Capital Misallocation and Secular Stagnation

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This version: February 2019
July 2018

Barcelona GSE Working Paper Series

Working Paper nº 1056

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February 26, 2019

Abstract

The widespread emergence of intangible technologies in recent decades may have significantly hurt output growth—even when these technologies replaced considerably less productive tangible technologies—because of low interest rates. After a shift toward intangible capital in production, the corporate sector becomes a net saver because intangible capital has a low collateral value. Firms' ability to purchase intangible capital is impaired by low interest rates because low rates slow down the accumulation of savings and increase the price of capital, worsening capital misallocation. Our model simulations reproduce key trends in the U.S. in the period from 1980 to 2015.

Keywords: Intangible Capital, Borrowing Constraints, Capital Reallocation, Secular Stagnation
JEL Classification: E22, E43, E44

* A previous version of this paper was entitled "Reallocation of Intangible Capital and Secular Stagnation". We thank Andrew Abel, Fiorella de Fiore (discussant), Wouter Den Haan (discussant), Andrea Eisfeldt, Antonio Falato, Maryam Farboodi (discussant), Simon Gilchrist, Adam Guren, Matteo Iacoviello, Arvind Krishnamurthy, Tim Landvoigt (discussant), Claudio Michelacci (discussant), Guillermo Ordonez, Enrico Perotti, Vincenzo Quadrini, and Stephen Terry, and seminar participants at Boston University, Boston College, the Federal Reserve Board, the Bank of Spain, the CREI macro lunch, the 7th Meeting of the Macro Finance Society (UCLA 2016), the 2016 Barcelona Summer Forum Workshop on Financial Markets and Asset Prices, the 2016 NBER SI Workshop on Macro, Money and Financial Frictions, the Midwest Macro Meetings, the Cleveland Fed Day-Ahead Meeting on Productivity, the BoE Workshop on Finance, Investment and Productivity, the Bank of Italy Workshop on Macroeconomic Dynamics, and the 2018 AEA for very helpful comments. We also thank Christoph Albert for excellent research assistance. Andrea Caggese acknowledges financial support from Resercaixa, and from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563). Ander Perez acknowledges financial support from the Ministry of Economics of Spain grant ECO2012-32434. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System. All errors are, of course, our own responsibility.

1 Introduction

Real interest rates have decreased in past decades, while economic growth has fallen short of previous trends, developments that have been linked to a process of 'secular stagnation' (*e.g.* Summers, 2015; Eichengreen, 2015). At the same time, the developed world has experienced a technological change toward a stronger importance of information technology and of knowledge, human, and organizational capital, which has gradually reduced the reliance on physical capital (Corrado and Hulten, 2010a) and has been linked to a significant decrease in corporate net borrowing (Falato et al., 2014; Döttling and Perotti, 2016).[1] This paper argues that the increased reliance on intangible capital and the low real interest rates interact to hurt capital reallocation and reduce productivity and output growth.

Aggregate productivity depends on an efficient reallocation of resources from contracting or exiting firms to new entrants or expanding firms. Capital reallocation is quantitatively significant and represents close to one third of total investment of U.S. listed firms (Eisfeldt and Shi, 2018). Existing research suggests that financial market imperfections are amongst the most important frictions preventing the efficient reallocation of capital (Eisfeldt and Rampini, 2006; Midrigan and Xu, 2014; Gopinath et al., 2017). The rise of intangible capital implies a growing importance of the reallocation of intangible assets such as organizational capital, human capital, brand equity, and research and development (R&D). These assets generally have a low collateral value, and their acquisition has to be financed mostly using retained earnings. As a result, the corporate sector borrows less, holds an increasing amount of cash, and switches from being a net borrower to a net saver. Lower interest rates decrease the speed at which firms can grow their accumulated savings to finance future expansions. In addition, lower interest rates increase the price of these intangible assets and further reduce the ability of credit-constrained expanding firms to purchase them. We show that the rise in intangibles, via these two effects, alters the dynamic relationship between interest rates and efficiency in the allocation of capital.

We formalize this intuition by developing a model of an economy with heterogeneous firms. We make three key assumptions, for which we provide strong empirical support in the paper. First, firms use tangible capital, intangible capital, and labor as complementary factors in the production of consumption goods. Second, a subset of firms have high productivity and suffer from financing constraints that prevent them from issuing equity or from borrowing any amount in excess of the collateral value of their holdings of tangible and intangible capital.

[1] The decrease in corporate net borrowing has translated into a shift in the net financial position of the nonfinancial corporate sector from a net borrowing position roughly before the year 2000 to a net saving position from 2000 onward (Armenter and Hnatkovska, 2016; Quadrini, 2016; Chen, Karabarbounis, and Neiman, 2016; Shourideh and Zetlin-Jones, 2016).

Third, we assume that firms can invest only occasionally.[2] In equilibrium, high productivity firms are constrained, save as much as possible in non-investing periods, and invest all of their accumulated net savings plus their maximum available borrowing in investing periods. The consumer sector is modeled as overlapping generations of households displaying a realistic life cycle, modeled in a way that enables us to obtain an equilibrium interest rate in the steady state that is not necessarily equal to the household rate of time preference.

We first inspect the analytical solution of a simplified version of the model to describe four channels through which lower interest rates interact with the intensity of intangible capital in firms' production function to affect the steady state equilibrium of our economy. First, a *net debtor channel* allows net borrowing high-productivity firms to pay down their debt more easily when interest rates are low and enables them to absorb more capital. Second, and conversely, a *savings channel* operates when the high-productivity firms are net savers: reductions in the interest rate decrease the speed of accumulation of savings, reduce the investment capacity of expanding firms, and hurt capital reallocation. Third, lower interest rates that increase the price of tangible and intangible assets reduce the amount of capital that high-productivity firms can purchase for a given amount of net worth and borrowing capacity—a *capital purchase price channel*. Fourth, a lower interest rate increases the present value of the collateral pledged next period, and reduces the size of the downpayment necessary to purchase capital, improving capital reallocation through a *collateral channel*. The analytical solution of the simplified model provides a clear illustration of the main theoretical finding of the paper: in an economy with a relatively low collateral value of capital, a drop in the interest rate worsens the allocation of resources and reduces aggregate investment, productivity, and output.

In the remaining sections of the paper, we calibrate and simulate our full general equilibrium model to study how parallel developments in the household and corporate sectors have interacted to generate aggregate patterns consistent with the secular stagnation hypothesis. In the household sector, we model a progressive decrease in individuals' rate of time preference and a progressive increase in their life expectancy, both of which put downward pressure on the equilibrium interest rate.[3] In the corporate sector, we introduce a gradual shift toward technologies that are more productive and more intensive in intangible capital.[4]

[2] It is a well-known stylized fact that productive plants typically have zero or small investment rates during most of their existence and experience infrequent, but very large, investment spikes (Doms and Dunne, 1998). Rather than modelling non-convex adjustment costs and state contingent investment decisions, our assumption of exogenous occasional investment opportunities has similar implications and is much more tractable, allowing for a closed-form solution.

[3] We interpret our exercise as a shortcut for a collection of different factors, such as population aging, wealth and income inequality, financial deepening, and foreign-sector developments, which have contributed to increase households' demand for savings in the past 40 years.

[4] We set the reliance on intangible capital to match its observed evolution from a pre-1980 value of 20%

We find that while the household sector developments in isolation and the corporate sector developments in isolation are both expansionary, the combination of both developments is contractionary. The drop in the interest rate increases high-productivity firms' ability to borrow (the *collateral channel*) and pay down their debt (the *net debtor channel*) while firms still rely strongly on tangible capital. As firms use increasingly more intangible capital and become net savers, low rates reduce the efficient allocation of capital by increasing capital prices (the *capital purchase price channel*) and by slowing the accumulation of corporate savings (the *savings channel*). The share of output produced by the high-productivity firms drops significantly. The lower corporate borrowing itself also puts downward pressure on interest rates, which amplifies the misallocation of capital. Despite the fact that the economy is shifting toward a higher reliance on a more productive type of capital, aggregate productivity falls cumulatively by 24.9%, and even though low rates encourage substantial capital creation, output is more than 5% lower than in a counterfactual scenario in which the allocation of capital remains unchanged.

We interpret this comparative static exercise as capturing the developments in the U.S. economy following the rise in the share of intangible capital and the rise in net household and foreign-sector savings in the past 40 years. In this respect, this model is remarkably consistent with a series of well-documented trends during this period: (i) net corporate savings increased as a fraction of gross domestic product (GDP), (ii) household leverage increased as a fraction of GDP, (iii) the real interest rate fell, (iv) intra-industry dispersion in productivity increased, and (v) output and productivity progressively declined relative to their previous trends. We provide a detailed discussion of these and other trends that motivate our paper in Section 2.

Overall, our results suggest that the interaction between low interest rates, intangible technologies, and corporate financing patterns might be an important factor behind secular stagnation.

Related Literature

The secular stagnation hypothesis as an explanation of recent economic trends has been proposed by, among others, Summers (2015) and Eichengreen (2015). Several recent theoretical papers, motivated by the slow recovery after the recent financial crisis, show that secular stagnation can be explained by a persistently binding zero lower bound (ZLB) in nominal interest rates. Prominent examples of a formalization of these ideas are Eggertsson and Mehrotra (2014) and Eggertsson, Mehrotra, and Robbins (2017), who show how a persistent tightening of the

of aggregate capital to a post-2010 value of 60% of aggregate capital (Corrado and Hulten, 2010a; Falato, Kadyrzhanova, and Sim, 2014; Döttling and Perotti, 2015). Since we assume that intangible capital is more productive than tangible capital, this gradual shift is consistent with the notion of the transition to intangible capital as a privately optimal choice of firms adopting technologies that are more productive.

debt limit facing households can reduce the equilibrium real interest rate and, in the presence of sticky prices and the ZLB, generate permanent reductions in output. Other prominent examples of papers that rely on the presence of a persistent ZLB are Bacchetta, Benhima, and Kalantzis (2016), who show that deleveraging shocks increase the money holdings of investors, crowding out investment, and Benigno and Fornaro (2015), who consider a framework with endogenous growth and permanent nominal rigidities. In their stagnation trap, weak growth keeps the interest rate against the ZLB, and low aggregate demand discourages firms' investment in innovation and further reduces growth.

Our paper has elements in common with this literature. As in Eggertsson, Mehrotra, and Robbins (2017), we consider a life-cycle model in order for the equilibrium interest rate not to be pinned down by the value of the discount factor β but, instead, to be affected by realistic household-side developments. As in Bacchetta, Benhima, and Kalantzis (2016), we consider entrepreneurs with occasional investment opportunities who save in a liquid instrument when they do not invest.

Our main contribution with respect to these studies is to identify and formalize a novel misallocation effect of endogenously low real interest rates. Our alternative explanation of the secular stagnation hypothesis does not rely on the ZLB or nominal rigidities, can account for a large drop in aggregate output and productivity, and is consistent with a broad set of well-documented trends. Importantly, while the ZLB in the nominal interest rate has been present in many countries since 2009, recent empirical evidence shows that the slowdown in productivity and output growth in both the U.S. and Europe started in the early 2000s, well before the financial crisis (*e.g.* Fernald, 2015; Cette, Fernald, and Mojon, 2016; Kahn and Rich, 2007, 2013).

The rising use of intangible capital has been documented by Corrado and Hulten (2010a), and its relation to the decrease in corporate borrowing and the rise in corporate cash holdings has been shown empirically by Bates et al. (2009). Giglio and Severo (2012), Falato et al. (2014), and Döttling and Perotti (2016) introduce models that describe how the rise in intangibles can lower the equilibrium interest rate by decreasing firms' net borrowing.[5] We add to this literature by describing a mechanism through which the rise in intangibles can have a negative effect on aggregate capital reallocation and growth.

Our paper is also closely related and complementary to the literature on financial frictions, firm dynamics, and misallocation (*e.g.* Buera, Kaboski, and Shin, 2011; Caggese and Cuñat,

[5] Other important implications of the increasing importance of intangible capital relate to the adequate measurement of aggregate and firm-level capital stocks (Corrado and Hulten, 2010; McGrattan and Prescott, 2010; Eisfeldt and Papanikolaou, 2014) and to asset pricing (Eisfeldt and Papanikolaou, 2013).

2013; Gilchrist, Sim and Zakrajsek, 2013; Moll, 2014; Midrigan and Xu, 2014; Buera and Moll, 2015). As in these papers, we consider steady state misallocation effects of financing frictions. Our contribution is to provide novel theoretical insights on the relationship between interest rates, the collateralizability of different types of capital, and misallocation.[6]

The rest of the paper is organized as follows. Section 2 introduces the empirical evidence that motivates this paper. We describe a very simple model in Section 3 that conveys the basic intuition of the mechanisms we introduce, and we develop a full-fledged general equilibrium extension in Section 4. The steady state and calibration of the general equilibrium model are described in Sections 5 and 6, respectively, and the simulation results are discussed in Section 7. Section 8 concludes.

2 Empirical Motivation

In this section, we summarize the key stylized facts that motivate our model.

1 ***- Productivity growth in advanced economies suffered a slowdown starting in the early 2000s.***

Fernald (2015) and Kahn and Rich (2007, 2013) estimate that growth in labor productivity and total factor productivity (TFP) in the U.S. switched from a high-growth to a low-growth regime in 2003 or 2004, suggesting that some of the causes of the productivity slowdown that continued after the financial crisis of 2008-2009 are not related to the crisis. Cette, Fernald, and Mojon (2016) report that Europe experienced a similar pre-crisis pattern. Also important is the fact that the most dramatic drops in the rate of productivity growth were in the IT-related sectors (Fernald, 2015).

2 ***- The real interest rate has fallen steadily in advanced economies since the 1980s, and demographic factors have played a key role in this trend.***

Nominal interest rates, both short- and long-term, have been falling since the early 1980s, while inflation expectations have remained largely unchanged, resulting in a fall in real interest rates (King and Low, 2014). Gagnon, Johannsen, and Lopez-Salido (2016) and Eggertsson,

[6] Gopinath et al. (2017) also consider a model with financial frictions and heterogeneous firms in which declining interest rates cause an increase in the dispersion in the productivity of capital. However, their mechanism is fundamentally different from ours. In their model, when the interest rate falls, all firms invest more and expand aggregate capital and output. Productivity dispersion increases because larger firms are able to grow more rapidly than smaller and more financially constrained ones. In our model, instead, low rates tighten financial constraints of high-productivity firms that utilize intangible capital, and reduce their investment.

Mehrotra, and Robbins (2017) perform a quantitative theoretical analysis based on realistic demographic changes in the U.S. in recent decades and both conclude that demographic factors—in particular, increased life expectancy and decreased fertility rates—can account for an important share of the real interest rate fall. Similar arguments have also been made by Baldwin and Teulings (2014), Rachel and Smith (2015), and Bean (2016).

3 - The U.S. and other developed economies are significantly more reliant on intangible capital now than in the 1980s.

The developed world has experienced a technological change toward a stronger importance of information technology and of knowledge, human, and organizational capital, which has gradually reduced the reliance on physical capital (Brown, Fazzari and Petersen, 2009; Corrado and Hulten, 2010a; Falato et al., 2014). In the United States, intangible capital as a share of total capital went from around 0.2 in the 1970s to 0.5 in the 2000s (Falato et al., 2014).

4 - The corporate sector in the U.S. has transitioned from a net debtor position to a net saver position.

In parallel to the trend toward a stronger reliance on intangible capital, there has been a shift in the net financial position of the nonfinancial corporate sector from a net borrowing position roughly before the year 2000 to a net saving position from 2000 onward (Armenter and Hnatkovska, 2016; Quadrini, 2016; Chen, Karabarbounis, and Neiman, 2016; Shourideh and Zetlin-Jones, 2016).

5 - Firms that rely on intangible capital are more financially constrained than those that rely on tangible capital, and this is partly responsible for the trend toward a net saving position of the corporate sector.

A large body of evidence shows that many firms, especially small and young ones, face financial frictions that increase the cost of equity and of unsecured debt relative to the cost of collateralized debt and of internal finance. Moreover, tangibility is one of the most important factors in determining the collateral value of firms' assets (Almeida and Campello, 2007). It follows that firms that rely more on intangible capital have lower borrowing capacity and use retained earnings more intensely to fund investment. Thus, financial frictions imply that trend 3 (the increased intangibles reliance) is an important factor in causing trend 4 (the shift toward a net saving corporate sector). Below we discuss evidence that supports this claim in three steps.

5.i) It is difficult to finance intangible capital with debt.

Hall (2002) documents, in an extensive survey of the literature, that "*R&D-intensive firms feature much lower leverage, on average, than less R&D-intensive firms*". She concludes that "*small and new innovative firms experience high costs of capital that are only partly mitigated by the presence of venture capital*". Brown, Fazzari, and Petersen (2009) document that U.S. firms finance most of their R&D expenditures out of retained earnings and equity issues. Gatchev, Spindt, and Tarhan (2009) document that, in addition to R&D, marketing expenses and product development are also mostly financed out of retained earnings and equity. Dell'Ariccia et al. (2017) document that the increased usage of intangible assets by firms helps explain why banks have shifted out of business lending and into residential real estate lending in the U.S. in recent decades. In contrast, tangible assets are mostly financed with debt.[7]

5.ii) Intangible capital can attract equity financing, but equity financing is costly, especially when used to finance intangible investment.

Lack of access to debt financing of firms that rely on intangible capital could be compensated by easy access to equity financing. A large body of evidence shows, however, that external equity financing is significantly costly (Altinkilic and Hansen, 2000; Gomes, 2001; Belo, Lin and Yang, 2016). Carpenter and Petersen (2002) argue that the cost of equity is likely to be even higher for intangibles firms, which typically suffer from highly skewed and uncertain returns and from substantial information asymmetries between entrepreneurs and potential investors. Partly to deal with these frictions, special types of equity financing for intangibles firms have been developed, such as venture capital. However, the amount of venture capital funds that a firm can attract is strongly positively associated to the firm's ownership of patents (Hall and Ziedonis, 2001, Baum and Silverman, 2004), suggesting that, even for this type of financing, the presence of somewhat collateralizable assets such as patents is important.

5.iii) Intangibles firms accumulate cash for precautionary reasons to avoid future financial shortages.

The process of technological change has been linked to an increase in the precautionary motives for cash accumulation to avoid future financial shortages (Bates et al., 2009; Falato et al., 2014; Falato and Sim, 2014; Döttling and Perotti, 2016; Begenau and Palazzo, 2016; Pinkowitz et al., 2016; Graham and Leary, 2017). Furthermore, firm-level empirical evidence suggests that the observed link between intangible intensity and high cash holdings is driven by financial frictions. Begenau and Palazzo (2016) introduce evidence showing that an important determinant of the increase in cash holdings of public firms is the increase in frequency of new

[7] Eisfeldt and Rampini (2009) report that a big share of machinery, equipment, buildings and other structures is financed with debt. Inventory investment and other tangible short-term assets attract substantial debt finance in the form of trade credit and bank credit lines (Petersen and Rajan, 1997; Sufi, 2009). Finally, investment in commercial real estate is primarily financed with mortgage loans (Benmelech, Garmaise, and Moskowitz, 2005).

firms that are very R&D intensive, and they suggest that these trends are consistent with a model in which cash holdings are driven by financial frictions. Similarly, Falato et al. (2014) show empirically that the relation between reliance on intangible capital and cash holdings is stronger among firms for which financing frictions are more severe.[8]

We make two final additional comments. First, we observe often that firms that use intangible assets and engage in innovation become large and financially unconstrained. In fact, technological changes associated with intangible assets have been identified as one cause of the trend toward the increasing domination in some industries by superstar firms with high profits (Autor, Dorn, Katz, Patterson, Van Reenen, 2017). In addition, intangibles firms might have more backloaded investment needs that necessitate less funding. Recent work by Dottling, Ladika, and Perotti (2017) using a sample of U.S. publicly-listed firms argues that intangibles firms in their sample of relatively unconstrained firms produce cash flows that can cover their tangible and intangible investment needs, on average. However, they still find in their sample of large firms that intangibles firms accumulate significantly larger amounts of cash and issue less debt than tangibles firms. While this evidence suggests that a subset of intangibles firms is able to grow out of their financial constraints and operate in a financially unconstrained fashion, the previous evidence convincingly shows that younger, smaller, and riskier intangibles firms suffer from significant financing constraints.

Second, we discuss some evidence of how the reallocation of intangible capital is financed. An important way through which intangible capital is reallocated is through merger and acquisitions (M&A) (Jovanovic and Rousseau, 2008, Bena and Li, 2014, Levine, 2017). The majority of M&A transactions, in turn, are financed using internal funds or using stock, while a minority are debt financed (Maksimovic, Phillips, and Yang, 2013, Custodio, 2014).

Taken together, these multiple pieces of empirical evidence are strongly consistent with the view that a substantial fraction of productive and expanding firms in intangible industries face some form of financial frictions, which generate an external finance premium and affect their savings and investment decisions.

6 - Productivity dispersion has increased in intangibles sectors during recent decades, while it has remained roughly constant in tangibles sectors. There is suggestive evidence that financial frictions are a contributing factor to this trend.

Kehrig (2015) analyzes establishment-level manufacturing data from the U.S. census and documents a significant increasing trend in the dispersion of productivity across firms within

[8] U.S. corporate tax rules that encourage cash retention abroad have been suggested as an important reason for U.S. corporate cash holdings (Harford et al., 2017). Pinkowitz et al. (2016) find, however, that higher cash holdings in U.S. firms seem to be mostly explained by the higher reliance on intangible assets.

sectors over the past 40 years.[9] Related evidence is documented by Hsieh and Klenow (2017) and Barth et al. (2016) but no other author, to the best of our knowledge, has explored the relation between the rise in intangibles and productivity dispersion. We perform this analysis using accounting data of 34,900 U.S. corporations obtained from Compustat, covering the period from 1980 to 2015, and containing 379,318 firm-year observations. We define intangible capital as the sum of knowledge capital and organizational capital. We consider two alternative productivity measures: labor productivity (y) and total factor productivity (TFP) (A) (see Appendix A for details). Productivity dispersion is computed as the standard deviation of the difference between the log of the productivity of firm i and the log of the aggregate productivity of the industry s in which firm i operates.

