is *individually rational* if and only if $\mu(v)P_v v$ for all $v \in V$. Moreover, we say that a pair (i, s) *blocks* the matching μ if and only if $sP_i\mu(i)$ and $iP_s\mu(s)$. Since in the problems we consider each school finds each student acceptable and likewise each student finds each school acceptable, every matching in $\mathcal{M}$ is individually rational. Therefore, we say that a matching is *stable* if there is no pair that blocks it. We denote by $S(P)$ the set of all stable matchings. The set of stable matchings is always non–empty (Gale and Shapley [13]). Moreover, for each problem, there is a student optimal stable matching that is weakly preferred by all students to any other stable matching in $S(P)$; similarly, there is a school optimal stable matching that is weakly preferred by all firms to any other stable matching in $S(P)$ (Gale and Shapley [13]).

3.2 General description of the extensive–form games

Given the definition of a school choice problem, we are now in a position to describe the extensive–form games agents play when confronted with the dynamic mechanisms, DI and DS. The extensive–form games are composed of the following elements:

- A finite set of players I, which corresponds to the set of students.

- A set of actions $A_i = S \cup \{i\}$ for each student i. In DI, this set includes all possible proposals that might potentially be made by i at some point in the game, including the "no school" option (modelled as proposing to oneself). In DS, choosing s means to hold the proposal from school s and to reject every other proposal that might have been received or held from a previous step, and to choose i means rejecting all proposals. We let $A \equiv \cup_{i \in I} A_i$.

- A set of nodes or histories X, where

    - the initial node or empty history x_0 is an element of X,
    - each $x \in X \setminus \{x_0\}$ takes the form $x = (a_1, a_2, ..., a_k)$ for some finitely many actions $a_i \in A$, and
    - if $(a_1, a_2, ..., a_k) \in X \setminus \{x_0\}$ for some $k > 1$, then $(a_1, a_2, ..., a_{k-1}) \in X \setminus \{x_0\}$.

  When $x = (x', a_1, \ldots, a_k)$, $k \geq 1$, we say that x' is a *predecessor* of x.

  We let $A(x) \equiv \{a \in A : (x, a) \in X\}$ denote the set of actions available to a student whose turn it is to move after history $x \in X$. Clearly, we have $A(x) \subseteq S \cup \{i\}$ for all $x \in X$ where it is student i's turn to move.

- A set of end nodes $E \equiv \{x \in X : (x, a) \notin X \text{ for all } a \in A\}$.

- A function $\iota : X \setminus E \to I$ that indicates whose turn it is to move at each decision node in $X \setminus E$.

  We let $X_i \equiv \{x \in X \setminus E : \iota(x) = i\}$ denote the set of decision nodes belonging to student i.

- A partition $\Im$ of the set of decision nodes $X \setminus E$, such that if x and x' are in the same element of the partition, then (*i*) $\iota(x) = \iota(x')$, and (*ii*) $A(x) = A(x')$. The information available to student $\iota(x)$ when, after history x, it is her turn to move is described by $\Im(x)$, the element of the partition $\Im$ that contains x.

  We let $\Im_i \equiv \{\Im(x) : x \in X \setminus E \text{ with } \iota(x) = i\}$ denote the set of information sets belonging to student i.

- An outcome function $g : E \to \mathcal{M}$ that associates each terminal history with a matching.

- For each student $i \in I$, a preference ordering P_i of schools.

  We are now in a position to give a definition of a strategy in this extensive–form game.

Definition 1: A (*pure*) *strategy* for student i in this extensive–form game is a function $\sigma_i : \Im_i \to A$ such that $\sigma_i(\Im(x)) \in A(x)$, for all $x \in X_i$.

Finally, even though we have provided a general description of the games induced by both DI and DS, they are clearly different mechanisms. For example, the set of actions available at a node x and the set of students allowed to move depend on the particular rules of the games induced by each mechanism. In Sections 3.3 and 3.4, we focus on each of these games in turn.

3.3   Game induced by DI

Two rules of DI, which we describe next, are of special importance.

Rule 1: In DI students cannot pass turns, i.e., if a student sends an application to "no school" at some step, she is not allowed to send applications at future steps.

Rule 2: In DI a student cannot send an application to a school more than once.

Whenever a student makes a new proposal it is because she was rejected and by Rule 2 she makes a new proposal to "no school" or to a school she has not proposed to before. Moreover, by Rule 1, after proposing to "no school" the student can no longer make proposals.

Therefore, in what the strategy of a student is concerned, her observed decisions induce an ordered list. Formally, consider student i, a profile of strategies $\sigma = (\sigma_i, \sigma_{-i})$ leading to $x \in E$, and let $x^1, \ldots, x^k, k \geq 1$, denote all the predecessors of x where student i proposes. Without loss of generality, let x^j be a predecessor of x^{j+1}, $j = 1, \ldots, k-1$.

Definition 2: In the game induced by DI, the *ordered list revealed by student i in the play of σ* is $Q_i^\sigma = \sigma_i(\Im(x^1)), \ldots, \sigma_i(\Im(x^k))$, if $\sigma_i(\Im(x^k)) = i$, and $Q_i^\sigma = \sigma_i(\Im(x^1)), \ldots, \sigma_i(\Im(x^k)), i$, otherwise.

Note that no two elements of this list are the same, so that this is a strict partial order of schools, having i as a last element. In fact, by Rule 1, i can only be the last element of this list and, by Rule 2, no school can appear twice.

Claim 1: The outcome of the game induced by DI when students use σ coincides with the outcome of the game induced by CI when students submit the ordered lists revealed in the play of σ.

Proof. Immediate from the definition of ordered lists revealed in the play of a strategy profile and from the fact that DI emulates CI. □

Still, the information revealed in the play of a strategy leading to a particular terminal node may only provide a partial order of schools, so that several (complete) orders may be compatible with it. Moreover, the strategy space is clearly much wider in DI than in CI, so that one strategy in DI may lead to different revealed ordered lists.

A class of strategies that deserves attention for its simplicity is the class of preference strategies. A student using a preference strategy always proposes to the best ranked school that she has not proposed to yet according to some preference ordering. Formally, consider student i, let Q_i be an ordered list of preferences over the elements of $S \cup \{i\}$, and let $\max_{Q_i}(S')$ be the best ranked element according to Q_i in the set $S' \subseteq S \cup \{i\}$, with $i \in S'$.

