Incomplete Insurance against Endogenous Idiosyncratic Uncertainty

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This version: January 2018
(November 2012)

Barcelona GSE Working Paper Series

Working Paper nº 670

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January 2018

Abstract

I study the effect of the market arrangement on competitive allocations in a model in which the distribution of idiosyncratic uncertainty is determined endogenously. The particular application I consider is a search model of the labor market embedded in a general equilibrium model with production and asset accumulation. It is shown that costly search with incomplete markets introduces a wealth effect at low levels

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of wealth such that poor agents do not search much or even find optimal not to look for a job. The combination of this effect with the usual one at higher wealth levels delivers equilibrium allocations that are remarkably different from the one that obtains under complete markets. I also use numerical methods to obtain quantitative predictions in a calibrated version of the model. The effect of the market arrangement remains dramatically large due to search externalities.

KEYWORDS: Search, incomplete markets, general equilibrium.

JEL CLASSIFICATION: D52, D58, E24, J22

1 Introduction

In current policy evaluation exercises it is customary to use an appropriate version of a general equilibrium model characterized by incomplete markets against idiosyncratic shocks in the vein of Aiyagari (1994), Huggett (1993), and Krusell and Smith (1998) amongst others. The key ingredient in this family of models is that agents are subject to uninsurable idiosyncratic shocks, and in most of the literature the effect of different policies is conducted under the assumption that distributions of probability describing idiosyncratic uncertainty are *policy invariant*.[1] The assumption of policy invariant shocks seems fine if shocks are meant to represent purely exogenous states, like the state of nature, or states which are beyond the influence of any agent's decision, perhaps as an aggregate shock to technology. At the individual level, however, it is likely that states such as employment, marital status and health or the education level -to mention a few examples that are usually considered in the literature- are not purely determined exogenously, but they also depend on the decisions of the individuals. If one adopts this view it seems obvious that having access to different policies (say with respect to unemployment benefits, the coverage of medical treatments, or property and wealth division in case of divorce), may have sizable effects on the related decisions about search for jobs, expenditure in prevention of health shocks, and on human capital accumulation and on saving decisions. All these decisions are likely to influence the distribution of idiosyncratic

[1] Examples of this line of research include Conesa and Krueger (1999), Athreya (2002), Li and Sarte (2006), Livshits, MacGee, and Tertilt (2007) and Choi (2010) among many others.

uncertainty and thus are likely to have an impact on the equilibrium allocations. The purpose of this paper is to investigate the importance of alleviating the policy invariant assumption.

Allowing policy effects on idiosyncratic uncertainty is likely to have sizable consequences in a wide range of situations involving contracts and insurance. For concreteness I choose to develop these ideas in a search and matching model of the labor market embedded in a general equilibrium model with production and asset accumulation. In particular, I follow the approach in Ríos-Rull (1994) and think of the market arrangement as a policy variable which allows me to revisit the differences between competitive allocations under complete and incomplete markets. The motivation for this exercise comes from the usual wisdom that at the aggregate level the differences across market arrangements are rather small. Essentially, the marginal propensity to save is slightly larger when agents face uninsurable uncertainty and thus output is also larger due to the precautionary saving under incomplete markets (Aiyagari 1994, Krusell and Smith 1998, among others). These basic results hold in models in which labor supply (labor productivity or employment status) is exogenously given as a random idiosyncratic endowment hence both the individual and the aggregate endowment is the same irrespectively of the market arrangement, i.e., they are policy invariant. To overcome this limitation I study a model in which once unemployed, an individual is allowed to engage in active search in order to increase the probability of finding a job in the following period (thus the model can be seen as a version of that in Merz 1995, and Andolfatto 1996). The results in

this paper challenge the common wisdom because once the policy invariant assumption is removed the differences across market arrangements can be dramatically large.

The two key ingredients of the model are the frictions in the labor market, captured with a matching function, and costly search in terms of consumption goods, which introduces a wealth effect on the decisions of search. At the individual level, therefore, the model captures the trade off between the cost of improving the odds of employment and the prize of being employed. Among other things, I show that with incomplete markets this trade off is far from being monotone in wealth (as it is with perfect insurance).[2] In Section 2 I use a two-periods version of the model to show that at both high and low levels of wealth, an unemployed worker chooses optimally not to search. The usual wealth effect explains why a wealthy unemployed worker decides not to work in the second period: the wage rate does not compensate for the loss in leisure. At low levels of wealth, however, the unemployed household finds that the cost of searching, as well as of saving, is arbitrarily large. And yet, saving offers a sure return, while searching is a risky investment because the probability of finding a job is well below one. At low levels of wealth, therefore, the cost of search exceeds its expected benefit, hence the agent finds optimal no to search. This effect at low levels of wealth under incomplete markets is new in the literature, and it stands in stark contrast with respect to the complete markets counterpart. When insurance markets are available a poor unemployed agent would always choose to search and to

[2] In a related empirical paper Lenz (2009) develops a model able to deliver non monotone search.

buy an insurance policy. That is, with complete markets the wealth effect at low levels of wealth disappears (and at high levels of wealth the agent would choose not to search and no to buy insurance, as she would under incomplete markets).

In Section 3 I study the long run equilibrium of the infinite horizon case. I provide an analytical example in which due to the wealth effect at low levels, the steady state is such that the distribution of wealth is polarized and thus there is only rich and poor agents that do not work, i.e., there is no production. It is worth emphasizing that the result is obtained in an otherwise standard model in which the result is driven by the assumption that if the agent does not search, then the probability of finding a job is zero.[3] In particular, had the markets been complete output would be positive. Hence this result suggests that the wealth effect at low wealth levels due to the market arrangement through the indirect channel of endogenous idiosyncratic uncertainty can be dramatically large.

In the final section of the paper I use quantitative methods to study the effects of the market arrangement in a version of the U.S. economy in which the wealth effect at low levels is absent.[4] I still find large differences across market arrangements steaming from equilibrium wealth effects on search effort. In particular, as a consequence of the lack of insurance markets search effort from the unemployed agents at the aggregate level is large. This large

[3]This is the usual assumption in the literature, see for instance Merz (1995) and Andolfatto (1996), but also Pissarides (2000).

[4]Poor agents can still find optimal not to search but there is always a small probability of receiving a job offer, say due to network effects as in Calvó-Armengol and Jackson (2004).

search represents a negative externality that the model captures by means of the matching function, as a large aggregate search reduces the job finding rate for each unemployed worker. Furthermore, the equilibrium amount of available capital for production is small given the large expenditure in search. Thus firms find optimal to open just a few vacancies which otherwise are filled very soon. The result is a steady state with a low level of production relative to the complete markets counterpart: depending on the parametrization the equilibrium output under incomplete markets may represent less than 50% of that under complete markets.[5] Finally I recalibrate the model to roughly match the stocks of employment, unemployment and not in the labor force observed in the U.S. and then I check the ability of the model to replicate the flows among these three states observed in the data. The model is able to account for the transitions starting from the states of employment and not in the labor force. However the transition from unemployment to employment is understated and the probability of remaining unemployed is overstated. Given the simplicity of the model, this finding suggests additional features of the data could be successfully incorporated to improve its predictions.

The results in this paper are ultimately due to the connection between wealth and idiosyncratic uncertainty through the decisions taken at the individual level. Hence similar results are likely to apply in other situations of interest. Informally it seems clear that health and education, for instance, are positively correlated with wealth.[6] This link in the labor market is less clear

[5]These large differences appear to be robust in a sensitivity analysis I report in the Appendix.

[6]The formal literature is too large to be summarized here. For the particular case

because unfortunately it is difficult to come up with direct evidence. All in all, looking for a job may not be expensive but is not free either: in the current tax code in the U.S. certain expenses related to job seeking activities (including employment and outplacement agency fees, expenses related to preparing and sending the résumé and travel and transportation expenses) are included in Miscellaneous Deductions.[7] To provide some additional support for the hypothesis I follow the indirect method in Shimer (2004) and I use PSID data from 2003 to 2011 on several measures of wealth from unemployed agents and the number of different methods they use to search for a job. The results, reported in the Appendix, suggest that poorer and richer agents use fewer methods than agents in the intermediate wealth bins. Interestingly, the unemployed agents using the largest number of search methods are never in the poorest bin.

Section 4 concludes the paper with a discussion about extensions of the current line of research, and the final Appendix contains proofs and additional quantitative results.

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of schools attendance and wealth around the world see for instance Filmer and Lant (1998). With respect to the link between wealth and health De Nardi, Pashchenko and Porapakkarm (2017), Jung and Tran (2016), Krueger and Ludwig (2016), and Conesa et al. (2017).

[7] See Publication 529 (2016), p. 5, Department of the Treasury. Unfortunately the actual amounts deduced are only recorded for the inspected tax declarations, and they are not publicly available. I would like to thank Sean Lowry at the Congressional Research Service, Government & Finance Division, for helpful explanations about these issues.

2 Wealth effects on search in the labor market

Consider a two-period version of a model of search in the labor market, in which the probability of receiving a job offer depends on the *active* search activity previously exerted by the worker. Specifically, assume that in the first period the agent is endowed with $a > 0$ units of goods which can be consumed, saved (to be consumed in the future) and devoted to search in order to find a job in the second period. Let $R > 1$ be the gross return on saving, and denote by $\xi > 0$ the cost per unit of search in terms of consumption goods. I assume that all jobs are identical in terms of wage rate ($w > 0$) and disutility of working.

The amount of search s is related to the probability of receiving a job offer through a constant returns to scale technology: let $\lambda \in (0, 1)$ be the probability of receiving a job offer per unit of search intensity, and assume that the probability of receiving an offer in the second period when s was the amount of search in the first is given by λs (thus searching in the second period is useless). Once the uncertainty in the second period is resolved, if the agent is employed then she/he must supply a unit of time as labor (hence the adjustment in the intensive margin is precluded). Finally, assume that the agent derives utility from consumption and from leisure as given by $u(c) + n(l)$ which satisfies the following assumption:[8]

A1: u is continuous on R_{++}, strictly increasing, strictly concave, differen-

[8]The formulation stated above is essentially a two periods version of the model in Merz (1995). Searching in the market could also represent a cost in terms of leisure. As of now I disregard this possibility and adopt the indivisible labor assumption to better isolate the wealth effects on the participation decision.

tiable, and satisfies the usual Inada conditions: $\lim_{c\to 0} u'(c) \to +\infty$, and $\lim_{c\to+\infty} u'(c) = 0$; n is such that the value of leisure in case of unemployment is normalized to zero, and in case of employment is normalized to $-m < 0$.

The decision problem of the agent is to choose a' and s in order to

$$\begin{aligned} \max_{(a',s)} v(a) = \quad & u(c) + \beta\{\lambda s(u(c_e) - m) + (1 - \lambda s)u(c_u)\} \\ \text{s. to} \quad & c + a' + \xi s = a, \\ & c_e = w + Ra', \\ & c_u = Ra',\ c, a', s \geq 0, \text{ and } s \leq 1/\lambda, \end{aligned}$$

taking as given a, w, R, and λ, and where $\beta \in (0, 1)$ is the discount factor. Notice that assets in the second period cannot fall below zero, hence there is a borrowing constraint. Since the possibility of writing contingent contracts is exogenously precluded, the borrowing constraint prevents having access to perfect insurance against unemployment by issuing an arbitrarily large amount of debt. Notice, however, that in principle it is possible to remove uncertainty by choosing $s = 1/\lambda$ and by choosing $s = 0$. Leaving aside for a moment these possibilities, the lack of insurance markets presumed in this subsection is consistent with the extreme view that search effort is a piece of private information that is too costly to verify.

