Recovering the Principle of Minimum Differentiation. An Iceberg Approach

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Recovering the Principle of Minimum Differentiation. An iceberg approach.*

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Abstract

The clustering of competitor outlets is a pervasive phenomenon in our cities. However, Hotelling's principle of minimum differentiation is well-known not to hold. The attempts to modify Hotelling's model are numerous in the literature on spatial competition, but mostly unsuccessful. We provide a new approach by endogenizing the transportation cost. In particular, we inherit a modeling technique from the literature of international trade. This is the iceberg formulation. With it, we are able to give a rationale to the agglomeration of firms in the middle of a Hotelling linear market.

Keywords: Iceberg transport costs, Principle of minimum differentiation, Hotelling.

JEL Classification: L12, D42, R32

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1 Introduction.

Back in 1929, Hotelling obtained his celebrated "Principle of Minimum Differentiation " (a term coined by Boulding, 1955). The result is appealing because it conforms with the common observation in many cities of the concentration of outlets of competitor firms very close to each other[1]. Also, it is a result widely used in models of electoral competition since the work of Downs (1957). However, d'Aspremont et al. (1979) showed that Hotelling's analysis was flawed due to the lack of quasi-concavity of the profit functions of the competing firms. Since then a large literature developed proposing variations of Hotelling's original set-up to recover the Principle. These attempts range from using mixed strategies to different conjectural variations; from "playing" with the distribution of consumers on the space to using probabilistic (logit) models. Generically, all these attempts share a common feature. This is the separation of the consumption good market from the transportation market. The latter characterized by a parametric price.

This paper can be viewed as a revision of Hotelling's spatial model of oligopolistic competition where the distinctive feature is the integration of transportation and consumption. Our proposal is motivated on two grounds. First, there are many sectors of the economy where transportation is supplied under oligopolistic conditions. Second, there are also markets showing a discrepancy between the amount delivered at the point of production and the amount received at the point of consumption. Some examples will help to illustrate our point.

The first one is borrowed from the energy sector. To satisfy the demand of electricity of an individual living at some distance from the power plant, the latter has to produce an extra amount of electricity to account for the power transmission losses due to the heating of the high- and low-tension wires from the power plant to the point of consumption. This is the so-called Joule effect. The overall losses between the power plant and consumers are between 8% and 15%.

[1]Excellent surveys of this literature are Anderson, *et al* (1992), Beckman and Thisse (1986) and Gabszewicz and Thisse (1986).

A second example taken from the agriculture sector considers the demand of water of a farmer in need to water a field located at a given distance from a lake or a water marsh providing the water supply. The supplier in satisfying the demand of the farmer has to take into account the water that will be lost during its transportation along the pipes joining the reservoir of water and the farm, due to evaporation, filtering, and other factors.

Finally, a third example inspired in the market for perishable goods refers to the post-harvest losses of vegetables, grains, fruits, flowers and the like. Usually, these are transported in refrigerated trucks with the refrigeration system consuming fuel from the tank of the truck.

In more general terms, we can think of this phenomenon as a loss of quality or, in a temporal interpretation, as a lag between the buying of the commodity and its consumption. This latter view is consistent with a reduced form of the evaluation of the time an individual spends in the shopping decision (comparing prices, traveling to the shop and back, etc.) before actually consuming the good. Note that this expenditure is evaluated at the price of the good consumed. This conforms with the observation that individuals take longer (think more thoroughly) when acquiring expensive goods as compared with cheap ones. This interpretation leads us to the literature of price dependent preferences à la Pollak (1977), in the particular case with "normal prices" as the only relevant ones, since the budget constraint ("market prices") is not binding as all consumers have high enough willingness to pay and their decision is to choose what provider to patronize.

The approach we propose to tackle this issue has been extensively used in urban economics (see e.g. Fujita, 1995, Abdel-Raman, 1994, Abdel-Rahman and Anas, 2004) or in general equilibrium models of international trade (see e.g. Krugman, 1991a,b, 1992, Helpman and Krugman, 1988). Curiously enough, in those areas the way to cope with the difference between the amount of good delivered and consumed has been different. There, transport costs are formulated in terms of the transported commodity. This modeling was formalized by Samuelson (1954, 1983) as "iceberg transport costs" taking up an idea originated in von Thünen (1930). Excellent surveys of this literature are Fujita and Krugman (2004), Fujita et al. (2000), Neary (2001), Ottaviano and Thisse (2004) and Schmutzler (1999). A usual interpretation of the iceberg transport cost in the so-called New Economic Geography, is that it embodies information costs, institutional and trade barriers, quality standards, and cultural and linguistic differences[2], in addition to the pure transport costs (Ottaviano and Thisse, 2004; Fujita *et al.*, 1999).

A common technical feature of the iceberg and traditional transport technologies is that both give rise to a delivered price function convex with distance. However, empirical evidence points towards concave delivered price schemes rather than convex with distance. This leads McCann (2005) to raise a warning against the use of such transport costs functions to provide real-world insights as the properties of such transport costs are largely implausible when compared with the available empirical evidence. In this respect de Frutos *et al.* (2002) show that for any convex transport cost function there exists a concave one such that the location-then-price games induced by these functions are strategically equivalent. Our proposal also overcomes McCann's criticism as it is robust to concave and convex transport cost functions.

A different approach to the use of iceberg transport cost functions follows Krugman (1998) where the spatial iceberg assumption is considered as a pure technical device for avoiding the need to model a two-sector economy with a consumption good industry and a transport industry. Following this line of reasoning, it can be argued that the iceberg approach vis-a-vis the Hotelling approach in the modeling of transport costs allows for an endogenous determination of the provision of transportation services. Also, as the price of transportation per unit of distance is assumed constant, implicitly it amounts to assume that the transport sector is perfectly competitive. No empirical evidence supports such assumption. However,

[2]This interpretation of the iceberg formulation of transport costs may not be very rigorous. The iceberg formulation implies that transport costs are dependent on the price of the transported commodity. It is not always easy to establish the link between some of those arguments and the price of the transported good.

as we will see, the technical simplicity of Hotelling analysis contrasts with the complexity of the iceberg formulation.

We aim at providing a direct comparison of the "costs" and "benefits" of using both approaches in a common framework. Thus, we propose to study price and location equilibria in a duopoly model à la Hotelling where transport costs are modeled in the iceberg tradition. In this sense, we want to contribute to the debate between the modeling of transport costs in the spatial location and price competition models and in the new economic geography tradition.

The iceberg formulation has the property of generating a quasi-concave profit function in the price competition stage as in d'Aspremont *et al.* (1979) when considering quadratic transport costs. However, and in contrast with d'Aspremont *et al.*, our model shows that in the subgame perfect equilibrium, firms have incentives to locate as close as possible, thus recovering the Principle of Minimum Differentiation. In d'Aspremont *et al.* analysis, it turns out that quadratic transport cost induce such a harsh price competition, that firms soften it by optimally locating as far apart as possible (that is by differentiating the most). In our case, the reason for the optimal location pattern relies on the fact that measuring transport costs in units of the consumption good gives firms an absolute competitive advantage in their respective hinterlands. Accordingly, firms have incentives to minimize the market area where there is direct competition. They do so by locating as close as possible. We introduce a restriction in the analysis by considering symmetric locations only. While greatly simplifying the computations, it is not essential for the analysis (as in d'Aspremont *et al.*).

Our analysis contributes to providing a rationale to the prevalent observation of concentration of selling points in many markets such as bank branches, bakeries, restaurants, among many others.

The paper is structured as follows: section 2 introduces the concepts of melting function and melting rate associated with the iceberg transport technology. It clarifies the difference induced in demand as compared to the traditional modeling of transport costs. Section 3 illustrates the effects of the melting function approach in a competitive framework defined by a symmetric duopoly choosing sequentially first locations and then prices. Section 4 studies the price competition stage. Next, in section 5 the location decisions are analyzed. A section with conclusions closes the paper.