[FIGURE 1 ABOUT HERE]

[FIGURE 2 ABOUT HERE]

Figures 1 and 2 plot the dispersion of labor productivity and TFP, respectively, in 2-digit SIC industries over time (normalized by the value in 1980). In both figures, the left panel shows average dispersion for all sectors, and it replicates the upward-sloping trend already documented by Kehrig (2015) using establishment-level data. In the right panel of both figures, the red dashed (blue solid) line displays the sales-weighted mean of the dispersion measure across industries in the top 50% (bottom 50%) of the distribution of the industry-wide ratio of intangible capital to total capital averaged across years.[10] Both figures show that the constant rise in the within-industry dispersion of productivity is driven by the sectors with higher average shares of intangible capital. Appendix A discusses two additional exercises that provide robustness to this result and show that the difference between the two trends is statistically significant and that the increase in dispersion for intangible sectors is not caused by a higher

[9] It is important to note that this paper, like Kehrig (2015), analyzes the dynamics of the cross-sectional dispersion of productivity, not the dispersion of business growth rates. Davis et al. (2006) focus on the latter and, using both firm- and establishment-level data, document a negative trend instead. These opposite trends are consistent with the findings of our model, in which a decline in the growth rate of expanding firms reduces reallocation of capital and increases steady state productivity differences.

[10] The sectors with high shares of intangible capital are: Chemicals and Allied Products; Industrial and Commercial Machinery and Computer Equipment; Electronic & Other Electrical Equipment & Components; Transportation Equipment; Measuring, Photographic, Medical, & Optical Goods, & Clocks; Miscellaneous Manufacturing Industries; Wholesale Trade - Durable Goods; Home Furniture, Furnishings and Equipment Stores; Miscellaneous Retail Business Services; and Engineering, Accounting, Research, and Management Services.

The sectors with low shares of intangible capital are: Oil and Gas Extraction; Food and Kindred Products; Paper and Allied Products; Rubber and Miscellaneous Plastic Products; Stone, Clay, Glass, and Concrete Products; Primary Metal Industries; Fabricated Metal Products; Wholesale Trade - Nondurable Goods; General Merchandise Stores; Food Stores; Apparel and Accessory Stores; and Eating and Drinking Places.

average industry growth. Sectors that grew faster, on average, in the 1980-2015 period had a lower increase in productivity dispersion than the other sectors.

This finding could be driven by an increase in frictions or distortions that hinder factor reallocation in intangibles sector or, instead, could reflect an increase in the dispersion of shocks to intangibles sectors. Our analysis does not disentangle these two factors. Nonetheless, Haltiwanger, Jarmin, and Miranda (2016a, 2016b) document that the increased firm-level dispersion over time is due to a decrease in the responsiveness to idiosyncratic productivity shocks, rather than to lower volatility of such shocks, particularly in the high-tech sector after the year 2000. They conclude that their evidence is consistent with an increase in frictions or distortions in the U.S. economy that prevent the optimal reallocation of resources, and mention financial frictions as one of the leading candidate explanations.

Specific evidence that financial frictions might be behind the rise in productivity dispersion is also contained in Gilchrist, Sim and Zakrajsek (2013). They report that the dispersion of firm-level borrowing costs—which they justify theoretically to be a proxy for capital misallocation due to financing constraints—has increased significantly in the U.S. in recent decades, and especially since the early 2000s. They do not explore how this measure varies across industries. An important caveat to the link between financing frictions and increased productivity dispersion is that the efficiency of financial markets is likely to have, all else equal, increased during recent decades. What we claim may have happened is related to our empirical observation 3-5 discussed earlier: there has been a compositional shift toward a stronger relevance of firms (intangibles firms) that are more financially constrained because of the nature of their technology. Our model provides an explanation of why financial frictions might have increased for these intangible sectors, and is able to generate different trends in productivity dispersion, depending on the degree of tangibility of firms assets, consistent with the evidence in Figures 1 and 2.

The rest of the paper introduces a model that can explain these six key stylized facts and that describes how they might be related. In particular, our mechanism explains the fall in productivity and output (trend 1) as a result of the decline in the real interest rate (trend 2) and the rise in intangible capital (trend 3), through a mechanism that operates by increasing resource misallocation (trend 6) as a result of worsening financial constraints in intangibles sectors (trends 4 and 5). In Section 7.3 we discuss how the observed timing of these trends—in particular, the timing of the post-2000 slowdown in productivity and output emphasized in the secular stagnation debate—is consistent with the endogenous timing that arises from our theoretical framework.

3 Simple and Intuitive Explanation of the Mechanisms

The objective of this section is to develop the simplest possible model that can describe our proposed mechanisms and deliver analytical results. To this end, we introduce a series of simplifying assumptions that will later be relaxed in the full-fledged general equilibrium setup of Section 4, which is used for realistic quantitative analysis. We introduce two channels that, under certain conditions, generate a *positive* relationship between interest rates and efficiency in the allocation of capital, and contrast these new channels with the traditional channels that predict a standard negative relationship between interest rates and efficiency.

3.1 The Savings Channel

Consider an infinite-horizon partial equilibrium setup in which infinitely-lived firms have access to a decreasing returns to scale production function that transforms capital k_{t+1} invested in period t into consumption goods $f(k_{t+1})$ in period $t+1$. All firms operate with the same technology, and decreasing returns to scale imply that the most efficient allocation of resources is for all firms to produce with the same amount of capital. One unit of capital can be produced instantaneously using one unit of the consumption good, and it depreciates every period at the rate $\delta \leq 1$. Assume, as in Woodford (1990) and Kiyotaki and Moore (2012), that investment opportunities only arrive occasionally: for simplicity, consider that firms can only invest every other period. In the analysis in this section, even periods $(t-2, t, ...)$ refer to investing periods and odd periods $(t-1, t+1, ...)$ refer to non-investing periods. The exogenous interest rate between periods t and $t+1$ is r_{t+1}, and there is perfect foresight about aggregate and idiosyncratic variables.

Firms maximize the present value of dividends paid out to shareholders. Consider first a financially unconstrained firm. Under the Modigliani-Miller theorem, maximizing dividends is equivalent to maximizing profits for a financially unconstrained firm. Moreover, since capital can be adjusted frictionlessly, the multi-period optimization problem can be decomposed into a sequence of one-period problems. Therefore, in period t, the firm chooses k_{t+1} to maximize $\pi_{t+1} = f(k_{t+1}) - (r_{t+1} + \delta) k_{t+1}$, and its optimal investment k^*_{t+1} will be determined by the neoclassical investment rule:

$$f'(k^*_{t+1}) = r_{t+1} + \delta. \tag{1}$$

Consider, instead, a financially constrained firm. For simplicity, assume that the firm cannot access any type of external finance and that it is forced to pay out as dividends every period a fraction λ of revenues $f(k_{t+1})$. If λ is high enough, the firm will be permanently constrained

and unable to attain the unconstrained investment level k^*_{t+1}.[11] Instead, it will invest all of its available internal funds in generic investment period t to achieve a level of capital k^c_{t+1} (where subscript c stands for 'constrained') given by:

$$k^c_{t+1} = (1-\lambda)\, f(k^c_{t-1})(1+r_t) + (1-\lambda)\, f((1-\delta)\, k^c_{t-1}) + (1-\delta)^2\, k^c_{t-1} \tag{2}$$

where $(1-\lambda)\, f(k^c_{t-1})$ are the revenues, net of dividend payments, received in period $t-1$ (a non-investing period), and $(1+r_t)$ is the return to saving those internal funds until the current investing period t. The firm can also produce $f((1-\delta)\, k^c_{t-1})$ in period t using its stock of undepreciated capital and holds an amount $(1-\delta)^2\, k^c_{t-1}$ of undepreciated capital.

Set $f(k_{t+1}) = k^{\alpha}_{t+1}$, with $0 < \alpha < 1$, and assume, without loss of generality, that $\delta = 1$. In the steady state of this economy, the investment levels of unconstrained and constrained firms (k^* and k^c) will be, respectively,

$$k^* = \left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}, \quad \text{and} \tag{3}$$

$$k^c = [(1-\lambda)\,(1+r)]^{\frac{1}{1-\alpha}}\,. \tag{4}$$

How do variations in the exogenous interest rate r affect the investment level of each type of firm? The standard *user cost of capital channel* in the case of the unconstrained firm introduces the usual negative relationship between k^* and r. For a constrained firm, however, k^c is a positive function of r because a higher r enables it to accumulate more internal savings and supports a higher level of investment: this is what we call throughout the rest of the paper the *savings channel*.

A corollary of this result is that, in an economy with constrained and unconstrained firms, decreases in interest rates might increase the misallocation of capital. Under our assumptions, it is the case that $f'(k^*) < f'(k^c)$ and that the degree of misallocation (measured by $f'(k^c) - f'(k^*)$) will increase when r goes down. In other words, the lower the interest rate, the lower will be aggregate productivity and output relative to the efficient allocation of resources.

3.2 The Capital Price Channel

In the previous example, we assumed that the price of capital in units of consumption goods is equal to 1. Consider now, instead, that capital is purchased in a capital market in which

[11] The intuition for this result is as follows. A firm with a low level of capital has a high marginal return $f'(k_{t+1})$ and generates positive retained earnings, after paying dividends, which are reinvested in capital. However, if λ is high enough, dividend payments grow fast as the firm accumulates more capital and, as a result, the firm is unable to grow beyond a level of capital k^c_{t+1} that is below the optimal unconstrained level k^*_{t+1}.

capital producers sell capital at price q_t. The investment rules in the steady state, in which $q_t = q_{t+1} = q$, now become

$$k^* = \left(\frac{\alpha}{q\,(1+r)}\right)^{\frac{1}{1-\alpha}}, \quad \text{and} \tag{5}$$

$$k^c = \left[\frac{(1-\lambda)\,(1+r)}{q}\right]^{\frac{1}{1-\alpha}}. \tag{6}$$

Consider the general case in which $\frac{\partial q}{\partial r} < 0$. This relationship between the interest rate and the price of capital is endogenized in the full model of Section 4, but we take it as given for now. A lower r, through its positive impact on q, will reduce the ability of constrained firms to invest. We call this partial effect ($\frac{\partial k^c}{\partial q}\frac{\partial q}{\partial r} > 0$) the *capital price channel*.

What are the implications of this channel for misallocation in an economy with constrained and unconstrained firms? Notice from (5) that the capital price channel also affects unconstrained firms and that $\frac{\partial k^*}{\partial q}\frac{\partial q}{\partial r}$ is also positive: unconstrained firms will choose to purchase less capital when q goes up. Therefore, from a partial equilibrium perspective, the effect of changes in r on misallocation through the capital price channel is ambiguous.[12]

3.3 The Collateral Channel and the Net Debtor Channel

Consider now that constrained firms have some debt capacity and can borrow an amount b_t subject to a collateral constraint. To provide a justification for the constraint, we relax the assumption that $\delta = 1$, and assume that firms can borrow up to a fraction θ of the present value of undepreciated capital next period, or

$$b_t \leq \bar{b}_t \equiv \theta \frac{(1-\delta)\,k^c_{t+1}}{1+r_{t+1}}, \tag{8}$$

where, for simplicity, we're assuming that the price of capital q_t–which does not play a role in the channels in this subsection–is constant and equal to 1.

Under the assumption, again, that the dividend payout ratio λ is high enough so that constrained firms are unable to attain the unconstrained investment level k^*_{t+1}, we have that (8) is binding and that the amount of investment k^c_{t+1} that a financially constrained firm can

[12] The condition under which

$$\frac{d\left[f'(k^c) - f'(k^*)\right]}{dq} > 0 \tag{7}$$

and, thus, misallocation increases, is given by $(1-\lambda)\,(1+r)^2 < \alpha$. For a given α, misallocation is likely to increase following an increase in q when λ is high and r is low, which is an environment in which constraints are very severe (low r worsens financial constraints through the savings channel described in Section 3.1) and the existing misallocation is high. Intuitively, a given change in investment has a stronger impact on the dispersion in marginal productivities when the investment level (marginal product of capital) of financially constrained firms is very low (very high) relative to that of unconstrained firms.

achieve in an investing period t is

$$k_{t+1}^c = (1-\lambda)\, f((1-\delta)\, k_{t-1}^c) + (1-\delta)^2\, k_{t-1}^c - (1+r_t)\, b_{t-1} + \overline{b}_t, \tag{9}$$

where the borrowing b_{t-1} (or saving if $b_{t-1} < 0$) it incurred in the previous period (non-investing period $t-1$) is:

$$b_{t-1} = (1+r_{t-1})\, \overline{b}_{t-2} - (1-\lambda)\, f(k_{t-1}^c). \tag{10}$$

The budget constraint (9) is similar to (2) but with the added terms that refer to firm borrowing. The first term in (9) is the output, net of dividend payments, that the firm produces in t using the undepreciated capital it had in period $t-1$. The second term captures any remaining undepreciated capital, $(1-\delta)^2\, k_{t-1}^c$, the firm has in period t. Finally, the firm borrows funds $\overline{b}_t$, and repays the debt $(1+r_t)\, b_{t-1}$ (if $b_{t-1} > 0$) that it might have incurred in non-investing period $t-1$. If $b_{t-1} \leq 0$, the firm had surplus funds in non-investing period $t-1$ and saved them at the same rate r_t. This net financial position b_{t-1} in period $t-1$ is given by (10). We assume that the equilibrium is such that $(1+r_{t-1})\, \overline{b}_{t-2} - (1-\lambda)\, f(k_{t-1}^c) \leq \theta \frac{(1-\delta)^2 k_{t-1}^c}{1+r_{t+1}}$ (i.e., it satisfies constraint (8) adjusted for a non-investing period) so that the firm does not need to liquidate capital to repay its liabilities in a non-investing period.[13]

We combine (9), the binding version of (8), and (10), and make some further simplifications to arrive at an expression for k^c in the steady state given by:

$$k^c = \frac{[(1-\lambda)\, f(k^c) - \theta\,(1-\delta)\, k^c]\,(1+r) + (1-\lambda)\, f((1-\delta)\, k^c) + (1-\delta)^2\, k^c}{1 - \theta \frac{1-\delta}{1+r}}. \tag{11}$$

The term $1 - \theta \frac{1-\delta}{1+r}$ in the denominator of (11), which is the downpayment necessary to purchase one unit of capital, is increasing in r. This is a standard *collateral channel*, by which decreases in r enable financially constrained firms to loosen their financial constraint, borrow more, and invest more.

Moreover, the interest rate r also multiplies the term $[(1-\lambda)\, f(k^c) - \theta q\,(1-\delta)\, k^c]$ in the numerator, which is the net financial position $(-b_{t-1})$ carried over from $t-1$. When θ is sufficiently large, financially constrained firms will carry a net debtor position $((1-\lambda)\, f(k^c) - \theta q\,(1-\delta)\, k^c < 0)$ over to the investing period and, in those cases, instead of a *savings channel* they experience a *net debtor channel* of the opposite sign. Lower interest rates reduce their interest expenses and provide them with more resources to invest in period t.

What are the implications of these channels for misallocation in an economy with constrained and unconstrained firms in which the constrained firms are net debtors $((1-\lambda)\, f(k^c) -$

[13] Notice that b_{t-1} does not have a straight bar above it, indicating that the firm need not be constrained in a non-investing period, even if it is always constrained in investing periods.

$\theta q\,(1-\delta)\,k^c < 0$)? Notice that these two channels are not operating for unconstrained firms, which still follow the neoclassical rule for their capital choice. Constrained firms follow equation (11), and a reduction in r increases their investment k^c because it reduces their interest expenses and increases their borrowing capacity. Therefore, $f'(k^c)$ decreases when r drops and, as a result, misallocation through these two channels in isolation decreases.

3.4 Discussion

This very simple partial equilibrium model yields the following predictions on the effects of a reduction in interest rates on the equilibrium allocation of resources. When capital has no (or low) collateral value—as is the case with intangible capital—, the savings channel predicts an unambiguous large increase in misallocation. When capital has a high collateral value—as is the case with tangible capital—and the firm borrows heavily to invest, the collateral channel and the net debtor channel reverse this result and imply that misallocation declines when r falls, consistent with the conventional intuition.

These results rely on two nonstandard assumptions about dividend payments and intermittent investment. However, these assumptions will be relaxed in the full model and replaced with a more standard firm dynamics setting with firm entry and exit and optimal dividend decisions.

4 General Equilibrium Model

We introduce an infinite-horizon, discrete-time economy populated by an intermediate sector that produces capital; by a final good sector in which firms use labor and capital to produce consumption goods; and by households, which provide labor and own both sectors. There are several important extensions to the simple model analyzed in Section 3, and we describe the main ones here. We introduce an intermediate capital-producing sector that allows us to endogenize in equilibrium the price and the aggregate stock of capital. In the final good sector, we model explicitly tangible and intangible capital, and we derive endogenously the accumulation of financial and physical assets of firms that live multiple periods. The household sector is modeled as a life-cycle framework, which allows us to endogenize the interest rate and study how it is affected by demographic changes and other demand-side factors.

4.1 Capital-Producing Sector

A representative firm in this sector chooses investment in tangible and intangible capital, respectively I_t^T and I_t^I, in order to maximize profits:

$$\max_{I^J} q_{J,t} I_t^J - e_t^J \left(\frac{I_t^J}{\varphi} \right)^{\varphi}, \tag{12}$$

where $\varphi > 1$ and $e_t^J > 0$ are exogenous parameters, and $q_{J,t}$ is the price of the type of capital $J \in \{T, I\}$. We allow for e_t^T and e_t^I to be time varying in order to capture trends in the evolution of the relative price of capital. The first order condition implies that the value of one new unit of capital $q_{J,t}$ is equal to the marginal cost of producing it: $e_t^J \left(\frac{I_t^J}{\varphi} \right)^{\varphi-1}$. Capital producers are not financially constrained and they optimally equalize the marginal cost and marginal return of capital. Solving, we obtain optimal investment $I_t^J = \varphi \left(\frac{q_{J,t}}{e_t^J} \right)^{\frac{1}{\varphi-1}}$ and profits $\pi_t^J = \frac{q_{J,t}^{\frac{\varphi}{\varphi-1}}}{(e_t^J)^{\frac{1}{\varphi-1}}} (\varphi - 1)$.

At the beginning of period t, total capital available is $\overline{K}_t^T$ and $\overline{K}_t^I$. New capital I_t^T and I_t^I is produced and sold in period t so that the aggregate dividends generated by the capital-producing sectors are

$$D_t^k = \pi_t^T + \pi_t^I. \tag{13}$$

During period t, tangible capital and intangible capital depreciate at the rates $0 \leq \delta < 1$. And the law of motion of aggregate capital is

$$\overline{K}_{t+1}^J = I_t^J + (1 - \delta) \overline{K}_t^J, \tag{14}$$

with $J \in \{T, I\}$.

4.2 Final Good Sector

As in the simple model analyzed in Section 3, financial frictions affect the equilibrium allocation of resources between constrained and unconstrained agents. In the literature, there are notable examples in which a similar equilibrium is obtained by assuming that a fraction of agents in the economy have productive investment opportunities, but informational or contractual frictions imply that they are financially constrained in equilibrium (among others, see Kiyotaki and Moore, 1997 and 2012; Del Negro et al, 2017). Another approach is, instead, to assume that all firms have the same production technology but that the presence of persistent idiosyncratic shocks and/or decreasing returns to scale implies that some firms—typically the younger ones—are endogenously more productive and financially constrained, and other firms—typically the

older ones—are less productive and financially unconstrained thanks to past accumulated savings (e.g. Buera, Kaboski and Shin, 2011; Kahn and Thomas, 2013). For simplicity, we adopt the former approach and assume that there are two types of final-good-producing firms: high-productivity and low-productivity. However, all of the results derived here could be generalized in a more complicated model following the latter approach.

4.2.1 High-Productivity Firms

There is a continuum of mass 1 of high-productivity firms.

Technology and financing opportunities

High-productivity firms produce a final good using a constant-returns-to-scale production function that is Cobb-Douglas in labor and capital. The firms use two different types of complementary capital, tangible and intangible. For simplicity, we assume that they are perfect complements. The production function takes the following form:

$$y_t^p = z_t(\mu)\, n_t^{(1-\alpha)} \left[\min\left(\frac{k_{T,t}}{1-\mu}, \frac{k_{I,t}}{\mu}\right)\right]^{\alpha}, \tag{15}$$

where $0 < \alpha \leq 1$ and $0 < \mu < 1$. The terms $k_{T,t}$ and $k_{I,t}$ represent tangible and intangible capital installed in period $t-1$ that produce output in period t, and n_t is labor. We adopt a Leontief production function for convenience, because it implies that all firms choose the same intangible capital share of total capital, and this facilitates aggregation.[14] The productivity term $z_t(\mu)$ is increasing in the share of intangible capital and captures the higher productivity of more intangibles-intensive technologies. The positive dependence of z_t on μ is not only unnecessary for our results, but in fact makes it harder for our mechanism to generate a contraction when the share of intangibles rises. It is introduced for empirical realism and also for the shift to intangibles, which we take as exogenous, to be consistent with a privately optimal choice of firms. Our results are robust to considering a z_t that does not depend on μ (and that is either constant or linearly increasing), as we report in Appendix E**.** We drop from now on reference to the dependence of z_t on μ for ease of notation and defer discussion of their relationship to the calibration section.

The budget constraint for high-productivity firms is given by the following dividend equation:

$$d_t = y_t^p + (1+r_t)a_{f,t} - a_{f,t+1} - q_{T,t}\left(k_{T,t+1} - (1-\delta)k_{T,t}\right) - q_{I,t}(k_{I,t+1} - (1-\delta)k_{I,t}) - w_t n_t, \tag{16}$$

[14] Using a more standard Cobb-Douglas production function would imply that the optimal ratio between tangible and intangible capital varies with the intensity of financial frictions. More constrained firms would use more intensely tangible capital, because its higher collateral value becomes more attractive, and this would create an additional distortion in the allocation of resources across firms. See Perez-Orive (2016) for a study of this type of distortion.

where r_t is the interest rate paid or received in date t; $q_{T,t,}$ and $q_{I,t}$ are the prices of tangible and intangible capital, respectively; and w_t is the wage. The term $a_{f,t} > 0$ indicates that the firm is a net saver, and $a_{f,t} < 0$ indicates that the firm is a net borrower.

High-productivity firms are subject to frictions in their access to external finance. First, they can issue one-period riskless debt, subject to the constraint that they can pledge, as collateral, the fractions θ^T and θ^I of tangible capital and intangible capital, respectively. This constraint translates into the following inequality:

$$a_{f,t+1} \geq -\frac{\theta^T q_{T,t+1} k_{T,t+1} + \theta^I q_{I,t+1} k_{I,t+1}}{1 + r_{t+1}}, \tag{17}$$

where $0 < \theta^I < \theta^T \leq 1$. We still assume, as in constraint (8) in the simple model of Section 3, that only the holdings of undepreciated capital next period are pledgeable. To simplify our equations, however, we adopt a notation that considers that depreciation is included in the terms θ^I and θ^T.

Second, high-productivity firms are are unable to issue equity, which means that dividends are subject to a non-negativity constraint:

$$d_t \geq 0. \tag{18}$$

In reality, firms finance part of their investment with equity issues, which could be captured in the model by assuming that dividends can be negative up to a fraction of the firm's value. However, rather than complicating the model further, in the calibration section we consider equity financing by assuming larger values of θ^T and θ^I than are normally assumed in the literature. This assumption is without loss of generality, because assuming instead negative dividends proportional to the firm's value and lower collateral values of capital would not change our qualitative and quantitative results.