The FOC with respect to assets and search are stated respectively as

$$-u'(c) + \beta R\{\lambda s u'(c_e) + (1 - \lambda s)u'(c_u)\} \leq 0 \text{ and } a' \geq 0, \tag{1}$$

$$-u'(c)\xi + \beta\lambda\{(u(c_e) - m) - u(c_u)\} \leq 0 \text{ and } s \geq 0, \tag{2}$$

which hold with complementary slackness. Given the choice for s, Equation (1) is the usual Euler condition describing optimal assets to carry over the second period. The Inada condition at the origin implies that $a' > 0$ and thus Equation (1) holds with equality. Equation (2) describes the optimal choice for s given a'.[9] The first term of Equation (2) measures the utility cost of increasing marginally the search effort, and the second term measures the present value of expected gains in utility derived from such an increase. Thus the optimal choice for s entails that benefits cannot exceed costs. Finally, notice that the objective function is not strictly concave, hence in principle the above FOC are necessary for optimality but may not be sufficient.[10]

The goal in this section is to understand how wealth effects interact with the searching decision in an incomplete markets environment in which there is a safe asset. I need to introduce assumption A2 for the previous problem to be interesting:

A2: $\xi R < \lambda w$.

A2 requires that the sure return of economizing on search costs is smaller than the expected return of searching. Let $a(a)$ and $s(a)$ denote the optimal choices for a' and s that solve the utility maximization problem. Lemma 1 below states that without A2 the optimal decision for search is $s(a) = 0$ for

[9] Notice that in spite of the linear probability and cost of search we are implicitly assuming that the constraint $\lambda s \leq 1$ is not binding. In the quantitative analysis I introduce the appropriate assumptions to preclude this possibility.

[10] The results developed below rely only on FOC, which must hold in any case. Furthermore, even in simple examples commonly used in the literature, like the log-utility case, it is possible to show that the FOC are both necessary and sufficient.

all $a > 0$.

Lemma 1: Assume A1 and $\xi R \geq \lambda w$. Then $s(a) = 0$ for all $a > 0$.

Proof: See the Appendix.

The first result shows that search effort is subject to a wealth effect that resembles that on labor supply when leisure is a normal good: if the agent is sufficiently rich, then there is no point in searching to find a job in the next period (or that when consumption is sufficiently high, then it is best to also enjoy high leisure).

Proposition 1: Assume A1, A2. There is $\bar{a}$ such that if $a \geq \bar{a}$ then $s(a) = 0$.

Proof: See the Appendix.

In the next result I look at the search effort when the agent is poor. Proposition 2 introduces additional restrictions on preferences to show that for a sufficiently poor agent it is optimal *not to search* in the labor market.

Proposition 2: Assume A1, A2, and that $u(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$ for $0 < \gamma \neq 1$, with $u(c) = \log c$ when $\gamma = 1$. There is $\underline{a} > 0$ such that if $a \in (0, \underline{a})$ then $s(a) = 0$.

Proof: See the Appendix.

With costly search it is not surprising that search decreases as wealth decreases, at least in some range. The striking result is that the agent may optimally choose not to search even at some positive but low level of wealth. The intuition to explain why both a rich and a poor agent may decide not to search is related to each other but is not the same. A rich agent decides not to search because at high levels of consumption the increase that would

bring in the additional labor income does not compensate the utility loss due to reduced leisure. For a poor household, however, the problem is that the utility value of the loss in current consumption (due to additional search) is too large to compensate for the small increase in the odds of receiving an offer in the following period. Hence, assuming that leisure is valued is critical to obtain the wealth effect at high levels of wealth, but it is irrelevant to obtain the wealth effect at the low levels described in Proposition 2. What matters for this result is that search and saving represent competing uses of available resources, that searching is risky -whereas saving is not- and that the utility value of a marginal unit of consumption is arbitrarily large at low levels of consumption.

The previous conclusion about the wealth effect on search effort when the agent is poor and insurance markets are incomplete stands in remarkable contrast when markets are complete. To see this, suppose that search effort can be perfectly monitored at no cost. Suppose in addition that under this conditions there is an insurance market against unemployment which opens in the beginning of the second period, and that this market is perfectly competitive. It is well known that in such an environment the agent chooses to fully insure against unemployment, and that the price of the insurance policy is given by $1 - \lambda s$ (remember that s is publicly observable). Using these facts one can write the utility maximization problem under complete markets as:

$$\begin{aligned} \max_{(a',s)} v(a) = \quad & u(c) + \beta\{u(c') - \lambda sm\} \\ \text{s. to} \quad & c + a' + \xi s = a, \\ & c' = w\lambda s + Ra', \\ & c, a', s \geq 0, \text{ and } s \leq 1/\lambda, \end{aligned}$$

given $a > 0$, and we assume that prices are the same as under incomplete markets. The FOC with respect to capital accumulation and search in this case read, respectively, as:

$$-u'(c) + \beta R\, u'(c') \leq 0 \text{ and } a' \geq 0, \tag{3}$$

$$-u'(c)\xi + \beta\lambda\{u'(c')w - m\} \leq 0 \text{ and } s \geq 0, \tag{4}$$

which hold with complementary slackness. The Inada condition at the origin implies that either $a' > 0$, or $s > 0$, or both, provided that $a > 0$. It is possible to show that under complete markets there is a wealth effect on search effort when wealth is large such that the optimal choice for search effort is zero. However, there is no wealth effect playing the same role as under incomplete markets when the agent is poor. This is the content of the following proposition.

Proposition 3: Assume A1, A2, and that there are complete markets against unemployment. Then there is $\breve{a} > 0$ such that $s(a) = 0 \Rightarrow a \geq \breve{a}$.

Proof: See the Appendix.

The differences for search effort according to Propositions 2 and 3 depending on the market arrangement are striking. The usual intuition is that

providing insurance may be inefficient because it may reduce the incentives to search. However, the results above show that at low levels of wealth it is precisely the lack of insurance what may disincentive search effort. Hence, the implications of proposition 2 are particularly relevant in the assessment of the differences across market arrangements. The reason is that with incomplete markets unemployed agents tend to smooth out consumption by consuming part of their precautionary stock of assets. How large can be the differences between complete and incomplete markets in general equilibrium is the content of the following sections. Before we turn to this issue, however, I briefly discuss the implications of relaxing the assumption that the probability of finding a job depends only on the amount of forgone consumption. In particular, assume for a moment that the probability of finding a job is given by a function $\lambda(s, 1-l)$, where $1-l$ is the amount of time devoted to search in the first period, and that the partial derivatives λ_s and λ_{1-l} are both non negative. Keeping all previous assumptions as before, it is straightforward to verify that Proposition 2 goes through as long as $\lim_{s\to 0} \lambda_s < +\infty$. Hence, allowing for time spent searching does not necessarily suppress the wealth effect at low levels of wealth.[11]

[11] What is critical for the result in Proposition 2 is to allow expenditure in s (i.e., forgone consumption) to affect the probability of finding a job in the following period. In particular, if one assumes $\lambda_s = 0$ for all s in the previous specification, then $s = 0$ is the optimal choice for search expenditure, and we are left with the usual wealth effect on leisure: the poorer the agent the larger is the time spent searching for a job. This implication, however, seems to be at odds with the empirical evidence discussed in Krueger and Mueller (2011) suggesting that the time spent searching for a job decreases over the unemployment spell.

3 General equilibrium with production and infinite horizon

Consider an infinite horizon version of the previous economy in which a production sector and a matching technology are explicitly added. The model economy in this section is similar to the model in Gomes et al. (2001), Krusell et al. (2007), and Nakajima (2008): it is a search model of the labor market extended to include endogenous search effort and asset accumulation.[12] To simplify the analysis the focus is on steady states, in which dates are irrelevant because all aggregate variables and prices remain constant over time.

3.1 Labor market

Assume as before that there are frictions in the labor market such that agents need to search for a job before becoming employed. Likewise, firms in the demand side of the market and willing to employ a worker need to post vacancies a period in advance, which once filled, become productive matches. Even if there are frictions in this market, there is also commitment: the worker commits to supply a pre-specified amount of time as labor in the employment state, and firms in exchange commit to pay the market wage rate w to their employed workers.

The labor force in the economy has mass one, and E will denote the mass of employed agents. The remaining fraction $1 - E$ of agents is non employed.

[12] Hence the model can also be seen as an incomplete markets version of the models in Hansen (1985), Merz (1995) and Andolfato (1996) abstracting from aggregate fluctuations.

Since it is likely that some agents without a job choose not to search, they will be considered *not in the labor* force and will be labeled N. Finally, those agents without a job but actively searching for one are labeled *unemployed*, and represent a mass U. Hence we have that $E + U + N = 1$ must hold at all times. The number of newly created matches is given by a matching function relating vacancies V and aggregate search intensity S:

$$M = M(V, S) \tag{5}$$

where M is the number of matches and where $M(V, S)$ is the increasing, concave and differentiable matching function which displays constant returns to scale. Below aggregate search intensity may include not only the search effort exerted by unemployed agents but also passive search and network effects from agents not in the labor force.

Existing matches are destroyed at the exogenous separation rate σ. Under these assumptions the mass of employed agents in any given period satisfies

$$E = M/\sigma.$$

The formulation of the labor market delivers endogenously the probability of finding a job per unit of search intensity, and also the probability of filling a posted vacancy, which are given respectively by

$$\lambda_w = \frac{M}{S}, \text{ and } \lambda_f = \frac{M}{V}. \tag{6}$$

It is straight forward to show that at a steady state the employment rate

satisfies:

$$\sigma E = \lambda_w S. \tag{7}$$

Assume as before that all contracts in the labor market are identical and are such that employed workers supply the same hours of work in exchange of the same wage rate w. This is discussed below when the production side of the economy is introduced.

3.2 Asset markets

I follow Krusell et al. (2007) and assume that there are only two assets in the economy: capital k, which is used in the production of goods as an input together with labor, and shares of a representative firm x, which represent claims on future profits. For these two assets to be valuable in equilibrium it must be the case that the following no arbitrage condition holds:

$$p = \frac{d + p}{R}, \tag{8}$$

where p is the price of shares, d is the dividend, and R is the net return to capital (i.e., think of R as $1 + r - \delta$, where r is the rental rate of capital, and where $\delta \in (0, 1)$ is its depreciation rate). The no-arbitrage condition in Equation (8) makes the composition of any portfolio irrelevant from the perspective of a household. It will be useful in the formulation of the households problem to define assets as:

$$a \equiv Rk + (p + d)x. \tag{9}$$

3.3 Households

At any point in time a given household is either employed or non employed, which is denoted by an element of $O = \{e, u\}$.[13] The problem in each of these states is discussed below.

3.3.1 Employed households

A household in the employment state supplies the requested labor in exchange of the given wage rate w. In the following period the household will continue being employed with probability $(1 - \sigma)$, but her match with the firm may be destroyed with probability σ.[14] There is no point in searching while in the employment state, provided that search is costly and that all jobs are identical.

The problem of the employed household consists of choosing current consumption c and assets in the following period a' (by choice of k' and x'), subject to her budget constraints and taking as given prices and probabilities of transition from employment to non employment. The budget constraint of the employed agent reads $c + k' + px' = w + Rk + (p + d)x$, which by virtue of Equation (8) and the definition of assets in Equation (9) we rewrite as $c + a'/R = w + a$. The problem of the employed household written in

[13] In the non employment state the agent will be classified either as *unemployed*, or as *not in the labor force*, depending on her search effort. This classification is relevant for the outcome in the labor market, but all agents in the non employment state will have access to the same markets and technologies.