2 Melting versus transport costs

Let δ be the distance between a consumer x and a firm. We have to distinguish between the demand that an individual addresses to a firm from his (her) consumption. Denote by $q(\delta;\mu)$ the quantity of the commodity a consumer located at a distance δ of the firm needs to buy to consume exactly one unit of the good. It is assumed that:

$$q(\delta;\mu) \geq 1,\ \frac{\partial q}{\partial \delta} > 0,\ \frac{\partial q}{\partial \mu} > 0,\ \frac{\partial^2 q}{\partial \delta^2} \geq 0 \text{ and } \frac{\partial^2 q}{\partial \mu^2} \geq 0. \tag{1}$$

The parameter μ captures the "speed" at which melting occurs and thus, determines (together with distance), the individual's additional demand guaranteeing the consumption of one unit. When $\mu = 0$ there is no melting, or equivalently we can adopt the convention of interpreting that the melting technology has achieved the highest possible efficiency. This gives the lower bound to the value of μ. Generically, when $\mu = 1$ melting is proportional to a function of the distance, while if $\mu > 1$ the melting is more than proportional. We can interpret that the efficiency of the melting technology is negatively related to the value of μ. For instance, a R&D investment to accomplish more efficient cooling systems in transporting perishable goods would be reflected in μ.[3]

To further ease a proper understanding of the role of the melting in the modeling, Figure 1 illustrates the standard transportation cost and the melting function approaches.

[3] A detailed analysis of the properties of the iceberg transport cost functions is found in McCann (2005).

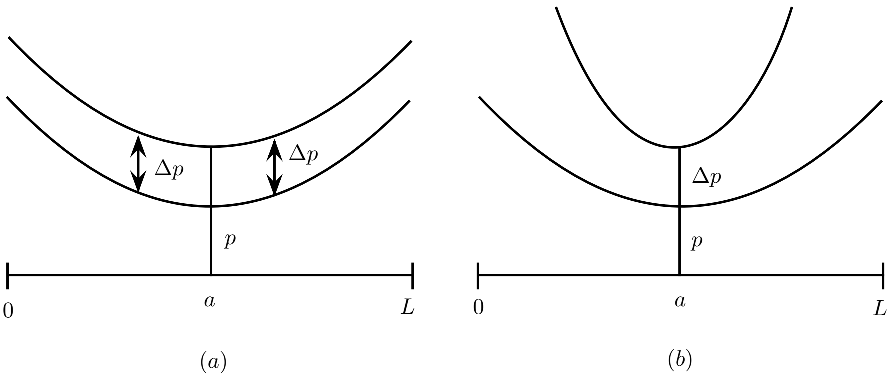

Figure 1: Delivered prices in a spatial model.

Consider a standard Hotelling-like spatial model with convex transport costs. The unit price at distance δ of the firm is given by $P(\delta; t, p) = p + t\delta^{\alpha}$, where p denotes the unit f.o.b. price and $\alpha > 1$. Note that in this case an increase in p translates in exactly the same way to all consumers, i.e. the impact is positive but independent of δ. This situation is depicted in part (a) of Figure 1 for a firm located in a and consumers located between 0 and L.

When there is melting the price paid by a consumer located at a distance δ of the firm is given, according to the proposed general melting function, by $P(\delta; \mu, p) = q(\delta; \mu)p$. We observe that the impact on $P(\delta; \mu, p)$ of an increase in p now is a positive function of δ. That is, $\frac{\partial^2 P(\delta;\mu,p)}{\partial p \partial \delta} > 0$. Generically, this situation is presented in part (b) of Figure 1.

In the standard spatial model, a decrease in price increases demand because the demand is downward sloping with respect to price. When transport costs are modeled in the iceberg fashion, a decrease in price has an additional effect on demand. Demand increases not only because it is negatively related to the price but because the extra quantity demanded ("transport cost") is also cheaper. Although both approaches are not directly comparable, we can illustrate one difference by saying that the iceberg transport technology induces a more elastic demand system than the traditional Hotelling model because now transport cost represents a transfer of resources to the firms.

Note that production and transportation are two different activities with different technologies. Although independent, they are linked through consumers' gross demand. Firms use the same technology to produce all units of the good. However, the additional demand due to melting depends on the transport technology and not on the production technology.

From the perspective of understanding distance as utility loss, the melting set up can be interpreted as the utility function depending on the "distance" between the individual's best-preferred variety and the one consumed, *and* of its price. Moreover, the loss of utility when the price increases is greater the greater the aforementioned distance. In other words, the individual is more price-sensitive the further away from his (her) best-preferred variety. The intuition for this phenomenon is that paying a higher price for a variety different from the first best option conveys a burden that is increasing in the distance between the two varieties. Technically, utility is not separable in distance and price (or income left to buy other goods). In this sense, melting is related to the literature on price dependent preferences à la Pollak (1977).[4]

3 Melting in oligopolistic markets

Consider a spatial market à la Hotelling described by a line segment of unit length (without loss of generality). Consumers are evenly distributed on the market with unit density. They are identical in all respects but for their location. A consumer is denoted by $x \in [0, 1]$. All consumers have a common reservation price $\overline{p}$. We assume $\overline{p}$ to be high enough (but finite) so that all consumers can afford purchasing from one of the firms (in other words, the market is fully covered). Consumers adjust their demands so that, they consume exactly one unit of the commodity.

Two firms a, and b using a constant marginal cost technology, are located at points a and b (measured from zero) where $a \in [0, b)$. Let p_a and p_b denote their respective

[4]Balasko (2003) shows the extension of price dependent preferences to a general equilibrium model.

prices, and $\bar{p}$ the common reservation price for consumers. We assume that first firms decide simultaneously their locations and then, they compete in prices by selecting simultaneously their mill prices.

The assumptions on $q(\delta; \mu)$ given by (1) imply that the demand addressed by consumer x is a symmetric and increasing function around the firm's location and convex in both δ and μ. There are different ways to give a functional form to $q(\delta; \mu)$ that is to define the melting technology. For example, melting may occur at a constant rate or may be proportional to the quantity bought and the distance traveled, or may be proportional to the distance traveled only. In our analysis, we will use the latter. To be precise,

Definition 1 (Melting with Distance: MD)**.** *We refer to MD melting as the situation where the melting is proportional to a power (α) of the distance. Formally:*

$$q(\delta; \mu) - \mu\delta^{\alpha} = 1 \text{ or } q(\delta; \mu) = 1 + \mu\delta^{\alpha}.$$

Under this definition of melting μ is proportional to an exponential of the distance. To illustrate, let $\alpha = 1$. Then, if $\mu = 1$ the volume of commodity lost along the way is proportional to the distance traveled. When $\mu > 1$ the melting is more than proportional. Note also, that $\mu\delta^{\alpha}$ represents the additional demand needed by a consumer located at a distance δ from the firm to be able to *consume* one unit of the commodity.

Let $\alpha = 1$ in the MD melting function[5] and $\mu > 0$. Then, the (delivered) price paid by a consumer located at a distance δ from firm i is given by $P_i = p_i(1+\mu\delta)$.

Remark 1. *Note that*

$$\frac{\partial P_i}{\partial \delta} = p_i\mu > 0, \text{ and } \frac{\partial^2 P_i}{\partial \delta \partial p_i} = \mu > 0$$

so that the delivered price is increasing in distance and its slope is increasing in the firm's mill price.