From the Leontief structure of the production function, it follows that $k_{T,t} = \frac{1-\mu}{\mu} k_{I,t}$. Therefore, from now on, we use this result to express all equations as a function of intangible capital only. At the beginning of each period, both types of capital are predetermined and in their optimal ratio $k_{T,t} = \frac{1-\mu}{\mu} k_{I,t}$; therefore, the production function can be written as

$$y_t^p = z_t n_t^{(1-\alpha)} \left(\frac{k_{I,t}}{\mu} \right)^{\alpha}. \tag{19}$$

After producing, the firm's technology becomes obsolete with probability ψ. In this case, the firm liquidates all of its capital, pays out as dividends all of its savings, including the liquidation value of capital, and exits. Exiting firms are replaced with newborn ones, with

initial endowment W_0. We follow Kiyotaki and Moore (2012) and assume that high-productivity firms can only invest each period with probability η. This assumption allows firms to have the opportunity to accumulate significant amounts of liquid savings when they do not invest, in line with the empirical evidence. The assumption is realistic, since many empirical papers document that firm investment is lumpy (Caballero, 1999). Because of the presence of non-convex adjustment costs, plants typically have zero or small investment rates during most of their existence, and experience few infrequent very large investment spikes (Doms and Dunne, 1998). Rather than modelling non-convex adjustment costs and state contingent investment decisions, our assumption of exogenous investment opportunities has similar implications but is much more tractable and allows for a closed-form solution.

Optimization

Firms choose their investment and savings in order to maximize the net present value of their dividends. We define the value function conditional on having an investment opportunity, denoted $V^+(k_{I,t}, a_{f,t})$, as follows:

$$\begin{aligned} V_t^+(k_{I,t}, a_{f,t}) &= \max_{n_t, dt, a_{f,t+1}, k_{I,t+1}} (1+\lambda_t)d_t + \vartheta_t \left(a_{f,t+1} + \frac{\theta^T q_{T,t+1} k_{T,t+1} + \theta^I q_{I,t+1} k_{I,t+1}}{1+r_{t+1}} \right) \\ &\quad + \frac{1}{1+r_{t+1}} \left[(1-\psi) V_{t+1}(k_{I,t+1}, a_{f,t+1}) + \psi d_{t+1}^{exit} \right], \end{aligned} \tag{20}$$

where λ_t and ϑ_t are the Lagrange multipliers of constraints (18) and (17), respectively, and

$$d_t^{exit} = y_t^p + (1+r_t)a_{f,t} + (1-\delta)q_{T,t}\frac{1-\mu}{\mu}k_{I,t} + (1-\delta)q_{I,t}k_{I,t} - w_t. \tag{21}$$

$V_{t+1}(k_{I,t+1}, a_{f,t+1})$ is the value function conditional on continuation but before the investment shock is realized:

$$V_{t+1}(k_{I,t+1}, a_{f,t+1}) = \eta V^+(k_{I,t+1}, a_{f,t+1}) + (1-\eta)V^-(k_{I,t+1}, a_{f,t+1}). \tag{22}$$

The value function of a non-investing firm, denoted $V^-(k_{I,t}, a_{f,t})$, is identical to $V^+(k_{I,t}, a_{f,t})$ but does not offer the opportunity to choose $k_{I,t+1}$.

The firm solves (20) (or its non-investing counterpart) subject to (16), (18), and (17). We next provide a characterization of high-productivity firms' optimal choice under the assumption that they are permanently financially constrained. We claim − and check later in our calibrated simulations − that, in equilibrium, the marginal return on capital for high-productivity firms

is always higher than their user cost:

$$\frac{\partial y^p_{t+1}}{\partial k_{I,t+1}} = \frac{\alpha z_{t+1} n_{t+1}^{(1-\alpha)}}{\mu}\left(\frac{k_{I,t+1}}{\mu}\right)^{\alpha-1} > \left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right) - \frac{(1-\delta)\left(q_{T,t+1}\frac{1-\mu}{\mu} + q_{I,t+1}\right)}{1+r_{t+1}}. \tag{23}$$

The first term on the right hand side of equation (23) is the total cost of one unit of $k_{I,t+1}$ and $\frac{1-\mu}{\mu}$ units of $k_{T,t+1}$. The second term is their residual value next period after production. The implication of assumption (23) for investing firms is that the borrowing constraint (17) is binding, and that firms choose not to pay dividends, so the equity constraint (18) is also binding. Making $d_t = 0$ in budget constraint (16), using (16) to substitute for $a_{f,t+1}$ in (17), assuming (17) is binding, and solving for $k_{I,t+1}$, we obtain their level of investment:

$$(k_{I,t+1} \mid \text{invest}) = \frac{y^p_t - w_t n_t + (1+r_t)a_{f,t} + (1-\delta)\left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right)k_{I,t}}{q_{T,t}\frac{1-\mu}{\mu} + q_{I,t} - \left(\theta^T \frac{q_{T,t+1}}{1+r_{t+1}}\frac{1-\mu}{\mu} + \theta^I \frac{q_{I,t+1}}{1+r_{t+1}}\right)}. \tag{24}$$

The right-hand side of equation (24) is the maximum feasible investment in intangible capital for a firm. The numerator is the total wealth available to invest. The first term is current profits $y^p_t - w_t n_t$, the second term is the net financial position from the previous period $(1+r_t)a_{f,t}$, and the last term is the residual value of tangible and intangible capital. The denominator captures the downpayment necessary to purchase one unit of $k_{I,t+1}$ and $\frac{1-\mu}{\mu}$ units of $k_{T,t+1}$. This is the total cost $q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}$ minus the term $\theta^T \frac{q_{T,t+1}}{1+r_{t+1}}\frac{1-\mu}{\mu} + \theta^I \frac{q_{I,t+1}}{1+r_{t+1}}$, which is the amount that can be financed by borrowing.

Investing firms in equilibrium borrow as much as possible, and

$$(a_{f,t+1} \mid \text{invest}) = -\left(\theta^T \frac{q_{T,t+1}}{1+r_{t+1}}\frac{1-\mu}{\mu} + \theta^I \frac{q_{I,t+1}}{1+r_{t+1}}\right)k_{I,t+1} < 0. \tag{25}$$

The implication of assumption (23) for non-investing firms is that they will not sell any of their capital, and, for these firms, the law of motion of capital is

$$(k_{I,t+1} \mid \text{not invest}) = (1-\delta)k_{I,t}. \tag{26}$$

Non-investing firms always retain all earnings and select $d_t = 0$ because they face a positive probability of being financially constrained in the future, and hence the value of cash inside the firm is always higher than its opportunity cost (see Appendix C for a formal proof). Substituting $d_t = 0$ and (26) in (16):

$$(a_{f,t+1} \mid \text{not invest}) = y^p_t + (1+r_t)a_{f,t} - w_t n_t. \tag{27}$$

Equations (25) and (27) determine the wealth dynamics of firms. A firm that invested in period $t-1$ but is not investing in period t has debt equal to $-a_{f,t} = \left(\theta^T \frac{q_{T,t}}{1+r_t}\frac{1-\mu}{\mu} + \theta^I \frac{q_{I,t}}{1+r_t}\right) k_{I,t}$. It uses current profits $y_t^p - w_t n_t$ to pay the interest rate on debt $-r_t a_{f,t}$ and to reduce the debt itself. As long as the firm is not investing, the debt $-a_{f,t}$ decreases until the firm becomes a net saver and has $a_{f,t} > 0$. At this point, wealth accumulation is driven both by profits $y_t^p - w_t n_t$ and by interest on savings $r_t a_{f,t}$, until the firm has an investment opportunity and its accumulated wealth $(1+r_t)a_{f,t}$ is used to purchase capital (see equation (24)). This discussion clarifies that a lower interest rate r_t helps the non-investing firm repay existing debt (the net debtor channel), but it slows down the accumulation of savings after the firm has repaid the debt (the savings channel).

Finally, the first order condition for n_t, for both investing and non-investing firms, implies that given the wage w_t and its predetermined capital $k_{I,t}$, a firm will choose the profit-maximizing level of labor, which determines the optimal capital-labor ratio:

$$\frac{k_{I,t}}{n_t} = \mu \left[\frac{w_t}{(1-\alpha)\, z_t}\right]^{\frac{1}{\alpha}}. \tag{28}$$

4.2.2 Low-Productivity Firms

There is a mass 1 of identical low-productivity firms that have access to two production functions. Each production function combines capital $k_{uJ,t}$ with specialized labor $n_{uJ,t}$ using a constant-returns-to-scale technology, where $J = \{I, T\}$ captures the tangibility of the capital used. The total amount $y_t^{\mathbf{u}}$ of the homogeneous final good produced is then

$$y_t^u = z_t^{u,I} n_{uI,t}^{1-\alpha} k_{uI,t}^{\alpha} + z_t^{u,T} n_{uT,t}^{1-\alpha} k_{uT,t}^{\alpha}, \tag{29}$$

where α determines the capital share.

There are two differences with respect to the high-productivity firms, which we introduce for tractability. First, we do not introduce the assumption of perfect complementarity between tangible and intangible capital that we have for the high-productivity firms in order to gain tractability in the pricing of capital. As will be shown in the next section, the low-productivity firms price capital in equilibrium, and therefore if we assumed a Leontief production function also for them, the relative price of tangible and intangible capital would be constrained by the Leontief parameter μ, and the simulations in Section 7 would be unable to match a realistic evolution of such relative price over the 1980-2015 period. Second, and as a consequence of the first difference, we do not make an assumption about how the shares of tangible and intangible

capital evolve for the low-productivity firms or about how the productivity of low-productivity firms evolves over time. These differences are without loss of generality, as shown later in the calibration and results sections of the paper.

This sector is assumed to be able to finance capital with equity from the household sector and to pay out all profits as dividends d_t^u to households every period:

$$d_t^u = y_t^u - w_t^{uI} n_{uI,t} - w_t^{uT} n_{uT,t} - q_{I,t}\left(k_{I,t+1}^u - (1-\delta)k_{I,t}^u\right) - q_{T,t}\left(k_{T,t+1}^u - (1-\delta)k_{T,t}^u\right). \quad (30)$$

Since d_t^u is allowed be negative, these low-productivity firms are able to issue equity and are therefore never financially constrained. An alternative approach would be to assume that they are subject to the same constraint (expression (17)) as high-productivity firms, but that they are relatively patient and sufficiently long-lived to be able to accumulate enough savings to become financially unconstrained (*e.g.* see Kiyotaki and Moore, 1997). This alternative approach would produce analogous quantitative and qualitative results.

In addition, the low-productivity firms sector is able to remunerate households for their labor services ($w_t^{uI} n_{uI,t} + w_t^{uT} n_{uT,t}$).

The first order conditions for the two types of labor imply that given wages w_t^{uI} and $w_t^{uT}$and a firm's predetermined capital stocks $k_{I,t}^u$ and $k_{T,t}^u$, a low-productivity firm will choose the profit-maximizing level of each type of labor, which determines the optimal capital-labor ratio:

$$\frac{k_{uJ,t}}{n_{uJ,t}} = \left[\frac{w_t^{uJ}}{(1-\alpha)\, z_t^{u,J}}\right]^{\frac{1}{\alpha}}. \quad (31)$$

Given that low-productivity firms are financially unconstrained, and provided that their marginal return on each of the two types of capital is lower than for high-productivity firms, low-productivity firms are willing to absorb all of the capital not demanded by high-productivity firms, at a price equal to their marginal return on capital.

4.2.3 Aggregation of the Firm Sector

We assume (see Section 4.3) that the aggregate supply of all types of labor is normalized to $N = N_{uI} = N_{uT} = 1$. Since all high-productivity firms produce at the optimal capital-labor ratio determined by equation (28), and the production function is constant returns to scale, we can aggregate production across firms to obtain

$$Y_t^p = z_t \left(\frac{K_{I,t}}{\mu}\right)^{\alpha}. \quad (32)$$

The wage is determined in competitive markets by the marginal return of labor:

$$w_t = (1-\alpha)\, z_t \left(\frac{K_{I,t}}{\mu}\right)^{\alpha}. \tag{33}$$

Furthermore, we can aggregate the output of low-productivity firms, substituting labor supply $N_{uI} = N_{uT} = 1$, and obtain

$$Y_t^u = z_t^{u,I}\left(\overline{K}_t^I - K_{I,t}\right)^{\alpha} + z_t^{u,T}\left(\overline{K}_t^T - K_{T,t}\right)^{\alpha}, \tag{34}$$

$$w_t^{uJ} = (1-\alpha)\, z_t^{u,J}\left(\overline{K}_t^J - K_{J,t}\right)^{\alpha}, \tag{35}$$

with $J = \{I, T\}$. We defer the description of aggregate capital $K_{I,t}$, aggregate financial assets $A_{f,t+1}$ and pricing of assets to Section 5. For their derivation, see Appendix C.

The assumption that labor is specialized and does not move across sectors simplifies our analysis, but it is not essential for our findings. More precisely, relaxing it would likely increase the magnitude of misallocation. Intuitively, an increase in the financing constraints of high productivity firms implies that their capital stock $K_{I,t}$ falls. From equation (33), it follows that these firms pay lower wages, thus mitigating financial frictions. Instead, if labor types were not sector specific, the equilibrium wage would have to equalize across sectors, and this mitigating effect would disappear.

4.3 Households

We consider a life-cycle model with two types of households, young and old—with measures H^y and H^o, respectively—whose sum is normalized to 1. As in Eggertsson, Mehrotra, and Robbins (2017), we consider a life cycle model so that the equilibrium interest rate is not pinned down by the value of the discount factor β, but is instead affected both by household-side developments, such as realistic demographic changes in the U.S. in recent decades, as well as firm-side developments, such as the rise in firms' cash holdings.

Both young and old households have log utility. Young households supply three types of differentiated labor: high-productivity firm labor (in exchange for wage w_t), low-productivity intangible technology labor (in exchange for wage w_t^{uI}), and low-productivity tangible technology labor (in exchange for wage w_t^{uT}). There is an inelastic aggregate supply of one unit of each type of labor. The assumption that labor is differentiated across sectors is made for simplicity. If we had assumed that households can choose in which sector to supply their labor, then in equilibrium wages would be equal across sectors. Therefore, our main finding that low interest rates generate a misallocation of resources away from high-productivity firms toward

low-productivity firms would be reinforced: as low-productivity firms absorb more capital, their labor productivity increases and they end up absorbing also a larger share of the labor supply. Nevertheless, we decided to leave the analysis of this additional effect to further research.

Young households receive a fraction γ of the aggregate dividends. Households remain young for N periods and become old after $N+1$ periods, so that there is a constant fraction $\phi = \frac{1}{N}$ of young households for every age between 1 and N, and, every period, a measure ϕH^y of households becomes old. Old households cannot work, receive a fraction $(1-\gamma)$ of aggregate dividends, and die with probability ϱ. The measure of old households H^o is determined as follows:

$$H^o = (1-\varrho)H^o + \phi H^y. \tag{36}$$

At the same time, the measure of young households is

$$H^y = (1-\phi)H^y + N^y, \tag{37}$$

where N^y is the constant measure of newborn households. From the assumption that $H_t^o + H_t^y = 1$, it follows that $N^y = \frac{\phi\varrho}{\phi+\varrho}$, $H_t^o = \frac{\phi}{\phi+\varrho}$, and $H_t^y = \frac{\varrho}{\phi+\varrho}$.

We follow Blanchard (1985) and Yaari (1965) in assuming that households participate in a life insurance scheme when old. For the detailed solution of the households' maximization problem, see Appendix D**.**

5 Steady State

5.1 Equilibrium

We consider a steady state equilibrium and drop reference to the time subscript t. Total output of the high-productivity and low-productivity firms is, respectively,

$$Y^p = z\left(\frac{K_I}{\mu}\right)^{\alpha}, \tag{38}$$

and

$$Y^u = z^{u,I}\left(\overline{K}^I - K_I\right)^{\alpha} + z^{u,T}\left(\overline{K}^T - K_T\right)^{\alpha}. \tag{39}$$

Dividends d are given by $d = D^p + D^u + D^k$. The dividends of low-productivity firms and capital-producing firms are, respectively, D^u and D^k. They are equal to their respective steady state profits:

$$D^u = Y^u - w^{uI} - w^{uT} - q_I\delta\left(\overline{K}^I - K_I\right) - q_T\delta\left(\overline{K}^T - K_T\right), \tag{40}$$

and

$$D^k = \frac{q_{T,t}^{\frac{\varphi}{\varphi-1}}}{e^{T\frac{1}{\varphi-1}}}(\varphi-1) + \frac{q_{I,t}^{\frac{\varphi}{\varphi-1}}}{e^{I\frac{1}{\varphi-1}}}(\varphi-1). \tag{41}$$

High-productivity firms do not distribute dividends while in operation. Their aggregate dividends D^p are equal to the savings distributed by the fraction ψ of exiting firms minus the endowment of new firms ψW_0:

$$D^p = \psi\left(\alpha z\left(\frac{K_I}{\mu}\right)^{\alpha} + (1+r)A_f + \left(q_T\frac{1-\mu}{\mu} + q_I\right)K_I\right) - \psi W_0. \tag{42}$$

By combining (32) and (33) with (62) (see Appendix C), we obtain

$$A_f = \frac{(1-\psi)\,\alpha z_t\left(\frac{K_I}{\mu}\right)^{\alpha} + \psi W_0 - \left(q_T\frac{1-\mu}{\mu} + q_I\right)[\psi + \delta(1-\psi)]\,K_I}{[1-(1-\psi)(1+r)]}. \tag{43}$$

Equation (43) determines financial wealth A_f, which is equal to the net earnings of the high-productivity firms, in the numerator, multiplied by a multiplicative factor $\frac{1}{1-(1-\psi)(1+r)}$, which measures the future value of one unit of wealth saved today by these firms. The net earnings are the endowment of the new firms ψW_0 plus the net earnings of continuing firms. The term $(1-\psi)\,\alpha z_t\left(\frac{K_I}{\mu}\right)^{\alpha}$ is retained earnings, net of wage payments. The term $\left(q_T\frac{1-\mu}{\mu} + q_I\right)[\psi + \delta(1-\psi)]\,K_I$ is total expenditures to replace the depreciated capital of continuing firms $\delta(1-\psi)K_I$, and the capital liquidated by exiting firms ψK_I.

In order to determine the aggregate capital stock of the high-productivity firms, we use (54) to express (56) in the steady state as

$$K_I = \frac{\eta(1-\psi)\left(\alpha z_t\left(\frac{K_I}{\mu}\right)^{\alpha} + (1+r)A_f\right) + \eta\psi W_0}{\left[q_T\left(1-\frac{\theta^T}{1+r}\right)\frac{1-\mu}{\mu} + q_I\left(1-\frac{\theta^I}{1+r}\right)\right][\delta + \psi(1-\delta)] - \left(q_T\frac{\theta^T}{1+r}\frac{1-\mu}{\mu} + q_I\frac{\theta^I}{1+r}\right)\eta(1-\delta)(1-\psi)}, \tag{44}$$

which has an intuitive explanation. The numerator is the aggregate amount of liquid resources of investing firms. The denominator is the downpayment necessary to support one unit of capital in the steady state. It requires the replacement of the depreciated capital and the lost capital of exiting firms (a fraction $\delta + \psi(1-\delta)$) and can benefit from using existing capital held by the investing firms as collateral (fraction $\eta(1-\delta)(1-\psi)$).

The prices of capital are determined by recursively iterating forward equations (59) and (60):

$$q_I = \frac{1}{r+\delta}z^{u,I}\alpha\left(\overline{K}^I - K_I\right)^{\alpha-\mathbf{1}} \tag{45}$$

and

$$q_T = \frac{1}{r+\delta}z^{u,T}\alpha\left(\overline{K}^T - K_T\right)^{\alpha-\mathbf{1}}, \tag{46}$$

where aggregate capital and investment are given by

$$\overline{K}^J = \frac{I^J}{\delta} \tag{47}$$

and

$$I^J = \varphi \left(\frac{q_J}{e_t^J} \right)^{\frac{1}{\varphi - 1}}, \tag{48}$$

respectively, for $J \in \{I, T\}$.

Finally, aggregate borrowing is equal to aggregate savings, or

$$A_f = B, \tag{49}$$

where B is aggregate household borrowing, which we derive in detail in Appendix D. By Walras' Law, the aggregate resource constraint is satisfied.

The steady state values of A_f, B, K_I, q_I, q_T, and r are jointly determined by equations (43), (44), (45), (46), (49), and (93).

5.2 Discussion

In this section, we briefly discuss two key features of the equilibrium described above.

First, a shift toward more intangible technologies reduces the borrowing capacity of the high-productivity firms. To see this, assume for simplicity that $q_T = q_I = q$. The collateral value of one unit of capital is $\frac{q}{1+r} \frac{1}{\mu} \left[(1-\mu)\, \theta^T + \mu \theta^I \right]$. Since $\theta^T > \theta^I$, a technology that relies more on tangible capital (lower μ) places a higher weight on the collateral value of tangible capital θ^T, thus increasing the overall collateral value of the firms' capital. Such an economy has a lower downpayment in the denominator of (44) and more capital K_I for a given total wealth in the numerator.

Second, net financial wealth A_f changes sign when moving from a tangible to an intangible economy. The denominator of A_f in Equation (43) is always positive. Moreover, since the endowment of new firms ψW_0 is in equilibrium small, the sign of the numerator is determined by the first term, which is positive, concave in K_I, and represents earnings net of wage payments, and the last term, which is negative, linear in K_I, and represents total expenditures to keep capital constant. A high average collateral value of capital in a tangible economy increases K_I until the negative term dominates, and it makes A_f negative: the high-productivity firms are, on aggregate, net borrowers. Conversely, in an intangible (high μ) economy, A_f is likely to be positive. The previous discussion clarifies that the exogenous assumptions made in the simple model in Section 3 are endogenously derived in the full general equilibrium model. Moreover,

even though a change in the interest rate affects aggregate capital K_I in (44) through the same four channels identified in the simple model in Section 3, it is important to emphasize that the endogeneity of financial assets amplifies the strength of the savings channel. When A_f is positive, a reduction in the interest rate reduces investment both through a reduction in the return on savings rA_f and through the multiplicative factor $\frac{1}{1-(1-\psi)(1+r)}$.