[14] That is, the separation rate σ is exogenously given, hence there is nothing the employed household can do to affect the probability of becoming non employed in the following period.

recursive form is:

$$W(a,e) = \max_{c,a' \in \Gamma(a,e)} \{u(c) - m + \beta[(1-\sigma)W(a',e) + \sigma W(a',u)]\} \qquad (10)$$

where

$$\Gamma(a,e) = \{(c,a') \in R_+ \times A : c + a'/R = w + a\}.$$

The set A in $\Gamma(a,e)$ is given by $[\underline{A}, \bar{A}]$. Thus the set A introduces a lower bound $\underline{A} \leq 0$ on assets holdings that prevents Ponzi schemes.[15] The FOC associated to the previous problem reads

$$-u'(w + a - a'/R) + \beta R[(1-\sigma)W'(a',e) + \sigma W'(a',u)] \leq 0, \qquad (11)$$

which holds with equality whenever $a' > \underline{A}$ (as usual, W' stands for the derivative of W with respect to a'). Once the match is lost the agent becomes non employed, and she spends one period (at least) in the non employment state.

3.3.2 Non employed households

During a period of non employment the household decides her search intensity, which increases the probability of finding a job in the following period. To be more general than in the two-period model of the previous section let $\pi(s)$ be the effective units of search when the agent exerts a search effort s. The function $\pi(s)$ is assumed to be increasing, concave and differentiable,

[15] The lower bound $\underline{A}$ is characterized more sharply below. Likewise, the upper bound $\bar{A}$ is needed for technical reasons. As of now it suffices to think of it as a large positive number.

with $0 \leq \pi(s)$ and $0 \leq \pi(s)' < +\infty$ for all $s \geq 0$. Hence $\lambda_w \pi(s)$ is the probability of finding a job in the following period when the agent chooses s in the current one. The assumptions on $\pi(s)$ will enable the possibility of $\lambda_w \pi(0) > 0$ and $\lambda_w \pi(s) < 1$ for all s.

The non employed agent also chooses current consumption and assets to be carried over the next period:

$$W(a,u) = \max_{(c,s,a') \in \Gamma(a,u)} \{u(c) + \beta[\lambda_w \pi(s) W(a',e) + (1 - \lambda_w \pi(s)) W(a',u)]\} \tag{12}$$

with

$$\Gamma(a,u) = \{(c,s,a') \in R_+ \times [0, \lambda_w^{-1}] \times A : c + a'/R + \xi s = a + \epsilon\}.$$

The term $\epsilon > 0$ in the budget constraint of the non employed agent reflects the fact that she has access to a home production technology which delivers some small amount of consumption goods. In this state the FOC with respect to assets reads

$$-u'(a - a'/R - \xi s) + \beta R[\lambda_w \pi(s) W'(a',e) + (1 - \lambda_w \pi(s)) W'(a',u)] \leq 0, \tag{13}$$

with equality whenever $a' > \underline{A}$. The FOC with respect to s is given by

$$-\xi u'(a + \epsilon - a'/R - \xi s) + \beta \lambda_w \pi'(s)[W(a',e) - W(a',u)] \leq 0, \tag{14}$$

which holds with equality whenever $s > 0$ (hence it is assumed that the constraint $\lambda_w \pi(s) \leq 1$ is not binding, otherwise the corresponding multiplier

should be taken into account). Given the optimal choice for assets in the next period, the FOC above states that the optimal amount of search effort is such that its marginal cost equals the present value of its benefit (in expected terms), which is measure as the difference in value of being employed and not employed. As it is explained above, a non employed agent that chooses not to search is classified as *not in the labor force*, hence only if the agent chooses to incur in some search cost will be labeled as *unemployed*.

Under A1 and the assumption that the utility function is bounded, standard results in Dynamic Programing can be invoked to assert that unique value functions satisfying Eq. (10) and (12) exist. I will proceed under the assumption that the necessary conditions in Eq. (11), (13) and (14) are also sufficient, thus there exist policy functions for consumption, assets and search that are continuous in a that attain the value functions.[16]

Once the problem of the worker in each state has been introduced, it is worth to state the connections with the problem studied in Krusell et al. (2011). Krusell et al. assume that a worker in the employment state (or "island") can choose whether to effectively supply labor or not, and in the unemployment state there is nothing she can do to alter the probability of receiving a job offer. Thus, in that model search is passive and all the action in the labor market takes place through employed workers. In contrast, in the model in this paper agents in the employment state cannot choose not to work, the separation rate is exogenously given, and agents in the non employment

[16]These results follow from the contraction mapping theorem and the theorem of the maximum. See also Theorem 3 and Corollary 2 in Denardo (1967). The assumption that the FOC are also necessary is sufficient for continuity of policy functions, and in the numerical examples I report later it is always satisfied.

state are the ones that may choose their search intensity. Hence, in the current model all the action in the labor market takes place through the non employed workers. As it will be seen the wealth effect at high levels of wealth is present in both approaches, but the wealth effect at low levels is only present in the current approach.

3.4 Firms

I assume that there is a single representative firm which has access to a constant returns to scale technology in capital and labor $F(K, L)$. The problem of the firm is dynamic in nature because a fraction σ of productive matches are destroyed in every period, and so it needs to create vacancies one period in advance. Furthermore, posting a vacancy has a fixed cost ϕ. For simplicity, I also assume that once a match is formed the firm commits to pay to the worker the market wage rate w in exchange of her unit of labor time, until the match is randomly destroyed.

In the recursive formulation of the problem of the firm the number of filled vacancies is the state variable, and is denoted L. The capital market is competitive and the firm hires capital at the market interest rate r. Finally, I assume that the firm discounts future profits at the factor $\tilde{\beta} = 1/R$ (the market net interest rate), and formulate the present value maximization of profits as

$$V_f(L) = \max_{(K,V,L') \in \Gamma_f(L)} \{[F(K, L) - rK - wL - \phi V] + \tilde{\beta} V_f(L')\} \qquad (15)$$

where

$$\Gamma_f(L) = \{(K, V, L') \in R^3_+ : L' = (1-\sigma)L + \lambda_f V, \},$$

and where the probabilities σ and λ_f are taken as given. In the previous problem $V_f(L)$ is the value of the firm, and the constraint is the law of motion for matches. Isolating V from that constraint and substituting the resulting expression in the objective function we then proceed to characterize optimal choices for K and L', respectively as follows:

$$F_K - r = 0, \tag{16}$$

and

$$-\phi + \lambda_f \tilde{\beta} V_f'(L') = 0.$$

The first condition above states that the firm hires capital in the market up to the point where its marginal product equals its marginal cost. Similarly, the second condition asserts that the optimal number of active matches is the one than equates its (sure) marginal cost, with its (expected) marginal present value. I rewrite this last condition by using the envelope theorem as:

$$-\phi + \lambda_f \tilde{\beta} \left[F_{L'} - w + \phi \frac{(1-\sigma)}{\lambda_f} \right] = 0. \tag{17}$$

It follows from the FOC above that the optimal amount of actual labor is such that its marginal value is larger than it marginal cost, hence profits will be positive.[17] It will be assumed that the profits of the firm are all

[17] To see this notice that Eq. (17) implies $F_L L + \phi(1-\sigma)L/\lambda_f > wL$ hence dividends d satisfy $d = F(K, L) - rK - wL - \phi V > \phi\phi(1-\sigma)L/\lambda_f - \phi V = \phi L/\lambda_f > 0$, which follows

distributed as dividends amongst the stockholders, but for this to be possibly sustained as an equilibrium the possibility of creating new vacancies must be somehow restricted.

3.5 Dividend rules and equilibrium

It is customary in the related literature about frictions in the labor market to close the model by assuming that a wage bargaining process takes place when an unemployed worker meets with a potential employer and that there is free entry of firms.[18] Since in the current model both firms and workers act competitively in the labor market, the wage rate and the level of employment will be determined simultaneously by means of a clearing condition for a competitive market with frictions. To close the model the number of vacancies that the firm can create will be indirectly restricted by means of a *dividend rule*, denoted $\bar{d}$, requesting $d \leq \bar{d}$. These two assumptions help to isolate the effects of the endogenous determination of idiosyncratic uncertainty that are the main interest in this paper. The following discussion is in order:

Discussion:

1. The formulation of the problem of the firm simplifies the analysis because it neglects any action from wealth effects in the bargaining problem between the worker and the firm which are typically present in the matching literature (e.g., Pissarides 1985, and Mortensen 1994). I adopt this strategy

thanks to the CRS of $F(K, H)$ and the law of motion of labor in the steady state.

[18]The usual approach is to obtain the equilibrium wage rate as the result of a Generalized Nash Bargaining problem (see, however, the criticisms in Shimer 2005, Costain and Reiter 2008, and Hagedorn and Manovskii 2008).

to better isolate the wealth effects on the search intensity problem leaving aside strategic interactions with bargaining power.

2. Krusell et al. (2007) study a related economy in which wealth does not affect search but it does affect the outcome of the bargaining problem between the firm and the worker. They show that at a steady state the action due to this wealth effect is negligible and it only plays some role at very low levels of wealth. If anything, the firm is able to pay *lower* wages when it is matched to an extremely poor agent. In view of the results in the previous section this effect of wealth would, in any case, reinforce the results on search intensity.

3. Related to this issue, Nakajima (2007) studies a centralized version of the bargaining problem in an environment in which the negotiation takes place between a representative firm and a "representative worker", i.e., an agent that behaves "as if" she was the owner of average wealth in the economy. My approach is related to Nakajima's in the sense that all employed workers receive the same wage, irrespectively of their wealth level.

4. One can postulate a wide range of dividend rules, such as arbitrary constants, functions of the aggregate state variables, and even rules depending on expectations. Interestingly, it is also possible to fix a dividend rule such that the equilibrium allocation under complete markets coincides with the efficient allocation (see Proposition 4 below). That is, a convenient choice of the dividends rule is able to undo the perverse effects of frictions and externalities in the labor market. Hence, Proposition 4 introduces conditions that produce similar effects to those stated in Hosios (1990), Merz (1995), and Andolfatto (1996), under which the competitive equilibrium with Nash

bargaining is efficient.

To state the equilibrium concept a definition of the aggregate state and its evolution is required. The position of an agent in every period can be described by a point $z \in Z = A \times O$. Hence the aggregate state is the distribution of agents over asset levels and employment status in the labor market: a probability measure Ψ defined on $\mathcal{Z}$, the Borel subsets of Z. The function $P(z, C)$ denotes the transition function that gives the probability of an agent currently in state z ends up with a state z' in a set $C \in \mathcal{Z}$ in the following period. Finally, the discussion in the previous subsection suggests that we could use the dividend rule to index competitive equilibria. Since we focus on stationary equilibrium, we restrict attention to constant dividend rules.

Definition: Given a dividend rule $\bar{d} > 0$, a Stationary Recursive Competitive Equilibrium (SRCE) is a list of value functions $W(z), V_f(L)$, a list of policy functions for consumption, assets, and search intensity (respectively $c(z), a(z), s(z)$); aggregate values for capital, employment, unemployment, not in the labor force, vacancies, and search (denoted respectively $\tilde{K}, \tilde{L}, \tilde{U}, \tilde{N}, \tilde{V}, \tilde{S}$), rental prices for capital and labor $\{r, w\}$, with $R = 1 + r - \delta$, price of shares p, probabilities $\{\lambda_w, \lambda_f\}$, a distribution Ψ, and a transition function P for z such that:

1) Households optimize: given $\{r, w, \lambda_w, \sigma\}$, the FOC in Eq. (11) to (14) are satisfied by $c = c(z)$, $a' = a(z)$, and $s = s(z)$ ($s(a, e) = 0$), and $W(a, e)$ and $W(a, u)$ satisfy respectively Eq. (10) and (12).

2) Firms optimize: given $\{r, w, \lambda_f, \sigma, \bar{d}\}$, $\tilde{K}$, $\tilde{L}$, and $\tilde{V}$ are such that Eq. (16)

and Eq. (17) hold, the constraint in $\Gamma_f(L)$ is satisfied with $L = L' = \tilde{L}$, $V_f(L)$ satisfies Eq. (15), and $d = F(\tilde{K}, \tilde{L}) - r\tilde{K} - w\tilde{L} - \phi\tilde{V} = \bar{d}$.

3) p satisfies the no arbitrage condition in Eq. (8).