Definition 3: In the game induced by DI, σ_i is a *preference strategy* if there exists an ordered list Q_i for student i such that for all $x \in X_i$, $\sigma_i(\Im(x)) = \max_{Q_i}(A(x))$. In this case, we say that σ_i is the *preference strategy consistent with Q_i*.

Remark 1: When student i uses the preference strategy consistent with Q_i, the ordered list that is revealed (at least partially) in any play of the game is Q_i.

Claim 2: Every Nash equilibrium outcome in the game induced by CI is a Nash equilibrium outcome in the game induced by DI.

Proof. See the Appendix. □

Haeringer and Klijn [15] show that every stable and possibly some unstable matchings (when the priority structure exhibits so-called Ergin cycles) can be sustained in Nash equilibrium in the game induced by CI. It therefore follows from Claim 2 that the same result holds in the game induced by DI.

Finally, whereas submitting the true preferences to the static mechanism is a weakly dominant strategy (Dubins and Freedman [7] and Roth [23]), the preference strategy consistent with the true preferences is not weakly dominant in the game induced by DI. In fact, when the other players use strategies that are not preference strategies, deviating from the preference strategy consistent with the true preferences may trigger the revelation of other, more favorable ordered lists by the other students and lead to a better outcome.

Claim 3: In the game induced by DI, the preference strategy consistent with the true preferences is in general not a weakly dominant strategy.

Proof. See the Appendix. □

Additionally, in the game induced by DI, due to the lack of information on others' moves (as the game proceeds a student remembers her own proposals and schools' reactions to these proposals), there are very few proper subgames, and those that do exist appear only at the end of the tree. It follows that the concept of subgame perfect Nash equilibrium does not allow us to refine the set of predicted outcomes either and our best prediction for the obtained matchings is a superset of the Nash equilibrium outcomes in the game induced by CI.

3.4   Game induced by DS

It is a well known fact that the set of Nash equilibrium outcomes of the game induced by CS when each school has one seat coincides with the set of stable matchings (Gale and Sotomayor [13], Roth [24]). In the following Claim, we show that the set of Nash equilibrium outcomes in the game induced by DS coincides with the set of stable matchings and, therefore, with the set of Nash equilibrium outcomes in the game induced by CS.

Claim 4: The set of Nash equilibrium outcomes in the game induced by DS coincides with the set of stable matchings.

Proof. See the Appendix. □

The following rule of DS is of special importance.

Rule 3: In DS once a student holds the proposal of a school, she can only reject it upon receiving and holding another school's proposal.

Since in the game induced by DS a student can receive a proposal from a school at most once, we can guarantee that the decisions made by a student along the path of a play reveal a strict partial order of schools. Moreover, each school whose proposal is held by a student is revealed to be better than being unmatched in which case, by Rule 3, being unmatched cannot be revealed preferred to being assigned to that school. Therefore, the rules of the game guarantee that transitivity can never be violated. Formally, consider student i, a profile of strategies $\sigma = (\sigma_i, \sigma_{-i})$ leading to $x \in E$, and let S' be the set of schools held by student i in the course of play of σ.

Definition 4: The *ordered list that is revealed by student i in the play of σ* is $Q_i^\sigma = i$, when $S' = \emptyset$, and is $Q_i^\sigma = s'_k, \ldots, s'_1, i$, where $s'_j \in S'$, $j = 1, \ldots, k$, is the jth school whose proposal is held by i in the course of play of σ.

Claim 5: The outcome of the game induced by DS when students use σ coincides with the outcome of the game induced by CS when students submit the ordered lists revealed in the play of σ.

Proof. Immediate from the definition of ordered lists revealed in the play of a strategy profile and from the fact that DS emulates CS. □

As it happens in the game induced by DI, the class of preference strategies is of particular interest. A student using a preference strategy always holds the proposal from the best ranked school in some ordered list of schools among those schools that propose at some step and the school whose proposal may be held from a previous step (if any). Formally, consider student i and let Q_i be an ordered list over the elements of $S \cup \{i\}$.

Definition 5: In the game induced by DS, σ_i is a *preference strategy* for student i if there exists an ordered list Q_i such that for all $x \in X_i$, $\sigma_i(\Im(x)) = \max_{Q_i}(A(x))$. In this case, we say that σ_i is the *preference strategy consistent with Q_i*

Preference strategies are a natural form of behavior and Claim 6 refers to a particular subclass, the subclass of preference strategies consistent with a truncation of the true preferences.

In the game induced by CS, a student employs a truncation strategy if she removes a tail of least preferred partners from the true preference list. For example, if the true ranking is s_2, s_3, s_4, s_1, then s_2, s_3 is a truncation strategy, but s_2, s_4 is not. In the game induced by CS, truncations may play an important role given that, for each strategy a student may use, the induced match can be replicated or improved upon by some truncation of the true preferences of the student (Roth and Vande Vate [25]). This fact, together with Claim 5 imply that, in the game induced by DS, given a profile of preference strategies for the students other than i, there is a best reply for i that consists of a preference strategy consistent with a truncation of the true preferences.

Nevertheless, for some profiles of strategies of the other players (including strategies other than preference strategies), there is no strategy consistent with a truncation of the true preferences that can be a best reply in DS. Therefore, contrary to what happens in CS, when choosing a best reply, agents cannot restrict to the class of truncations.

Claim 6: Consider the game induced by DS. Given a strategy–profile of all but one student i, there need not be a best reply of student i that is a preference strategy consistent with a truncation of her true preferences.

Proof. See the Appendix. ☐

Finally, we can also show that, as it happens in DI, given the low level of information on other players' chosen strategies, the game induced by DS has very few subgames and subgame perfect Nash equilibrium does not give us sharper predictions on the possible outcomes. Therefore, in the game induced by DS, the set of subgame perfect Nash equilibrium outcomes coincides with the set of Nash equilibrium outcomes.

4   Experimental results

We now present the results of our experiment. First, we analyze the behavior of the subjects (Section 4.1). Afterwards, we show how behavior affects stability (Section 4.2) and welfare (Section 4.3).

4.1   Behavior

Dubins and Freedman [7] and Roth [23] show that subjects have incentives to reveal preferences truthfully in treatment CI. Nevertheless, it is well documented in the experimental literature that agents do not always realize that it is in their best interest to reveal their true preferences when confronted with centralized strategy–proof mechanisms (see, e.g., Calsamiglia *et al.* [3], Chen and Kesten [4], Chen and Sönmez [6], Featherstone and Niederle [11], Guillen and Hing [14], Klijn *et al.* [19],[20], and Pais and Pintér [22]). We analyze whether a dynamic implementation of the student proposing DA algorithm where, as the algorithm proceeds, subjects receive feedback about their previous applications and corresponding decisions, mitigates or aggravates this problem.