6 Calibration

[TABLE 1 ABOUT HERE]

For the purpose of evaluating the qualitative and quantitative importance of the channels explained earlier for the real economy, we calibrate the model to be broadly in line with recent U.S. data. We simulate the evolution of the economy from 1980 to the present as a sequence of steady states. Since we analyze the effects of developments that occurred slowly over a relatively long period of time, we believe that this comparative statics exercise is a suitable approximation, for our purposes, of the transition dynamics of the economy during this period. Our benchmark calibration is illustrated in Table 1. Our calibration strategy is twofold. We set most of our parameters to match the average value, over the 1980-2015 period, of selected empirical moments. A subset of parameters — those that are key to the mechanisms introduced in our model — are the basis of our comparative statics exercises and are set to change according to their observed variation or the observed variation of some direct moment they influence during the 1980-2015 period. In this latter group we include the share of intangibles (μ), the cost of producing capital (driven by parameters e_t^T and e_t^I), the rate of time preference of households (β), and the longevity of households (driven by ϱ).

We start discussing the calibration of parameters that remain constant across the different steady states. In the firm sector, the elasticity of output with respect to capital α is set equal to 0.4 for both types of firms, a common value used in most of the literature.[15] The measures of high-productivity and low-productivity firms are assumed to be equal. This assumed share of high-productivity firms, which are financially constrained in our model, matches the observed share of credit-constrained firms in the United States, estimated by Farre-Mensa and Ljungqvist (2015) to be roughly 50%.[16]

The pledgeability parameters of tangible capital θ^T and intangible capital θ^I are equal to

[15] See King and Rebelo (1999) or Corrado, Hulten and Sichel (2009).

[16] They find that roughly three quarters of privately held firms are financially constrained. Within the sample of publicly listed firms, they report different estimates of the share of financially constrained firms that range between 10% and 45%. Given these estimates, we set the share of high productivity firms to be 50% in our simulations.

1.00 and 0.35, respectively. As discussed in Section 4.2.1, these parameters also include the depreciation of the tangible and intangible capital. We assume tangible capital to be fully collateralizable, in line with Falato et al. (2014). Moreover, we calibrate θ^I to generate net leverage in the high-productivity firms on average equal to 6.4%, in line with the average net leverage ratio for Compustat publicly-listed firms.[17] We set θ^I relatively high compared with the literature to accommodate for the fact that we only allow firms to issue collateralized debt. As discussed in Section 2, in reality, firms finance their acquisitions in part with equity issues and other forms of external financing beyond collateralized debt.[18]

In order to calibrate the exit probability ψ and the investment probability η, we interpret them as shocks that generate creative destruction. Therefore, even though we do not model explicitly heterogeneous products, we interpret ψ as the probability that the firm's technology becomes obsolete because a competing firm enters the market and produces an improved version of its product. Moreover, we interpret η as the arrival probability of an investment opportunity to produce a new product. According to this interpretation, we set $\psi = \eta = 13\%$, which generates yearly capital reallocation of 6% of total capital (tangible plus intangible). This is consistent with David (2014), which measures reallocation of capital generated by mergers and acquisitions to be around 5% of total capital in the past few decades, and with the reallocation data from Eisfeldt and Rampini (2006).[19] The intuition is that when a firm's technology becomes obsolete, it sells its capital to the new and more productive firms.

The TFP of low-productivity firms, $z^{u,T}$ and $z^{u,I}$, is normalized to 10. The TFP of high-productivity firms z_t is modeled as:

$$z_t = [1 + (\mu - 0.2)\kappa]\, z, \tag{50}$$

so that for the early 1980s value of $\mu = 0.2$, $z_t = z$ for simplicity. We set $z = 25$ to match the average interquartile productivity differential of the firms, which in our simulations is 2.54 over the 1980-present period, a number consistent with the cross-sectional dispersion in productivity for U.S. firms identified in Syverson (2004) for a similar time period.[20] The parameter κ

[17] Bates et al. (2009) using data from 1980 to 2006, compute a value of 7.9%. They calculate net leverage as the ratio of total debt minus cash holdings to the book value of total assets, which maps in the model to $-A_f / (q_T K_T + q_I K_I)$.

[18] Notice that, since we are assuming that θ_I and θ_T also include the depreciation of capital, it follows that the effective collateral value of the depreciated tangible capital is larger than 100%. An alternative approach could have been to assume a value of θ_T equal to $1 - \delta$, and a value of θ_I much closer to zero, in line with Falato, Kadyrzhanova, and Sim (2014), and introduce equity issues by allowing dividends d_t to be negative, with an associated equity issuance cost proportional to the amount financed. This approach would have slightly complicated the model and yielded very similar quantitative results.

[19] Using capital reallocation data available at Andrea Eisfeldt's website (https://sites.google.com/site/andrealeisfeldt/reallocation_data_eisfeldt.xlsx), we compute an average capital reallocation of 5.8% of total capital over the 1980-2013 period.

[20] Syverson (2004) examines plant-level data from 1977 and finds an average interquartile difference in labor

measures the increase in TFP associated with a stronger intensity of intangible capital in the production function. We choose a benchmark value of $\kappa = 0.1$, and we analyze the sensitivity of our results to values between 0 and 0.2. A positive value of κ is not only unnecessary for our results, but in fact makes it harder for our mechanism to generate a contraction when the share of intangibles rises. However, it allows us to interpret the rise of intangible capital as the process of adopting technologies that are more productive but require more intangible assets, such as R&D and human and organizational capital, in the production process. It is also consistent with the notion that adopting these new technologies is a privately optimal choice of firms, and allows us to be able to make conservative and robust statements about the potential for negative effects of the shift to intangibles.[21] In Appendix E, we check that our main insights are robust to considering a z_t that does not depend on μ (and that is either constant or linearly increasing), to considering any positive dependence between z_t and μ, and to considering that the TFP of low-productivity firms' intangible capital (driven by $z^{u,T}$) also increases at the same rate as z_t.

The depreciation factor δ is set equal to 15%. This value is consistent with the depreciation rates used for the perpetual inventory method in Section 2.[22] The initial endowment of newborn firms W_0 is equal to 5, and is the only one not to be calibrated to match a specific moment due to a lack of a clear empirical counterpart. It corresponds to 2% of average firm annual output. Our results show very little sensitivity to variations in our choice of W_0 in the range 0.1%-20%.

The parameters associated to capital production are φ, e_t^T and e_t^I. The parameter φ determines the elasticity of the capital stock to the price of capital (see equation (48)), and we calibrate it so that the elasticity of the stock of capital to the user cost of capital is in line with the empirical evidence. Caballero, Engel and Haltiwanger (1995) estimate the short run elasticity of the capital stock to the user cost of capital to be between 0 and -0.1, and the long run elasticity to be between -0.3 and -1 for most 2-digit sectors. Since we do not model taxes, and the price of the consumption good is normalized to 1, the user cost of tangible capital in

productivity around 2 for 4-digit U.S. manufacturing sectors. Since the dispersion of productivity is larger for less narrowly defined sectors, a value of 2.54 is probably a very conservative estimate of the dispersion of productivity across all firms.

[21] The benchmark value of $\kappa = 0.1$ implies that an increase in μ is privately optimal at the steady state equilibria obtained for most values of μ.Our results are robust to setting κ high enough so that increases in μ are always privately optimal. Our benchmark calibration, however, reflects the possibility that some of the technological changes that have driven an increase in the intensity of intangible capital are not always endogenous firm choices but the consequence of structural economic changes, such as secular changes in the sectoral specialization of different countries.

[22] For tangible capital, this value is appropriate since we interpret it as a combination of more durable assets, such as equipment and structures, and less durable ones, such as inventories. For intangible capital, this value is consistent with existing literature regarding intangible and tangible capital, while possibly too low for other intangible assets such as computerized information and brand equity (Corrado, Hulten and Sichel, 2006). Assuming higher depreciation rate for intangible capital does not significantly change the results presented in the following sections.

our model is $(r+\delta)\,q^T$. We consider changes in the user cost of capital driven by exogenous changes in the interest rate. Our production sector implies that a decrease in r increases q^T(see equation (46)). However the user cost of capital falls in equilibrium, because the increase in q^T does not fully compensate the reduction in r. We choose a value of $\varphi = 9$, which generates an elasticity equal to -0.23. Given the value of φ, the initial values of e_t^T and e_t^I determine the aggregate supply of tangible and intangible capital and their equilibrium prices. We calibrate them so that the relative price of tangible to intangible capital is normalized to 1 in our early 1980s steady state simulation, and so that output of the high-productivity firms is roughly 50% of total output.

There are two household sector parameters that we keep constant across comparative statics. The share of dividends that are paid to the working-age population, γ, is set to 40% in order to target a real interest rate of $r = 6\%$ in our simulation of the early 1980s, consistent with the real rate in that period. We set the number of years households remain young to $N = 40$, which corresponds to a working-age period between the ages of 25 and 65 years.

Finally, we discuss the parameters that we vary in our comparative statics exercises. We follow Falato et al. (2014) in setting μ, the reliance on intangible capital of firms, at 0.2 in our exercise for the early 1980s, so that the share of intangible capital over total capital is 20%. We introduce a gradual linear shift in μ so that our simulation matches the observed intangible to total capital ratio of 60% ($\mu = 0.6$) in the 2010s (Corrado and Hulten, 2010a; Falato et al., 2014; Döttling and Perotti, 2016).

The household sector parameters that we vary across our simulations are the discount factor β and the probability of death after the age of 65, ϱ. First, we vary ϱ so that we match changes in the life expectancy in the U.S. between the 1980s and the present.[23] We vary the rate of time preference β to match the evolution of real interest rate from around $r = 6\%$ in the 1980s to around 0% in the present, and so that the value of β on average over our comparative statics exercises is in line with values used in the literature.[24]

The parameters e_t^I and e_t^T in the capital production function (12) capture the observed evolution of capital prices. In the model, equations (45)-(48) jointly determine how the capital stock and capital prices react to changes in the equilibrium interest rates driven by changes in household preferences. A drop in r increases q_T and q_I, for a given stock of capital in the unproductive sector (see equations (45) and (46)). Higher prices stimulate investment

[23] The Centers for Disease Control and Prevention (https://www.cdc.gov.htm) reports that life expectancy was around 70 years in 1970 and 78 years in 2016.

[24] Common values used in the literature range from 0.93 used in Jermann and Quadrini (2012) to 0.97 in Christiano, Eichembaum and Evans (2005). We set β to range from 0.9425 in the early 1980s to 0.9805 in recent years.

(equation (48)), which increases the stock total capital, as well as the part of it employed in low-productivity firms. This increases the denominator of equations (45) and (46), and thus dampen the initial increase in prices. This discussion clarifies that, if we keep e_t^I and e_t^T constant in our comparative statics exercise, a drop in the interest rate tends to increase capital prices in equilibrium, thus increasing misallocation through the capital price channel described in Section 3.

We think this channel is interesting from a theoretical standpoint, and we analyze its implications in counterfactual simulations in Section 7.2. However, for our main analysis in Section 7.3 we chose to vary e_t^I and e_t^T in order to deliver a realistic decline in tangible capital prices, and constant intangible capital prices.

For tangible capital, it is well know that it has experienced a significant decrease in its relative price. Karabarbounis and Neiman (2013) estimate that the price of capital has fallen approximately by 30% between the late 1970s and the 2000s, and we match this trend by decreasing e_t^T accordingly. Reliable measures of the change in the relative price of intangible capital are not available, however.[25] Some authors have used instead the GDP deflator, which implies by construction no change in the relative price of intangible capital (Corrado, Hulten and Sichel, 2009). Other authors use an input cost approach. An important factor in the production of intangible capital is skilled labor (Dougherty, Inklaar, McGuckin, and van Ark, 2007; Robbins, Belay, Donahoe, and Lee, 2012), which has experienced an important increase in its relative cost since the 1980s (Lemieux, 2008). An increase in input costs however might translate into lower intangible capital prices if the productivity of capital production increases substantially. This is the case for R&D, one of the types of intangible capital: Robbins, Belay, Donahoe, and Lee (2012) estimate an annual fall in the relative price of R&D of around 1.2% between 1998 and 2007 despite an increase in input costs. Computerized information, on the other hand, is estimated by Byrne and Corrado (2016) to have experienced an average annual real price change of -1% in the 1963-87 period, and of around -4% in the 1987-2015 period. Putting this evidence together, we change e_t^I over time so that the relative price of intangible capital remains roughly constant over time.

7 Simulation Results

In this section, we first validate our calibration, by verifying that it implies a realistic dispersion in the shadow cost of external finance across firms. Then, we introduce two comparative static

[25] See Corrado, Haskel, Iommi, and Jona-Lasinio (2012) for a detailed description of the challenges in obtaining a general price deflator for intangible capital.

exercises that capture how parallel developments in the household and the corporate sector have interacted to generate aggregate patterns consistent with the secular stagnation hypothesis. In the first exercise, we explore how an expansion of households' savings affects economic outcomes in a tangibles-intensive economy compared to an intangibles-intensive one. In the second exercise, we introduce a simulation that replicates key trends in the United States between 1980 and 2015, a period characterized by an increase in households' incentive to save and by a rise in the reliance on intangible capital.

7.1 Dispersion in the Cost of External Finance

In Section 2, we described extensive empirical evidence consistent with the main assumption of the model—that a significant fraction of firms face financial frictions and additional costs to access external finance. Gilchrist, Sim, and Zakrajsek (2013) show that these frictions increase the cross-sectional dispersion in the marginal return to investment, which can be approximated empirically with the dispersion in borrowing costs across firms. Here, we prove that the same mapping exists in our model and show that the intensity of financial frictions implied by our benchmark calibration is consistent with the empirical evidence.

In our model, we capture the dispersion in productivity as the standard deviation of the logarithm of the marginal return to investment (MRK), where MRK is given by

$$MRK \equiv \frac{\frac{\alpha z}{\mu}\left(\frac{K_I}{\mu}\right)^{\alpha-1} + (1-\delta)\left(q_I + \frac{1-\mu}{\mu}q_T\right)}{q_I + \frac{1-\mu}{\mu}q_T} \tag{51}$$

for high-productivity firms and by

$$MRK^U \equiv \frac{z^{u,I}\alpha\left(\overline{K}^I - K_I\right)^{\boldsymbol{\alpha-1}} + \frac{1-\mu}{\mu}z^{u,T}\alpha\left(\overline{K}^T - K_T\right)^{\boldsymbol{\alpha-1}} + (1-\delta)\left(q_I + \frac{1-\mu}{\mu}q_T\right)}{q_I + \frac{1-\mu}{\mu}q_T} \tag{52}$$

for low-productivity firms. Recall that high-productivity firms invest in a bundle of $\frac{1-\mu}{\mu}$ units of tangible capital for each unit of intangible capital. MRK is the marginal return from spending one additional unit of wealth in this bundle. MRK^U is the same return for the low productivity firms. Condition (58), which implies that high-productivity firms are more constrained in equilibrium, also implies that $MRK > MRK^U$. Moreover, using the equilibrium conditions (45) and (46), it follows that $MRK^U = 1 + r$. Since low-productivity firms are indifferent between saving and investing, they equalize the marginal return and the marginal cost of capital. Therefore, $MRK - MRK^U = \Delta$ can be interpreted as the shadow cost of external finance for high productivity firms. It represents the extra cost these firms would be willing to spend

to obtain one additional unit of external finance. We denote the dispersion in the marginal return to investment as $s.d.(\log \Delta)$, as in Gilchrist, Sim, and Zakrajsek (2013). These authors estimate that this measure of misallocation in the data was around 0.2 in the 1980s. For our benchmark calibration for the same period, we find a value of 0.03. We believe this difference is plausible, for two reasons. First, because in the data there is much more heterogeneity in borrowing costs than in the model, where we have only two types of firms. Second, because the empirical dispersion in borrowing cost can be driven both by differences in riskiness and by financial frictions, but our model only captures the latter component. Nonetheless, in our main comparative analysis exercise below we will consider, for robustness, a range of values of $s.d.(\log \Delta)$.

7.2 Comparative Statics Exercise 1: The Effect of a Rise in Households' Propensity to Save

In order to clarify the different effects at play, we first conduct a counterfactual exercise in which households' propensity to save and life expectancy both gradually increase, reducing the equilibrium interest rate. We run two simulations: one in which the share of intangible capital is kept constant at $\mu = 0.05$ (a tangibles economy), and another in which it is kept constant at $\mu = 0.65$ (an intangibles economy). The expansion in household savings is achieved by decreasing the rate of household time preference (increasing β) and by increasing life expectancy (lowering ϱ) to generate a decline in the interest rate from 6% to around 1%.[26] All the other parameters are kept constant at the benchmark level, including e_t^I and e_t^T. The sequence of steady states associated to the set of different values of β, ϱ and μ is displayed in Figure 3.

[FIGURE 3 ABOUT HERE]

The left panel in the middle row of Figure 3 shows that the net leverage of high-productivity firms is positive in the tangibles economy and firms are on average net borrowers. Corporate net leverage is instead negative in the intangibles economy and firms are on average net savers. Correspondingly, households are net savers (borrowers) in a tangibles (intangibles) economy, as shown in the top-right panel. Household sector developments encourage households to save more in a tangibles economy and borrow less in an intangibles economy, pushing down the interest rate in both cases. The drop in the interest rate increases the price of capital and encourages capital creation, so that aggregate tangible and intangible capital stocks increase.

[26] All parameters are identical in the two cases except for the discount factor β, which is set so that in both cases the comparative statics exercise starts with a value of $r = 6\%$. Therefore, while in the tangibles economy β changes from 0.9425 to 0.9805, in the intangibles economy it changes from 0.9375 to 0.9755. To avoid confusion we do not report these different values of β on the x-axis.

The left and middle panels in the last row of Figure 3 analyze the changes in the allocation of capital and in efficiency. In the tangibles economy, capital allocation improves and there is an expansion of capital and output of high-productivity firms. High-productivity firms have a high leverage and the decline in the interest rate benefits them, both because it is easier to pay back debt (the net debtor channel) and because they can borrow more when they invest (the collateral value channel).[27] These two channels prevail over the capital price channel, which operates in the opposite direction, and imply that the drop in r benefits high-productivity firms; they can absorb a higher share of existing capital, thus improving the allocation of resources.[28] Conversely, in the intangibles economy, firms are net savers. As explained in Section 3, in this case the decline in the interest rate hurts their accumulation of wealth (the savings channel), and the collateral value channel is very weak because firms' borrowing capacity is limited, so that a lower rate is strongly contractionary.

The last panel shows that, overall, output increases by around 1.5% in the tangibles economy, both because of the positive reallocation effect and because of the increase in the aggregate capital stock, while it declines by around 1.5% in the intangibles economy, because the contraction in the allocation of capital to the high productivity firms offsets the positive effect of the increase in aggregate capital.

[FIGURE 4 ABOUT HERE]

Figure 4 shows that the dispersion in the marginal productivity of capital increases with lower rates in the intangibles economy, while it falls moderately in the tangibles economy. The dispersion in TFP shows similar diverging trends as well. The values of μ chosen for the tangibles and intangibles economies correspond to the 5% and 95% percentiles, respectively, of the cross sectional distribution of the average share of intangible capital in 2-digit U.S. industrial sectors over the 1980-2015 period. Since interest rate movements are almost identical in both simulations, these can be interpreted as two sectors in an economy where capital and labor are sector specific. In this respect, the simulated trends shown in Figure 4 are fully consistent with the empirical trends shown in Figure 2.

[27] The drop in the interest rate increases corporate leverage in the tangibles economy via the collateral value channel and via a second, less intuitive, mechanism. As q_T and q_I go up and the productivity of capital in the high-productivity firms goes down because of the decreasing marginal product of capital and a fixed aggregate labor supply, the share of capital financed by debt as opposed to by the accumulation of past output goes up.

[28] Another indirect benefit of this positive reallocation is that since the low-productivity firms absorb less capital, their marginal return is higher, relative to the intangibles economy, driving up capital prices (middle row, middle graph), and stimulating capital production (middle row, left graph).

7.3 Comparative Statics Exercise 2: The Simultaneous Rise in Households' Propensity to Save and in Intangible Capital (1980-2015)

In Section 7.2 we explored an expansion in household savings but kept the intensity of intangible capital constant. In this section, in contrast, we reproduce in a comparative statics exercise the simultaneous rise in the propensity of households to save and in the reliance on intangible capital observed during the period from 1980 to 2015. To increase our understanding of the interaction between both developments, we also describe a sequence of steady states in which we only increase the reliance on intangible capital. Our results are displayed in Figure 5. Since we abstract from long-run growth considerations, the graphs that show relative changes in total output should be interpreted as deviations from long-run trends.

We first focus on the exercise that explores the rise in intangibles in isolation. The gradual increase in μ pushes high-productivity firms to demand progressively more intangible capital and less tangible capital. Intangible capital attracts less external finance, which tightens firms' borrowing constraints significantly and decreases corporate leverage. High-productivity firms switch from being net borrowers to being net lenders, consistent with evidence in the United States for corporations (Armenter and Hnatkovska, 2016; Quadrini, 2016; Chen, Karabarbounis and Neiman, 2016). The increase in net corporate savings reduces interest rates moderately, by about 1%, to ensure that households borrow more and absorb the excess savings. Capital misallocation increases substantially, as the aggregate capital stock increases but the capital stock held by high-productivity firms decreases. This misallocation is mostly the consequence of the increased reliance on a type of capital that firms cannot finance externally, and to a lesser extent the consequence of lower interest rates. Aggregate output rises initially driven by higher productivity of intangible capital (see equation (50)) and by the increase in the capital stock, but eventually levels off and falls slightly as the negative misallocation effects of a decrease in corporate borrowing and lower interest rates dominate. Overall, a shift to intangibles is expansionary.

[FIGURE 5 ABOUT HERE]

When we consider corporate and household developments simultaneously, we observe instead a fall in aggregate output. The interest rate falls from 6% to around 0% and capital prices are generally higher than in the simulation of the rise in intangibles in isolation. Aggregate capital in the high-productivity firms (third row, middle panel) is roughly constant in the initial 1980-1990 period, while leverage is still positive and the increase in productivity driven by the rise of intangibles compensates the negative effects of the lower borrowing capacity. In this

period, total output (bottom panel) expands by 2% until around the mid-1990s, thanks to the increase in productivity and aggregate capital.[29] During the 1990s and 2000s, however, capital and output of high-productivity firms both fall substantially because their borrowing capacity declines further and the economy becomes more similar to the intangibles economy described in Section 7.2, an economy in which a decline in the interest rate causes a large contraction of the output of high productivity firms. By 2015, their output has fallen by 18%, compared to a fall of 10% in the economy in which only the rise in intangibles occurs (third row, right panel). Lower rates damage the high productivity firms both because of the savings channel, which becomes stronger the larger are their net savings, and because low rates imply relatively higher capital prices, which hurt firms through the capital price channel. Thus, the reduction in interest rates, which is expansionary for highly leveraged high-productivity firms, hurts capital reallocation and growth once the economy relies more on intangible and less collateralizable capital.