[5]We can develop the analysis for a concave transport function taking, for instance, $\alpha = 1/2$ with analogous results. Namely, the firm quoting the lower price would be able to capture a non-connected market share. In this respect, our analysis is robust to McCann (2005) criticisms. Note also that $\alpha = 1/2$ corresponds to the transport cost function proposed in McCann (1993, 1998).

3.1 Indifferent consumers

To construct the contingent demand system, we will distinguish three regions in the market and define their corresponding indifferent consumers. A first region is given by the segment $[0, a]$ (a's hinterland). Denote an indifferent consumer in that region as x_1. The second region is the segment $[a, b]$, and the corresponding indifferent consumer will be denoted as x_2. Finally, the interval $[b, 1]$ (b's hinterland) describes the third region and x_3 denotes its indifferent consumer.

Indifferent consumer x_1. A consumer in $[0, a]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(a - x)) = p_b(1 + \mu(b - x))$$

or

$$x_1(p_a, p_b; a, b) = \frac{p_b(1 + \mu b) - p_a(1 + \mu a)}{\mu(p_b - p_a)} \in [0, a] \tag{2}$$

Note that firm b may only capture consumers in firm a's hinterland if the consumer in 0 prefers to buy from b rather than to buy from a, that is, if $p_b(1 + \mu b) < p_a(1 + \mu a)$. In this case $p_b < p_a$ and x_1 is well-defined.

Indifferent consumer x_2. A consumer in $[a, b]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(x - a)) = p_b(1 + \mu(b - x))$$

or

$$x_2(p_a, p_b; a, b) = \frac{p_b(1 + \mu b) - p_a(1 - \mu a)}{\mu(p_b + p_a)} \in [a, b] \tag{3}$$

Indifferent consumer x_3. A consumer in $[b, 1]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(x - a)) = p_b(1 + \mu(x - b))$$

or

$$x_3(p_a, p_b; a, b) = \frac{-p_b(1 - \mu b) + p_a(1 - \mu a)}{\mu(p_b - p_a)} \in [b, 1] \tag{4}$$

Note that firm a may only capture consumers in firm b's hinterland if the consumer in 1 prefers to buy from a rather than to buy from b, that is, if

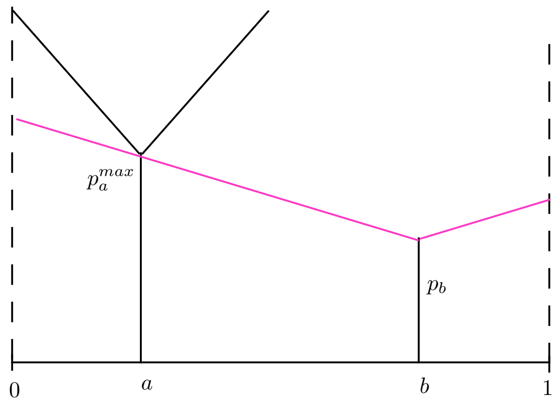

Figure 2: Firm a captures no demand.

$p_b(1+\mu(1-b)) > p_a(1+\mu(1-a)) \Rightarrow \mu(p_b-p_a) > p_a(1-\mu a)-p_b(1-\mu b)$. In this case x_3 is well-defined.

To easy notation we will refer to the indifferent consumers as x_1, x_2, x_3 when such notation will not induce confusion.

Remark 2. *Note that indifferent consumer functions are continuous at a and b. That is,*

$$x_1|_a = x_2|_a \qquad \textit{and} \qquad x_2|_b = x_3|_b.$$

3.2 Firm a's contingent demand

Before computing the contingent demand captured by firm a, let us identify four critical prices.

Firm a will capture no demand at all when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_1 = a$. Naturally, for even higher prices firm a will remain inactive in the market. Let us denote such price as p_a^{max}. Its expression is given by the solution of $x_1 = a$, i.e.

$$p_a^{max} = p_b(1 + \mu(b - a)). \tag{5}$$

Figure 2 illustrates this scenario.

As firm a lowers the price, it starts capturing consumers in the neighborhood of its

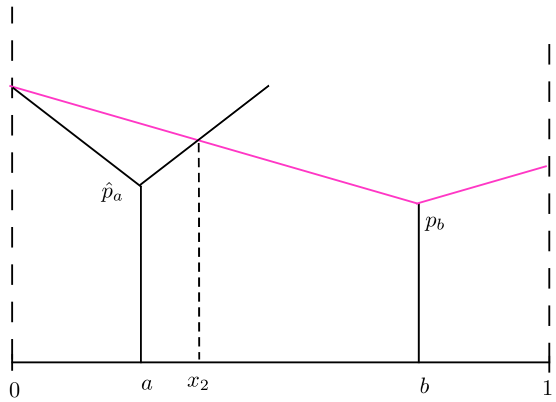

Figure 3: Firm a captures its hinterland.

location a. Firm a captures all consumers in its hinterland when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_1 = 0$. Let us denote such price as $\hat{p}_a$. It is given by,

$$\hat{p}_a = p_b \frac{1 + \mu b}{1 + \mu a}. \tag{6}$$

Note that $\hat{p}_a > p_b$. Figure 3 illustrates.

Further reductions of the price, allows to increase demand from consumers located to the right to a. Also, at a price $\tilde{p}_a$ (given p_b), firm a will start to capture consumers in the hinterland of firm b. This price is given by the solution of $x_3 = 1$, and its expression is,

$$\tilde{p}_a = p_b \frac{1 + \mu(1 - b)}{1 + \mu(1 - a)}. \tag{7}$$

Note that, $\tilde{p}_a < p_b$. Figure 4 illustrates.

Finally, firm a captures all consumers in the market when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_3 = b$. Naturally, for even lower prices firm a's demand will not expand as all consumers are already patronizing it. Let us denote such price as p_a^{min}. Its expression is given by the solution of $x_3 = 1$, i.e.

$$p_a^{min} = p_b \frac{1}{1 + \mu(b - a)}. \tag{8}$$

Figure 5 illustrates.

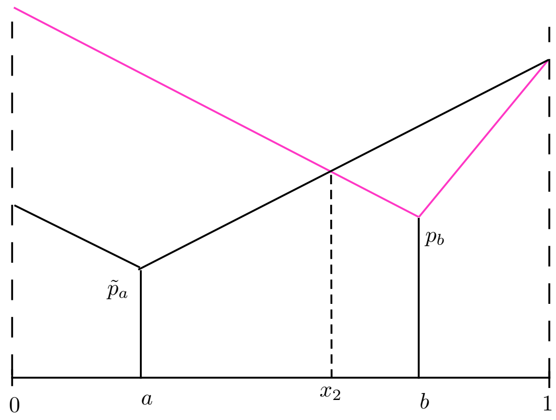

Figure 4: Firm a's demand at $\tilde{p}_a$.

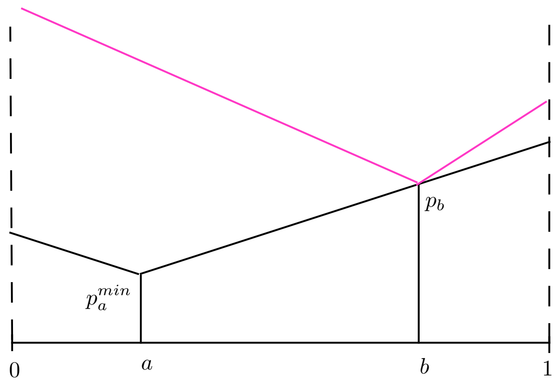

Figure 5: Firm a captures all the market.