Given some student's true preferences, we write s^k for the school that is ranked in k-th position (from best to worst), i.e., the k-th best school. For example, if the student's true ranking is s_2, s_3, s_4, s_1, then $s^4 = s_1$.

Consider the mechanism CI. Haeringer and Klijn [15] show that a strategy that does not rank all schools is weakly dominated. Now suppose a student ranks all schools. Haeringer and Klijn [15] show that a strategy that for some $i < j$ ranks school s^j somewhere above/before school s^i is weakly dominated by a strategy that is obtained by switching s^j and s^i back in the (true) relative order. Therefore, the number of such switches can be employed to measure a subject's degree of

(ir)rationality. For example, if the true ranking is s_2, s_3, s_4, s_1 and a subject reports in treatment CI the list s_4, s_2, she switches schools s^1 and s^3 and schools s^2 and s^3. But we do not have any information on how she for instance relates schools s^2 and s^4.

Now consider the mechanism DI. Obviously the induced game has a much richer structure than that of CI. In particular, strategies can be much more complex. And, in fact, by Claim 3, the preference strategy consistent with the true preferences need not be weakly dominant. For this reason, we expect at least as many switches —i.e., situations in which a student first proposes to a school s^j and in a later step to a better school s^i ($i < j$)— as in the case of CI, except that now we should take into account that there is only limited information about the student's strategy. More precisely, we will often not be able to determine whether a subject would make an irrational switch, simply because the game has not reached a situation in which this could happen/be observed.

HYPOTHESIS 1A: *Subjects switch pairs weakly more often in treatment DI than in treatment CI.*

Table 2 presents for each i, j with $i < j$ in row $s^i - s^j$ the frequency with which (the better) school s^i is switched with (the worse) school s^j.

	CI	Problem				**DI**	Problem			
	All	1	2	3	4	All	1	2	3	4
$s^1 - s^2$	*0.32*	0.42	0.32	0.27	0.32	*0.28*	0.30	0.22	0.24	0.35
$s^1 - s^3$	*0.25*	**0.34**	**0.21**	0.15	0.28	*0.10*	0.08	0.04	0.05	0.23
$s^1 - s^4$	*0.14*	**0.20**	0.14	**0.10**	0.12	*0.08*	0.11	0.05	0.00	0.14
$s^2 - s^3$	*0.19*	0.27	0.15	0.18	0.17	*0.13*	0.10	0.07	0.08	0.26
$s^2 - s^4$	*0.11*	0.17	0.08	0.12	0.07	*0.11*	0.15	0.10	0.02	0.17
$s^3 - s^4$	*0.08*	0.15	0.07	0.04	0.05	***0.30***	**0.51**	**0.38**	0.15	**0.17**
Truth–telling	*0.51*	0.42	0.55	0.53	0.53	*0.62*	0.59	0.72	0.72	0.46

Table 2: Frequencies that two schools are switched under the static (left) and dynamic (right) student proposing DA mechanisms. Significant treatment effects obtained from panel data regressions with individual random effects are indicated in boldface for the treatment with the higher frequency. We also display the frequency of subjects that cannot be rejected to tell the truth.

The left part of the table shows that subjects often switch pairs in treatment CI. Also, in each of the four problems, the switching frequency is highest for the pair $s^1 - s^2$ and lowest for the pair $s^3 - s^4$. A comparison with the right part of the table reveals that switching frequencies tend to be lower in treatment DI. For example, in Problems 1-3 there are very few subjects who apply to their third best or worst school without having applied before to their most preferred or second most preferred school. Consequently, it is no surprise that the switching frequencies for several of the pairs involving s^1 are significantly higher in treatment CI. On the other hand, we can also see that in Problems 1,2, and 4, subjects switch the pair $s^3 - s^4$ significantly more often in treatment

DI than in treatment CI. Finally, and with respect to the aggregated data, we find that the pairs $s^1 - s^2$, $s^1 - s^3$, $s^1 - s^4$, and $s^2 - s^3$ are switched more often in treatment CI than in treatment DI, while the pair $s^3 - s^4$ is switched more often in treatment DI than in treatment CI.

Table 2 also shows the frequencies with which subjects cannot be rejected to tell the truth. In treatment CI this is equal to determining the frequencies that subjects submit lists equal to their true preferences, while we calculate in treatment DI the frequencies whether subjects "go down the list" (a subject applies first to her most preferred school; then, if she got rejected at some point of the game, an application to the second most preferred school is submitted; and so forth, until the game stops). One observes higher frequencies in treatment DI for Problems 1-3, which is due to two effects: subjects switch less pairs in these problems in treatment DI and frequencies are *ex ante* biased towards treatment DI since not all of the possible comparisons are observed in this treatment.

Result 1A: *With the exception of the pairs $s^2 - s^4$ and $s^3 - s^4$, subjects switch pairs more often in treatment CI than in treatment DI. Moreover, in Problems 1-3 there are substantially more subjects who cannot be rejected to tell the truth in treatment DI than in treatment CI.*

The static school proposing mechanism differs from the static student proposing mechanism in that students have now incentives to manipulate. In the dynamic implementation of the school proposing mechanism we would like to find out whether subjects understand that truth–telling might not always be optimal. More specifically, as a natural counterpart to Hypothesis 1A, we now seek to study whether and how a dynamic implementation of the school proposing DA algorithm may change subjects' behavior.

Consider the mechanism CS. As mentioned in Section 3.4, a notable class of manipulations is that of strategies that involve a truncation of the true preferences. Obviously, truncation strategies have a very simple structure relative to the true preferences. But maybe more importantly, the set of truncation strategies is known to be strategically exhaustive, i.e., each school a student is matched to when manipulating her preferences can be obtained or strictly improved upon by using some truncation strategy (Roth and Vande Vate [25]). Truncation strategies have been observed in practice, for instance, in the sorority rush (Mongell and Roth [21]). Therefore, it is of interest to determine to which extent subjects employ truncation strategies in our experiment.

Now consider the mechanism DS. Similarly to the previous discussion on CI and DI, the mechanism DS induces a much richer game than CS. Additionally, according to Claim 6, the class of preference strategies consistent with truncations need not be strategically exhaustive. Therefore, we expect subjects to truncate weakly less often in treatment DS than in treatment CS. We do note however that there is only limited information about the students' strategies in treatment DS. More precisely, we are often not able to determine whether a subject employs a strategy based on a truncated list, simply because the game has not reached a situation in which this could be concluded.