It is important to note that while a decline in interest rates caused by household developments expands aggregate output in a tangibles economy (see Figure 3), and a shift to intangibles also expands it, the combination of the two developments is overall contractionary, with output in 2015 around 1% lower than in 1980 (bottom panel in Figure 5).

[FIGURE **6** ABOUT HERE]

The contraction in output happens—in spite of our assumption that intangible capital is more productive—because of a strong misallocation effect in which too many resources are absorbed by low-productivity firms. The gradual increase in misallocation is reflected in the gradual increase in productivity dispersion shown in Figure 6.

[FIGURE 7 ABOUT HERE]

These results are robust to alternative specifications of the relationship between productivity z and the intensity of intangible capital μ, as shown in Appendix E. Nonetheless, they are not entirely consistent with the stylized fact 1 mentioned in Section 2—that the productivity slowdown emphasized in the secular stagnation debate started in the early 2000s, as shown by Fernald (2015) and others. Our simulations generate an accelerated decline in TFP in 1995-2015 relative 1980-1995. But Fernald (2015) also shows that TFP actually grew at a faster rate during 1995-2003. Even though it is beyond the scope of this paper, because of its relatively

[29] The values of e_t^T and e_t^I, which are calibrated to an empirically realistic evolution of capital prices in the simulation with both household and corporate developments, are also an important factor driving the increase in capital stock.

stylized nature, to match the exact quantitative evolution of productivity, we show in Appendix F that our model is qualitatively consistent with the Fernald (2015) evidence once we assume an evolution of z_t and of the real interest rate that is more in line with the empirical evidence.

Finally, Figure 7 replicates the evolution of aggregate output in the benchmark case with both developments (bottom panel in Figure 5), and compares it to a counterfactual simulation in which the net debtor/savings channel is eliminated. This counterfactual is constructed by assuming that interest rate changes can affect capital prices and the collateral constraint, but that firms' interest rate on debt or return on savings is kept constant at the initial value of 6%. In the simulated period in which firms are net borrowers (1980-1995), the net debtor channel implies that lower rates benefit high productivity firms, and shutting it down (the dashed line in Figure 7) lowers output relative to the benchmark. However, once firms become net savers, the savings channel implies that lower rates worsen reallocation, and significantly contributes to the decline in aggregate output.

7.3.1 Quantifying the Misallocation Effects

In this section, we quantify more precisely the misallocation implications of the simultaneous rise in households' propensity to save and in intangible capital analyzed in the previous section. Moreover, we analyze their sensitivity to two key elements: the initial dispersion in marginal returns across the high- and low-productivity firms $s.d.(\log \Delta)$, which measures the intensity of financial frictions, and the productivity gap between intangible and tangible capital.

For the value of $s.d.(\log \Delta)$, we consider a range of values between 0.02 and 0.04 in our comparative static exercise, around the midpoint of 0.03 obtained in our benchmark calibration. In our model, the dispersion in marginal returns is driven by the dispersion in total factor productivity across firms ($z - z^U$), and by the collateralizability of capital θ^I and θ^T. That is, increases in $z - z^U$ for a given θ^I and θ^T, and it decreases in θ^I and θ^T for a given $z - z^U$. For this exercise we chose to keep z and z^U constant and, instead, vary θ^I and θ^T to change the financial frictions of the high-productivity firms. The lower bound value of $s.d.(\log \Delta) = 0.02$ is obtained with $\theta^I = 0.4$ and $\theta^T = 1.05$. The upper bound value of $s.d.(\log \Delta) = 0.04$ with $\theta^I = 0.29$ and $\theta^T = 0.87$.

The productivity gap between intangible and tangible capital is captured by κ in (50). The higher its value, the higher is the productivity improvements generated of more intangible technologies. Given the difficulty in measuring this object precisely in the data, we consider a range of values between 0 and 0.2, around the benchmark calibration of $\kappa = 0.1$.

[TABLE 2 ABOUT HERE]

Table 2 shows the percentage change in aggregate total factor productivity (TFP) between the initial (1980) and final (2015) periods of our comparative statics exercise. Aggregate TFP is measured as the weighted average between TFP in the high-productivity and low-productivity sectors, where the weights are the share of total capital $\overline{K}^T + \overline{K}^I$ used in each sector. The middle part considers the benchmark value of $\kappa = 0.1$, while the other columns display the results of assuming values $\kappa = 0$ and $\kappa = 0.2$. Odd columns report the actual % increase in TFP, while even columns show "$\%TFP^{CF}_{80-15}$", the percentage changes in TFP in a counterfactual scenario in which the allocation of resources does not worsen following the rise in intangibles and propensity to save. That is, we impose that in 2015 there is the same share of total capital used in each sector than in 1980, thus eliminating the misallocation effects that endogenously reduce the capital of high-productivity firms.[30] Finally, for each panel we report three rows, each with a different initial (1980s) value of $s.d.(\log \Delta)$.

Panel A of Table 2 considers the effects of a drop in interest rate r in isolation, without the contemporaneous rise in the intangible ratio μ, which is kept constant at the 1980 value of 0.2. In this case, the value of κ is irrelevant (see equation 50). In the least constrained simulation with $s.d.(\log \Delta) = 0.2$, high-productivity firms have more borrowing capacity, are more leveraged, and lower interest rates help them borrow and expand more. However, lower rates increase the supply of capital at a rate faster than the expansion of the high-productivity firms, and in equilibrium relatively more capital is absorbed by the low-productivity firms in 2015 than in 1980, thus explaining the small 2% drop in TFP. Consistently with this explanation, the drop in TFP is larger the larger is the initial level of financial frictions (higher $s.d.(\log \Delta)$).

Panel B of Table 2 displays the results of a rise in intangible capital in isolation. For $\kappa = 0$, the $\%TFP^{CF}_{80-15}$ column is always equal to zero, because $z'(\mu) = 0$. For κ equal to 0.1 and 0.2, the $\%TFP^{CF}_{80-15}$ column is around 2.3% and 4.6%, respectively. This is the increase in TFP that would take place in an economy in which the rise of intangibles improves productivity without any negative misallocation effect.[31] The actual change in TFP is instead strongly negative, between -14.7% and -17.8% in the benchmark case of $\kappa = 0.1$. The drop is larger the less constrained is the economy ($s.d.(\log \Delta) = 0.02$). This is because, in this simulation, high-productivity firms are more leveraged in the initial 1980 simulation, thanks to a higher value of

[30] This counterfactual scenario is constructed by changing the pledgeability parameters θ_T and θ_I so that the allocation of capital remains constant at the 1980 level.

[31] The value changes slightly for different values of $s.d.(\log \Delta)$, which determine slightly different initial weights in the calculation of TFP.

θ_T relative to θ_I, and the rise of intangibles determines a larger drop in the collateralizability of total capital, and thus a larger increase in financial frictions.

Panel C of Table 2 considers both the rise of intangibles and the drop in the interest rate. Interestingly, the drop in TFP is significantly larger than the sum of the changes in Panels A and B. For example, for the benchmark values of $\kappa = 0.1$ and $s.d.(\log \Delta) = 0.03$, the drop of TFP is -3.6% in Panel A, -18.8% in Panel B, and as much as -24.9% in Panel C. Intuitively, the decline in interest rates has much stronger effects when, because of the rise of intangibles, the corporate sector becomes a net saver, thus magnifying the misallocation effects of the savings channel. The value of -24.9% also implies that, in the 1980-2015 period, these misallocation effects reduced TFP growth by 0.71% yearly. This value seems very large, and to put it in perspective, we relate it to the corresponding increase in financial frictions.

[TABLE 3 ABOUT HERE]

In Table 3 we report the value of $s.d.(\log \Delta)$ in 2015 for each simulation. For the benchmark initial value of $s.d.(\log \Delta) = 0.03$ in 1980, the rise in intangibles in Panel B determines $s.d.(\log \Delta)_{2015} = 0.13$, and the joint corporate and household developments, displayed in Panel C, result in $s.d.(\log \Delta)_{2015} = 0.19$. These changes are also large, but broadly consistent with Gilchrist, Sim, and Zakrajsek (2013), who estimate an increase in $s.d.(\log \Delta)$ from 0.2 in 1980 to 0.5 in 2012.

[TABLE 4 ABOUT HERE]

Finally, in Table 4 we analyze aggregate output. There is a large increase in output, of the order of a cumulative 8-10% increase, when only household sector developments are allowed, as shown in Panel A. As explained before, lower rates stimulate the production of capital. However, the increase is much smaller in Panel B and especially in Panel C, where it becomes negative for some simulations. When intangible capital is as productive as intangible capital ($\kappa = 0$ in the first pair of columns), the rise in intangibles in isolation is contractionary (the $\%Y_{80-15}$ column). This effect occurs through the increased misallocation of capital: if we control for misallocation effects (the $\%Y^{CF}_{80-15}$ column), the rise in intangibles is expansionary by lowering interest rates and encouraging capital creation. This pattern is even stronger when we allow for household sector developments to occur simultaneously. Even if intangible capital is substantially more productive than tangible capital ($\kappa = 0.1$ and $\kappa = 0.2$ in the second and third sets of columns), the strong misallocation effects in Panel C, which range from 4.5% ($= 3.7\% - (-0.8\%)$) to 6.9% ($= 8.9\% - 2.0\%$) in terms of the cumulative negative impact on

output, mean that these developments are contractionary or only very mildly expansionary. In our benchmark calibration, output falls by 0.8%, and is 5.2% lower than it would be in the absence of misallocation effects.

8 Conclusion

This paper highlights a novel misallocation effect of endogenously low interest rates that has potentially important policy implications. Our results are consistent with several developments that have taken place in the past 40 years: (i) net corporate savings increased as a fraction of GDP, (ii) household leverage increased as a fraction of GDP, (iii) the real interest rate fell, (iv) intra-industry dispersion in productivity increased, and (v) output and productivity progressively declined relative to their previous trends. Interestingly, the model shows that even though the shift to intangible technologies was already taking place in the 1970s, its net negative effects on output growth only started to gather pace from the mid-1980s onward. This finding is consistent with studies that show a decline in dynamism of U.S. businesses starting in the mid-1980s and gathering speed especially from 2000 onward (Haltiwanger, 2015).

More broadly, our results suggest that the changes in firms' financing behavior brought about by technological evolution might help explain the subpar growth experienced in recent years, because they have occurred during a period of low interest rates. Our insights could be extended to develop interesting policy implications. On the one hand, the mechanisms described in this paper, operating mostly through the endogenous reaction of interest rates, suggest that the rise in intangibles might have important implications for monetary policy. On the other hand, the negative externality in households' and firms' excessive saving decisions might introduce a role for a fiscal policy that discourages such saving.

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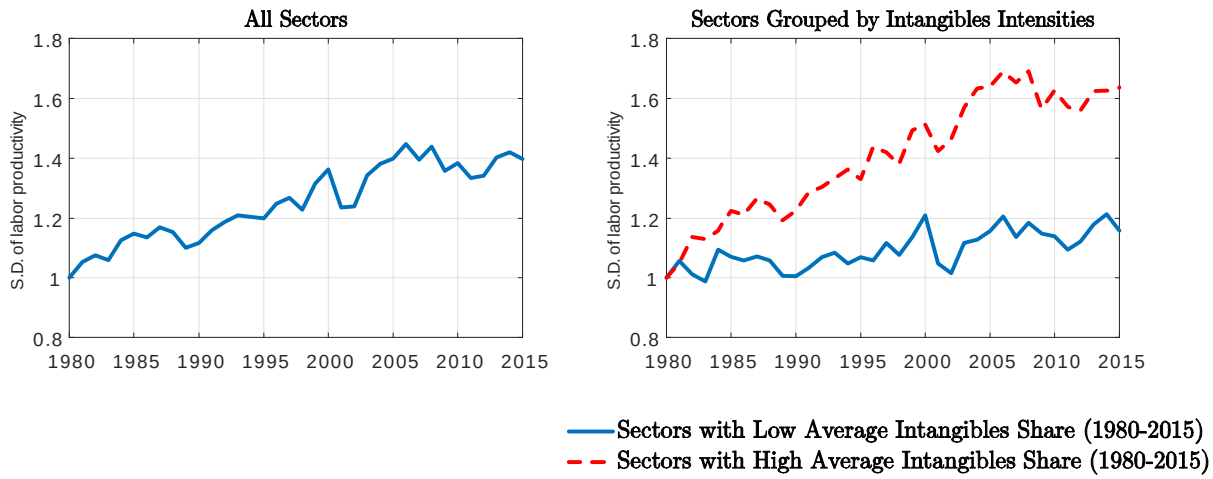

Figure 1: Within-Industry Dispersion in Firm-Level Labor Productivity (*Source: Compustat data, own calculations*)

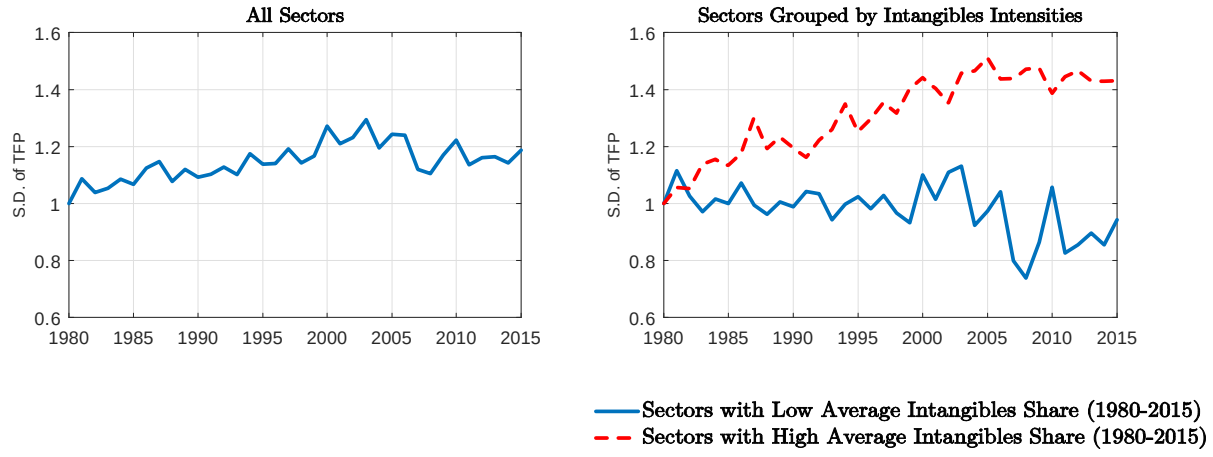

Figure 2: Within-Industry Dispersion in Firm-Level TFP (*Source: Compustat data, own calculations*)

Parameters that remain constant across comparative statics

Parameter	**Symbol**	**Value**
Capital share	α	0.4
Low-productivity firms, TFP tangible technology	$z_t^{u,T}$	10
Low-productivity firms, TFP tangible technology	$z_t^{u,I}$	10
Years households remain young	N	40
High-productivity firms, TFP	z	25
Collateral value of tangible capital	θ^T	1
Collateral value of intangible capital	θ^I	0.35
Probability of an investment opportunity	η	0.13
Additional productivity of intangible capital	κ	0.1
Adjustment cost convexity	φ	9
Exit probability of high-productivity firms	ψ	0.13
Endowment of new firms	W_0	5
Depreciation of capital	δ	0.15
Share of dividends to young households	γ	40%

Parameters that change across comparative statics

Parameter	**Symbol**	**Value in 1980 − in 2015**
Discount factor	β	$0.9425 - 0.9805$
Intangible share of total capital	μ	$0.2 - 0.6$
Probability of death of old households	ϱ	$0.170 - 0.075$
Adjustment cost parameter (intangible)	e^I	$3.2 * 10^{-6} - 15.3 * 10^{-6}$
Adjustment cost parameter (tangible)	e^T	$2.1 * 10^{-10} - 0.6 * 10^{-10}$

Table 1: Benchmark Calibration - Parameter Choices

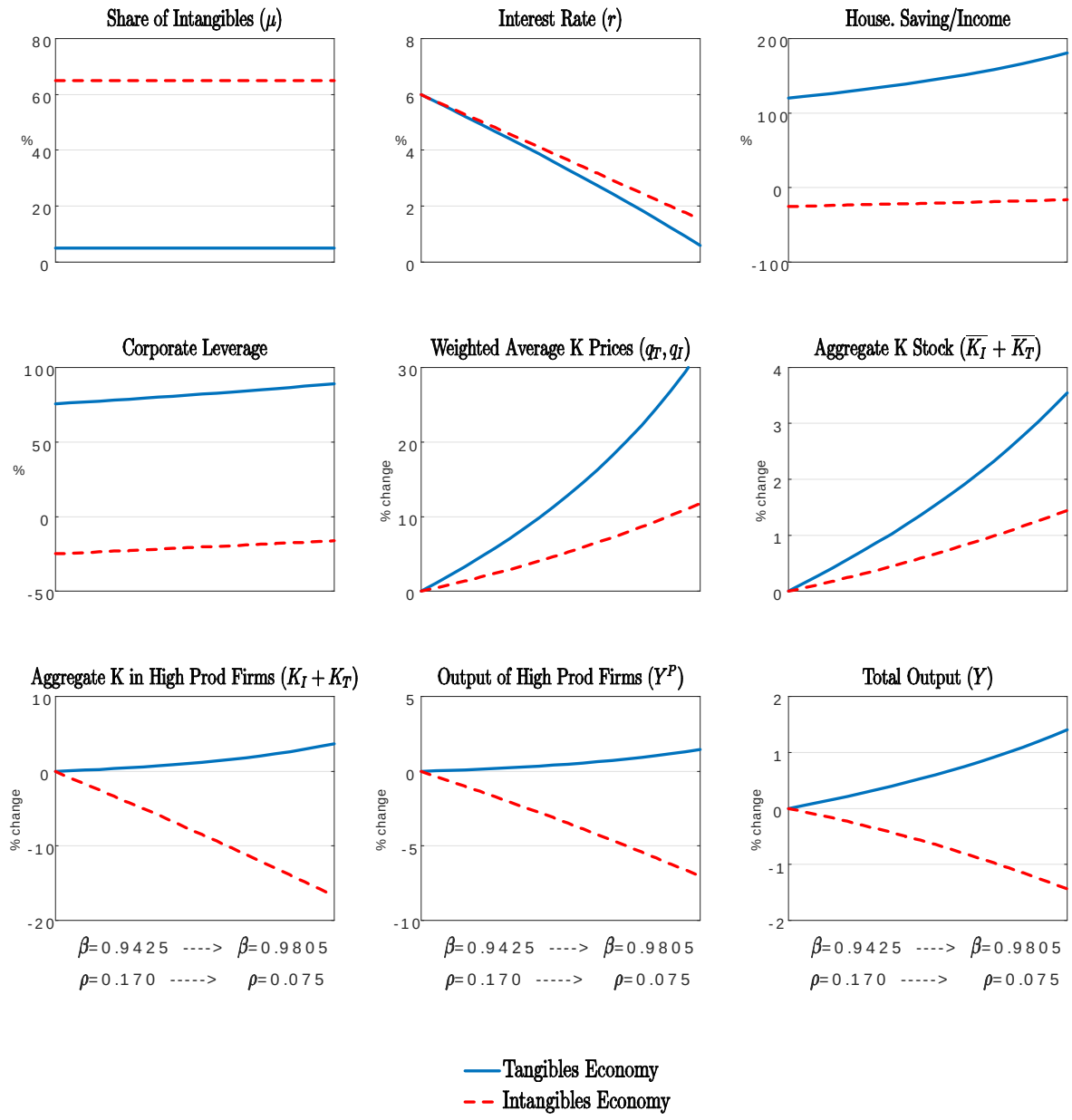

Figure 3: Simulation Exercise: households' propensity to save gradually increases because of (i) a decrease in the rate of time preference (β increases) and (ii) a decrease in the likelihood of death of old households (ϱ decreases) - comparison of the effects of the expansion in households savings in a tangibles economy ($\mu = 0.05$) and an intangibles economy ($\mu = 0.65$).

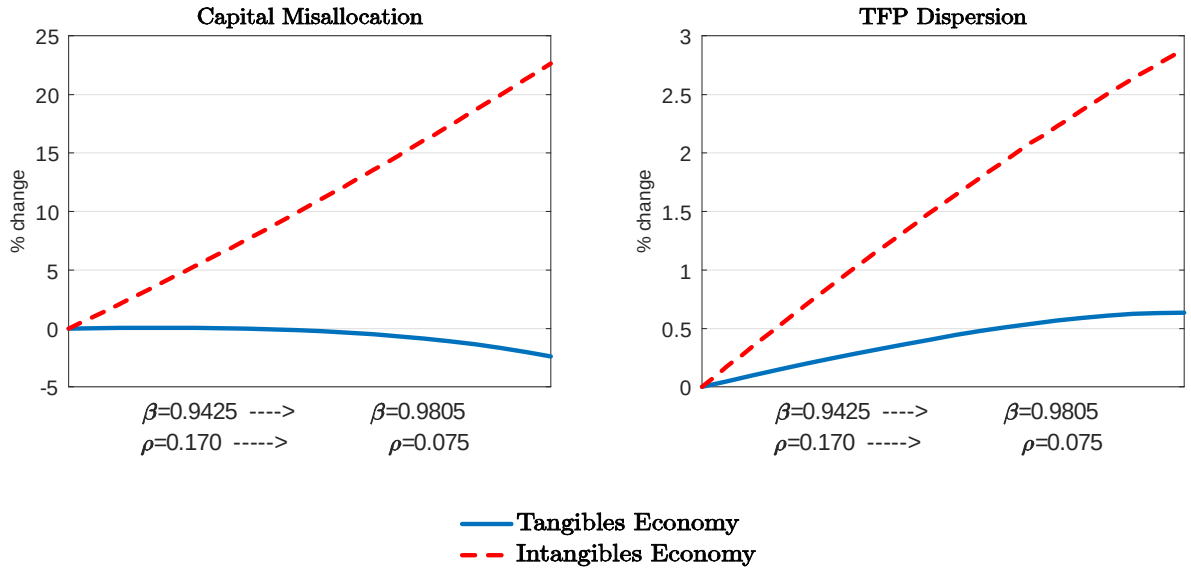

Figure 4: Simulation Exercise: households' propensity to save gradually increases because of (i) a decrease in the rate of time preference (β increases) and (ii) a decrease in the likelihood of death of old households (ϱ decreases) - comparison of capital misallocation and TFP dispersion in a tangibles economy ($\mu = 0.05$) and an intangibles economy ($\mu = 0.65$).