To construct the (contingent) demand of firm a, lets us start by assuming that it quotes a price $p_a \geq p_a^{max}$. Then,

$$D_a(p_a, p_b) = 0, \text{ if } p_a \geq p_a^{max}$$

For prices below p_a^{max}, firm a starts capturing consumers on both sides of a. At $\hat{p}_a$, it will capture all consumers in $[0, a]$ (and, of course some more consumers to its right). Therefore, the demand function in this domain of prices will be,

$$D_a(p_a, p_b) = \int_{x_1}^{a} (1 + \mu(a - s))ds + \int_{a}^{x_2} (1 + \mu(s - a))ds, \text{ if } p_a^{max} \geq p_a \geq \hat{p}_a.$$

As firm a quotes prices lower than $\hat{p}_a$ its demand expands from its right hand side in $[a, b]$ only up to the price $\tilde{p}_a$. Therefore, for prices until $\tilde{p}_a$ demand is given

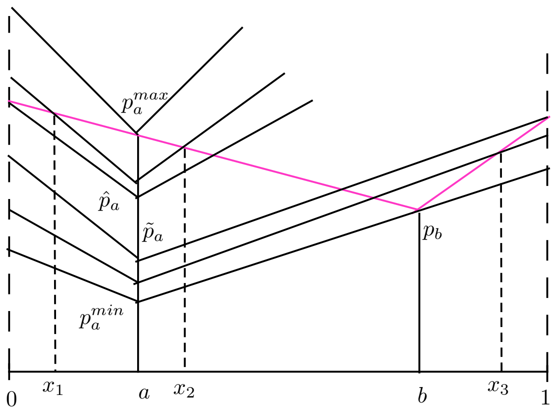

Figure 6: Constructing firm a's contingent demand.

by,

$$D_a(p_a, p_b) = \int_0^a (1 + \mu(a - s))ds + \int_a^{x_2} (1 + \mu(s - a))ds, \text{ if } \hat{p}_a \geq p_a \geq \tilde{p}_a.$$

From this point, lower prices allows firm a to start stealing consumers in firm b's hinterland, so that its demand will be given by

$$D_a(p_a, p_b) = \int_0^a (1+\mu(a-s))ds + \int_a^{x_2} (1+\mu(s-a))ds + \int_{x_3}^1 (1+\mu(s-a))ds,$$
$$\text{if } \tilde{p}_a \geq p_a \geq p_a^{min}.$$

At p_a^{min} firm a captures all the market, so that further reductions of its price does not increase demand. Thus,

$$D_a(p_a, p_b) = \int_0^a (1 + \mu(a - s))ds + \int_a^1 (1 + \mu(s - a))ds, \text{ if } p_a \leq p_a^{min}$$

Figure 6 summarizes the discussion.

Formally, the contingent demand of firm a is,

$$D_a(p_a, p_b) = \begin{cases} 0, & \text{if } p_a \geq p_a^{max} \\ \int_{x_1}^a (1 + \mu(a - s))ds + \int_a^{x_2} (1 + \mu(s - a))ds, & \text{if } p_a^{max} \geq p_a \geq \hat{p}_a \\ \int_0^a (1 + \mu(a - s))ds + \int_a^{x_2} (1 + \mu(s - a))ds, & \text{if } \hat{p}_a \geq p_a \geq \tilde{p}_a \\ \int_0^a (1 + \mu(a - s))ds + \int_a^{x_2} (1 + \mu(s - a))ds & \\ \qquad + \int_{x_3}^1 (1 + \mu(s - a))ds, & \text{if } \tilde{p}_a \geq p_a \geq p_a^{min} \\ \int_0^a (1 + \mu(a - s))ds + \int_a^1 (1 + \mu(s - a))ds, & \text{if } p_a^{min} \geq p_a \end{cases} \tag{9}$$

It is straightforward to verify the continuity of this contingent demand from the continuity of the indifferent consumer functions (see Remark 2).

As mentioned earlier, the aim of the analysis is to verify whether the consideration of an iceberg transport cost function in a formulation that makes our model as close as possible to Hotelling's proposal, allows for recovering the *principle of minimum differentiation*. In this way we could reconcile the model with the pervasive casual observation of the agglomeration of competitors in the market space. To this end, we consider the subset of symmetric locations in a two-stage game where after identifying the non-cooperative equilibrium prices, we study the optimal symmetric location pattern.

4 Price equilibrium

We now study the price equilibrium when firms locations are symmetric: $b = 1-a$. Given the demand function just identified, if a symmetric equilibrium exists, it must lie in the domain of prices $(\hat{p}_a, \tilde{p}_a)$. In this domain, the relevant piece of the demand function is,

$$D_a(p_a, p_b) = \int_0^a (1 + \mu(a - s))ds + \int_a^{x_2} (1 + \mu(s - a))ds, \text{ if } \hat{p}_a \geq p_a \geq \tilde{p}_a.$$

or

$$D_a(p_a, p_b) = a + \frac{a^2\mu}{2} - \frac{(p_a + (-1 - \mu + 2a\mu)p_b)(p_a + (3 + \mu - 2a\mu)p_b)}{2\mu(p_a + p_b)^2} \quad (10)$$

Assume without loss of generality that marginal production costs are constant and denoted by $c > 0$ for both firms. This cost can be thought of encompassing not only production cost but also other external sources of cost. Suppose that production conveys some pollution. Then, a regulator may impose on the firm an environmental tax per unit produced, or equivalently force firms to buy emission rights proportional to the level of production.[6]

[6]The power and heat generation sector and energy-intensive industry sectors (for example, including oil refineries, steel works and production of iron, aluminum, metals, cement, lime, glass, ceramics, pulp, paper, cardboard, acids and bulk organic chemicals) as heavy emitters of carbon dioxide are subject to this policy to encourage reduction of emissions.

Firm a's profits in the domain of prices relevant to the analysis are

$$\Pi_a(p_a, p_b) = (p_a - c)D_a(p_a, p_b)$$
$$= (p_a - c)\Big(a + \frac{a^2\mu}{2} - \frac{(p_a + (-1-\mu+2a\mu)p_b)(p_a + (3+\mu-2a\mu)p_b)}{2\mu(p_a+p_b)^2}\Big)$$

The profit function of each firm is continuous. The consideration of linear MD melting does not imply a discontinuity in the profit function (however, there is a discontinuity in the profit function for the case of linear transport costs in the traditional Hotelling model, as it is shown in d'Aspremont *et al.*, 1979). The first order condition is given by

$$\frac{\partial \Pi_a}{\partial p_a} = \frac{1}{2\mu(p_a+p_b)^3}\Big((p_a^3 + 3p_a^2 p_b)(a^2\mu^2 + 2a\mu - 1)$$
$$+ p_a p_b^2(-a^2\mu^2 + 4a\mu^2 + 14a\mu - \mu^2 - 4\mu - 7)$$
$$+ p_b^2(4 - 8a\mu + 4\mu + \mu^2 - 4a\mu^2 + 4a^2\mu^2)2c + p_b^3(5a^2\mu^2 - 4a\mu^2 - 6a\mu + \mu^2 + 4\mu + 3)\Big) = 0 \quad (11)$$

The second order condition for a maximum is $\frac{\partial^2 \Pi_a}{\partial p_a^2} < 0$. We have:

$$\frac{\partial^2 \Pi_a}{\partial p_a^2} = -\frac{(3c - p_a + 2p_b)(2a\mu - \mu - 2)^2 p_b^2}{\mu (p_a + p_b)^4}$$

Note that

$$\tilde{p}_a = p_b \frac{1+\mu a}{1+\mu(1-a)} < p_a < \hat{p}_a = p_b \frac{1+\mu(1-a)}{1+\mu a}$$

so that we have

$$3c - p_a + 2p_b > 3c - p_b \frac{1+\mu(1-a)}{1+\mu a} + 2p_b = 3c + p_b \frac{1+\mu(3a-1)}{1+\mu a}.$$

This allows for a characterization of the second order condition as follows:

Remark 3. *Characterization of the second order condition:*

- *If* $a > \frac{1}{3}$, *the second order condition is always satisfied.*
- *If* $a < \frac{1}{3}$, *then* $\mu < \frac{1}{1-3a}$ *is a sufficient condition to satisfy the second order condition.*

Given the symmetry in locations, the natural candidate for a price equilibrium is a symmetric one, that is a price equilibrium where $p_b = p_a$. Then, expression (11) simplifies to

$$\frac{\partial \Pi_a}{\partial p_a}\Big|_{p_b=p_a} = \frac{c(2-2a\mu+\mu)^2}{8p_a\mu} + \frac{a^2\mu^2+2a\mu-1}{2\mu},$$

so that the (candidate) symmetric equilibrium price is

$$p^* = c\frac{(2-2a\mu+\mu)^2}{4(1-2a\mu-a^2\mu^2)} \tag{12}$$

Remark 4. *The (candidate) symmetric equilibrium price is above the marginal cost c.*

Note that

$$\frac{c(2-2a\mu+\mu)^2}{4(1-2a\mu-a^2\mu^2)} > c \Longleftrightarrow 4\mu + 4a^2\mu^2 + \mu^2(1-2a)^2 > 0$$

that is always verified (as long as $c > 0$).

Remark 5. *The (candidate) symmetric equilibrium price is well-defined if and only if $a\mu < \sqrt{2}-1$.*

The candidate symmetric equilibrium price is well-defined if and only if the denominator of (12) is positive. That is,

$$f(a) \equiv 1-2a\mu-a^2\mu^2 > 0 \iff a\mu < \sqrt{2}-1 \tag{13}$$

Note, that $f(a)$ is strictly decreasing and strictly concave in a. Thus, $f(a)$ attains its minimum value at $a = \frac{1}{2}$:

$$\min f(a) = f(\frac{1}{2}) = 1-\mu-\frac{1}{4}\mu^2 \begin{cases} > 0 \text{ if } \mu < 2(\sqrt{2}-1) \\ < 0 \text{ if } \mu > 2(\sqrt{2}-1) \end{cases}$$

When (13) *is fulfilled for* $a \leq \frac{1}{2}$ it follows that $f(\frac{1}{2}) > 0$ and thus, $f(a) > 0, \forall a$.

Combining remark 3 and remark 5, we conclude that for,

$$\mu < \begin{cases} \min\{\frac{1}{1-3a}, \frac{\sqrt{2}-1}{a}\}, & \text{if } a < \frac{1}{3} \\ \frac{\sqrt{2}-1}{a}, & \text{if } a > \frac{1}{3} \end{cases} \tag{14}$$

the price p^* is the only equilibrium candidate.

Note that

$$\frac{1}{1-3a} = \frac{\sqrt{2}-1}{a} \text{ at } \hat{a} = \frac{\sqrt{2}-1}{3\sqrt{2}-2} \approx 0.1847$$

Note also that

$$\frac{d(\frac{1}{1-3a})}{da} > 0 \text{ and } \frac{d(\frac{\sqrt{2}-1)}{a}}{da} < 0.$$

Thus, $\frac{1}{1-3a} - \frac{\sqrt{2}-1}{a}$ is increasing in a.

These features, allow us to re-state (14) as

Proposition 1. *Let*

$$\mu < \mu(a) = \begin{cases} \frac{1}{1-3a}, & \text{if } a \in [0, \hat{a}] \\ \frac{\sqrt{2}-1}{a}, & \text{if } a \in [\hat{a}, \frac{1}{2}] \end{cases} \tag{15}$$

Then, if a symmetric price equilibrium exists, it is unique. It is given by,

$$p^* = c\frac{(2-2a\mu+\mu)^2}{4(1-2a\mu-a^2\mu^2)}. \tag{16}$$

Corollary 1. *Let $\mu \leq 2(\sqrt{2}-1)$. Then, if a symmetric price equilibrium exists, it is defined for all values of a.*

Proof. Consider first $\mu(a)$ when $a \in [0, \hat{a}]$. Then,

$$\mu(0) = 1, \text{ and } \mu(\hat{a}) = 3\sqrt{2}-2 \equiv \hat{\mu}.$$
$$\frac{d\mu(a)}{da} = \frac{3}{(1-3a)^2} > 0, \text{ and } \frac{d^2\mu(a)}{da^2} = \frac{18}{(1-3a)^3} > 0.$$

Next study $\mu(a)$ when $a \in [\hat{a}, \frac{1}{2}]$. Then,

$$\mu(\hat{a}) = 3\sqrt{2}-2 \equiv \hat{\mu}, \text{ and } \mu(\frac{1}{2}) = 2(\sqrt{2}-1) \equiv \tilde{\mu}.$$
$$\frac{d\mu(a)}{da} = -\frac{\sqrt{2}-1}{a^2} < 0, \text{ and } \frac{d^2\mu(a)}{da^2} = \frac{2(\sqrt{2}-1)}{a^3} > 0.$$

so that the function $\mu(a)$ is continuous in a and differentiable except at $\hat{a}$. It is increasing and convex in $a \in [0, \hat{a}]$ while it is decreasing and convex in $a \in [\hat{a}, \frac{1}{2}]$. Since $\tilde{\mu} < 1$, the subset (μ, a) containing points in all the domain of a is described by a rectangle whose upper side is at $\tilde{\mu}$.

Figure 7 illustrates. □

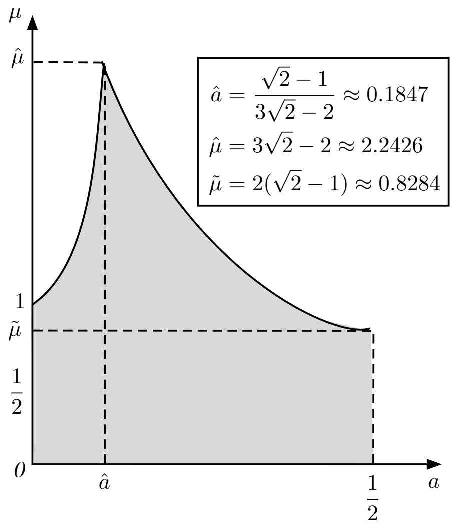

Figure 7: Space (μ, a) satisfying (15)

Remark 6. *The candidate equilibrium price, p^* is increasing in μ.*

Recall that the consumers' reservation price is high enough to guarantee that the market is always covered, and that consumers adjust their total demand so as to consume just one unit. Therefore, an increase of μ means that consumers need to buy additional amounts of the good to be able to consume one unit. Firms experience this increase of demand and react increasing their prices.[7]

Remark 7. *At the (candidate) symmetric equilibrium price, p^*, firms share the market evenly and demand is given by*

$$D_a(p^*) = \frac{1}{8}(8a\mu^2 - 4a\mu + \mu + 4)$$

Then, the profits of each firm evaluated at the (candidate) symmetric equilibrium price are,

$$\Pi(p^*;\mu) = (p^* - c)D_a(p^*) = \frac{c\mu(4 + \mu - 4a\mu + 8a^2\mu)^2}{32(1 - 2a\mu - a^2\mu^2)} \tag{17}$$

Note that from Remark 5 this profit function is well defined only when p^* is well defined, that is, if and only if (13) holds.

[7] A formal proof of this statement can be found in the appendix.

Note that a direct observation of (17) tells us that $\Pi(p^*; \mu)$ is increasing in μ. The denominator decreases with μ while the numerator is increasing in μ because

$$\frac{d(4 + \mu - 4a\mu + 8a^2\mu)}{d\mu} = 1 - 4a + 8a^2 > 0.$$

This should not be surprising. Remark 6 already indicates that the equilibrium price is increasing with μ. Also, the greater the value of μ the more extra amount has to be acquired to guarantee the consumption of one unit of the commodity. Thus, the two forces are aligned in the assessment of the impact of the degree of melting on profits. Moreover, $\Pi(p^*; \mu)$ is increasing in c.