HYPOTHESIS 1B: *Subjects truncate weakly less often in treatment DS than in treatment CS.*

First, it turns out that under CS subjects switch pairs with frequencies that are similar to those presented in Table 2 for treatment CI, while under DS almost no subject accepts an offer from a school s^j after having temporarily accepted before an offer from a more preferred school s^i ($i < j$). In fact, aggregated over all problems and pairs $s^i - s^j$ ($i < j$), the switching frequency is 25% in treatment CS but only 2% in treatment DS.

We now present in Table 3 the frequencies that an offer from a school is acceptable. More precisely, for CS we consider the frequency that a school is ranked in the submitted list. For DS we consider the frequency that an offer from a school is accepted in some step of the algorithm conditional on that the subject does not hold an offer from another school.

CS		Problem				DS		Problem			
	All	1	2	3	4		All	1	2	3	4
s^1	*0.96*	0.97	0.98	0.92	0.96		*0.99*	0.99	1.00	0.99	1.00
s^2	***0.96***	**0.97**	0.93	**0.99**	0.93		*0.90*	0.82	0.98	0.82	0.98
s^3	***0.87***	**0.95**	**0.86**	**0.80**	0.88		*0.67*	0.62	0.55	0.56	0.93
s^4	***0.78***	**0.84**	**0.75**	**0.74**	0.80		*0.44*	0.45	0.44	0.02	0.83
Truncation	*0.46*	0.31	0.43	0.52	0.58		*0.98*	0.96	0.99	0.97	0.99

Table 3: Frequencies that a school is acceptable under the static (left) and dynamic (right) school proposing DA mechanisms. Significant treatment effects obtained from panel data regressions with individual random effects are indicated in boldface for the treatment with the higher frequency. We also display the frequency of subjects that cannot be rejected to use a truncation strategy.

The table shows that only a few schools are declared unacceptable in the static school proposing DA mechanism. Also, the data for this mechanism is consistent in the sense that for all $i < j$, the frequency that school s^i is accepted is greater than or equal to the frequency with which school s^j is accepted (the only exception is the pair $s^1 - s^2$ in Problem 3). This consistency can also be found in treatment DS, but, most importantly, the acceptance rates of s^2, s^3, and s^4 are substantially lower in Problems 1-3. Hence, it is no surprise that the aggregated acceptance rates for those schools are significantly lower in treatment DS than in treatment CS.

In the last row of Table 3 we display the frequency of subjects that cannot be rejected to use a truncation strategy (which nonetheless take different meanings in the treatments CS and DS). We see that only a few subjects can be rejected to play truncation strategies in treatment DS. Recall that the information we can collect from the decisions of the subjects in treatment DS is limited due to the nature of the mechanism. For example, if a subject initially receives only an offer from s^2 that she accepts and rejects later on offers from s^3 and s^4, we can only conclude that the ordered list of preferences revealed by this subject in the play of the game includes s^2 and that s^2 is better ranked than both s^3 and s^4, but we do not know how s^3 and s^4 compare and whether these two schools would be accepted in case the subject was not holding s^2's proposal in the first

place. Since there are many truncation strategies that are consistent with the supposed behavior, we cannot reject that she behaves according to a truncation. This is different for a subject who rejects an offer from s^2 in the first step of the algorithm and temporarily accepts an offer from s^3 in the second step. Such a subject's strategy is clearly not a preference strategy consistent with a truncation of the true preferences.

Result 1B: *In Problems 1-3 subjects accept offers from low ranked schools significantly less often in treatment DS than in treatment CS. Moreover, there are substantially more subjects who cannot be rejected to play truncation strategies in treatment DS than in treatment CS.*

4.2 Stability

Stability is the central solution concept in matching markets. In the problems we consider, a matching is said to be "blocked" if there is a student that prefers to be assigned to some school with a slot that is either available or occupied by another student with a lower priority. Then, a matching is "stable" if no student can form a blocking pair with any school. Problem 1 has four stable matchings. In the student optimal stable matching, which is obtained in treatments CI and DI when all students reveal their preferences truthfully, each student is assigned to her top school. Similarly, the school optimal stable matching, the matching obtained in treatments CS and DS under truth–telling, is such that each student is assigned to her worst school. In the two "compromise" stable matchings all students are assigned to their second most and third most preferred school, respectively. Problem 2 has 3 and Problem 3 has 2 stable matchings. Finally, there is one stable matching in Problem 4 (where for each $k = 1, \ldots, 4$, student i_k is matched to school s_k) and all four mechanisms coincide under truth–telling.

There is no reason to expect that stability is higher under the dynamic implementations of the mechanisms. In fact, according to Claim 2, all Nash equilibrium outcomes in CI (which includes all stable and possibly some unstable matchings) can also be sustained in Nash equilibrium in the game induced by DI. Moreover, according to Claim 4, the set of Nash equilibrium outcomes of the game induced by DS is the set of stable matchings which hence coincides with the set of Nash equilibrium outcomes in CS.

HYPOTHESIS 2: *The frequency of stable matchings is not higher under the dynamic implementations.*

Nevertheless, we have already seen that, for Problems 1-3, the likelihood of switching pairs of schools in the student proposing mechanisms and of behaving in a way consistent with a truncation of the true preferences in the school proposing mechanisms depends on whether we consider the static or the dynamic implementations. We now analyze whether and how these different behaviors affect the rates of stable outcomes.