4) Factor markets clear:

$\tilde{S} = \int_Z \pi(s(z))d\Psi$, $\tilde{U} = \int_Z \mathcal{I}_{s(a,u)>0}d\Psi$ and $\tilde{N} = \int_Z \mathcal{I}_{s(a,u)=0}d\Psi$ hold, and

4.1) $\tilde{L} = \lambda_w\tilde{S}/(\sigma + \lambda_w\tilde{S})$, and $\tilde{L} + \tilde{U} + \tilde{N} = 1$;

4.2) $\tilde{K} = (\int_Z a(z)d\Psi - p - \bar{d})/R$;

5) Probabilities λ_w, λ_f satisfy Eq. (6) with $\tilde{U}$ and $\tilde{S}$, and $\tilde{V}$.

6) $P(z, C) = prob\{(a(z), y') \in C|y\}$, for $y \in O$ and $\forall C \in \mathcal{Z}$.

7) The distribution is stationary: $\Psi(C) = \int_Z P(z, C)d\Psi, \forall C \in \mathcal{Z}$.

The functions $\mathcal{I}_{s(a,u)>0}$ and $\mathcal{I}_{s(a,u)=0}$ are, respectively, the indicator functions that take the value one only when $s(a, u) > 0$ and when $s(a, u) = 0$, and take the value zero in all other cases. The next Proposition 4 states the connection between efficient allocations and the competitive allocation under complete markets.

Proposition 4: Let $y^* = \{c^*, s^*, l^*, k^*, v^*\} \in R^5_{++}$ be the stationary solution of the planner's problem corresponding to the previous economy (i.e., y^* constitutes the efficient allocation). Consider the analogous market economy with complete markets in which $\bar{d} = \phi[(1 - \tilde{\beta}^*(1 - \sigma))/(\lambda^*_f\tilde{\beta}^*) - v^*/l^*]$. Then the competitive allocation, y_c, is such that $y_c = y^*$.

Proof: See the Appendix.

4 Results

4.1 An analytical example

I start by looking at the consequences of the wealth effects on search in an infinite horizon version of the model in the previous section.

Proposition 5: Assume A1, u is bounded, $\pi(s) = s$ and that the borrowing limit is close enough to the *natural limit*.[19] Then any RSCE of incomplete markets is such that $\beta R = 1$ and $\tilde{S} = 0$, hence output reduces to home production.

Proof: See the Appendix.

In Proposition 5 A1 and u bounded help to use standard results from dynamic programing. The assumptions that the borrowing limit is sufficiently large allows for unlucky households that remain unemployed to reduce their asset holdings to smooth out consumption, which however converges to a small level. Finally, $\pi(s) = s$ is the same assumption as in Merz (1995) and as discussed in the previous section, it produces that sufficiently rich and poor agents both choose $s = 0$, even in an infinite horizon context. The usual result that $\beta R < 1$ in a RSCE with incomplete markets cannot hold in this case because being borrowing constrained becomes and absorving state (hence all agents would be borrowing constrained), and $\beta R > 1$ can be ruled out by the usual reasons. Hence necessarily $\beta R = 1$, in which case there are only rich agents holding the debt of poor agents, no agent works

[19]The natural debt limit assures present value budget balance and is equivalent to the largest amount of debt that the agent is able to repay with probability one in any following period, and hence, consumption in that period would be zero. See Aiyagari (1994).

and no production takes place.

Comments:

1. The result in Proposition 5 stands in stark contrast to the model economy with complete markets, such as Merz (1995), in which production would be positive in the deterministic steady state. These differences between complete and incomplete markets are strikingly larger than the ones found elsewhere in the literature neglecting the effect of the market arrangement on the distribution of idiosyncratic shocks (Aiyagari 1994, Huggett 1997, and Krusell and Smith 1998, amongst others). Hence, the results above should warn us against too quick comparisons across market arrangements, or policies, ignoring the feed-back effect on the distribution of idiosyncratic uncertainty.

2. In addition to the no production result of the economy with incomplete markets, any competitive equilibrium of the economy described above is characterized with polarization of the distribution of assets and with no mobility. Furthermore, there are many distributions Ψ that are compatible with equilibrium. These features are similar to the ones studied in Marcet and Obiols-Homs (2017) of an economy considering the effect of the so-called "nutrition curve".

3. The result in Proposition 5 follows in spite of the fact that the economy is very competitive. The main problem is that at low levels of assets, workers give up searching and find it optimal to stop looking for a job (i.e., competing in the labor market). This suggests that usual view that additional competition may be a way to eliminate inefficiencies and thus to increase

welfare, may not be desirable for discouraged agents. Hence, there seems to be ample room for welfare improvements through policy interventions and a welfare state.

Figure 1 displays a representation of decision rules for assets in the employment and in the non employment state, and also of the support of equilibrium distributions (the solid line in the horizontal axis). In the employment state the agent always faces uncertainty, and since $\beta R = 1$ it always pays off to increase assets. Hence, $a(a, e)$ only intersects the 45 degrees line at $\bar{A}$. The shape of $a(a, u)$ reveals the optimal decisions on search. Above $\bar{a}$ the wealth effect implies that $s(a, u) = 0$. This means that the agent faces no uncertainty, and since the interest factor is equal to the rate of time preference, she can sustain a large consumption level forever. Likewise, below $\underline{a}$ the wealth effect implies that $s(a, u) = 0$, hence agents face again no uncertainty and maintain the consumption level corresponding to keeping constant assets forever. Between the threshold levels for assets the agent in the unemployment state does search (and it may even be that $\lambda_w s(a, u) = 1$, so that the agent may fully insure the probability of finding a job in the following period). The decision rule for $a(a, u)$ is below the 45 degrees line precisely because the agent is smoothing consumption. Over time, either the agent converges to an asset level larger or equal than $\bar{a}$, or it converges to $\underline{a}$ (if the agent starts with an asset level below $\underline{a}$ then she stays there forever).

Figure 1 (at the end of the paper)

There are several policy options that in principle could avoid the catas-

trophic equilibrium described in Proposition 5. For instance, if the borrowing limit is stringent enough then agents with assets at that low level will find optimal to continue searching. Such a borrowing limit (or minimum saving) will rule out the "no search" state as an absorbing state, and thus, the stationary equilibrium will be similar to the one usually described in the literature (which is characterized by *i)* $\beta R < 1$, *ii)* no polarization of wealth, and *iii)* full mobility). Another possibility would be to implement a system of taxes at high income levels and subsidies for low income workers along the lines in Conesa, Kitao and Krueger (2009). Policies along these lines still have an impact on the search intensity of individuals, and so, the effect of such policies on the equilibrium is likely to be sizable. We use quantitative methods to investigate related issues in the next section.

4.2 Quantitative results

In the benchmark calibration described below I try to stay as close as possible to the previous studies that use a similar model. In light of Proposition 4 I choose to calibrate the complete markets version of the economy and then I use the efficient $\bar{d}$ in the incomplete markets economy to better isolate the effects of the market arrangement.

In the quantitative exercises reported below the utility function is given by $u(c_t, l_t) = \log c_t + n_0(1 - l_t)$, with l_t acting as an indicator function which takes the value 0 when the agent is working and the value 1 otherwise. Thus n_0 measures the desutility of working and the value of leisure when not working is normalized to zero. I also assume that the production function

is the usual Cobb-Douglas, $F(K,L) = f_0 K^{\alpha} L^{1-\alpha}$, and that the matching technology is given by $M(V,S) = VS/(V^{\gamma}S^{\gamma})^{(1/\gamma)}$.[20] In the examples I report below effective search by a non employed agent with $s \geq 0$ is given by

$$\pi(s) = s_0 + s_1 \frac{s}{1+s}. \tag{18}$$

Notice therefore that with $s_0 > 0$ effective search (hence the probability of finding a job) is positive even if the agent chooses $s = 0$, thus the catastrophic steady states described in the analytical example are not possible. Furthermore s_1 will be appropriately restricted so that the probability of finding a job will always be smaller than 1.

With these choices I need to calibrate β, n_0, f_0, α, δ, the cost parameters ξ and ϕ, the parameters directly related to the labor market: $\gamma, s_0, s_1, \sigma, \epsilon$, and the dividend rule $\bar{d}$.

I follow Krusell et al. and I calibrate the model to monthly frequency, hence I fix $\beta = 0.9967$, $\alpha = 0.3$ and $\delta = 0.0067$ which match the usual targets. With complete markets it is optimal to have either unemployed agents or agents not in the labor force, but not both categories at the same time. Hence I proceed under the assumption that there are only unemployed agents and choose $\sigma = .038$, which is consistent with the observed average probability of leaving employment in the U.S. for the period 1994-2007 at monthly frequency reported in Krusell et al. Consistently with this choice I target a job finding rate of 0.32, which again is consistent with the data

[20] This matching technology displays constant returns to scale and was introduced by Den Haan et al. (2000). The main advantage for our purposes is that both M/V and M/S are always smaller than 1.

reported in Krusell et al. Notice that these choices determine the efficient level of employment and unemployment at the steady state of the model. Given these values and in order to facilitate the comparison across market arrangements I normalize output under complete markets to one, which is obtained with $f_0 = 0.39$.

In the labor market I need to fix the values of s_0, s_1 and γ consistently with the target for the job finding rate of 0.32. I borrow $\gamma = 1.27$ from Den Haan et al. (2000). There is no clear empirical counterpart to fix the values for s_0 and s_1, hence I proceed as follows. If an agent in the model chooses not to search then her probability of finding a job in the following period would be given by $\lambda_w s_0$, where λ_w is the probability of finding a job per unit of search effort. Given this I fix $s_0 = 0.1$, a relatively small value, and target $\lambda_w s_0 = 0.044$, which is the empirical counter part reported in Krusell et al. for the U.S. Given the values for λ_w and s_0 I impose that even if an agent spends an infinite amount of resources searching the corresponding probability of finding a job will still be below one. In particular, I impose $\lambda_w(s_0 + s_1) = 0.95$ which then delivers $s_1 = 2.06$.[21] Next I choose the values for cost parameters ϕ, ξ and for ϵ. I fix $\epsilon = 0.311$ which is roughly equal to 40% of the individual labor income (the same as in Krusell et al.). I follow Hall and Milgrom (2008) and I target a cost of posting a vacancy of about 13% of the wage rate. To match this target I fix $\phi = 0.101$ and to be consistent with the previous targets this choice implies that $\xi = 0.034$ and that $n_0 = -0.549$. Finally, I assume $\underline{A} = 0$, and the $\bar{d}$ that renders the

[21] For completeness I report also the results from a sensitivity analysis to the choices for s_0 and s_1.

competitive equilibrium to be efficient is $\bar{d} = 4.7/10^4$. This concludes the benchmark calibration and it is summarized in Table 2.

*** Table 2 about here ***

Table 3 reports the equilibrium allocation under complete and incomplete markets (CM and IM from now on) under the assumption that no borrowing is permitted (hence $\underline{A} = 0$). Contrary to the results with exogenous idiosyncratic uncertainty, in the model with endogenous uncertainty several aggregates appear to be substantially smaller under IM than under CM. In particular, under IM output, the stock of capital and employment appear to be about three times smaller than under CM. On the other hand, the fraction of unemployed is about 70% under IM and about 10% in CM, search effort is ten times larger under IM, and the capital labor ratio which is about 18% larger under IM. The rough picture that emerges is that the lack of insurance markets leads unemployed agents to larger search which then collapses the labor market and precludes the possibility of capital accumulation for precautionary reasons: the scarcity of capital implies that firms do not find attractive to post a large number of vacancies, which otherwise are filled with almost probability one, and the job finding rate dramatically decreases. The congestion of the labor market is a general equilibrium effect steaming from the endogenous nature of the search process. Figure 2 portrays a representation of the decision rule for search. Interestingly, the decision rule displays an inverted u-shape and eventually equals zero (for asset levels above 86 units). Furthermore, since borrowing is not permitted search effort remains at relatively high levels even for borrowing constrained

agents. The results under this standard calibration for the complete markets arrangement suggests that the lack of perfect insurance may have large effects on equilibrium allocations, and that these effects may run counter to the usual results stressed in the literature.