4.1 Existence of a symmetric price equilibrium

So far we have characterized the properties that a symmetric price equilibrium would satisfy. There remains to verify the conditions of existence of such symmetric price equilibrium given by (16). As in d'Aspremont et al. (1979), it may be the case that when locations are close enough firms have incentives to undercut the rival's price.

Let us study firm a's incentives to undercut firm b's price (the analysis would be analogous for the incentives to undercut by firm b). First, let us note that firm a does not want to deviate from p^* to $p_a \in [\tilde{p}_a, p^*)$ as we have already shown that p^* is the best reply of firm a in the interval $[\hat{p}_a, \tilde{p}_a]$ when firm b sets $p_b = p^*$. Moreover, firm a prefers to set price p_a^{min} rather than to set any price below p_a^{min}.

Consider now the possibility of a deviation by firm a from p^* to p_a^{min}. Then,

$$p_a^{min}(p_b^*) = p_b^* \frac{1}{1 + \mu(1 - 2a)} = \frac{c(2 - 2a\mu + \mu)^2}{4(1 - 2a\mu - a^2\mu^2)(1 + \mu(1 - 2a))}$$

so that the associated profits are

$$\Pi_a(p_a^{min}, p_b^*) = \Big(p_a^{min}(p_b^*) - c\Big)\Big(\int_0^a (1+\mu(a-s))ds + \int_a^1 (1+\mu(s-a))ds\Big) =$$
$$= \Big(p_a^{min}(p_b^*) - c\Big)\Big(a + \frac{a^2\mu}{2} + (1-a) + \frac{\mu(1-a)^2}{2}\Big) =$$
$$= \Big(\frac{c(2-2a\mu+\mu)^2}{4(1-2a\mu-a^2\mu^2)(1+\mu(1-2a))} - c\Big)\Big(1 + \frac{\mu(a^2+(1-a)^2}{2}\Big) =$$
$$= \Big(\frac{c(\mu^2+8a\mu)}{4(1+\mu(1-2a))}\Big)\Big(1 + \frac{\mu(a^2+(1-a)^2}{2}\Big) =$$
$$= \frac{c\mu(\mu+8a)(2+\mu(1-2a+2a^2))}{8(1+\mu(1-2a))} \quad (18)$$

Next, compute the variation of profits at p^* and at $p_a^{min}(p_b^*)$. Using (18) and (17), we obtain,

$$\triangle\,\Pi_a = \Pi_a(p_a^{min}, p_b^*) - \Pi_a(p^*, p^*) =$$
$$\frac{c\mu(\mu+8a)(2+\mu(1-2a+2a^2))}{8(1+\mu(1-2a))} - \frac{c\mu(4+\mu-4a\mu+8a^2\mu)^2}{32(1-2a\mu-a^2\mu^2)} =$$
$$\frac{\Big(4c\mu(\mu+8a)(2+\mu-2\mu a+2\mu a^2))(1-2a\mu-a^2\mu^2)\Big)}{32(1+\mu(1-2a))(1-2a\mu-a^2\mu^2)} -$$
$$- \frac{\Big(c\mu(4+\mu-4a\mu+8a^2\mu)^2(1+\mu-2\mu a))\Big)}{32(1+\mu(1-2a))(1-2a\mu-a^2\mu^2)} \quad (19)$$

The sign of (19) is given by the sign of the difference of numerators, since the denominator is positive. Let $H(a,\mu)$ denote that difference of numerators. Then (given that $c\mu > 0$),

$$\operatorname{sgn}\triangle\Pi_a = \operatorname{sgn}\Big(4(\mu+8a)(2+\mu-2\mu a+2\mu a^2)(1-2a\mu-a^2\mu^2) -$$
$$(4+\mu-4a\mu+8a^2\mu)^2(1+\mu-2\mu a)\Big) \equiv \operatorname{sgn} H(a,\mu).$$

Lemma 1. *$H(a,\mu)$ is decreasing in μ.*

Proof. See appendix. □

Lemma 2. *$H(a,\mu)$ is strictly concave in a*

Proof. See appendix. □

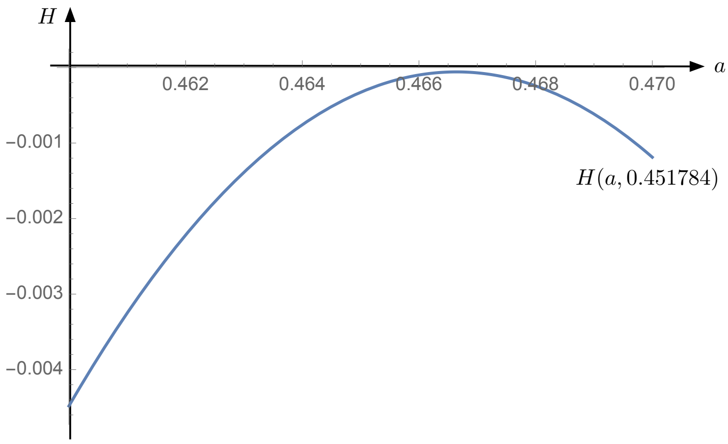

Figure 8: $H(a, \bar{\mu})$ for $a \leq \frac{1}{2}$.

Lemma 3. *Let* $\bar{\mu} = 0.451784$*. If* $\mu \geq \bar{\mu}$ *then* $\triangle\Pi_a < 0, \forall a$.

Proof. As $H(a, \mu)$ is decreasing in μ, we want to identify the value $\bar{\mu}$ such that $H(a, \bar{\mu})$ is tangent to the a-axis and the function $H(a, \bar{\mu})$ is zero at a value $\bar{a} < \frac{1}{2}$. Formally, we look for the solution of

$$H(a, \mu) = 0$$
$$\frac{\partial H(a, \mu)}{\partial a} = 0$$

where

$$\frac{\partial H(a, \mu)}{\partial a} = 2(48\mu - 256a\mu + 16\mu^2 + \mu^3 - 216a\mu^2 + 96a^2\mu - 40a\mu^3 - 4a\mu^4$$
$$+384a^2\mu^2 + 120a^2\mu^3 - 384a^3\mu^2 + 12a^2\mu^4 - 256a^3\mu^3 - 16a^3\mu^4 + 160a^4\mu^3 + 32)$$

The numeric solution to the system of equations is: $\bar{\mu} = 0.451784,\ \bar{a} = 0.46664$. Accordingly, $H(a, \bar{\mu})$ is tangent to the horizontal axis at $\bar{a} = 0.46664$, and $H(\bar{a}, \bar{\mu}) = 0$. Figure 8 shows $H(a, \bar{\mu}) < 0$ for $a \leq \frac{1}{2}$: □

Lemma 3 says that when $\mu \geq \bar{\mu}$ profits evaluated at p_a^{min} are lower than profits evaluated at p^*. It is left to verify that profits in the range of prices $[p_a^{min}, \tilde{p}_a]$ and in $[\hat{p}_a, p_a^{max}]$ are below profits evaluated at p^*.

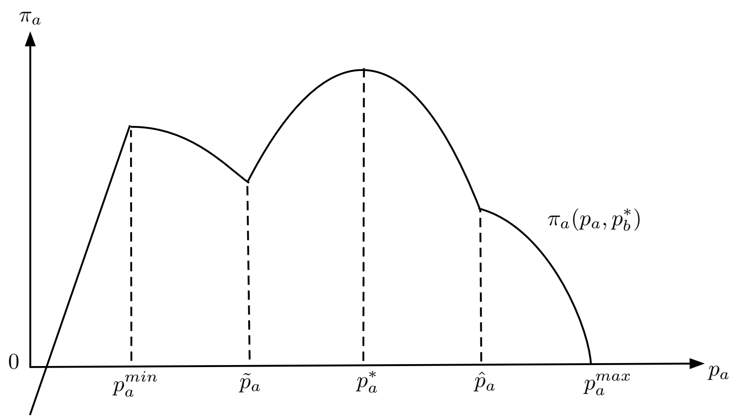

Figure 9: Firm a's profit function.