Figure 1 plots the frequency with which a stable matching is reached over the course of the experiment.
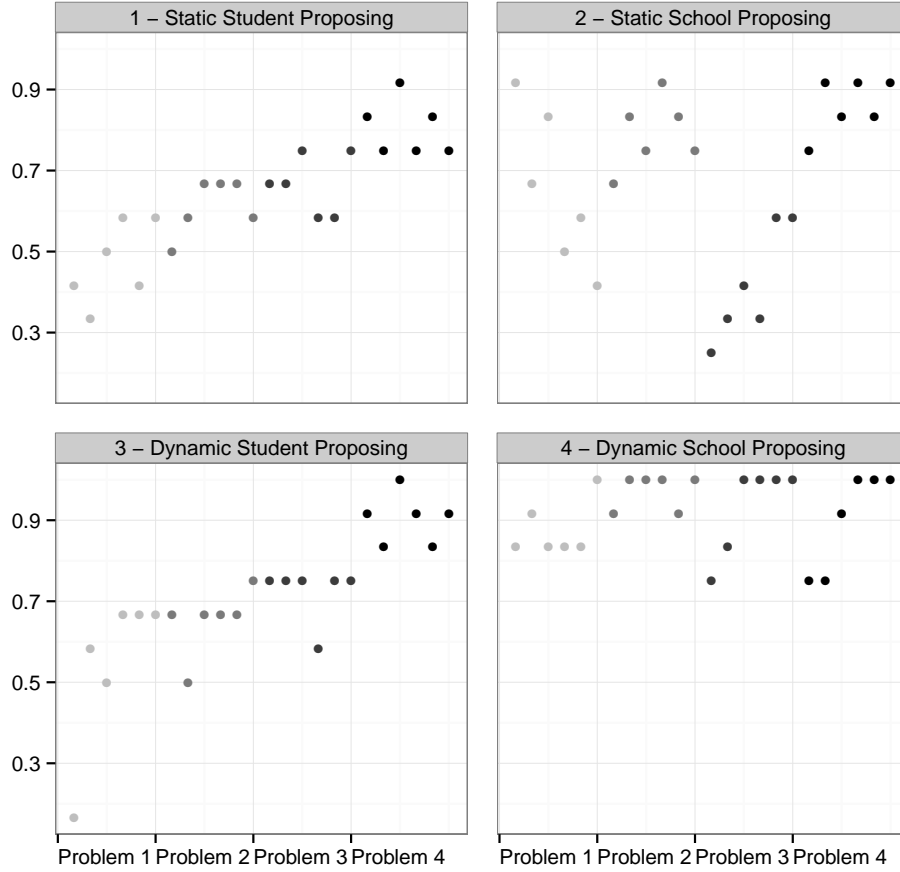
Figure 1: Frequencies of stable matchings. Since the number of stable matchings ranges from 1 (Problem 4) to 4 (Problem 1), comparisons across problems are meaningless.

There are stark differences between the treatments. The dynamic school proposing DA mechanism seems to perform best among all four mechanisms. In Problems 1-3 it visibly outclasses not only its natural peer, the static school proposing DA mechanism, but also the two student proposing DA mechanisms. On the other hand, there are hardly any differences between the two student proposing DA mechanisms. This intuition is confirmed if we look at the overall frequencies of stable matchings (63.89% in treatment CI, 68.08% in treatment DI, 70.49% in treatment CS, and 90.01% in treatment DS).

We run four random effects estimations, one for each problem, to assess treatment differences (Table 4). Our explanatory variables are three dummy variables that identify the different treatments: the variable "Student proposing" (β_2) takes value 1 in treatments CI and DI and 0 in treatments CS and DS, the variable "Static" (β_3) takes value 1 in treatments CI and CS and 0 in treatments DI and DS, and the variable "Student proposing $\times$ Static" (β_4) takes value 1 in treatment CI and 0 in the other three treatments. Since each problem is played six times, we also include the variable "Repetition" (which takes values from 1 to 6) in order to account for learn-

ing effects within a problem. In this setting, differences across treatments are evaluated against treatment DS which serves as a benchmark. In particular, β_3 measures the differences between treatments CS and DS, while $\beta_3 + \beta_4$ assesses the difference between treatments CI and DI.

	Problem 1	Problem 2	Problem 3	Problem 4
Constant (β_0)	0.8354***	0.9139***	0.8326***	0.8444***
	(0.0862)	(0.0487)	(0.0640)	(0.0069)
Repetition (β_1)	0.0113	0.0167	0.0280*	0.0017
	(0.0203)	(0.0126)	(0.0127)	(0.0013)
Student proposing (β_2)	-0.3333***	-0.3194**	-0.2083	-0.0001
	(0.0927)	(0.1183)	(0.1107)	(0.0058)
Static (β_3)	-0.2222**	-0.1806**	-0.5139***	-0.0042
	(0.0697)	(0.0620)	(0.1039)	(0.0068)
Student proposing $\times$ Static (β_4)	0.1528	0.1389	0.4583*	-0.0055
	(0.1340)	(0.1605)	(0.1856)	(0.0112)
R^2	0.0648	0.0458	0.0701	0.0155
Observations	1152	1152	1152	1152

Table 4: Random effects estimations on the frequency of reaching a stable matching. Errors are clustered on matching groups. *** indicates significance at $p = 0.001$ (two-sided), ** indicates significance at $p = 0.01$ (two-sided), and * indicates significance at $p = 0.05$ (two-sided). The qualitative insights remain the same if we employ probit estimations.

In consistence with Figure 1, the value of β_3 in Table 4 is significantly smaller than 0 in Problems 1-3; that is, the likelihood of reaching a stable matching in treatment DS is higher than in treatment CS (by 22 percentage points in Problem 1, 18 percentage points in Problem 2, and 52 percentage points in Problem 3). Next, we apply χ^2-tests to see whether the value of $\beta_3 + \beta_4$ is significantly different from zero. Since this hypothesis is rejected at $p = 0.05$ in all four problems, there are no treatment differences between CI and DI.

Mechanism	Overall	Problem 1	Problem 2	Problem 3	Problem 4
Static Student Proposing	*2.87%*	4.43%	3.21%	2.26%	1.56%
Dynamic Student Proposing	*3.80%*	5.90%	5.12%	2.78%	1.39%
Static School Proposing	*4.02%*	3.91%	3.30%	6.60%	2.25%
Dynamic School Proposing	*1.41%*	1.56%	0.69%	1.56%	1.83%

Table 5: Frequencies of blocking pairs.

We conclude this part of our analysis by presenting in Table 5 the percentage of student–school

pairs that can block a resulting matching. The data shows that fewer blocking pairs can be formed under the static student proposing DA mechanism than under the dynamic student proposing DA mechanism (the only exception is Problem 4) and that the dynamic implementation outperforms the static implementation when the school proposing DA mechanism is employed (the only exception is again Problem 4). The overall difference between the static and the dynamic implementation is considerably larger for the school than for the student proposing DA mechanism (2.61 percentage points versus 0.93 percentage points), which reaffirms our findings from the regressions that differences are only significant for the school proposing mechanism.