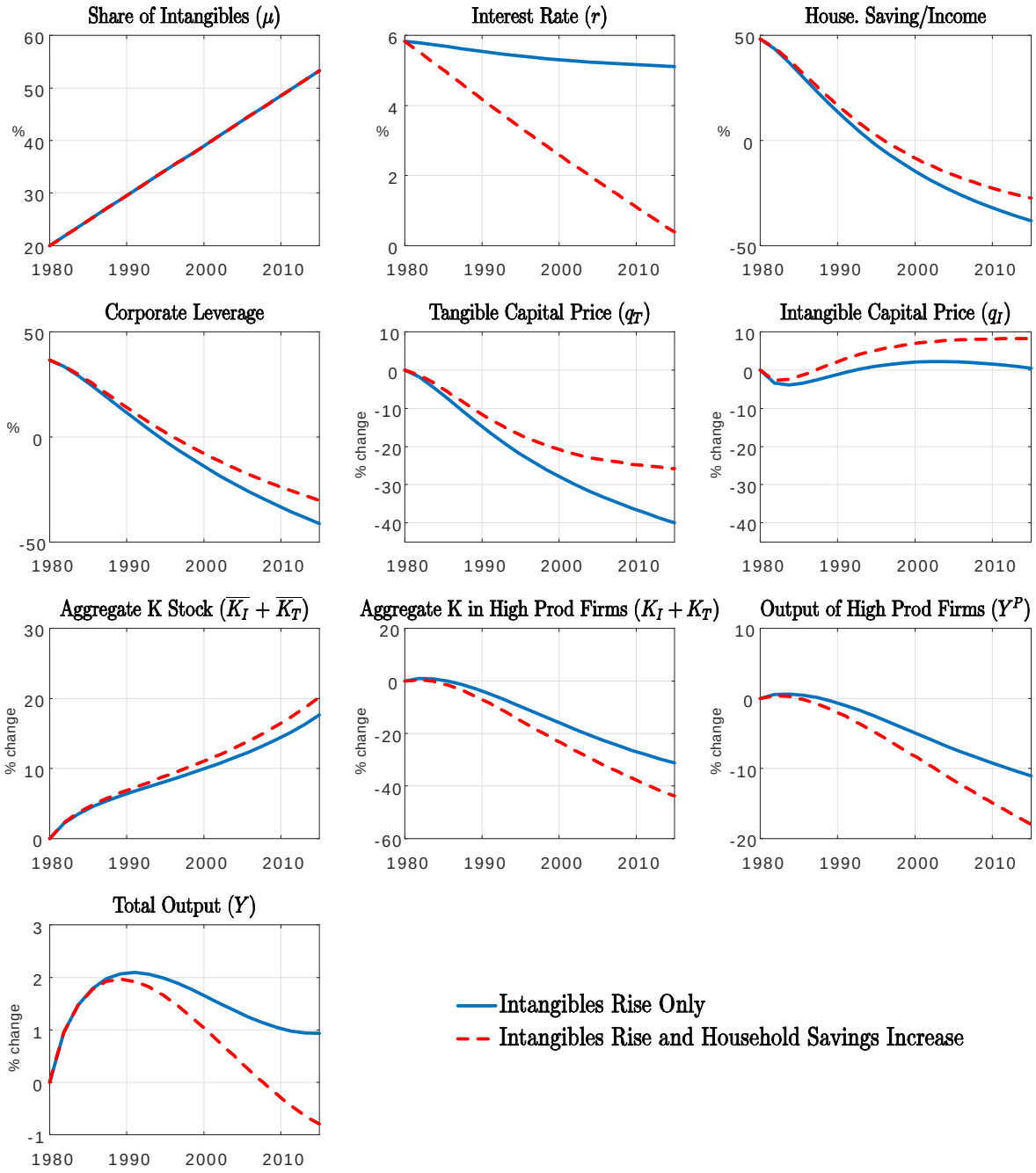

Figure 5: Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of effects when both trends occur and when only the increase in the share of intangible capital occurs

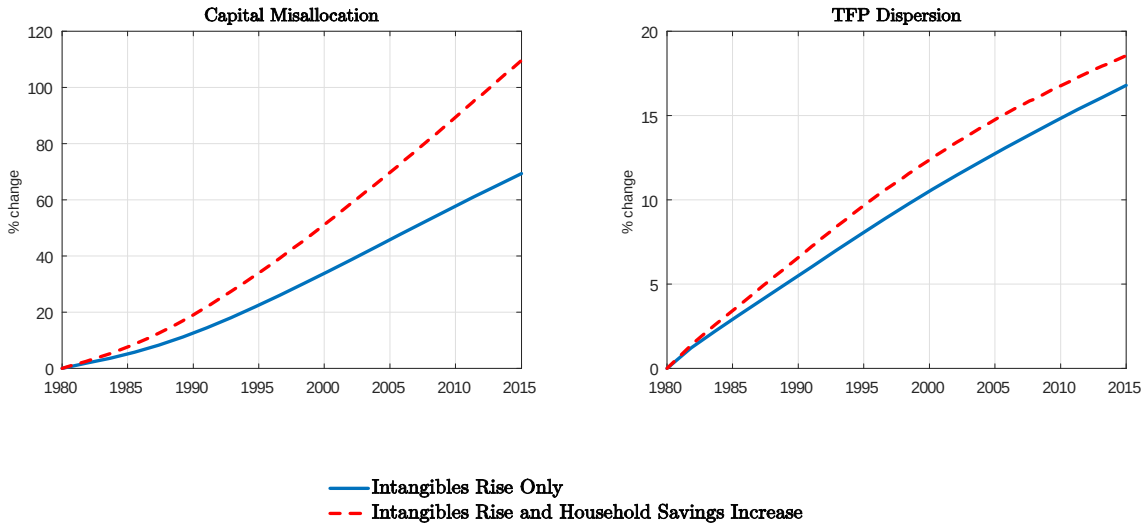

Figure 6: Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of capital misallocation and TFP dispersion when both trends occur and when only the increase in the share of intangible capital occurs.

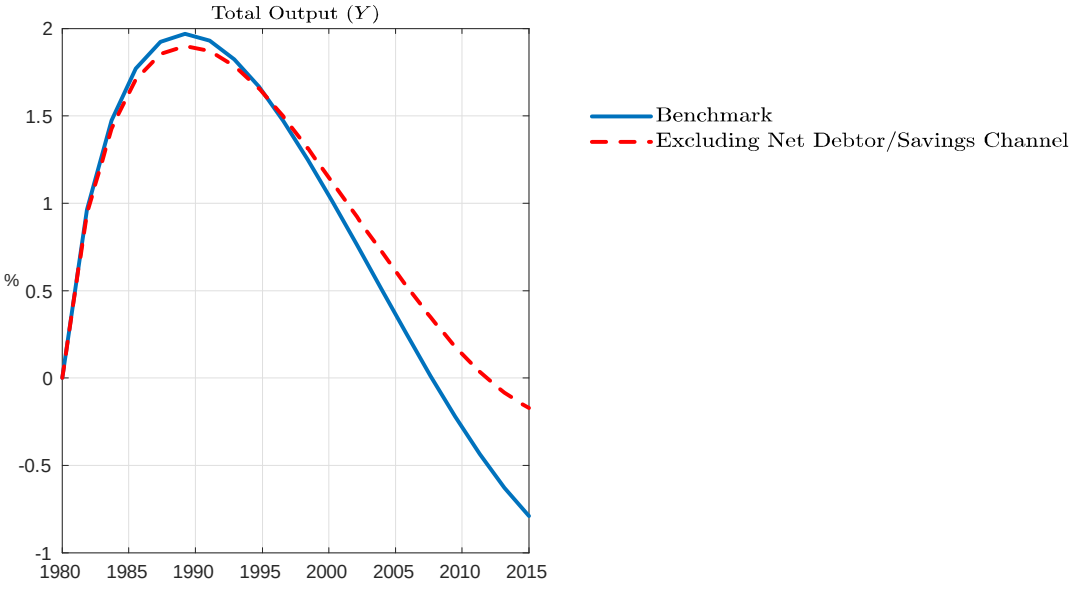

Figure 7: Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of effects when we shut down the net debtor/savings channel

Panel A: Rise in Households' Propensity to Save

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$
	0.02	-2.0%	0.0%	-2.0%	0.0%	-2.0%	0.0%
$s.d.(\log \Delta)$	0.03	-3.6%	0.0%	-3.6%	0.0%	-3.6%	0.0%
	0.04	-5.7%	0.0%	-5.7%	0.0%	-5.7%	0.0%

Panel B: Rise in Intangible Capital

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$
	0.02	-21.7%	0.0%	-19.8%	2.4%	-17.8%	4.7%
$s.d.(\log \Delta)$	0.03	-20.8%	0.0%	-18.8%	2.3%	-16.8%	4.6%
	0.04	-18.9%	0.0%	-16.8%	2.3%	-14.7%	4.5%

Panel C: Both Trends Simultaneously

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$	$\%TFP_{80-15}$	$\%TFP^{CF}_{80-15}$
	0.02	-27.2%	0.0%	-25.5%	2.4%	-23.8%	4.7%
$s.d.(\log \Delta)$	0.03	-26.6%	0.0%	-24.9%	2.3%	-23.2%	4.6%
	0.04	-24.7%	0.0%	-22.9%	2.3%	-21.1%	4.5%

Table 2: Change in aggregate total factor productivity (TFP) over the simulated period.

Panel A: Rise in Households' Propensity to Save

		$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$
		$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$
$s.d.(\log \Delta)$	0.02	0.04	0.04	0.04
	0.03	0.06	0.06	0.06
	0.04	0.10	0.10	0.10

Panel B: Rise in Intangible Capital

		$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$
		$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$
$s.d.(\log \Delta)$	0.02	0.12	0.12	0.12
	0.03	0.13	0.13	0.13
	0.04	0.14	0.14	0.15

Panel C: Both Trends Simultaneously

		$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$
		$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$	$s.d.(\log \Delta)_{15}$
$s.d.(\log \Delta)$	0.02	0.17	0.17	0.17
	0.03	0.19	0.19	0.19
	0.04	0.20	0.20	0.20

Table 3: Level of misallocation (log(r)) at the end of the simulated period.

Panel A: Rise in Households' Propensity to Save

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$
$s.d.(\log \Delta)$	0.02	9.6%	10.0%	9.6%	10.0%	9.6%	10.0%
	0.03	9.1%	9.9%	9.1%	9.9%	9.1%	9.9%
	0.04	8.2%	9.8%	8.2%	9.8%	8.2%	9.8%

Panel B: Rise in Intangible Capital

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$
$s.d.(\log \Delta)$	0.02	−1.5%	0.5%	0.7%	2.7%	2.9%	5.0%
	0.03	−1.2%	1.2%	1.0%	3.5%	3.2%	5.8%
	0.04	−0.5%	2.5%	1.8%	4.8%	4.0%	7.2%

Panel C: Both Trends Simultaneously

		$\kappa = 0$		$\kappa = 0.1$		$\kappa = 0.2$	
		$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$	$\%Y_{80-15}$	$\%Y^{CF}_{80-15}$
$s.d.(\log \Delta)$	0.02	−3.0%	1.2%	−0.8%	3.7%	1.3%	6.3%
	0.03	−2.9%	2.0%	−0.8%	4.4%	1.4%	6.9%
	0.04	−2.4%	4.0%	−0.2%	6.4%	2.0%	8.9%

Table 4: Change in output (Y) over the simulated period.

APPENDICES
For Online Publication Only

A Robustness of Dispersion Evidence

A.1 Construction of the Intangible Capital Measure

We define intangible capital as the sum of knowledge capital and organizational capital.[32] Following Falato et al. (2014), we measure the former by capitalizing R&D expenses and the latter by capitalizing selling, general and administrative (SG&A) expenses weighted by 0.2.[33] The expenditures are capitalized by applying the perpetual inventory method with a depreciation rate of 15% for R&D and 20% for SG&A. In order to get a measure for tangible capital, we also use the perpetual inventory method to capitalize tangible capital expenses with a depreciation rate of 15%. We drop firms that are observed only once and firms that are not observed in a continuous time period, and we exclude regulated, financial, and public service firms. We consider sectors at the 2-digit Standard Industrial Classification (SIC) level and drop those with less than 500 firm-year observations over the sample period. We measure output by sales, labor input by the number of employees, and total capital by the sum of capitalized tangible and intangible capital.

TFP is defined as the residual of a Cobb-Douglas production function with a capital share of income equal to 0.40. Labour productivity is sales over labour cost. To control for outliers, we drop firms in the 1st and 99th percentiles of the distribution of labor productivity.

A.2 Robustness Checks

One alternative explanation of the results shown in Figures 1 and 2 in Section 2 could be that the sectors with a high intangibles share do not have a worse allocation of resources, but rather are more dynamic and fast growing, and that the increase in dispersion of productivity reflects this higher dynamism. However, in Figure A, we show that sectors with high average sales growth have lower productivity dispersion in the whole sample period.

[FIGURE A ABOUT HERE]

[TABLE A ABOUT HERE]

Furthermore, Table A shows regression results where the dependent variable is a measure of productivity dispersion for each 2-digit sector-year observation. The regressors we consider are as follows: the dummy *High share*, which is equal to 1 if the sector belongs to the 50% 2-digit industries with the highest average intangible share and which is equal to 0 otherwise; a time trend; and year and sector fixed effects. In columns 1 and 2, the dependent variable is the dispersion in TFP. Column 1 includes year fixed effects and shows that the dispersion is significantly larger for sectors with higher intangible share. Column 2 includes a time trend, interacted with the *High share* variable, and both sector and time fixed effects. It shows that the

[32] Falato et al. (2014) also consider informational capital. However, they state that their results do not depend on its inclusion. As informational capital can be measured only at the industry level but not at the firm level using Compustat data, we choose not to include this type of capital.

[33] A portion of SG&A expenses captures expenditures that increase the value of intangible capital items such as brand names and knowledge capital. Part of SG&A expenditures, however, does not affect the value of intangible capital, so Falato et al. (2014) follow Corrado, Hulten, and Sichel (2009) and assume that the portion relevant to intangible capital is around 0.2.

trend in dispersion over time is significantly more positive in the 50% most intangible sectors than in the other sectors, confirming the significance of the result shown in Figures 1 and 2. Similar results, across the two groups of high and low intangibles sectors, are obtained using labor productivity, as shown in columns 3 and 4.

B Optimal Dividend and Cash Accumulation Policy

Given equation (20), the first order condition for cash holdings $a_{f,t+1}$ for non investing firms is:

$$(1+\lambda_t) = (1-\psi)\left[\eta(1+\lambda^+_{t+1}+\vartheta_t) + (1-\eta)(1+\lambda^-_{t+1}+\vartheta_t)\right] + \psi. \tag{53}$$

If we substitute (53) recursively forward, it is clear that if the firm expects ϑ_t to be positive now or in the future, then $\lambda_t > 0$, and a non-investing firm will always retain all earnings and $d_t = 0$. It is important to note that this is so because there is no cost of holding cash.

C Aggregation of the Firm Sector - Derivations

Aggregate wealth W_t of the high-productivity firms at the beginning of period t is

$$W_t \equiv Y^p_t - w_t + (1+r_t)A_{f,t} + (1-\delta)\left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right)K_{I,t}. \tag{54}$$

Aggregate capital is determined as follows. A fraction $(1-\psi)$ of high-productivity firms continue activity, and a fraction η of those have an investment opportunity. They have a fraction $(1-\psi)\eta$ of total wealth W_t, which they use to buy the amount of capital given by equation (24). A fraction ψ of high-productivity firms exit, and are replaced by an equal number of firms with an initial endowment of W_0 and no capital. A fraction η of new entrants invest. Therefore, we define total intangible capital in the hands of investing agents at the end of period t, expressed in aggregate terms, as $\eta K^{INV}_{I,t+1}$, where $K^{INV}_{I,t+1}$ is

$$K^{INV}_{I,t+1} = \frac{(1-\psi)W_t + \psi W_0}{\left(q_{T,t} - \theta^T \frac{q_{T,t+1}}{1+r_{t+1}}\right)\frac{1-\mu}{\mu} + q_{I,t} - \theta^I \frac{q^I_{I,t+1}}{1+r_{t+1}}}. \tag{55}$$

The $(1-\eta)$ fraction of surviving firms that do not have an investment opportunity continue to hold their depreciated capital. Therefore, aggregate capital for the next period is equal to

$$K_{I,t+1} = \eta K^{INV}_{I,t+1} + (1-\delta)(1-\psi)(1-\eta)K_{I,t} \tag{56}$$

and

$$K_{T,t+1} = \frac{1-\mu}{\mu}K_{I,t+1}. \tag{57}$$

The marginal return of capital in the high-productivity firms is as follows. In order obtain a marginal increase $\frac{\partial Y^p_t}{\partial K_{I,t}} = \frac{\alpha}{\mu}z_t\left(\frac{K_{I,t}}{\mu}\right)^{\alpha-1}$, these firms purchase one unit of intangible capital and $\frac{1-\mu}{\mu}$ units of tangible capital. The equilibrium described earlier requires that the high-productivity firms have the highest return on capital, or

$$\frac{\alpha}{\mu}z_t\left(\frac{K_{I,t+1}}{\mu}\right)^{\alpha-1} > z^{u,I}_t\alpha\left(\overline{K}^I - K_{I,t}\right)^{\alpha-1} + \frac{1-\mu}{\mu}z^{u,T}_t\alpha\left(\overline{K}^T - K_{T,t}\right)^{\alpha-1}, \tag{58}$$

where the right-hand side of this inequality captures the marginal return of one unit of tangible capital and $\frac{1-\mu}{\mu}$ units of intangible capital in the low-productivity firms.

If condition (58) is satisfied, then it follows immediately that the prices of capital are

$$q_{I,t} = z_I^{u,I}\alpha\left(\overline{K}^I - K_{I,t}\right)^{\alpha-\mathbf{1}} + \frac{1-\delta}{1+r_{t+1}}q_{I,t+1}, \tag{59}$$

and

$$q_{T,t} = z_t^{u,T}\alpha\left(\overline{K}^T - K_{T,t}\right)^{\alpha-\mathbf{1}} + \frac{1-\delta}{1+r_{t+1}}q_{T,t+1}. \tag{60}$$

If we substitute (59) and (60) into (58), it follows that

$$\frac{\alpha}{\mu}z_t\left(\frac{K_{I,t+1}}{\mu}\right)^{\alpha-1} > q_{I,t} - \frac{1-\delta}{1+r_{t+1}}q_{I,t+1} + \frac{1-\mu}{\mu}\left(q_{T,t} - \frac{1-\delta}{1+r_{t+1}}q_{T,t+1}\right), \tag{61}$$

which implies that the claim (23) is correct. Aggregate financial assets of the high-productivity firms ($A_{f,t+1}$) are equal to the assets saved from the previous period by continuing firms, $(1-\psi)\left((1+r_t)A_{f,t}\right)$, plus their current retained earnings, $(1-\psi)\left(Y_t^p - w_t\right)$, plus the endowments of new firms (ψW_0) minus total investment, $\left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right)\left(K_{I,t+1} - (1-\delta)(1-\psi)K_{I,t}\right)$:

$$\begin{aligned} A_{f,t+1} &= (1-\psi)\left(Y_t^p - w_t + (1+r_t)A_{f,t}\right) + \psi W_0 \\ &\quad - \left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right)\left(K_{I,t+1} - (1-\delta)(1-\psi)K_{I,t}\right). \end{aligned} \tag{62}$$

Finally, total dividends paid out by exiting high-productivity firms to households are equal to

$$D_t^p = \psi\left(Y_t^p - w_t + (1+r_t)A_{f,t} + \left(q_{T,t}\frac{1-\mu}{\mu} + q_{I,t}\right)K_{I,t}\right) - \psi W_0, \tag{63}$$

and the dividends paid by the low-productivity firms are**:**

$$D_t^u = Y_t^u - w_t^{uI} - w_t^{uT} - q_{I,t}\left[\left(\overline{K}^I - K_{I,t+1}\right) - \left(\overline{K}^I - K_{I,t}\right)\right] - q_{T,t}\left[\left(\overline{K}^T - K_{T,t+1}\right) - \left(\overline{K}^T - K_{T,t}\right)\right]. \tag{64}$$

D Households

We derive below the solution of households' optimization problem under the steady state, so that wages, dividends and interest rate are constant. Households have log utility. A representative old household still living at time t maximizes the following objective function:

$$V_t^o\left(b_t^o\right) = \max_{c_t^o, b_{t+1}^o}\sum_{j=0}^{\infty}(1-\varrho)^j\beta^j\log\left(c_{t+j}\right) \tag{65}$$

subject to

$$c_t^o = b_{t+1}^o + (1-\gamma)d - \frac{(1+r)}{(1-\varrho)}b_t^o.$$

Working backward, we next consider the optimization problem of a young agent of age N

in period t, who will become old in period $t+1$[34]:

$$V^y_{t,N}\left(b^y_{t,N}\right) = \max_{c^y_{t,N}, b^o_{t+1}} u\left(c^y_{t,N}\right) + \beta(1-\varrho)V^o_{t+1}\left(b^o_{t+1}\right) \tag{66}$$

subject to

$$c^y_{t,N} = \gamma d + w^{TOT} - (1+r)b^y_{t,N} + b^o_{t+1}. \tag{67}$$

where w^{TOT} is defined as:

$$w^{TOT} \equiv w + w^{uI} + w^{uT}$$

Then we consider the optimization problem for a young household of age $j < N$:

$$V^y_{t,j}\left(b^y_{t,j}\right) = \max_{c^y_{t,j}, b^y_{t+1,j+1}} u\left(c^y_{t,j}\right) + \beta V^y_{t+1,j+1}\left(b^y_{t+1,j+1}\right) \tag{68}$$

subject to

$$c^y_{t,j} = \gamma d + w^{TOT} - (1+r)b^y_{t,j} + b^y_{t+1,j+1} \tag{69}$$

D.1 Individual Problem of Old Households

We follow Blanchard (1985) and Yaari (1965) in assuming that households participate in a life insurance scheme when old. The insurance scheme works within a cohort so that the survivors within a cohort pay the debt of the dying (if they are in debt) or, alternatively, receive the savings of the dying. An old household begins a period with net debt $(1+r_t)b^o_t$. The insurance contract specifies that the ϱ fraction of old households that die transfer their assets (or debt) $(1+r_t)b^o_t$ to the life insurer. Among the fraction $(1-\varrho)$ of households that survive, if they are net savers ($b^o_t < 0$), then they receive a return $\frac{1}{1-\varrho}(1+r_t)b^o_t$ on their assets, while, if they are net debtors ($b^o_t > 0$), they make a payment of $\frac{1}{1-\varrho}(1+r_t)b^o_t$ to the life insurer.