Numerical simulations show that profits are decreasing in $[p_a^{min}, \tilde{p}_a]$ and in $[\hat{p}_a, p_a^{max}]$. Generically, the shape of firm a's profit function (at p_b^*) is depicted in figure 9. The following proposition summarizes the discussion

Proposition 2. *Let $\mu \in [\bar{\mu}, \tilde{\mu}]$, where $\bar{\mu} = 0.451784$ and $\tilde{\mu} = 2(\sqrt{2} - 1)$. Then, p^* as defined in (12) is the unique price equilibrium in the whole domain of a.*

Proof. Straightforward from Corollary 1, lemma 3, and simulations. □

Proposition 2 characterizes the space of (a, μ) for which the equilibrium price is defined over the entire domain of a. It also informs us about the efficiency of the technology of melting used by the firms ($\mu \in (0.45, 0.83)$ approx). If the technology would be "too" efficient ($\mu < 0.45$) price competition would be so fierce that not even locations as far apart as possible ($a = 0$) would avoid the incentives for firms to deviate to p_a^{min}.

Summarizing, as long as firms are "sufficiently far apart", the price equilibrium exists and it is unique. When firms get "too close", the equilibrium price can only be sustained with an additional condition on the range of feasible values of μ.

5 Location

Having characterized the price competition stage, the study of the optimal symmetric location needs to restrict the values of μ so that p^* is defined in the whole domain of a. Proposition 2 tells us that the set of feasible values of μ is $\mu \in [\bar{\mu}, \tilde{\mu}]$. Thus, hereafter we assume it as the domain of values of μ.

Recall the expressions of the equilibrium price and demand, as a function of the location parameter a, are given by

$$\begin{aligned} p(a) =& c\frac{(2-2a\mu+\mu)^2}{4(1-2a\mu-a^2\mu^2)} \\ D_a(a) =& \frac{1}{8}(8a\mu^2 - 4a\mu + \mu + 4) \end{aligned}$$

Lemma 4. *The symmetric equilibrium price is increasing in a.*

Proof. To verify it, compute

$$\frac{dp}{da} = \frac{(4a + a\mu + 1)(2 + \mu - 2a\mu)\, c\mu^2}{2(1 - 2a\mu - a^2\mu^2)^2}$$

Note that

$$\operatorname{sgn}\frac{dp}{da} = \operatorname{sgn}(2 + \mu - 2a\mu)$$

Then,

$$2 + \mu - 2a\mu > 0 \iff a < \frac{2+\mu}{2\mu}.$$

Given that

$$\lim_{\mu\to\infty} \frac{2+\mu}{2\mu} = \frac{1}{2}, \text{ and } \lim_{\mu\to 0} \frac{2+\mu}{2\mu} = \infty,$$

it follows that

$$a < \frac{2+\mu}{2\mu}$$

and

$$\frac{dp}{da} > 0.$$

□

Corollary 2. *The margin of the price over the marginal cost is increasing in a.*

Proof. Straightforward. □

The intuition for these results stems from the fact that, given that the market is covered, as the hinterlands of the firms increase, so do their monopoly power. The Lerner index is

$$L_a = 1 - \frac{c}{p(a)}$$

and

$$\frac{dL_a}{da} = \frac{c}{p^2}\frac{dp(a)}{da} > 0$$

In other words, the effect of firms' competition for consumers located between them is offset by their larger hinterlands.

Lemma 5. *The demand function,$D_a(a)$ is U-shaped in a*

Proof. Recall that the demand of an individual located at x to consume one unit of the good is $q(a) = 1 + \mu\delta$. The aggregate additional demand of consumers patronizing firm a is given by

$$D_a(a) = \int_0^a (1+\mu(a-x))dx + \int_a^{\frac{1}{2}} (1+\mu(x-a))dx = \frac{1}{8}\left(4 + \mu - 4a\mu + 8a^2\mu\right) \tag{20}$$

Note that

$$\frac{dDem_a}{da} = \frac{\mu}{2}(4a-1), \quad \text{and} \quad \frac{d^2 D_a}{da^2} = 2\mu$$

so that $D_a(a)$ is a convex function in $a[0, \frac{1}{2}]$. It reaches its maximum value, $((4 + \mu)/8)$, at both $a = 0$ and $a = 1/2$. The minimum value, $((8 + \mu)/16)$, appears at $a = 1/4$. It is decreasing in $a \in [0, 1/4)$ and increasing in $a \in (1/4, 1/2]$. □

The profit level as a function of the location parameter a is,

$$\Pi(a) = (p - c)D_a(a) = \frac{c\mu(4 + \mu - 4a\mu + 8a^2\mu)^2}{32(1 - 2a\mu - a^2\mu^2)} \tag{21}$$

Proposition 3. *In a symmetric location equilibrium, firms locate at the center of the market.*

Proof. Let

$$A(a,\mu) \equiv 4 + \mu(1 - 4a + 8a^2) > 0, \forall a \in [0, \frac{1}{2}], \mu \in [\bar{\mu}, \tilde{\mu}] \tag{22}$$

$$B(a,\mu) \equiv 1 - 2a\mu - a^2\mu^2 > 0, \forall a \in [0, \frac{1}{2}], \mu \in [\bar{\mu}, \tilde{\mu}] \tag{23}$$

The first order condition of the location subgame is

$$\frac{\partial \Pi^*}{\partial a} = \frac{c\mu^2 A(a,\mu)}{16(B(a,\mu))^2}\Big(\mu + 8a\mu + a\mu^2 + 16a - 24a^2\mu - 8a^3\mu^2\Big)$$

The first term of the product is positive. Therefore, the sign of this first order condition is given by the sign of the expression in brackets.

Consider the terms $16a - 24a^2\mu - 8a^3\mu^2$. Using (13) we can verify that

$$\begin{aligned} 24a^2\mu + 8a^3\mu^2 < 24a^2\mu + 8a^2\mu(\sqrt{2}-1) = 8a^2\mu(2+\sqrt{2}) \\ < 8a(2+\sqrt{2})(\sqrt{2}-1) = 8a\sqrt{2} < 16a. \end{aligned}$$

Therefore, the expression in brackets is positive and, accordingly profits are increasing in a. □

Remark 8. *The profit function* (21) *is increasing in c and μ. The denominator is positive from* (13). *Therefore, the positive relation between profits and the marginal cost c is trivial. Regarding μ, the proof is relegated to the appendix.*

Proposition 3 provides a new rationale for the principle of minimum differentiation.

The intuition behind this result goes as follows: Proposition 3 tells us that

$$\frac{d\Pi}{da} = \frac{d(p-c)}{da} D_a(a) + (p-c)\frac{dD_a}{da} > 0 \tag{24}$$

or

$$\left| \frac{\dfrac{d(p-c)/da}{p-c}}{\dfrac{dD_a/da}{D_a}} \right| > 1$$

This is a measure of the relative variation of the price-cost margin with respect to the relative variation of demand as location changes. To be more precise, note that the first term in the right-hand side of (24) is positive as it is, using Corollary (2), the product of two positive terms; the second term, using lemma 5 is positive for $a \in (\frac{1}{4}, \frac{1}{2}]$ and negative for $a \in [0, \frac{1}{4})$. As before, in the range $a \in (\frac{1}{4}, \frac{1}{2}]$, the second term is also the product of two positive terms, so that the derivative is the sum of two positive terms, and thus positive. In the range $a \in [0, \frac{1}{4})$, the second term of the derivative of the profit function is negative. However, the overall relative positive variation of the price-cost margin offsets the relative negative variation of the demand, and yields the positive relation between profits and location.