Result 2: *The frequency of stable matchings is the same for treatments CI and DI. In Problems 1-3, the frequency of stable matchings is significantly higher in treatment DS than in treatment CS.*

4.3 Welfare

For the dynamic mechanisms to be reasonable alternatives for the standard static mechanisms, it is crucial that decisions under the former mechanisms also translate into higher payoffs. And, given the results above that compare the set of Nash equilibrium outcomes in the dynamic and static mechanisms, there is *a priori* no reason to expect that the dynamic implementations present an improvement over their static counterparts in this dimension. In fact, switching the order of schools is a weakly dominated strategy in CI and the unique Nash equilibrium outcome in undominated strategies in this mechanism is the student optimal stable matching, which maximizes total payoffs in all problems. By contrast, there are no weakly dominant strategies in DI and the set of equilibrium outcomes is a superset of the set of stable matchings (Claim 2), thus containing inefficient matchings. On the other hand, in what the school proposing mechanisms are concerned, by Claim 4, the set of Nash equilibrium outcomes in both mechanisms equals the set of stable matchings. Moreover, if all subjects use truncation strategies in CS —which are know to be strategically exhaustive under this mechanism— a stable matching is obtained as long as subjects do not truncate above their partners under the student optimal stable. In fact, by truncating, stable matchings that correspond to higher average payoffs may be obtained.

HYPOTHESIS 3: *Average payoffs are not higher under the dynamic implementations.*

Nevertheless, with respect to the school proposing DA mechanisms, there are only very few subjects who can be rejected to play a preference strategy consistent with a truncation of the true preferences in treatment DS compared to treatment CS (Result 1B). And, with respect to the student proposing DA mechanisms, we have seen in Result 1A that all pairs except for $s^2 - s^4$ and $s^3 - s^4$ are switched less often in treatment DI than in treatment CI. The fact that subjects actually switch less (or act according to the true preferences more often) under DI, may imply that the student optimal stable matching is obtained more often than what theoretic results imply, so that welfare levels may be higher than expected.

Figure 2 plots the relative payoffs over the course of the experiment. In this figure, a value of 0.7 indicates that a subject earned on average 70% of her maximal payoff of 24 ECU. Since it is not always possible to assign all students to their most preferred school, comparisons across problems should be taken with care.
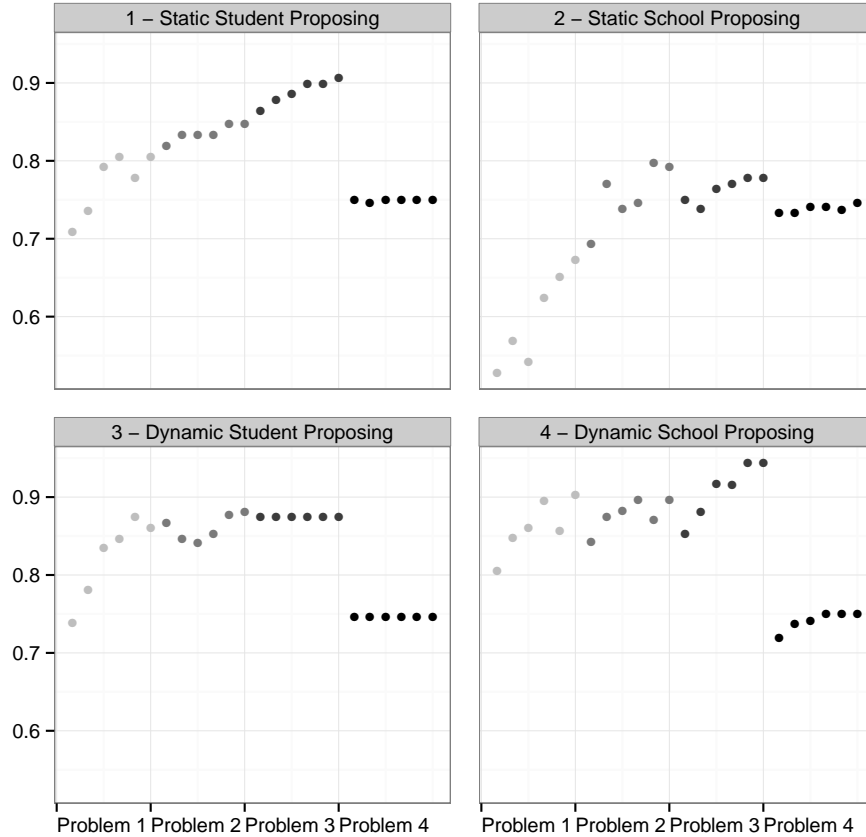

Figure 2: Relative payoffs per round in each of the four treatments. The student (school) optimal stable matching leads to a relative payoff of 1 (0.5) in Problem 1, 1 (0.67) in Problem 2, 0.92 (0.67) in Problem 3, and 0.75 (0.75) in Problem 4.

We can make three main observations. First, there are hardly any differences between the four mechanisms in Problem 4. Second, the dynamic school proposing DA mechanism leads to substantially higher payoffs than the static school proposing DA mechanism in Problems 1-3. And third, the two student proposing DA mechanisms perform similarly in Problems 1-3. Finally, aggregated over all problems, relative payoffs are 0.8112 in treatment CI, 0.8259 in treatment DI, 0.7139 in treatment CS, and 0.8472 in treatment DS. This is suggestive evidence that over the course of the experiment the dynamic implementation of the school proposing DA mechanism performs not only better than the static school proposing DA mechanism, but also does not yield lower payoffs than the two student proposing mechanisms.

To see whether these visual insights are confirmed, we run again random effects regressions, using the same explanatory variables as in the subsection on stability (Table 6).

	Problem 1	Problem 2	Problem 3	Problem 4
Constant (β_0)	0.7830***	0.8470***	0.8796***	0.7340***
	(0.0190)	(0.0182)	(0.0142)	(0.0280)
Repetition (β_1)	0.0224***	0.0086***	0.0085***	0.0021
	(0.0024)	(0.0020)	(0.0017)	(0.0015)
Student proposing (β_2)	−0.0387	−0.0159	−0.0350	0.0043
	(0.0241)	(0.0277)	(0.0204)	(0.0381)
Static (β_3)	−0.2635***	−0.1208***	−0.1462***	−0.0029
	(0.0198)	(0.0237)	(0.0167)	(0.0392)
Student proposing × Static (β_4)	0.2117***	0.0953***	0.1609***	0.0065
	(0.0295)	(0.0341)	(0.0278)	(0.0539)
R^2	0.2122	0.0485	0.0867	0.0025
Observations	1152	1152	1152	1152

Table 6: Random effects estimations on the relative payoff per subject. Errors are clustered on matching groups. *** indicates significance at $p = 0.001$ (two-sided), ** indicates significance at $p = 0.01$ (two-sided), and * indicates significance at $p = 0.05$ (two-sided).