*** Table 3 about here ***

*** Figure 2 about here ***

Before we turn to other exercises it is convenient to briefly report the results of several examples conducted by way of a sensitivity analysis to various modifications of the benchmark parameters, namely s_0, s_1 and n_0 (one at a time, holding everything else constant). Broadly speaking the results suggest that increasing s_0, and reducing n_0, tends to reduce search effort and eventually promotes to choose not to participate in the labor force under IM. With respect to increases in s_1 it tends to increase the return to search and thus search effort increases. Interestingly, in all the examples I have computed output and capital are always larger under CM compared to the IM case. A summary of these results can be found in a Table 4 in Appendix 6.3.

We saw in Figure 2 that as assets increase from the borrowing limit search initially increases, but then it decreases and it eventually vanishes at a relatively high asset level. Wealth effects, therefore, have sizable differences for the idiosyncratic uncertainty hold by heterogeneous agents, even if all them face the same prices. The optimal decision rule for search suggests that with a larger borrowing limit agents could possibly be poorer and thus the borrowing limit may have a sizable effect on equilibrium allocations. Figure 3

reports the equilibrium decision rule for several borrowing limits (expressed as a fraction of the natural debt limit implied by the corresponding equilibrium prices). In the reported examples even though search effort decreases for the poorer agents in each case, it is clear that they always find optimal to exert strictly positive search effort. At higher levels of wealth it is never optimal to search for a job (the sluggish shape for $B = .5B^*$ is due to a noisy graphical representation). A combination of a generous borrowing limit together with a smaller value for s_1 along the lines discussed in the previous paragraph is able to produce a positive mass of very poor agents choosing not to search in the labor market. For completeness, Table 5 in Appendix 6.3 reports the equilibrium allocations under the various borrowing limits.

The previous examples suggest that with endogenous probability of finding a job the effect of the market arrangement on equilibrium allocations is large. Since in the current model search creates a powerful externality it is natural to wonder if there are multiple equilibria. I investigate this issue by looking for equilibria in different regions of the non negative (k, θ) orthant. I find that the labor market effectively approximately clears at higher values for θ (remember that $\theta = V/S$) for which employment and unemployment under incomplete markets is substantially closer to the complete markets counterpart. However, in these cases I also find that the capital market is always far from clearing. Hence, the equilibria reported in the previous tables appear to be unique.

4.3 Matching stocks and flows in the labor market

A common feature of the examples reported above is that relative to the actual data from the U.S. economy the fraction of unemployed agents is too large and the fractions of employed and out of the labor force are too small. Thus I recalibrate the model under IM to roughly match the fractions of workers that are employed, unemployed and not in the labor force in the U.S., and then I assess the ability of the model to reproduce the flows between these three states.[22]

Tables 6 and 7 report the equilibrium allocation assuming the benchmark calibration but with $s_0 = 0.2$, $s_1 = 2.2$, $n_0 = -1.15$ and a borrowing limit equal to $0.999B^*$.[23] As is clear from Table 7 (top panel) the new calibration essentially reproduces the distribution of agents among the three states of the labor market, albeit the fraction of unemployed is slightly too large compared to the data. In Table 7 (lower panel) there is a closer look to the flow probabilities among E, U and N. Broadly speaking the model does a good job at explaining the transitions from E and from N. However it is also clear that the transitions starting from U are far from the empirical counterparts. For instance, in the model there is no way to go from U to N. This is due to a small inaccuracy in the integration by simulation: under the current calibration there are very poor agents that choose not to search, but their mass is negligible. Related to this, in the model there is a positive mass of agents that do transit from N to

[22] The spirit of this exercise is similar to that in Cole and Rogerson (1999).

[23] These modifications do not alter the goodness of fit with respect to other targets: ϵ still represents 40% of the equilibrium w, and the cost of creating a vacancy is about 13% of w.

U. This situation corresponds to rich unemployed agents (being employed for a long time period and eventually becoming unemployed), whom start their unemployment spell not participating in the labor market, hence they deplete assets, and only when they are sufficiently poor they start actively looking for a job.

There is not a unique combination of parameters that is consistent with the labor market stocks stressed in tables 6, 7. However, in all combinations I tried the transitions from the U state are off the empirical counterparts. The intuition for this finding is that one would need to increase the transition rate from U to E, but then the stock at U also increases and grossly overestimates the unemployment rate in the data.

5 Conclusion

This paper develops a general equilibrium model with production and frictions in the labor market to study the link between wealth, idiosyncratic uncertainty and the market arrangement which is taken as a policy variable. It is found that the lack of complete insurance markets gives rise a a wealth effect at low levels of wealth such that the poorer agents in the economy may find optimal not to participate in the labor market. In the quantitative exercises using a version of the model calibrated to the U.S. economy it is found that search externalities are larger under incomplete markets and thus the differences in the equilibrium allocations between complete and incomplete markets remain sizable. Finally, additional results suggest that the model is

able to account for some of the stocks and flows observed in the U.S. labor market, but not all.

The results in this paper suggest that there is a powerful mechanism linking wealth to uncertainty at the individual level and it is worth to continue exploring its deeper implications for the labor market outcomes but also away from its boundaries. With respect to the labor market it would be interesting to consider human capital that depreciates during unemployment spells, the effect of the spouses and also the equilibrium allocations when there is imperfect commitment. These extensions may help to rationalize the transitions from unemployment to the employment and to non participation that the current model fails to reproduce. Beyond the boundaries of the labor market the decisions about health prevention and education acquisition taking into account the sort of wealth effects on the distribution of idiosyncratic uncertainty are prime candidates of promising lines for future research. At the heart of this research agenda there is the concern for the mass of socially excluded individuals that for some reason choose not to participate in the labor market, the financial market, in health prevention programs and or in education activities. The mechanisms explored in this paper may provide a rationale for these choices and thus they may help to design better policy interventions. This is interesting work that will be conducted in the near future.

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6 Appendix

6.1 Some empirical evidence

Direct evidence on expenditures incurred to find a job is not currently available, and thus I resort to an indirect method to provide some support for the non linear wealth effects addressed in this paper. In particular, I use data from the PSID for the waves 2003, 2005, 2007, 2009 and 2011, which includes not only the occupational status of the head of the household, but also her/his educational attainment, various measures of income and wealth, and several questions about what are the actual methods used to find a job.[24] Hence this data allows me to investigate if there is a link between the level of a given type of income/wealth and the number of methods used to find a job. To this end, for a given type of wealth I divide the Dollar distance between the largest and the smallest reported amount into 10 equal intervals, and I compute the average number of search methods used by the agents in each wealth interval.

Table 1 reports the average number of search methods used by unemployed household heads included in each interval and for several measures of income/wealth (total family income, checking/saving accounts, stocks, home equity, wealth net of debts, and wealth plus home equity). For instance, in the first raw I use total family income, and it is clear that the average number of methods initially increases, it reaches a top in the fifth interval, and

[24] In terms of methods of search the panel includes 8 options for the kind of activities realized during the last for weeks before the interview: contacting public employment agencies, private employment agencies, the current employer, other employers, talking to friends or relatives, placing or answering ads, other activities, and do nothing.

then it declines as income continues increasing. To rule out the possibility of low labor productivity being responsible for low search activity at low levels of income, in this calculation I only include household heads with an education level of tenth degree and higher.[25] Furthermore, I checked that similar results obtain when I decrease the minimum requirement to 9th or 8th degree, but more importantly, also when I increase it to 11th or 12th degree. In rows 2 to 6 of Table 1 I repeat the same calculations but using alternative types of wealth, and roughly speaking a similar inverted u-shape relationship between wealth and search effort/activity is obtained.[26] Even if some measures of wealth have unclear effects on search (as in the case of the Checking and Saving accounts), it is remarkable that in all cases the largest average number of search methods is never located in the lowest interval of wealth, but rather it is very often located in the fifth interval. Finally, the results reported for the higher levels of wealth are consistent with standard theory, which would predict that at higher levels of wealth agents would search less. I take these observations as suggesting that income/wealth effects on search effort may be non linear, and thus, relatively poor agents may end up searching less than relatively richer agents.

*** Table 1 about here ***

[25]The Completed Education Level is also collected in the PSID. Values in the range 1 - 16 represent the actual grade of school completed. A value of 17 indicates that the Head completed some postgraduate work. The average degree in the subsample (I only consider individuals for which there is actually information about income and wealth) is slightly above 12, and its standard deviation is about 3.

[26]In some intervals there are no observations because of the well known fact that the distribution of wealth is skewed to the right in the U.S.

6.2 Proofs

Proof of Lemma 1: We proceed by contradiction. Suppose that $\xi R \geq \lambda w h_0$ and that the solution to the decision problem entails $a(a)$ and $s(a) > 0$ for some $a > 0$. The value of such a policy, $v(a)$ satisfies that:

$$\begin{aligned} v(a) &= u(a - a(a) - \xi s) + \beta\{\lambda s(a)[u(a(a)R + wh_0) - m] \\ &\quad +(1 - \lambda s(a))u(a(a)R)\} \\ &\leq u(a - a(a) - \xi s(a)) + \beta\{u(a(a)R + \lambda s(a)wh_0) - \lambda s(a)m\}, \end{aligned}$$

since u increasing and concave by A1. Consider now the value in utility of an alternative policy, call it $\tilde{v}$, in which $s = 0$ and we let $\tilde{a} = a(a)+\xi s(a)$. Notice that the policy is feasible. Then, $\tilde{v} = u(a - \tilde{a}) + \beta u(\tilde{a}R)$. Finally, suppose toward a contradiction that $\tilde{v} \leq v(a)$. Since $u(a - \tilde{a}) = u(a - a(a) - \xi s(a))$ by construction, $\tilde{v} \leq v(a)$ requires that

$$u(\tilde{a}R) \leq u(a(a)R + \lambda s(a)wh_0) - \lambda s(a)m < u(a(a)R + \lambda s(a)wh_0),$$

hence $\tilde{a}R = (a(a) + \xi s(a))R < a(a)R + \lambda s(a)wh_0$ (by A1). This contradicts that $\xi R \geq \lambda w h_0$, and the proof is concluded ■

Proof of Proposition 1: We proceed in two steps. We show first that there is a level $\hat{a}$ for $a(a)$ such that the FOC in Eq. (2) cannot hold with equality. To see this, notice that $s(a) > 0$ requires $u(a(a)R + wh_0) - m - u(a(a)R) > 0$. It follows from A1 that $u(a(a)R + wh_0) - u(a(a)R) \leq u'(a(a)R)wh_0$. Since $u'(a(a)R)$ converges to zero as $a(a)$ diverges to infinity, then there is $\hat{a}$ such that $u(a(a)R + wh_0) - m - u(a(a)R) = 0$. Thus if $a(a) \geq \hat{a}$, then