Summarizing, proposition 3 holds because as firm a moves towards the center of the market, two dynamics appear. Starting at $a = 0$ and letting firm a move to the right, when the market to its left (hinterland) is smaller than its market to the right, the increase in price offsets the decrease in additional demand. When firm reaches location $a = 1/4$, and keeps on moving to the right, the hinterland becomes larger than the market on its right and the increase in price is reinforced by the increase in additional demand.

6 Conclusion

One of the most appealing results in address models of differentiation is Hotelling's *principle of minimum differentiation*. It postulates that market competitors will choose neighboring locations in equilibrium. Although d'Aspremont et al (1979) proved that the result is flawed, casual observation conforms with it.

In this paper, we offer a different approach to the ones in the literature trying to recover the *principle*. Our motivation is two-fold. On the one hand, address models of differentiation are characterized by the exogeneity of the transport cost function. Implicitly, this amounts to assume that transport services are delivered under perfectly competitive conditions. However, the transport market often is oligopolistic. On the other hand, there are many markets showing a discrepancy between the amount of commodity delivered at the point of production (sale) and the amount available for consumption. The energy market serves to illustrate the point. Inspired by Samuelson's modeling of transport technology in the international trade theory, we propose to apply the iceberg formulation to a Hotelling-like setup encompassing our two motivating features. We propose a particular melting function proportional to the distance traveled in order to keep the analysis as close as possible to the original Hotelling's linear transport cost. In this way, our results can be contrasted (not compared) with those of d'Aspremont *et al.* (1979). We show that the iceberg approach allows obtaining continuous delivered price functions, and also demands become more elastic. The ultimate goal of the study is to present a model where, in equilibrium, the principle of minimum differentiation holds. To this end we set-up a duopoly model of location-then-price competition, where we restrict firms to choose symmetric locations. We have chosen a particular type of melting function proportional to distance. We characterize a symmetric price equilibrium and show that it induces a symmetric location around the center of the market, thus reproducing the principle of minimum differentiation.

The iceberg formulation has the property of generating a quasi-concave profit function in the price competition stage. In contrast with d'Aspremont *et al.*, our model shows that in the subgame perfect equilibrium, firms have incentives to locate as close as possible, thus recovering the Principle of Minimum Differentiation. The reason for this optimal location pattern relies on the fact that measuring transport costs in units of the consumption good gives firms an absolute competitive advantage in their respective hinterlands. Accordingly, firms have incentives to minimize the market area where there is direct competition. They do so by locating as close as possible. We introduce a restriction in the analysis by considering symmetric locations only.

Finally, the iceberg approach endogenizes the transport cost in the consumer decision problem, so that consumer preferences are now price dependent. Although in a very particular formulation, the use of melting functions connects the analysis of spatial competition with the literature on price dependent preferences because an individual's utility function depends on the "distance" between the best-preferred varieties and the one consumed, *and* of its price. The in-depth analysis of this connection is left for future research.

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Appendix

Proof of Remark 6

Proof. The candidate equilibrium price is given by (12):

$$p^* = c\frac{(2 - 2a\mu + \mu)^2}{4(1 - 2a\mu - a^2\mu^2)}$$

Computing the derivative with respect to μ yields

$$\frac{\partial p^*}{\partial \mu} = \frac{c}{2}\frac{(1 - a\mu + 4a^2\mu)(2 - 2a\mu + \mu)}{(1 - 2a\mu - a^2\mu^2)^2} \tag{25}$$

so that we need only assess the sign of the numerator. From remark 4 we know that $p^* > c > 0$. Hence,

$$1 - 2a\mu - a^2\mu^2 > 0.$$

which implies

$$1 - a\mu > a\mu(1 + a\mu). \tag{26}$$

Using (26) we can write,

$$(1 - a\mu + 4a^2\mu > a\mu(1 + a\mu) + 4a^2\mu > 0.$$

Also, using (26) we can write

$$2-2a\mu+\mu = 1+\mu-a\mu+1-a\mu > 1+\mu-a\mu+a\mu(1+a\mu) = 1+\mu+a^2\mu^2 > 0.$$

Hence the two terms of the numerator of (25) are positive, implying that p^* is increasing in μ. $\square$

Proof of lemma 1

$H(a, \mu)$ is decreasing in μ

Proof.

$$\frac{d(H(a,\mu))}{d\mu} = -(10\mu-96a-64a\mu+256a^2-64a^3+3\mu^2-6a\mu^2+432a^2\mu-512a^3\mu+ 384a^4\mu+120a^2\mu^2+16a^2\mu^3-240a^3\mu^2-32a^3\mu^3+384a^4\mu^2+32a^4\mu^3-192a^5\mu^2+16) = -\Big((-96a + 256a^2 - 64a^3 + 16) + \mu(10 - 64a + 432a^2 - 512a^3 + 384a^4)+ \mu^2(3-6a+120a^2-240a^3+384a^4-192a^5)+\mu^3(16a^2-32a^3+32a^4)\Big) < 0$$

as all interior parenthesis are positive when $a \leq \frac{1}{2}$. Hence, the function $H(a, \mu)$ is decreasing in μ. $\square$

Proof of lemma 2

$H(a, \mu)$ is strictly concave in a.

Proof.

$$\frac{d(H(a,\mu))}{da} = 2(48\mu-256a\mu+16\mu^2+\mu^3-216a\mu^2+96a^2\mu-40a\mu^3-4a\mu^4+ 384a^2\mu^2+120a^2\mu^3-384a^3\mu^2+12a^2\mu^4-256a^3\mu^3-16a^3\mu^4+160a^4\mu^3+32)$$

and

$$\frac{d^2(H(a,\mu))}{da^2} = 8\mu\Big((48a-64)+\mu(-54+192a-288a^2)+$$
$$\mu^2(-10+60a-192a^2+160a^3)+\mu^3(-1+6a-12a^2)\Big) < 0$$

as all interior parenthesis are negative when $a \leq \frac{1}{2}$. Hence, the function $H(a,\mu)$ is concave in a. □

Proof of remark 8

$\Pi^*(a,\mu)$ is increasing in μ.

Proof. Recall

$$\Pi^*(a,\mu) = \frac{c\mu(4+\mu-4a\mu+8a^2\mu)^2}{32(1-2a\mu-a^2\mu^2)} = \frac{c\mu[A(a,\mu)]^2}{32B(a,\mu)}$$

Then,

$$\frac{\partial\Pi^*(a,\mu)}{\partial\mu} = \frac{cA(a,\mu)}{32[B(a,\mu)]^2}\Phi(a,\mu)$$

where

$$\Phi(a,\mu) = 3\mu-12a\mu-4a\mu^2+24a^2\mu+20a^2\mu^2-a^2\mu^3-32a^3\mu^2+4a^3\mu^3-8a^4\mu^3+4$$

Note that $a < 0.5$ and $\mu < 1$ imply that $A(a,\mu) > 0$.

Also, (13) implies $B(a,\mu) > 0$. Therefore, the sign of $\Phi(a,\mu)$ determines the sign of the derivative.

We can write,

$$\begin{aligned}\Phi(a,\mu) &> -4a\mu^2+24a^2\mu+3a^2\mu^2+1\\ &> -4a\mu^2+23a^2\mu+4a^2\mu^2+\mu^2\\ &= \mu^2(1-2a)^2+23a^2\mu > 0\end{aligned}$$

□