The first order condition with respect to b^o_{t+1} is

$$c^o_{t+1} = \beta(1+r)c^o_t. \tag{70}$$

We guess a consumption policy rule:

$$c^o_t = \Delta d + \Theta b^o_t,$$

and plug it into the FOC

$$\Delta d + \Theta\left[c^o_t - (1-\gamma)\,d + \frac{(1+r)}{(1-\varrho)}b^o_t\right] = \beta(1+r)\left(\Delta d + \Theta b^o_t\right)$$

$$c^o_t = \left[\frac{\beta(1+r)\Delta}{\Theta} + (1-\gamma) - \frac{\Delta}{\Theta}\right]d + (1+r)\left[\beta - \frac{1}{(1-\varrho)}\right]b^o_t,$$

and then solve for the unknown coefficients

$$\begin{aligned}\Delta &= \frac{\beta(1+r)\Delta}{\Theta} + (1-\gamma) - \frac{\Delta}{\Theta}\\ \Theta &= (1+r)\left[\beta - \frac{1}{(1-\varrho)}\right]\end{aligned}$$

[34] We assume that an agent can also die with probability ϱ in the transition between young and old.

$$\begin{aligned}\Delta &= \frac{\beta\Delta}{\left[\beta - \frac{1}{(1-\varrho)}\right]} + (1-\gamma) - \frac{\Delta}{(1+r)\left[\beta - \frac{1}{(1-\varrho)}\right]} \\ \Delta &= \frac{(1-\gamma)(1+r)[1-\beta(1-\varrho)]}{\varrho + r}\end{aligned}$$

The policy rule is:

$$c_t^o = (1-(1-\varrho)\beta)\left[\frac{(1-\gamma)(1+r)}{(\varrho+r)}d - \frac{(1+r)}{(1-\varrho)}b_t^o\right]. \tag{71}$$

and the evolution of the wealth of old households is given by

$$b_{t+1}^o = \frac{(1-\varrho)[1-\beta(1+r)]}{(\varrho+r)}(1-\gamma)d + (1+r)\beta b_t^o, \tag{72}$$

which says that old households slowly consume their savings if $\beta(1+r) < 1$, and do so at a faster rate the higher the dividends. In our simulations typically $\beta(1+r) < 1$.[35]

D.2 Individual Problem of Young Households

We first consider the optimization problem of an agent of age N in period t, who will become old in period $t+1$:

$$V_{t,N}^y\left(b_{t,N}^y\right) = \max_{c_{t,N}^y, b_{t+1}^o} u\left(c_{t,N}^y\right) + \beta(1-\varrho)V_{t+1}^o\left(b_{t+1}^o\right) \tag{73}$$

such that:

$$c_{t,N}^y = \gamma d + w^{TOT} - (1+r)b_{t,N}^y + b_{t+1}^o. \tag{74}$$

The first order condition implies that

$$\frac{1}{c_{t,N}^y} + \beta\left(\frac{\partial V_{t+1}^o\left(b_{t+1}^o\right)}{\partial b_{t+1}^o} + \frac{\partial V_{t+1}^o\left(b_{t+1}^o\right)}{\partial b_{t+2}^o}\frac{\partial b_{t+2}^o}{\partial b_{t+1}^o}\right) = 0.$$

And applying the envelope theorem we obtain:

$$c_{t+1}^o = \beta(1+r)c_{t,N}^y.$$

We substitute c_{t+1}^o using (71) and we obtain:

$$c_{t,N}^y = \left(\frac{1}{\beta} - (1-\varrho)\right)\left(\frac{(1-\gamma)d}{(\varrho+r)} - \frac{b_{t+1}^o}{(1-\varrho)}\right). \tag{75}$$

Then we consider the optimization problem for a young household of age $j<N$:

$$V_{t,j}^y\left(b_{t,j}^y\right) = \max_{c_{t,j}^y, b_{t+1,j+1}^y} u\left(c_{t,j}^y\right) + \beta V_{t+1,j+1}^y\left(b_{t+1,j+1}^y\right), \tag{76}$$

such that:

$$c_{t,j}^y = \gamma d + w^{TOT} - (1+r)b_{t,j}^y + b_{t+1,j+1}^y, \tag{77}$$

[35] To see this more clearly, denote $a = -b$ as savings, and write

$$a_{t+1}^o = (1+r)\beta a_t^o - \frac{(1-\varrho)[1-\beta(1+r)]}{(\varrho+r)}(1-\gamma)d.$$

which yields the standard Euler equation:

$$c_{t,j}^{y} = [\beta(1+r)]^{-(N-j)}\, c_{t+N-j,N}^{y}. \tag{78}$$

Equations (75) and (78) fully characterize the life-cycle path of consumption of a household as a function of its assets when entering old age in period $t+1$, b_{t+1}^{o}.

D.3 Value of Savings of Oldest Young: b_{t+1}^{o}

We use the above equations, the budget constraint (69), and the assumption that newborn households have no endowment ($b_{t,1}^{y} = 0$) to determine the value of savings for retirement $b_{t+1}^{o} \equiv b_{t+1,N+1}^{y}$.

We use the budget constraint for $j = 1$ (a young household of $age = 1$), in which the debt brought over, $b_{t,1}^{y}$, is zero:

$$b_{t,1}^{y} = \frac{\left(\gamma d + w^{TOT}\right) - c_{t,1}^{y} + b_{t+1,2}^{y}}{(1+r)} = 0,$$

and we solve forward:

$$\begin{aligned} b_{t,1}^{y} &= \frac{\left(\gamma d + w^{TOT}\right) - c_{t,1}^{y}}{(1+r)} + \frac{\left(\gamma d + w^{TOT}\right) - c_{t+1,2}^{y} + b_{t+2,3}^{y}}{(1+r)^{2}} \\ &= \left(\gamma d + w^{TOT}\right) \sum_{j=1}^{N} \frac{1}{(1+r)^{j}} - \sum_{j=1}^{N} \frac{c_{t+j-1,j}^{y}}{(1+r)^{j}} + \frac{b_{t+N,N+1}^{y}}{(1+r)^{N}} \end{aligned}$$

Making use of the FOC:

$$c_{t,j}^{y} = [\beta(1+r)]^{-(N-j)}\, c_{t+N-j,N}^{y} \tag{79}$$

we get

$$\begin{aligned} c_{t,j}^{y} &= [\beta(1+r)]^{-(N-j)}\, c_{t+N-j,N}^{y} \\ c_{t,1}^{y} &= [\beta(1+r)]^{-(N-1)}\, c_{t+N-1,N}^{y} \\ c_{t+1,2}^{y} &= [\beta(1+r)]^{-(N-2)}\, c_{t+1+N-2,N}^{y} = [\beta(1+r)]^{-(N-2)}\, c_{t+N-1,N}^{y} \\ c_{t+2,3}^{y} &= [\beta(1+r)]^{-(N-3)}\, c_{t+2+N-3,N}^{y} = [\beta(1+r)]^{-(N-3)}\, c_{t+N-1,N}^{y} \\ c_{t+N-1,N}^{y} &= [\beta(1+r)]^{-(N-N)}\, c_{t+N-1+N-N,N}^{y} = c_{t+N-1,N}^{y}, \end{aligned}$$

and plug in and simplify

$$\begin{aligned} \sum_{j=1}^{N} \frac{c_{t+j-1,j}^{y}}{(1+r)^{j}} &= \frac{c_{t,1}^{y}}{(1+r)} + \frac{c_{t+1,2}^{y}}{(1+r)^{2}} + \frac{c_{t+2,3}^{y}}{(1+r)^{3}} + ... + \frac{c_{t+N-1,N}^{y}}{(1+r)^{N}} \\ &= \frac{c_{t+N-1,N}^{y}}{(1+r)^{N}} \left[\sum_{j=0}^{N-1} \beta^{-(N-1-j)}\right] = -\frac{c_{t+N-1,N}^{y}}{(1+r)^{N}} \frac{\beta^{N}-1}{\beta^{N-1}(\beta-1)}. \end{aligned}$$

We substitute back in and keep simplifying

$$b_{t,1}^{y} = \left(\gamma d + w^{TOT}\right) \frac{1}{r}\left[1 - \frac{1}{(1+r)^{N}}\right] - \frac{c_{t+N-1,N}^{y}}{(1+r)^{N}} \frac{\beta^{N}-1}{\beta^{N-1}(\beta-1)} + \frac{b_{t+N,N+1}^{y}}{(1+r)^{N}}$$

$$0 = \left(\gamma d + w^{TOT}\right) \frac{1}{r}\left[(1+r)^N - 1\right] - \left(\frac{1}{\beta} - (1-\varrho)\right) \frac{\beta^N - 1}{\beta^{N-1}(\beta-1)} \frac{(1-\gamma)d}{(\varrho+r)}$$
$$+ \left[\left(\frac{1}{\beta} - (1-\varrho)\right) \frac{1}{(1-\varrho)} \frac{\beta^N - 1}{\beta^{N-1}(\beta-1)} + 1\right] b^y_{t+N,N+1}$$
$$\left(\frac{1}{\beta} - (1-\varrho)\right) \frac{\beta^N - 1}{\beta^{N-1}(\beta-1)} \frac{(1-\gamma)d}{(\varrho+r)} - \left(\gamma d + w^{TOT}\right) \frac{1}{r}\left[(1+r)^N - 1\right]$$
$$= \left[\left(\frac{1}{\beta} - (1-\varrho)\right) \frac{1}{(1-\varrho)} \frac{\beta^N - 1}{\beta^{N-1}(\beta-1)} + 1\right] b^y_{t+N,N+1}$$

Finally, we solve to get:

$$-b^y_{t+N,N+1} = \frac{\left(\gamma d + w^{TOT}\right) \frac{1}{r}\left[(1+r)^N - 1\right] - \Psi \frac{(1-\gamma)d}{(\varrho+r)}}{\frac{\Psi}{1-\varrho} + 1} \tag{80}$$

$$\Psi \equiv \left(\frac{1}{\beta} - (1-\varrho)\right) \frac{\beta^N - 1}{\beta^{N-1}(\beta-1)}$$

Equation (80) is very intuitive. Savings for retirement $-b^o_{t+1}$ increase in the difference between income before and after retirement. Moreover, an increase in life expectancy (a drop in ϱ) reduces the value of the term Ψ and therefore increases $-b^o_{t+1}$.

D.4 Aggregate Savings of the Young

The previous section determines a sequence of optimal consumption at every age, $c^y_{t,1}, ..., c^y_{t,N}$, and applying the budget constraint (69) we can determine a sequence of assets for every age $b^y_{t,2}, ..., b^y_{t,N}$, which is constant for every period t. In equilibrium there is a measure 1 of households, a fraction $\frac{\phi}{\phi+\varrho}$ old, and a fraction $\frac{\varrho}{\phi+\varrho}$ young. Moreover there is a measure $\frac{\varrho}{\phi+\varrho}\frac{1}{N}$ of young households for each age. Therefore, after dropping the subscript t, we can define aggregate savings of the young households as:

$$B^y = \frac{\varrho}{\phi+\varrho} \frac{1}{N} \sum_{j=1}^{N} b^y_{j+1}. \tag{81}$$

Savings of a young household are:

$$b^y_{j+1} = c^y_j - \gamma d - w^{TOT} + (1+r) b^y_j \tag{82}$$

We solve for b^y_N (from now on for simplicity omit the superscript y) :

$$b_N = \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r} c_N + \frac{1}{1+r} b_{N+1}, \tag{83}$$

where both b_{N+1} and c_N are determined by (75) and (80) above. At age $N-1$ (we use $c_t = [\beta(1+r)]^{-1} c_{t+1}$):

$$b_{N-1} = \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r} c_{N-1} + \frac{1}{1+r}\left[\frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r} c_N + \frac{1}{1+r} b_{N+1}\right]$$

$$
\begin{aligned}
b_{N-1} &= \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r}\left[\beta(1+r)\right]^{-1} c_N \\
&\quad + \frac{1}{1+r}\left[\frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r}c_N + \frac{1}{1+r}b_{N+1}\right] \\
&= \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) + \frac{1}{(1+r)^2}\left(\gamma d + w^{TOT}\right) \\
&\quad - \frac{1}{1+r}\left[\beta(1+r)\right]^{-1} c_N - \frac{1}{(1+r)^2}c_N + \frac{1}{(1+r)^2}b_{N+1} \\
&= \left(\frac{1}{1+r} + \frac{1}{(1+r)^2}\right)\left(\gamma d + w^{TOT}\right) - \left(\frac{1}{\beta}+1\right)\frac{c_N}{(1+r)^2} + \frac{1}{(1+r)^2}b_{N+1},
\end{aligned}
$$

and therefore at age $N-2$:

$$
\begin{aligned}
b_{N-2} &= \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r}c_{N-2} + \frac{1}{1+r}b_{N-1} \\
&= \frac{1}{1+r}\left(\gamma d + w^{TOT}\right) - \frac{1}{1+r}\left[\beta(1+r)\right]^{-1}\left[\beta(1+r)\right]^{-1} c_N \\
&\quad + \frac{1}{1+r}\left(\left(\frac{1}{1+r} + \frac{1}{(1+r)^2}\right)\left(\gamma d + w^{TOT}\right) - \left(\frac{1}{\beta\,(1+r)^2} + \frac{1}{(1+r)^2}\right)c_N + \frac{1}{(1+r)^2}b_{N+1}\right) \\
&= \left(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3}\right)\left(\gamma d + w^{TOT}\right) - \left(\frac{1}{\beta^2} + \frac{1}{\beta} + 1\right)\frac{c_N}{(1+r)^3} + \frac{1}{(1+r)^3}b_{N+1},
\end{aligned}
$$

and at a generic age $N-t$:

$$
b_{N-t} = \sum_{j=0}^{t} \frac{\gamma d + w^{TOT}}{(1+r)^{j+1}} - \frac{c_N}{(1+r)^{t+1}} \sum_{j=0}^{t} \frac{1}{\beta^j} + \frac{b_{N+1}}{(1+r)^{t+1}} \tag{84}
$$

We use general formulas: $\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$ and $\sum_{j=0}^{t} x^j = \left(1-x^{t+1}\right)\frac{1}{1-x}$, or $\sum_{j=0}^{t} \frac{1}{(1+r)^j} = \left(1 - \frac{1}{(1+r)^{t+1}}\right)\frac{1+r}{r}$ and $\sum_{j=0}^{t} \frac{1}{(1+r)^{j+1}} = \left(1 - \frac{1}{(1+r)^{t+1}}\right)\frac{1}{r}$, so that

$$
\begin{aligned}
\sum_{j=0}^{t} \frac{\gamma d + w^{TOT}}{(1+r)^{j+1}} &= \left(1 - \frac{1}{(1+r)^{t+1}}\right)\frac{\gamma d + w^{TOT}}{r} \\
\frac{c_N}{(1+r)^{t+1}} \sum_{j=0}^{t} \frac{1}{\beta^j} &= \frac{c_N}{(1+r)^{t+1}}\left(1 - \frac{1}{\beta^{t+1}}\right)\frac{\beta}{\beta-1}
\end{aligned}
$$

hence:

$$
b_{N-t} = \left(1 - \frac{1}{(1+r)^{t+1}}\right)\frac{\gamma d + w^{TOT}}{r} - \frac{c_N}{(1+r)^{t+1}}\left(1 - \frac{1}{\beta^{t+1}}\right)\frac{\beta}{\beta-1} + \frac{b_{N+1}}{(1+r)^{t+1}} \tag{85}
$$

Now we add up the savings/borrowing over all ages from b_2 to b_N to get:

$$\begin{aligned}\sum_{t=0}^{N-2} b_{N-t} &= \frac{\gamma d + w^{TOT}}{r}\sum_{t=0}^{N-2}\left(1-\frac{1}{(1+r)^{t+1}}\right) - \frac{\beta}{\beta-1}c_N\sum_{t=0}^{N-2}\left(\frac{1}{(1+r)^{t+1}}\left(1-\frac{1}{\beta^{t+1}}\right)\right)\\ &\quad + b_{N+1}\sum_{t=0}^{N-2}\left(\frac{1}{(1+r)^{t+1}}\right)\end{aligned}$$

The value of each summation term is:

$$\begin{aligned}\sum_{t=0}^{N-2}\left(1-\frac{1}{(1+r)^{t+1}}\right) &= -\frac{1}{r}\left(r-(r+1)^{-N+1}-Nr+1\right)\\ &= -\frac{1}{r}\left(1+r(1-N)-\frac{1}{(1+r)^{N-1}}\right)\\ &= \frac{1}{r}\left(\frac{1}{(1+r)^{N-1}}+r(N-1)-1\right)\end{aligned}$$

$$\begin{aligned}\sum_{t=0}^{N-2}\left(\frac{1}{(1+r)^{t+1}}\left(1-\frac{1}{\beta^{t+1}}\right)\right) &= \sum_{t=0}^{N-2}\frac{1}{(1+r)^{t+1}} - \sum_{t=0}^{N-2}\left(\frac{1}{[(1+r)\beta]^{t+1}}\right)\\ &= \frac{1-(1+r)^{-N+1}}{(1+r)-1} - \frac{1-[(1+r)\beta]^{-N+1}}{[(1+r)\beta]-1}\end{aligned}$$

$$\sum_{t=0}^{N-2}\left(\frac{1}{(1+r)^{t+1}}\right) = \frac{1-(1+r)^{-N+1}}{(1+r)-1}$$

Substituting them back:

$$\begin{aligned}\sum_{t=0}^{N-2} b_{N-t} &= \frac{\gamma d + w^{TOT}}{r}\frac{1}{r}\left(\frac{1}{(1+r)^{N-1}}+r(N-1)-1\right)\\ -\frac{\beta}{\beta-1}c_N&\left[\frac{1-(1+r)^{-N+1}}{(1+r)-1} - \frac{1-[(1+r)\beta]^{-N+1}}{(1+r)\beta-1}\right] + b_{N+1}\frac{1-(1+r)^{-N+1}}{(1+r)-1}\end{aligned}$$

We rename terms:

$$\sum_{t=0}^{N-2} b_{N-t} = A_1\frac{\gamma d + w^{TOT}}{r} - A_2 c_N + A_3 b_{N+1}$$

$$\begin{aligned}A_1 &\equiv \frac{1}{r}\left(\frac{1}{(1+r)^{N-1}}+r(N-1)-1\right)\\ A_2 &\equiv \frac{\beta}{\beta-1}\left[\frac{1-(1+r)^{-N+1}}{(1+r)-1} - \frac{1-[(1+r)\beta]^{-N+1}}{(1+r)\beta-1}\right]\\ A_3 &\equiv \frac{1-(1+r)^{-N+1}}{(1+r)-1}\end{aligned}$$

D.5 Aggregate Savings of the Old

In equilibrium there are $\frac{\varrho}{\phi+\varrho}\frac{1}{N}$ households that become old every period, and $\frac{\varrho}{\phi+\varrho}\frac{1}{N}(1-\varrho)^j$ households that survived for j periods . Therefore aggregate savings of the old households are:

$$B^o = \frac{\varrho}{\phi+\varrho}\frac{1}{N}\sum_{j=1}^{\infty}(1-\varrho)^j b_j^0 \tag{86a}$$

also note that b_1^0 is the initial savings from young age as defined in (80)

Recall that (from (72)):

$$b_{j+1}^o = A + \beta(1+r)b_j^o, \tag{87}$$

and

$$A \equiv \frac{(1-\varrho)\left[1-\beta(1+r)\right]}{(\varrho+r)}(1-\gamma)d,$$

so that:

$$b_2^o = A + \beta(1+r)b_1^o, \tag{88}$$

$$b_3^o = A + \beta(1+r)b_2^o = A + \beta(1+r)A + \beta^2(1+r)^2 b_1^o, \tag{89}$$

$$b_t^o = A + \beta(1+r)A + ... + \beta^{t-2}(1+r)^{t-2}A + \beta^{t-1}(1+r)^{t-1}b_1^o, \tag{90}$$

$$b_t^o = A\left[\sum_{j=0}^{t-2}\beta^j(1+r)^j\right] + \beta^{t-1}(1+r)^{t-1}b_1^o, \tag{91}$$

$$b_t^o = \frac{\left(\beta\left(1+r\right)\right)^{t-1}-1}{\beta\left(1+r\right)-1}A + \left[\beta(1+r)\right]^{t-1}b_1^o, \tag{92}$$

$$\begin{aligned}
\sum_{t=1}^{\infty}(1-\varrho)^t b_t^0 &= \sum_{t=1}^{\infty}(1-\varrho)^t\left[\frac{\left(\beta\left(r+1\right)\right)^{t-1}-1}{\beta\left(1+r\right)-1}A + \left[\beta(1+r)\right]^{t-1}b_1^o\right] \\
&= \frac{A}{\beta\left(1+r\right)-1}\sum_{t=1}^{\infty}(1-\varrho)^t\left[\left(\beta\left(r+1\right)\right)^{t-1}-1\right] + b_1^o\sum_{t=1}^{\infty}(1-\varrho)^t\left[\beta(1+r)\right]^{t-1} \\
&= \left[\frac{A}{\beta+r\beta-1}+b_1^o\right]\sum_{t=1}^{\infty}(1-\varrho)^t\left(\beta\left(r+1\right)\right)^{t-1} - \frac{A}{\beta+r\beta-1}\sum_{t=1}^{\infty}(1-\varrho)^t \\
&= \left[\frac{A}{\beta+r\beta-1}+b_1^o\right]\frac{1}{\beta(1+r)}\sum_{t=1}^{\infty}\left[(1-\varrho)\beta(1+r)\right]^t - \frac{A}{\beta+r\beta-1}\sum_{t=1}^{\infty}(1-\varrho)^t,
\end{aligned}$$

$$\sum_{t=1}^{\infty}\left[(1-\varrho)\beta(1+r)\right]^t = \frac{1}{1-(1-\varrho)\beta(1+r)} - 1 = \frac{(1-\varrho)\beta(1+r)}{1-(1-\varrho)\beta(1+r)},$$

and

$$\sum_{t=1}^{\infty}(1-\varrho)^t = \frac{1}{1-1+\varrho} - 1 = \frac{1-\varrho}{\varrho}.$$

Finally:

$$\sum_{t=1}^{\infty}(1-\varrho)^t b_t^0 = \left[\frac{A}{\beta+r\beta-1}+b_1^o\right]\frac{(1-\varrho)}{1-(1-\varrho)\beta(1+r)} - \frac{A}{\beta+r\beta-1}\frac{1-\varrho}{\varrho}$$

D.6 Summing Up Aggregate Household Borrowing

Aggregate household borrowing is:

$$B = B^o + B^y \tag{93}$$

where savings of the old is:

$$B^o = \frac{\varrho}{\phi+\varrho}\frac{1}{N}\left[\left(\frac{A}{\beta+r\beta-1}+b^{retirement}\right)\frac{(1-\varrho)}{1-(1-\varrho)\beta(1+r)}-\frac{A}{\beta+r\beta-1}\frac{1-\varrho}{\varrho}\right]$$

and

$$A \equiv \left(\frac{(1-\varrho)\left(1-(1+r)\beta\right)}{(\varrho+r)}\right)(1-\gamma)d,$$

$$b^{retirement} \equiv \frac{\Psi\frac{(1-\gamma)d}{(\varrho+r)}-\left(\gamma d+w^{TOT}\right)\frac{1}{r}\left[(1+r)^N-1\right]}{\frac{\Psi}{1-\varrho}+1},$$

and

$$\Psi \equiv \left(\frac{1}{\beta}-(1-\varrho)\right)\frac{\beta^N-1}{\beta^{N-1}(\beta-1)}.$$

And savings of the young is:

$$B^y = \frac{\varrho}{\phi+\varrho}\frac{1}{N}\sum_{t=0}^{N-2} b_{N-t} = \frac{\varrho}{\phi+\varrho}\frac{1}{N}\left[A_1\frac{\gamma d+w^{TOT}}{r}-A_2c_N+A_3b^{retirement}\right],$$

where

$$\begin{aligned}
A_1 &\equiv \frac{1}{r}\left(\frac{1}{(1+r)^{N-1}}+r(N-1)-1\right),\\
A_2 &\equiv \frac{\beta}{\beta-1}\left[\frac{1-(1+r)^{-N+1}}{(1+r)-1}-\frac{1-[(1+r)\beta]^{-N+1}}{(1+r)\beta-1}\right],\\
A_3 &\equiv \frac{1-(1+r)^{-N+1}}{(1+r)-1},
\end{aligned}$$

and

$$c_N = \left(\frac{1}{\beta}-(1-\varrho)\right)\left(\frac{(1-\gamma)d}{(\varrho+r)}-\frac{b^{retirement}}{(1-\varrho)}\right).$$

E Robustness: Relationship Between Productivity and Intangibles Intensity

In this appendix, we provide a robustness analysis of our assumption that the total factor productivity of high-productivity firms, captured by $z_t(\mu)$, is increasing in the share of intangible capital μ while the total factor productivity of low-productivity firms (determined by $z_t^{u,I}$ and $z_t^{u,T}$) is not. As discussed in Section 4, the positive dependence of z_t on μ is introduced for empirical realism and also for the exogenous shift to intangibles to be consistent with a privately optimal choice of firms. Keeping $z_t^{u,I}$ and $z_t^{u,T}$ constant, on the other hand, is done for tractability. In this appendix we show that these assumptions are without loss of generality and that our results go through when we relax them.