$u(a(a)R+wh_0)-m-u(a(a)R) \leq 0$, and therefore $s(a)=0$. In step two we show that there is $\bar{a}$ such that $a(\bar{a}) \geq \hat{a}$. To see this, suppose that $a(a) < \hat{a}$ for all a. Since consumption in the second period is strictly increasing in $a(a)$, then it follows from Eq. (1) that

$$\begin{aligned} u'(c(a)) &= \beta R\{\lambda s(a)u'(a(a)R+wh_0)+(1-\lambda s(a))u'(a(a)R)\} \\ &\geq \beta R u'(a(a)R+wh_0) \\ &> \beta R u'(\hat{a}R+wh_0), \end{aligned}$$

where the weak inequality uses that $s(a) \in (0, 1/\lambda]$ and that u is strictly concave, and the strict inequality uses that $a(a) < \hat{a}$. Hence $c(a) < \bar{c} = (u')^{-1}(\beta R u'(\hat{a}R + wh_0))$, i.e., consumption is bounded above. However, using in the budget constraint the hypothesis that $a(a) < \hat{a}$, and the constraint $0 \leq s(a) \leq 1/\lambda$, then consumption can be made arbitrarily large by choosing an arbitrarily large a. Hence if we define $\bar{a} = \hat{a} + \bar{c} + \xi/\lambda$, then it must be the case that $a(\bar{a}) > \hat{a}$. This contradicts the fact that $a(a) < \hat{a}$ for all a. Finally, we show that for $s(a) = 0$ for all $a > \bar{a}$. To this end, take any $a > \bar{a}$, and suppose that $s(a) > 0$. This necessarily requires that $u(a(a)R + wh_0) - m - u(a(a)R) > 0$, which means that $a(a) < \hat{a}$. By the same previous argument, we would conclude that $c(a) < \bar{c}$, and using again the budget constraint, we would find that $a(a) = a - c(a) - \xi s(a) > \bar{a} - \bar{c} - \xi/\lambda = \hat{a}$. This contradicts that $s(a) > 0$, and the proof is concluded

■

Proof of Proposition 2: We will show that the FOC in (2) holds with strict inequality for all positive a but smaller than some $\underline{a}$, and thus, $s = 0$ for

all $a \in (0, \underline{a})$. To see this, notice that $s > 0$ implies that $a' < a$ (hence $a'R < aR$) and that $c < a$. $s > 0$ also implies that the FOC in (2) holds with equality:

$$\xi u'(c) = \beta\lambda[u(c_e) - m - u(c_u)].$$

With u increasing and strictly concave the right hand side of the previous equation is monotonically decreasing in a', and if in addition u is bounded, then it converges to $\beta\lambda[u(wh_0) - m - u(0)]$ as a converges to zero. And yet, by A1 $u'(c)$ diverges to infinity as a converges to zero. Hence, with bounded u there is $\underline{a}$ that satisfies $\xi u'(\underline{a}) = \beta\lambda[u(wh_0) - m - u(0)]$ (hence we are implicitly assuming that at zero assets being employed is preferable to not having a job), and so, if $a \in (0, \underline{a})$ then $s = 0$. This argument also takes care of the CRRA case with $\gamma < 1$ (remember that γ is the parameter governing relative risk aversion). For the CRRA case with $\gamma > 1$, we combine the previous FOC with the FOC corresponding to assets accumulation to obtain:

$$\xi R\lambda s c_e^{-\gamma} + \xi R(1 - \lambda s)c_u^{-\gamma} = \lambda\frac{c_e^{1-\gamma}}{1-\gamma} - \lambda\frac{c_u^{1-\gamma}}{1-\gamma} - \lambda m.$$

Rewrite the previous expression and multiply both sides by c_u^{γ} to obtain:

$$\lambda\left(\frac{c_u}{c_e}\right)^{\gamma}\left(\xi Rs - \frac{c_e}{1-\gamma}\right) + \xi R(1 - \lambda s) + \lambda\frac{c_u}{1-\gamma} = -\lambda m c_u^{\gamma}.$$

Notice that as $a \to 0$ we have that $c_e \to wh_0$, $c_u \to 0$, and that $s \to 0$. Hence, it follows that the right hand side of the equation above can be made arbitrarily close to zero as $a \to 0$, and yet the left hand side converges

to $R\xi > 0$. Thus the FOC for s can not hold with equality when a is sufficiently small. The argument for the log case ($\gamma = 1$) is nearly identical. This concludes the proof ■

Proof of Proposition 3: Let $\breve{a} = (u')^{-1}(\beta R\lambda m/(\lambda w h_0 - \xi R))$. We have that $u'(c)\xi \geq \beta\lambda\{u'(c')wh_0 - m\} = \beta\lambda(u'(c)wh_0/(\beta R) - m)$, where the last equality follows from Eq. (3), which necessarily holds with equality when $s(a) = 0$. Rearranging we get that $u'(c) \leq \beta R\lambda m/(\lambda w h_0 - \xi R)$ (because of A2), which implies that $c \geq (u')^{-1}(\beta R\lambda m/(\lambda w h_0 - \xi R))$, by A1. Since $a - a(a) = c$, then the previous inequality can only happen if $a \geq \breve{a}$, as it was to be shown ■

Proof of Proposition 4: We consider first the efficient allocation corresponding to the economy in the main text. This allocation can be found by solving the following planner's problem:

$$\max_{\{c_t, s_t, v_t, k_{t+1}, n_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \{u(c_t) - m n_t\}$$

subject to

$$c_t + k_{t+1} + \xi s_t(1 - n_t) + \phi v_t = F(k_t, n_t) + (1 - \delta)k_t + \epsilon(1 - n_t), \qquad (19)$$

$$n_{t+1} = (1 - \sigma)n_t + M(v_t, s_t(1 - n_t)), \qquad (20)$$

the usual non negativity constraints, and taking k_0 and n_0 as given. The

FONC at a steady state corresponding to the previous problem read:

$$\begin{aligned}
c: &\quad u'(c) = \tilde{\mu} \\
s: &\quad \hat{\mu}\xi = \hat{\rho}M_2 \\
v: &\quad \hat{\mu}\phi = \hat{\rho}M_1 \\
k: &\quad 1 = \beta(F_1 + 1 - \delta) \\
n: &\quad \hat{\rho} = -\beta m + \beta\hat{\mu}[F_2 - \epsilon + \xi s_t] + \beta\hat{\rho}[1 - \sigma - M_2 s]
\end{aligned}$$

(Notice that we denote F_i and M_i, $i = 1, 2$, the partial derivatives of these functions with respect to the corresponding arguments). It is straightforward to show that the above FOC collapse into a single equation:

$$u'(c)\left\{\frac{\phi(1-\beta(1-\sigma))}{M_1} - \beta[F_2 - \epsilon]\right\} = -\beta m. \tag{21}$$

Hence, the stationary efficient allocation satisfies Equations (19), (20), and (21). We use * to denote the efficient allocation.

Consider next a market arrangement in which there is a representative household composed by a mass one of family members, and a representative firm. In this environment the family fully insures its members against consumption fluctuations due to unemployment shocks. The problem of the family consists of choosing the fraction of its members that work in each period, the amount of assets (equity plus capital, as given in Eq. (9)) to carry over the following period, search effort, and the amount of consumption of each

member in order to maximize present value of equally weighted utility:

$$\max_{\{c_t,s_t,a_{t+1},n_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \{u(c_t) - mn_t\}$$

subject to

$$c_t + a_{t+1}/R + \xi s_t(1-n_t) = wn_t + a_t + \epsilon(1-n_t), \tag{22}$$

$$n_{t+1} = (1-\sigma)n_t + \lambda_w s_t(1-n_t), \tag{23}$$

the usual non negativity constraints, and taking a_0, n_0, and R, w, σ and λ_w as given. The FONC at a steady state corresponding to the previous problem read:

$$\begin{aligned}
c: &\quad u'(c) = \mu \\
s: &\quad \mu\xi = \rho\lambda_w \\
a: &\quad 1 = \beta R \\
n: &\quad \rho = -\beta m + \beta\mu[w - \epsilon + \xi s] + \beta\rho[1 - \sigma - \lambda_w s].
\end{aligned}$$

As before, the above FOC collapse into

$$u'(c)\left\{\frac{\xi(1-\beta(1-\sigma))}{\lambda_w} - \beta[w-\epsilon]\right\} = -\beta m. \tag{24}$$

That is, given prices and probabilities of transition, the choices of the households satisfy stationary versions of Equations (22), (23), and (24).

The firm is assumed to act under perfect competition and thus it takes factor prices and probabilities of transition as given. In any given period the

representative firm chooses the amount of capital to be used in the current period, the number of vacancies to open for the following period, and the number of workers to employ in the following period, in order to maximize the present value of the firm (which is discounted at the rate $\tilde{\beta} = 1/R$):

$$\max_{\{K_t, V_t, N_{t+1}\}} \sum_{t=0}^{\infty} \tilde{\beta}^t \{F(K_t, N_t) - r_t K_t - w_t N_t - \phi V_t\}$$

subject to

$$N_{t+1} = (1 - \sigma) N_t + \lambda_F V_t, \tag{25}$$

the usual non negativity constraints, and taking N_0, and r, w, σ and λ_F as given. The FONC at a steady state corresponding to the previous problem read:

$$\begin{aligned} F: &\quad F_1 = r \\ V: &\quad \phi = \tilde{\rho} \lambda_F \\ N: &\quad \tilde{\rho}(1 - \tilde{\beta}(1 - \sigma)) = \tilde{\beta}[F_2 - w]. \end{aligned}$$

Combining the above FOC we get

$$\frac{\phi}{\lambda_F}(1 - \tilde{\beta}(1 - \sigma)) = \tilde{\beta}[F_2 - w]. \tag{26}$$

Hence, the solution of the firm's problem satisfies the stationary version of Eq. (25), and Eq. (26). [27]

[27] It is well known that the previous setting is observationally equivalent to one in which there is a continuum of single-agent households, each of them having access to a complete set of Arrow securities. Hence, the previous setting can be seen a complete markets economy in reduced form. Furthermore, we write the previous problems in sequence form, but standard arguments can be invoked to show that the solutions are the same ones we would obtain in a recursive formulation.

A stationary competitive equilibrium given a dividend rule $\hat{d}$ is a list (k_0, n_0), a list $y_c = \{c, n, s, k, x, v\}$, and a list $q_c = \{r, w, p\}$ such that the household and the firm optimize (i.e., equations (22)-(26) hold with $k = k_0$ and $n = n_0$), the no arbitrage condition in Eq. (8) holds, and such that all markets clear. We show next that with $\hat{d} = d^*$ and $(k_0, n_0) = (k^*, n^*)$, there are prices $q_c = \{r^*, w^*, p^*\}$ such that $y_c = y^*$. Specifically, with $(k_0, n_0) = (k^*, n^*)$ it is clear from the FOC of the firm that $r^* = F_1(k^*, n^*)$, and from the FOC of the household that $\beta R = 1$ and so $\tilde{\beta} = \beta$. Also, $d^* = \phi[(1 - \tilde{\beta}(1 - \sigma))n^*/(\lambda_F^* \tilde{\beta}) - v^*]$, with $\lambda_F^* = M(v^*, s^*(1 - n^*))/v^*$ and imposing $d = d^*$ (where $d = F(k^*, n^*) - r^* k^* - w^* n^* - \phi v^*$) implies that

$$w^* = F_2(k^*, n^*) - \frac{\phi}{\lambda_F^* \tilde{\beta}}[(1 - \tilde{\beta}(1 - \sigma)) = F_2^* - \frac{\phi}{\lambda_F^* \beta}[(1 - \beta(1 - \sigma)), \quad (27)$$

where we used the linear homogeneity of F. Equation (27) is nothing but Equation (26). That is, we used the FOC of the firm and the efficient allocation to find candidates to equilibrium prices for capital and labor. The no-arbitrage condition in the asset market requests that $p^* = \beta d^*/(1 - \beta)$. We show next that Equation (24) fed with the efficient allocation is satisfied at those prices. Inserting Eq. (27) into Eq. (22) and simplifying, it suffices to show that

$$\frac{\xi}{\lambda_w^*} + \frac{\phi}{\lambda_F^*} = \frac{\phi}{M_1^*}.$$

This is obvious once we realize that $\xi/\phi = M_2/M_1$ (from the FOC of the planner problem), that $M_1 = m'(\theta)$ (with $\theta = v^*/(s^*(1 - n^*))$ and $m(\theta) = M(v^*, s^*(1-n^*))/(s^*(1-n^*))$, $M_2 = m(\theta) - m'(\theta)\theta$, and that $\lambda_F^* = m(\theta)\theta^{-1}$, and $\lambda_w = m(\theta)$. Hence if we take $y_c = y^*$ the FOC associated to the

optimization problems of households and firms are all satisfied. Finally, we have already imposed market clearing capital and labor, the feasibility constraint is obviously satisfied, and the Walras law asserts that the shares market is also in equilibrium. This completes the proof ■

We now prove Proposition 5. The proposition is in fact a corollary of a series of lemmas, as follows. For convenience I introduce the following assumption:

A2': $wh_0 > \epsilon$.