Supporting the conclusions drawn from the observation of Figure 2, the expected payoff of a subject in Problem 4 is the same in all four treatments and very close to the expected payoff associated with the unique stable matching, namely 0.75 (=18 ECU). More importantly, we can see that in Problems 1-3, the value of β_3 is negative and significantly different from zero at $p = 0.001$; that is, payoffs in these problems are higher in treatment DS than in treatment CS (by 6.32 ECU in Problem 1, 2.90 ECU in Problem 2, and 3.51 ECU in Problem 3). The payoff difference between CS and DS in Problem 1 is particularly high. Finally, we apply χ^2-tests to see whether there are differences between treatments CI and DI. We find for Problem 1 that payoffs are significantly higher in treatment DI at $p = 0.05$. The corresponding payoff difference is 1.24 ECU. There are no significant payoff differences between CI and DI for Problems 2 and 3.

Result 3: *In Problems 1-3, payoffs are significantly higher in treatment DS than in treatment CS. In Problem 1, payoffs are significantly higher in treatment DI than in treatment CI.*

5   Conclusion

Abdulkadiroğlu and Sönmez [1] show that the student proposing DA mechanism has nice theoretical properties in that revealing one's preferences truthfully is a weakly dominant strategy in the

induced static game, so that the student optimal stable matching is obtained as a consequence. However, when the mechanism is brought to the laboratory it is commonly found that untrained subjects behave strategically even though they should not. It is thus the objective of this paper to analyze whether better experimental outcomes can be obtained if, instead of submitting lists, subjects make decisions dynamically by going through each of the underlying matching algorithms.

Our theoretical results for the dynamic student proposing DA mechanism show that if players use a "going down the list" strategy, that is, they take a ranking of schools (that may or may not coincide with their true preferences) and always send an application to the better ranked school they have not yet applied to, then the outcome is equal to the outcome of the static game when the corresponding lists are submitted to the central planner. But since the dynamic game allows for richer strategies, it is not true that going down the list with respect to one's true preferences is a weakly dominant strategy, so that in a laboratory experiment a wider range of behavior could be expected under the dynamic implementation. We nevertheless find experimentally that subjects tend to switch fewer pairs of schools at the top of their true preference under the dynamic implementation of the student proposing DA mechanism, even though these differences are not strong enough to substantially affect outcomes in terms of stability (63.89% in the static and 68.08% in the dynamic mechanism) and relative efficiency (0.8112 in the static and 0.8259 in the dynamic mechanism).

We also consider the static and the dynamic school proposing DA mechanisms. Students typically have incentives to manipulate the static mechanism, for instance by using truncation strategies, which are strategically exhaustive in this context. Since there are truncation strategies that are a best reply under the static but not under the dynamic mechanism, theoretically a wider range of behavior can again be supported for the dynamic implementation. Perhaps surprisingly, we find in our laboratory experiment that the dynamic school proposing DA mechanism does not only outperform the corresponding static implementation in terms of stability (70.49% in the static and 90.01% in the dynamic mechanism) and relative efficiency (0.7139 in the static and 0.8472 in the dynamic mechanism), but also the two student proposing DA mechanisms. This is because if an offer from some school is temporarily accepted under the dynamic implementation, subjects do not switch that school for a worse one later on, which is crucial for preventing unstable outcomes to occur. The overall switching frequency in the static implementation, on the other hand, is with 25% rather high and close to that of the static student proposing DA mechanism. Moreover, it is only under the dynamic school proposing DA mechanism that subjects learn that it can be worthwhile in terms of payoffs to reject offers from less desirable schools.

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Appendix – proofs

Proof of Claim 2

Let Q be a Nash equilibrium in the game induced by CI leading to μ. Let σ be the profile of preference strategies consistent with Q in the game induced by DI. By Claim 1 and Remark 1, σ leads to μ. We show that σ is a Nash equilibrium of the game induced by DI. By contradiction, assume that $\hat{\sigma}_i$ is a profitable deviation for some student i in the game induced by DI and let $\hat{Q}_i$ be the ordered list that is revealed by student i in the play of $(\hat{\sigma}_i, \sigma_{-i})$. By Claim 1, the outcome of the game induced by DI when students use $(\hat{\sigma}_i, \sigma_{-i})$ coincides with the outcome of the game induced by CI when students submit $(\hat{Q}_i, Q_{-i})$. Therefore, $\hat{Q}_i$ is a profitable deviation for i against Q_{-i} in the game induced by CI, which contradicts the initial assumption that Q is a Nash equilibrium in the game induced by CI. $\square$

Proof of Claim 3

Consider Problem 1. We show that the preference strategy for i_1 that is consistent with her true preferences is not a weakly dominant strategy. Let σ_{-i_1} be a preference profile of students $\{i_2, i_3, i_4\}$ such that

(1) if $j = i_2$ then σ_j prescribes to apply to s_2 in the first phase and whenever she gets rejected by s_2 to apply to s_3 (when rejection by s_2 takes place in first phase) and s_1 (when rejection by s_2 takes place in second or later phase);

(2) if $j = i_3$ then σ_j is the preference strategy consistent with $Q_{i_3} = s_1, s_2$;

(3) if $j = i_4$ then σ_j is the preference strategy consistent with $Q_{i_4} = s_4$.

Let σ_{i_1} be the preference strategy consistent with $Q_{i_1} = s_2$. Obviously, σ_{i_1} is not consistent with the true preferences P_{i_1}. Next, we compute the matching obtained in the game induced by DI when the students play strategy profile σ. Notation: $i \rightarrow s$ indicates that student i makes an offer to school s.

Phase 1:
offers: $i_1, i_2 \rightarrow s_2$, $i_3 \rightarrow s_1$ and $i_4 \rightarrow s_4$
tentative matching: $i_1 - s_2$, $i_3 - s_1$, $i_4 - s_4$
Phase 2:
offers: $i_2 \rightarrow s_3$
final matching: $i_1 - s_2$, $i_2 - s_3$, $i_3 - s_1$, $i_4 - s_4$

Let σ'_{i_1} be consistent with the true preferences $P_{i_1} = s_1, s_2, s_3, s_4$. We compute the matching obtained in the game induced by DI when the students play strategy profile $\sigma' \equiv (\sigma'_{i_1}, \sigma_{-i_1})$.