[FIGURE B ABOUT HERE]

[FIGURE C ABOUT HERE]

We report two alternative analyses. In both cases we reproduce the analysis of Section 7.3, in which we explore the simultaneous rise in the propensity of households to save and in firms' reliance on intangible capital observed during the period from 1980 to 2015. For clarity, we also report the exercise in which we only increase the reliance on intangible capital. In our first robustness exercise, we assume that z_t, $z_t^{u,I}$ and $z_t^{u,T}$ are always constant. The results are displayed in Figure B. The results are qualitatively similar to the ones reported in Figure 5. The main difference is that the rise in intangibles in isolation is now contractionary. But our main result—that the interaction between both trends is contractionary—remains and is of a similar magnitude. Our second robustness exercise assumes that z_t, $z_t^{u,I}$ and $z_t^{u,T}$ all grow at the same rate (and the same rate at which z_t grows in our exercise displayed in Figure 5). The results of this exercise are displayed in Figure C. As in the first exercise, the results are qualitatively similar in this case too, and the interaction between both trends is contractionary and of a similar magnitude.

[FIGURE D ABOUT HERE]

A related possible concern with our benchmark assumption is that the prediction of the model that an increase in μ induces greater dispersion in productivity is a consequence of the assumption that z_t increases with μ while $z_t^{u,I}$ and $z_t^{u,T}$ remain constant. Effectively, we would be increasing exogenously the dispersion of TFP between the high productive and low productive sector. The exercises described in Figures B and C already suggest that this is not the case, but to further remove concerns we display in Figure D the dispersion in the marginal productivity of capital and the dispersion in TFP in the three exercises displayed in Figures 5, B, and C. The increase in the dispersion of capital productivity is in fact lower in our benchmark exercise compared to the exercise with common productivity growth rates. The increase in the dispersion of TFP is of a similar magnitude in our benchmark exercise compared to the exercise with common productivity growth rates.

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A further concern with our assumptions is that they are not consistent with the available empirical evidence about the evolution over time of productivity and interest rates. In our comparative statics exercise, as well as in the robustness Appendix E, we assume that $z_t(\mu)$ is linear—that is, the rise in the share of intangible capital increased productivity at the same pace over the 1980-2015 period. However, Corrado, Hulten, and Sichel (2009) estimate that the contribution of the rise of intangible capital is responsible for a 0.84% annual increase in labor productivity in the 1995-2003 period, and only for a 0.43% in the earlier period between 1973 and 1995. This evidence is consistent with Fernald (2015) and several other authors who argue that new IT technologies had the biggest impact in the 1995-2003 period thanks to their adoption as general purpose technologies.

[FIGURE E ABOUT HERE]

Furthermore, so far we have assumed a gradual change in households' preferences that generates a gradual decrease in the real interest rate, while Rachel and Smith (2015) document

that two third of the drop in real interest rate from around 6% in the early 80s to 0% happened after 2000. Therefore, in Figure E we relax these assumptions and instead consider that z_t is constant until 1995 and increases at a yearly rate of 0.55% from 1995 on. Furthermore, we assume household sector developments such that two thirds of the drop in the interest rate happens after 2000. Figure E confirms the main result that the misallocation caused by the rise of intangibles reduces TFP over time. Moreover, it shows that the productivity rise from 1995 onward is able to generate positive TFP growth temporarily during 1995-2001, while afterward—despite the fact that z_t is still increasing by 0.55% yearly—TFP declines because of the additional misallocation caused by falling interest rates. This intraperiod evolution of TFP (low growth until 1995, higher growth between 1995 and 2001, and again low growth from 2001 onward) is qualitatively consistent with Fernald's (2015) estimates.

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(Appendix Tables and Figures)

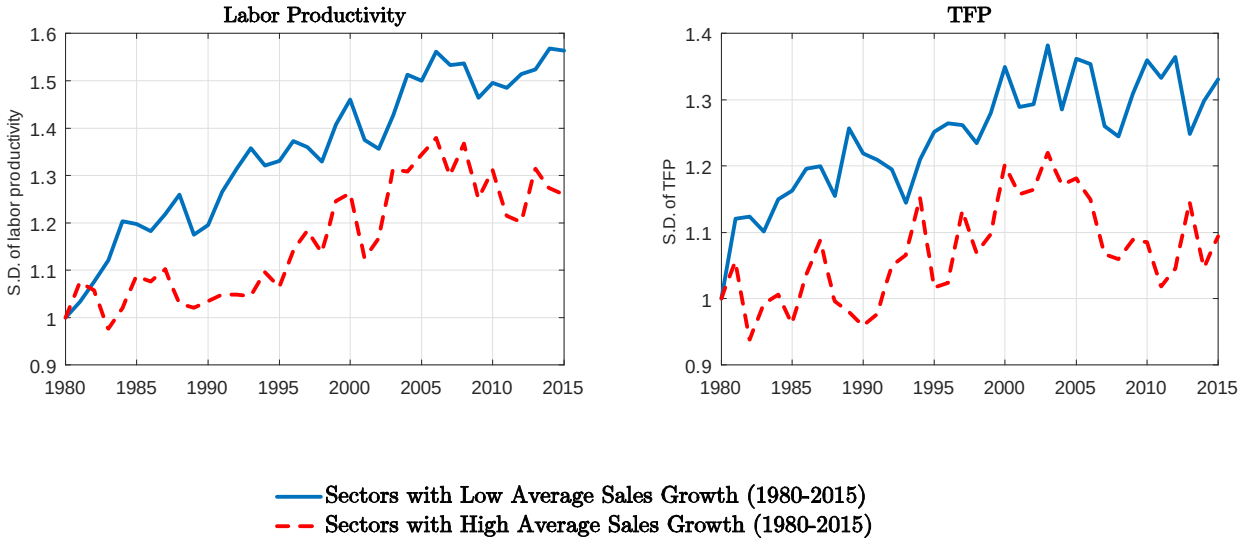

Figure A: Within-Industry Dispersion in Firm-Level Labor Productivity and TFP: Robustness Exercise that Groups Sectors According to Average Sales Growth Rates (*Source: Compustat data, own calculations*)

	(1)	(2)	(3)	(4)
Variables	*TFP*	*TFP*	*y*	*y*
Time trend		−0.000892		0.00316 * **
		(0.000837)		(0.000737)
Time trend*High share		0.00558 * **		0.00370 * **
		(0.000622)		(0.000548)
High share	0.0897 * **		0.110 * **	
	(0.00990)		(0.0144)	
Observations	828	828	828	828
R-squared	0.112	0.632	0.134	0.869
Industry FE	no	yes	no	yes
Year FE	yes	yes	yes	yes

Table A: Relationship Between the Intangible Share and the Dispersion in Productivity - Regression Analysis

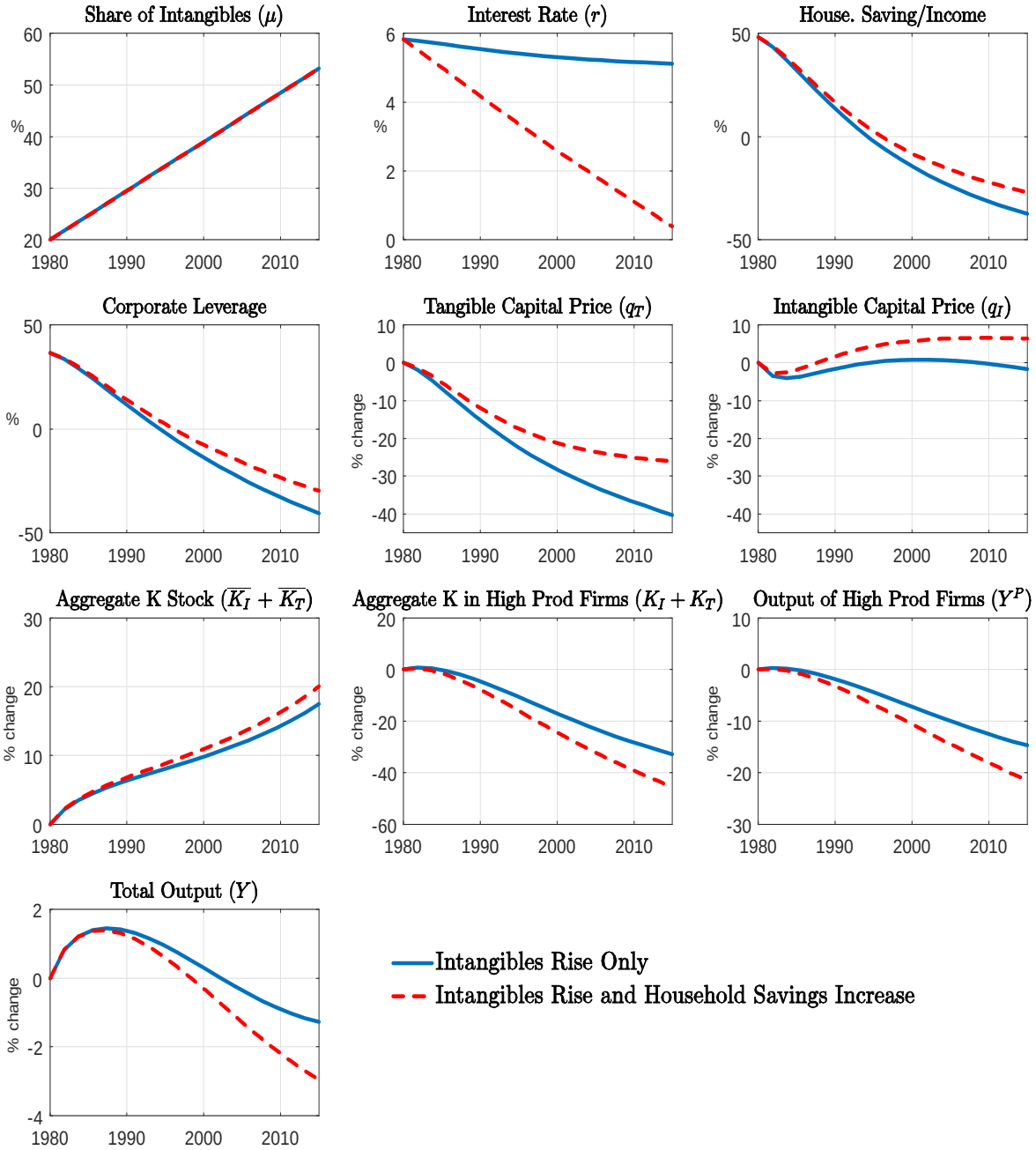

Figure B: Robustness Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of effects when both trends occur and when only the increase in the share of intangible capital occurs. In these simulations z_t, $z_t^{u,I}$ and $z_t^{u,T}$ are constant.

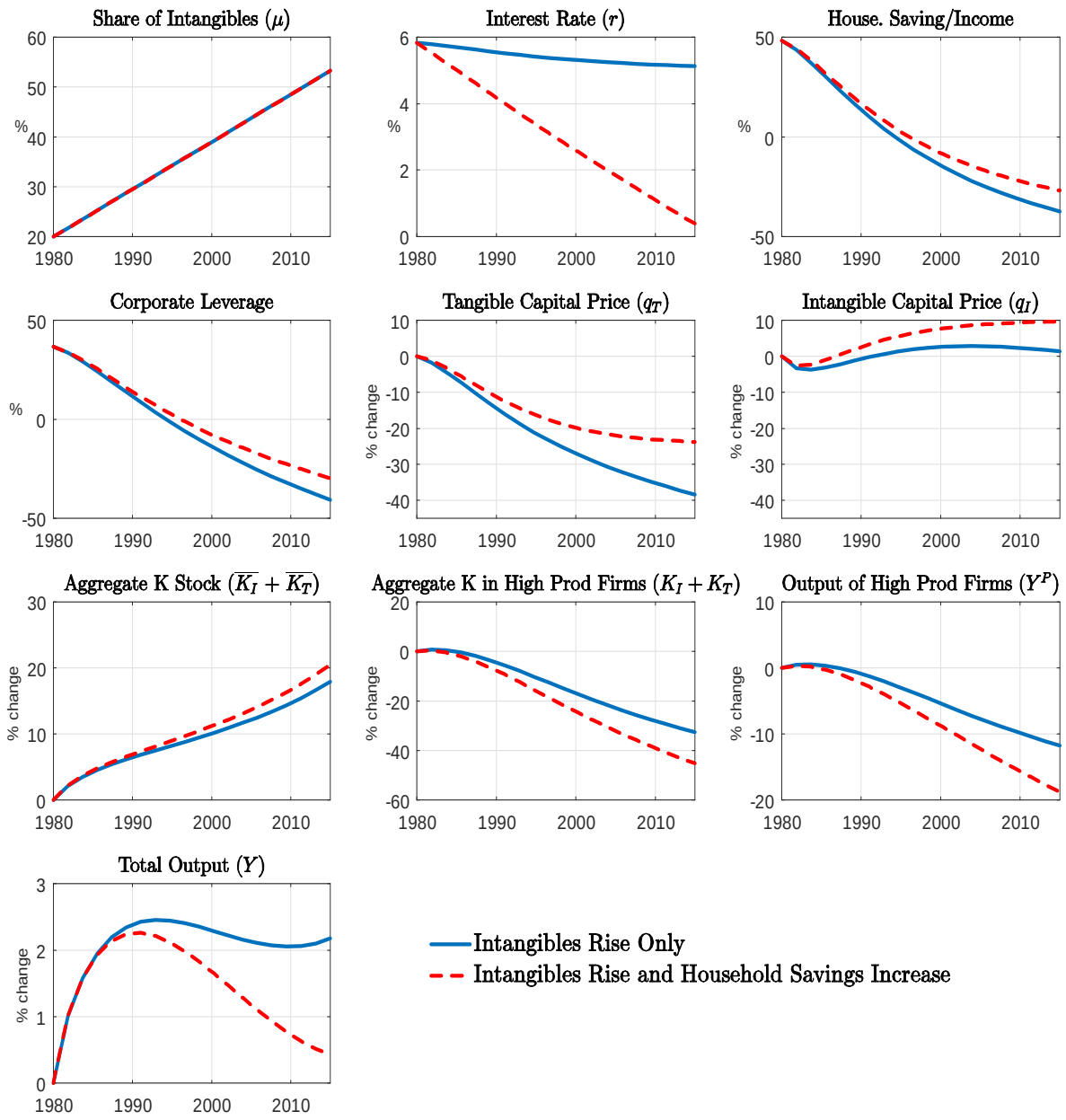

Figure C: Robustness Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of effects when both trends occur and when only the increase in the share of intangible capital occurs. In these simulations z_t, $z_t^{u,I}$ and $z_t^{u,T}$ all increase at the same rate.

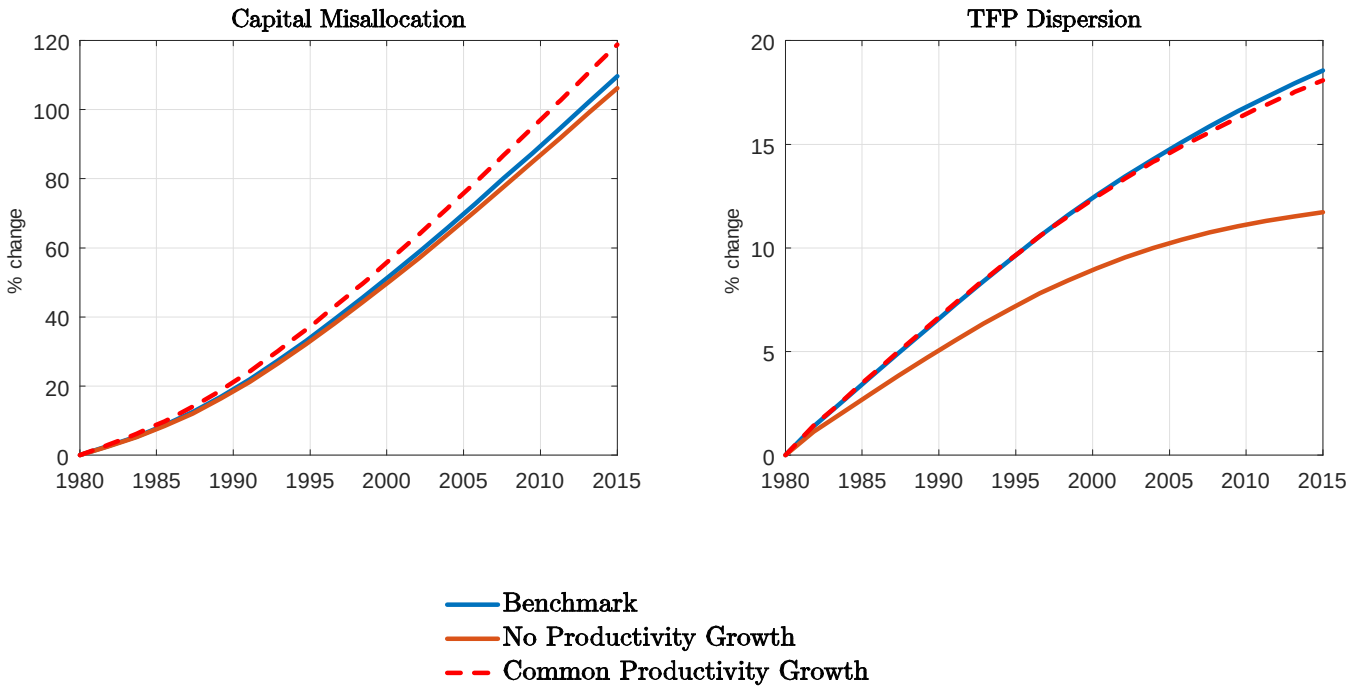

Figure D: Robustness Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of evolution of misallocation measures in three scenarios: benchmark analysis, z_t, $z_t^{u,I}$ and $z_t^{u,T}$ remain constant, and z_t, $z_t^{u,I}$ and $z_t^{u,T}$ all increase at the same rate.

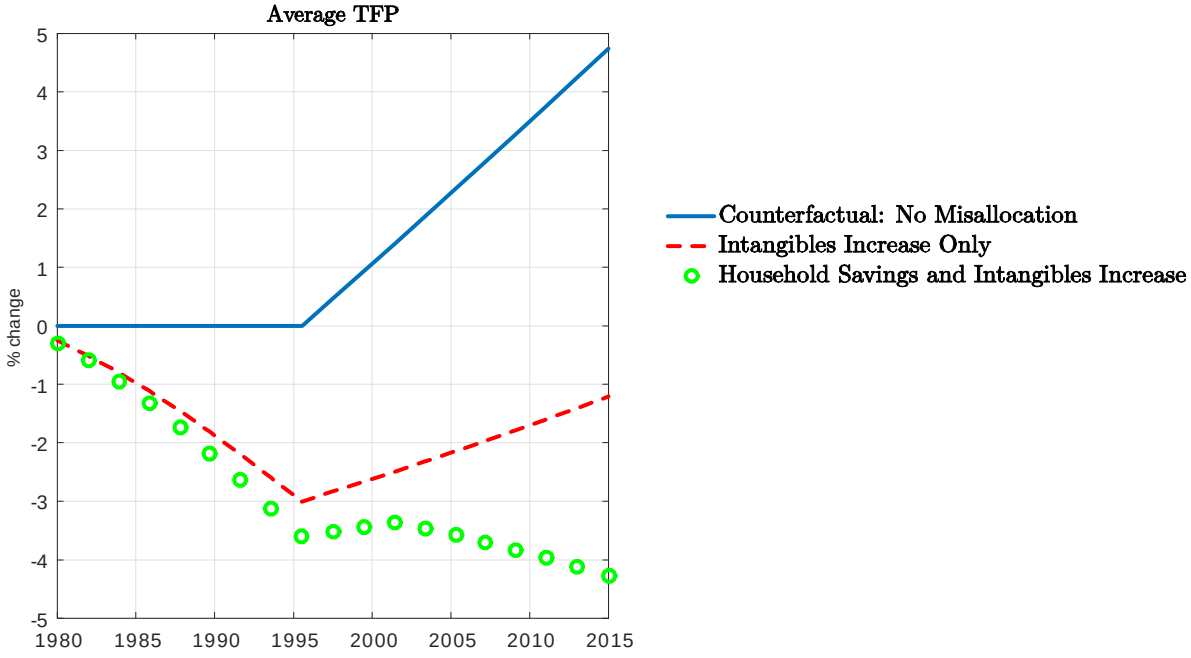

Figure E: Robustness Simulation Exercise: households' propensity to save and the share of intangible capital both gradually increase - comparison of effects on TFP when both trends occur, when only the increase in the share of intangible capital occurs, and in a counterfactual partial equilibrium scenario. Productivity parameter z_t is constant until 1995 and increases at a rate of 0.55% from 1995 on.