Lemma 5.1: Assume A1-A2', u is bounded, $R > 1$, and $\lambda_w > 0$. There is $\bar{a} > \underline{A}_N$ such that if $\bar{A} \geq \bar{a}$, then $s(\bar{A}, u) = 0$.

Proof of Lemma 5.1: Consider the utility level associated to keeping assets constant at arbitrary levels a, and doing no search in the unemployment state: $\tilde{c}(a, u) = a(R-1)R^{-1} + \epsilon$ (notice, in particular, that this policy is feasible). Also, the budget constraint in the employment state together with the borrowing constraint implies that $c(a, e) \leq a + wh_0 - \underline{A}/R = \tilde{c}(a, e)$. Finally, notice that since u is bounded, then by A1 there is $\bar{a}$ such that $\bar{u} - m - u(\tilde{c}(\bar{a}, u)) = 0$, where $\bar{u}$ is the upper bound of u. It follows that if $G(a) = u(\tilde{c}(a, e)) - m - u(\tilde{c}(a, u))$, then $G(a) \leq 0$ for $a \geq \bar{a}$. We will show that with $\bar{A} \geq \bar{a}$, then $s(\bar{A}, u) = 0$. To see this, assume $\bar{A} \geq \bar{a}$, and notice that

$$\begin{aligned} W(\bar{A}, e) &= u(c(\bar{A}, e)) - m + \beta[(1-\sigma)W(a(\bar{A}, e), e) + \sigma W(a(\bar{A}, e), u)] \\ &\leq u(\tilde{c}(\bar{A}, e)) - m + \beta[(1-\sigma)W(\bar{A}, e) + \sigma W(\bar{A}, u)], \end{aligned}$$

where the inequality uses $\tilde{c}(\bar{A}, e) \geq c(\bar{A}, e)$, and the fact that $a(\bar{A}, e) \leq \bar{A}$,

and so,

$$W(\bar{A}, e) \leq \frac{u(\tilde{c}(\bar{A}, e)) - m}{1 - \beta(1 - \sigma)} + \frac{\beta\sigma W(\bar{A}, u)}{1 - \beta(1 - \sigma)}. \tag{28}$$

Consider now the decision in case $a = \bar{A}$ and the agent is unemployed. A feasible choice is precisely to fix $\tilde{s}(\bar{A}, u) = 0$ and $\tilde{a}(\bar{A}, u) = \bar{A}$, in which case the value associated to these choices is $\tilde{W}(\bar{A}, u) = u(\tilde{c}(\bar{A}, u))/(1 - \beta)$. Suppose, toward a contradiction that the previous choices are suboptimal, i.e., that $s(\bar{A}, u) > 0$. If this is so, then it must necessarily be also the case that reaching the employment state is better than staying in the unemployment state, i.e., that $W(\bar{A}, u) < W(a(\bar{A}, u), e) \leq W(\bar{A}, e)$ where the weak inequality holds because $\bar{A} \geq a(\bar{A}, u)$, hence $\Gamma(\bar{A}, e) \supseteq \Gamma(a(\bar{A}, u), e)$. Combining the above inequality with Eq. (28) we get that:

$$W(\bar{A}, u) < \frac{u(c(\bar{A}, e)) - m}{1 - \beta} \leq \frac{u(\tilde{c}(\bar{A}, e)) - m}{1 - \beta} \leq \frac{u(\tilde{c}(\bar{A}, u))}{1 - \beta} = \tilde{W}(\bar{A}, u),$$

where the second weak inequality follows from $G(a) \leq 0$, and the equality follows from $\bar{A} \geq \bar{a}$. Hence, $s(\bar{A}, u) > 0$ cannot be optimal, since there is another feasible policy that improves the value of the objective. Thus, $s(\bar{A}, u) = \tilde{s}(\bar{A}, u) = 0$, and the proof is concluded ■

Hence, Lemma 5.1 can be seen as the usual wealth effect on labor supply. We also have that sufficiently poor agents choose not to search:

Lemma 5.2: Assume A1-A2, u is bounded, $R > 1$, and $\lambda_w > 0$. There is $\underline{a} > \underline{A}_N$ such that if $\underline{A}_N < \underline{A} < \underline{a}$, then $s(a, u) = 0$ for $a \in [\underline{A}, \underline{a}]$.

Proof of Lemma 5.2: We follow the insight from Proposition 3 and show that the FOC in Eq. (14) cannot hold for a sufficiently small a. Proceed by

contradiction and assume $s(a,u) > 0$, and thus,

$$\xi u'(a + \epsilon - a(a,u)/R - \xi s(a,u)) = \beta\lambda_w[W(a(a,u),e) - W(a(a,u),u)] \quad (29)$$

holds. It is straightforward to check that the natural debt limit is given by $\underline{A}_N = -\epsilon R/(R-1)$. We show first that the right hand side of the above equation remains bounded for all $a \in A = [\underline{A}_N, \bar{A}]$. To see this, notice that for all $a \in A$, $\Gamma(a,e) \subseteq \Gamma(\bar{A},e)$, hence by construction we have $W(a,e) \leq W(\bar{A},e) < u(\bar{c}_e)/(1-\beta)$, where $\bar{c}_e = wh_0 + \bar{A} - \underline{A}/R$. Likewise, for all $a \in A$, $W(a,u) \geq u(0)/(1-\beta)$. Both $u(\bar{A})$ and $u(0)$ are finite by hypothesis. Therefore, we conclude that the following inequality holds:

$$[W(a,e) - W(a,u)] \leq [u(\bar{A}) - u(0)]/(1-\beta) = M$$

for all $a \in A$, and for some $M < +\infty$. We argue now that the left hand side of Eq. (29) can be made arbitrarily large. This follows from the fact that u' is unbounded at the origin by A1, hence u' diverges to $+\infty$ as $a \to \underline{A}_N$. The implication is that there is $\underline{a} > \underline{A}_N$ such that

$$\xi u'(\underline{a} + \epsilon - \underline{A}_N/R) = \beta\lambda_w M.$$

It is straightforward to show that for $a \leq \underline{a}$, $u'(\underline{a} + \epsilon - \underline{A}_N/R) \leq u'(a + \epsilon - a(a,u)/R - s(a,u)\xi)$. Hence, for $a \leq \underline{a}$ the FOC cannot hold with equality, and thus, $s(a,u) = 0$ ■

Notice that Lemmas 5.1 and 5.2 do not hinge on βR being larger, equal, or smaller than 1. Irrespectively of this, then, there may be poor and rich

households that do not face uncertainty because they are better off by not searching. From now on we assume that the set A is such that the assumptions of the previous lemmas hold, which is summarize as

A3: A is such that $\underline{A}_N < \underline{A} < \underline{a}$ with $\underline{A} < 0$, and $\bar{A} \geq \bar{a}$.

Notice that the borrowing limit does allow some borrowing. Our next task is to characterize the gross return on assets in any SRCE.

Lemma 5.3: Assume A1-A3, and u is bounded. In any SRCE, $\beta R = 1$.

Proof of Lemma 5.3: The proof has two parts. In the first part we argue that $\beta R > 1$ is not possibly a SRCE. In the second part we rule out the case of $\beta R < 1$, hence the only possibility is that $\beta R = 1$. With respect to the first part, consider the FOC with respect to capital, which we write as

$$u'(c(z)) \geq \beta R E[u'(c(z'))|z]$$

and must hold for all agents. The assumption and u concave imply that

$$u'(c(z)) > u'(E[c(z')|z]),$$

hence $c(z)) < E[c(z')|z]$. Integrate both sides of the previous expression with respect to the invariant measure Ψ to obtain that

$$\int_Z c(z) d\Psi < \int_Z E[c(z')|z] d\Psi = \int_Z c(z) d\Psi,$$

which is a contradiction (the equality follows by Theorem 8.3 in Stokey and Lucas 1989).

For the second part of the proof suppose for a moment that the mass of agents that are borrowing constrained is zero, so that proceed as in the previous in the proof and integrating the FOC of the consumer problem we obtain

$$-(1-\beta R)\int_Z u'(c(z))d\Psi = 0,$$

which is impossible when $(1-\beta R) > 0$. Hence there is a subset $\underline{Z}$ of Z such that if $z \in \underline{Z}$ then $a(z) = \underline{A}$. In stationary equilibrium we must have that $\Psi(\underline{Z}) = 1$, because unemployed agents do not search (Proposition 6). That is, $z = (\underline{A}, u)$ is an absorbing state. Hence if $\beta R < 1$ then all agents are indebted and this is incompatible with market clearing. Thus the only possibility is that $\beta R = 1$ ■

Lemma 5.4: Assume A1-A3, and u is bounded. In any SRCE $\lambda_w = 0$.

Proof of Lemma 5.4: Under the maintained assumptions, in an SRCE we have that $\beta R = 1$. It follows from the FOC governing asset accumulation that $a(a,u) = a$ for $a \leq \underline{a}$, because $s(a,u) = 0$ by Proposition 5. This means that the following equation is satisfied for all $a \in A$:

$$u'(c(z)) + \Xi(z) = E[u'(c(z'))|z]. \tag{30}$$

Integrate both sides of the above equation using the invariant measure Ψ, and use Theorem 8.3 in Stokey and Lucas (1989) to get that $\int_Z \Xi(z)d\Psi = 0$. Hence we have that $u'(c(z)) = E[u'(c(z'))|z]$, so $u'(c(z))$ is a bounded, non-negative, martingale, and thus it converges. The arguments from Huggett (1997) apply also in the case of $\beta R = 1$ to show that $a(a,u) < a$ for $a \in (\underline{a}, \bar{a})$,

hence the only places u' can possibly converge are $\underline{u'} \geq u'(c(\underline{a}, u))$ and $\bar{u}' \leq u'(c(\bar{a}, u))$. In either case $s(a, u) = 0$ (by Lemmas 5.1 and 5.2), hence $\lambda_w = 0$ ■

Proof of Proposition 5: The proof follows directly from Lemmas 5.1-5.4 ■

6.3 Computing an Equilibrium

I approximate the policy functions that solve the agent's problem on the employment and unemployment state on a grid of 800 points (not equally spaced). Then I proceed as follows:

1. Let $z = (\tilde{k}, \theta)$ where $\tilde{k} = K/N$ and $\theta = V/S$, and guess initial values for z. Using θ I obtain the implied λ_w and λ_f from the corresponding M/S and M/V equation, which can be written as a function of θ.

Given $\tilde{k}$ and $\lambda_f(\theta)$ it is straightforward to use the FOC of the firm's problem to obtain the implied r, and the implied w:

$$r(\tilde{k}) = f_0 \alpha (\tilde{k})^{\alpha - 1}, \text{ and } w(\tilde{k}, \theta) = f_0 (1 - \alpha)(\tilde{k})^{\alpha} + \frac{\phi((1 - \sigma)\tilde{\beta} - 1)}{\lambda_f(\theta)\tilde{\beta}},$$

where $\tilde{\beta} = 1/(1 + r(\tilde{k}) - \delta)$

2. Given $\lambda_w(\theta)$, $R(\tilde{k}) = 1 + r(\tilde{k}) - \delta$ and $w(\tilde{k}, \theta)$, I use the FOC of the agent's problems to approximate policy functions for assets and search, denoted respectively by by $a(a, e)$, $a(a, u)$, and $s(a, u)$. A non standard feature of the agent's problem is that search intensity in the unemployment state depends on the actual values of the value functions) of being employed and unemployed. Thus I guess initial policies $a_0(a, e)$, $a_0(a, u)$, and $s_0(a, u)$, ob-

tain the implied $W_0'(a,e)$ and $W_0'(a,u)$, and I also iterate on each agents problem to get initial $W_0(a,e)$, $W_0(a,u)$. Using these initial values I then search on the grid, and allow for non grid choices for assets by using the FOC and interpolation between points.[28]. At the end of the first iteration I get new policies, so that I can obtain again the associated values of the value functions and their derivatives. That is, in this step I keep iterating from policies to value functions until a pre-specified accuracy criterion is satisfied.