Phase 1:
offers: $i_1, i_3 \rightarrow s_1$, $i_2 \rightarrow s_2$ and $i_4 \rightarrow s_4$
tentative matching: $i_2 - s_2$, $i_3 - s_1$, $i_4 - s_4$

Phase 2:

offers: $i_1 \rightarrow s_2$

tentative matching: $i_1 - s_2$, $i_3 - s_1$, $i_4 - s_4$

Phase 3:

offers: $i_2 \rightarrow s_1$

tentative matching: $i_1 - s_2$, $i_2 - s_1$, $i_4 - s_4$

Phase 4:

offers: $i_3 \rightarrow s_2$

tentative matching: $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_4$

Phase 5:

offers: $i_1 \rightarrow s_3$

final matching: $i_1 - s_3$, $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_4$

Since i_1 prefers s_2 (obtained at σ) to s_3 (obtained at σ'), the preference strategy for i_1 consistent with her true preferences is not a weakly dominant strategy in the extensive–form game. $\square$

Proof of Claim 4

Let us start by showing that every equilibrium outcome is stable. By contradiction, let a matching $\mu \notin S(P)$ be reached in equilibrium σ. Since all schools are acceptable for all students in the problems we consider, this means that there exists a blocking pair, i.e., a pair (s, i) such that $s P_i \mu(i)$ and $i P_s \mu(s)$. Given that proposals are issued according to the schools' preferences, it must have been the case that i received school s's proposal and rejected it at some point. Therefore, consider a strategy σ_i' for student i that coincides with the strategy σ_i, except for the fact that once s's proposal is received, it is held and every other proposal is rejected. Using this strategy against the strategies of the other students σ_{-i} guarantees that i is matched to s (the game proceeds in exactly the same way as before up to the point when student i does not reject the offer from s and thus ends up matched to s). Since $s P_i \mu(i)$, the strategy σ_i' is a profitable deviation for student i and σ is not an equilibrium.

To show that every matching $\mu \in S(P)$ can be reached in equilibrium consider the strategy profile σ where each student i holds $\mu(i)$'s proposal whenever received and rejects every other proposal. Clearly, σ leads to μ. Moreover, there is no profitable deviation for any student i. In fact, for every $s \in S$ such that $s P_i \mu(i)$, by stability of μ, we have $\mu(s) P_s i$. Hence, by definition of σ_{-i}, s proposes $\mu(s)$ before proposing i and $\mu(s)$ holds this proposal and ends up matched to s. $\square$

Proof of Claim 6

Consider Problem 3. Let $i = i_4$. Let σ_{-i} be a preference profile of students $\{i_1, i_2, i_3\}$ such that for any information set $\mathcal{I} \in \Im_j$ with $j \neq i$, σ_j prescribes that

(1) if $j = i_1$ and j receives only an offer from s_3, she keeps it;
    if $j = i_1$ and j receives only an offer from s_4, she rejects it;

if $j = i_1$ and j receives only an offer from both s_3 and s_4, she keeps the offer from s_4 (and rejects the offer from s_3);

(2) if $j = i_2$ receives or retains an offer from s_1, she keeps it (and rejects any other offer);

(3) if $j = i_3$ receives or retains an offer from s_2, she keeps it (and rejects any other offer).

Let σ_i be a strategy such that in any information set $\mathcal{I} \in \Im_i$, σ_i prescribes that

(4a) if i does not retain a school and receives only an offer from both s_2 and s_3, she keeps the offer from s_3 (and rejects the offer from s_2);

(4b) if i retains or receives only an offer from s_4, she keeps it (and rejects any other offer).

Since $s_2 P_i s_3$, it follows from (4a) that σ_i is *not* a preference strategy consistent with a truncation of the true preferences. Next, we compute the matching obtained in the game induced by DS when the students play strategy profile σ. Notation: $s \to i$ indicates that school s makes an offer to student i.

Phase 1:
offers: $s_1, s_4 \to i_2$ and $s_2, s_3 \to i_4$
tentative matching: $i_2 - s_1$, $i_4 - s_3$

Phase 2:
offers: $s_2 \to i_3$ and $s_4 \to i_1$
tentative matching: $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_3$

Phase 3:
offers: $s_4 \to i_3$
tentative matching: $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_3$

Phase 4:
offers: $s_4 \to i_4$
tentative matching: $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_4$

Phase 5:
offers: $s_3 \to i_1$
final matching: $i_1 - s_3$, $i_2 - s_1$, $i_3 - s_2$, $i_4 - s_4$

Note that at σ student i_4 is matched to her most preferred school.

Let σ_i' be a preference strategy consistent with a truncation P_i^t of student i's true preferences $P_i = s_4, s_1, s_2, s_3$. We compute the matching obtained in the game induced by DS when the students play strategy profile $\sigma' \equiv (\sigma_i', \sigma_{-i})$. We distinguish between two cases.

CASE 1: $P_i^t = s_4, s_1, s_2$ or $P_i^t = s_4, s_1, s_2, s_3$.
Phase 1:
offers: $s_1, s_4 \to i_2$ and $s_2, s_3 \to i_4$
tentative matching: $i_2 - s_1$, $i_4 - s_2$

Phase 2:

offers: $s_3 \to i_1$ and $s_4 \to i_1$

tentative matching: $i_1 - s_4$, $i_2 - s_1$, $i_4 - s_2$

Phase 3:

offers: $s_3 \to i_2$

tentative matching: $i_1 - s_4$, $i_2 - s_1$, $i_4 - s_2$

Phase 4:

offers: $s_3 \to i_3$

final matching: $i_1 - s_4$, $i_2 - s_1$, $i_4 - s_2$ or $i_1 - s_4$, $i_2 - s_1$, $i_3 - s_3$, $i_4 - s_2$

CASE 2: $P_i^t = \emptyset$, $P_i^t = s_4$, or $P_i^t = s_4, s_1$.

Phase 1:

offers: $s_1, s_4 \to i_2$ and $s_2, s_3 \to i_4$

tentative matching: $i_2 - s_1$

Phase 2:

offers: $s_2 \to i_3$ and $s_3, s_4 \to i_1$

tentative matching: $i_1 - s_4$, $i_2 - s_1$, $i_3 - s_2$

In the remainder of the algorithm s_3 will only make an offer to students i_2 and i_3. However, according to their strategies, students i_2 and i_3 will retain their schools s_1 and s_2. Hence, the final matching is given by $i_1 - s_4$, $i_2 - s_1$, $i_3 - s_2$.

In each of the two cases, student i_4 is *not* matched to her most preferred school, which completes the proof of the claim. □