3. I simulate the decision rules over a large number of periods, and compute averages over time of the variables of interest. By ergodicity, these averages coincide with cross sectional averages (in practice I use 900,000 time periods and discard the first 10,000 observations to reduce the effect of initial conditions). In particular, in this step I obtain values for aggregate assets $A(z)$, aggregate search $S(z)$, employment $E(z)$, unemployment $U(z)$, and not in the labor force $N(z)$.

4. Given $\bar{d}$ and the $R(z)$ implied by $\tilde{k}$, the no arbitrage condition for assets determines $p(z) = \bar{d}/(R(z)-1)$. This is used in the market clearing condition for assets to obtain $K(z) = (A(z) - p(z) - \bar{d})/R(z)$. Given this value for capital and the previously found value for $E(z)$ I compute the implied $\tilde{k}_n(z)$, hence I can check if the equilibrium condition:

$$r(z) - f_0\alpha(\tilde{k}_n(z)))^{\alpha-1} = 0 \tag{31}$$

[28]I used linear interpolation, but I also tried Schumaker quadratic splines. Hence, the method is similar to the one described in Huggett (1993)

holds. Next, I use the equation involving the dividends rule to obtain

$$V(z) = (f0\tilde{k}_n(z)^{\alpha} - r((z))\tilde{k}_n(z) - w((z)) - \bar{d})E/\phi$$

and then obtain

$$\theta_n(z) = V(z)/S(z),$$

so I can check if

$$\theta - \theta_n(z) = 0 \tag{32}$$

approximately holds.

I iterate following the previous scheme until the last two equations are approximately satisfied. Unfortunately Newton based methods or the Broydn method are impractical and in general did not converge, thus very often I had to nail down the equilibrium candidate by a "manual bisection".

I find that the accuracy of the equilibrium decreases as the borrowing limit is increased. I tried to gain accuracy by including a large number of points in the relevant region of the state space (the region where the agent will spend most time in the simulation step). One difficulty one needs to deal with is that in this family of problems the largest amount of capital the agent is willing to hold in the employment state is large, so I included a few points for assets to be able to reach around 400 units (which the agents optimally chooses not to over-pass). I checked that having the largest value for assets at 500, 600, or 1,000 units has literally no effect on decision rules. I tried several methods to approximate the decision rules, such as plain value function iterations and the Schumaker method (see Judd, 1998), but they

were not useful either because of severe problems of accuracy, or because they were very unstable. I have also obtained results under different grids (including step-grids with larger distance between points in different steps, and exponential grids). The magnitude of the differences between complete and incomplete markets is sensitive to the grid I use, but the results go all in the same direction as the ones reported in the text.

6.4 Sensitivity analysis

Table 4 reports the results of a few examples conducted to assess the sensitivity of the results to several parameter values. The first column reproduces the steady state allocation in the benchmark calibration as a reference and the other columns report the allocation when a single parameter value is modified. Specifically, the second column considers the effect of increasing s_0 from 0.1 to 0.2, the third column reports the effects of decreasing n_0 from -0.58 to -1., and fourth column considers the effects of increasing s_1 from 2.06 to 2.2.[29] Increasing s_0 and n_0 makes less profitable to search and thus it tends to decrease, to the extent that there is a positive mass of rich agents choosing not participating in the labor market. The opposite effect is monotonically observed if these parameter values are increased. Increasing (decreasing) s_1 makes search more profitable and thus search increases (decreases).

Table 5 reports the results under more generous borrowing limits (all other

[29] These choices are arbitrary and represent an illustration of the effects. An extensive and systematic examination of increasing/decreasing these parameters was conducted and is not reported for reasons of space.

parameters are as in the benchmark calibration). In these exercises B^* stands for the natural debt limit in equilibrium, an endogenously determined object. A larger borrowing limit tends to decrease equilibrium aggregates such as capital, employment (hence output) and to increase unemployment and not in the labor force (the case of $B = 0.95B^*$ is anomalous and it is due to the lack of accuracy of the solution).

6.5 The differences across market arrangements

Table 1: Wealth effects on the number of search methods

Int. Wealth	1	2	3	4	5	6	7	8	9	10
TFI	1.2	1.5	1.5	1.7	2.1	1.7	1.0	–	–	0.0
CDA	1.3	1.2	1.7	1.5	1.3	1.0	0.0	–	–	0.0
STK	1.3	1.4	1.2	0.0	1.8	–	0.0	3.0	–	1.0
HEQ	1.0	1.3	1.3	1.1	2.5	–	–	2.0	–	1.7
W+EQ	1.3	1.4	0.6	0.0	3.0	–	–	–	–	0.0
W-EQ	1.3	1.3	1.2	0.7	3.0	–	–	–	–	0.0

Average number of search methods used by the unemployed households in each of the income/wealth intervals, as given by: Total Family Income (TFI), Checking and Deposit accounts (CDA), Stocks (STK), Home Equity (HEQ), Wealth plus Home Equity (W+EQ) and Wealth minus Home Equity (W-EQ). Data from PSID waves 2003, 05, 07, 09 and 11.

Table 2: Parameter values of the benchmark calibration.

β	0.9967	α	0.36	δ	0.0067
f_0	0.39	s_0	0.1	s_1	2.06
σ	0.038	ξ	0.034	ϕ	0.101
ϵ	0.311	n_0	-0.549	$\bar{d}$	$4.7/10^4$

The calibration corresponds to the complete markets economy under the efficient dividends rule. ϵ^* equals 40% of labor the labor income in the competitive equilibrium with complete markets.

Table 3: Steady states under complete and incomplete markets.

	Incomplete Markets	Complete Markets
Y	0.346	1.
K	11.631	29.967
E	0.294	0.893
U	0.705	0.106
N	0.	0.
K/E	39.550	33.525
S	0.757	0.077
V	0.01	0.047
λ_w	0.013	0.44
λ_f	0.996	0.71

Table 4: Steady states with IM, sensitivity analysis.

	Bench.	$s_0 = .2$	$n_0 = -1$	$s_1 = 2.2$
Y	0.346	0.557	0.21	0.407
K	11.631	17.741	7.373	13.498
E	0.294	0.485	0.181	0.349
U	0.705	0.514	0.566	0.651
N	0.	0.0001	0.251	0.
K/E	39.550	36.569	40.28	38.68
S	0.757	0.755	0.333	0.782
V	0.01	0.017	0.006	0.125
λ_w	0.013	0.023	0.019	0.015
λ_f	0.996	0.993	0.994	0.995

Table 5: Steady states with IM and several borrowing limits.

	$B = .25B^*$	$B = .5B^*$	$B = .75B^*$	$B = .95B^*$
Y	0.417	0.323	0.217	0.316
K	12.828	9.857	6.613	9.607
E	0.369	0.287	0.193	0.28
U	0.629	0.706	0.782	0.706
N	0.0002	0.007	0.024	0.012
K/E	34.691	34.35	34.229	34.193
S	0.705	0.737	0.721	0.725
V	0.013	0.01	0.007	0.013
λ_w	0.019	0.014	0.009	0.014
λ_f	0.994	0.996	0.997	0.996

B^* is the natural debt limit corresponding to each equilibrium.

Table 6: Equilibrium allocation matching stocks in the labor market

Y	0.673
K	20.18
K/E	33.496
S	0.109
V	0.025
λ_w	0.207
λ_f	0.891

Table 7: Stocks and flows in the labor market

U.S. 1994-2007				Model			
$E = 0.632$				$E = 0.602$			
$U = 0.034$				$U = 0.08$			
$N = 0.334$				$N = 0.318$			
From	To			From	To		
	E	U	N		E	U	N
E	0.962	0.013	0.025	E	0.962	0.013	0.025
U	0.276	0.501	0.223	U	0.12	0.88	0
N	0.044	0.026	0.929	N	0.042	0.005	0.953

Data from the U.S. comes from Table 1 in Krusell et all (2011).

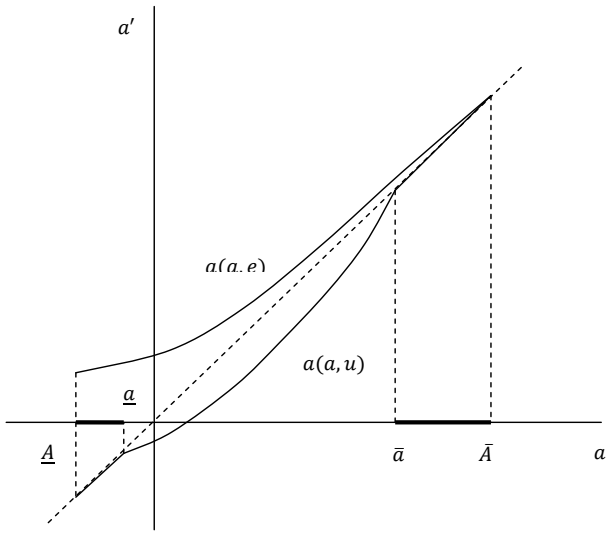

Figure 1: Decision rules for assets under incomplete markets when $\beta R = 1$. The solid line is the support of the equilibrium distribution.

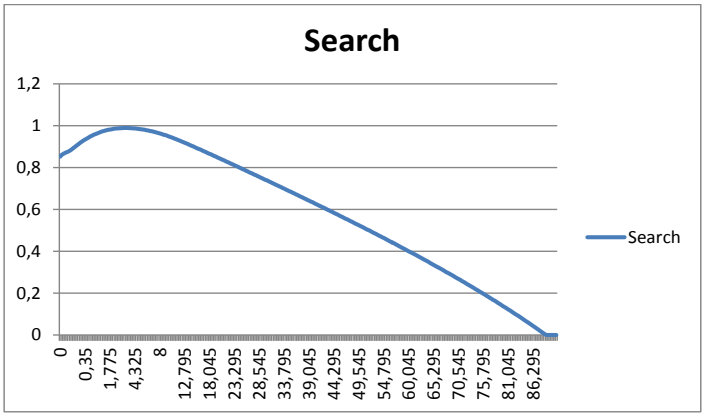

Figure 2: Equilibrium decision rule for search with $B = 0$ as a function of assets.

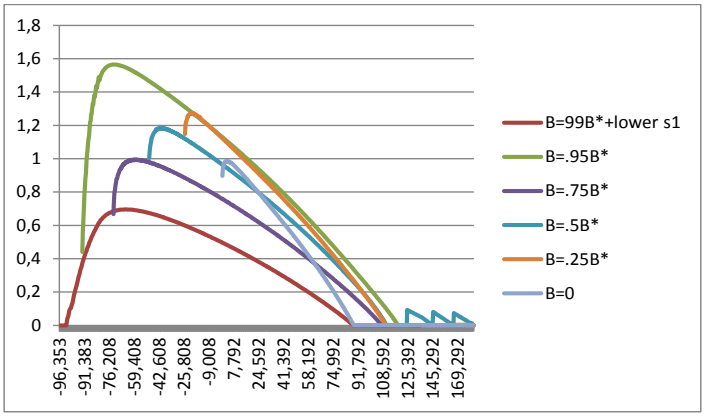

Figure 3: Equilibrium decision rules for search under several borrowing limits. B^* stands for the natural limit in the corresponding equilibrium.