The Macroeconomics of Rational Bubbles: A User's Guide

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Abstract

This paper provides a guide to macroeconomic applications of the theory of rational bubbles. It shows that rational bubbles can be easily incorporated into standard macroeconomic models, and illustrates how they can be used to account for important macroeconomic phenomena. It also discusses the welfare implications of rational bubbles and the role of policy in managing them. Finally, it provides a detailed review of the literature.

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Recent decades have been characterized by large swings in asset prices. To illustrate this, the top panels of Figures 1-4 depict the ratio of household net worth to GDP in Japan, the United States, Spain and Ireland at different times over the last three decades. Loosely speaking, this ratio captures the value of all assets in an economy and its evolution largely mirrors the behavior of real estate and stock prices. As the figures shows, all of these countries have experienced episodes of large increases in asset prices – entailing a creation of wealth of one GDP or more – followed by sharp declines.

Besides their direct effect on household wealth, these fluctuations in asset prices have had profound macroeconomic implications. This is illustrated in the middle and bottom panels of Figures 1-4, which respectively depict the current account balance and the growth rates of output, consumption, and the capital stock. There are two main messages from these panels. First, large asset price booms were in most instances accompanied by large capital inflows, and their collapse by sharp reversals in capital flows. Second, asset price booms were often accompanied by large upswings in real activity – as measured by the growth rates of output, consumption and the capital stock – and their collapse by economic busts. Indeed, most boom-bust episodes ended in economic recessions, as depicted by the shaded bars in Figures 1-4.

In light of these developments, macroeconomists have felt the need to develop new models to understand what drives these large swings in asset prices and how do they affect the macroeconomy. We review here one class of models that rely on two simple premises or working hypotheses. The first one is that asset prices are not only driven by fundamentals, but also by bubbles that respond to random and capricious shifts in market psychology. The second hypothesis is that these bubbles are consistent with individual rationality. In fact, shifts in market psychology can be easily incorporated into standard macroeconomic models that rely on rational expectations, individual maximization, and market clearing.

These two hypotheses define the research on the macroeconomics of rational bubbles. The key words are macroeconomic and rational. Macroeconomic in the sense that this research is not interested in explaining the causes and effects of pricing anomalies or pathologies in some specific market, e.g. tulips, but rather in understanding large, widespread fluctuations in asset prices in modern economies. Rational in the sense that one of its key insights is that there are multiple market psychologies that are consistent with individual rationality. Whereas macroeconomists typically focus on one such psychology, i.e., asset prices are equal to the fundamental, there is no compelling reason to do so. Other psychologies may provide a more natural account of important

macroeconomic phenomena.

Two caveats are in order. The first is that this is a user's guide, which is defined by the Oxford English Dictionary as "a handbook containing instructions on how to use a device". As such, it is more technical than the usual survey or literature review. We want to tell readers what bubbles do, but we also want to show them how and why. We do so through a series of simple models and examples.

A second caveat is that this guide reflects our personal views, which have evolved over more than a decade of working on the topic. Because of this, it draws heavily from our own research and reflects our general narrative. Needless to say, many other researchers have contributed substantially to the topic. We acknowledge this in a detailed literature review that explains how this line of research has evolved and how the different contributions relate to each other.

The rest of the paper is organized as follows. Section 1 uses a small open economy model to show how bubbles generate fluctuations in capital flows, investment, and output. This partial-equilibrium framework allows us to identify and analyze some of the most important macroeconomic effects of bubbles. Section 2 extends the analysis to a world economy model. This general-equilibrium setting enables us to analyze the conditions for the existence of bubbles. It also allows us to explore the interaction between financial globalization and bubbles, and to derive the welfare and policy implications of bubbles. Section 3 contains the literature review. Finally, section 4 provides a brief discussion of the main challenges that this research program is facing.

1   Bubbly boom-bust cycles

The popular notion of a bubble or bubbly episode refers to a situation in which, for no really good reason, asset values and credit start growing rapidly. This marks the beginning of a period in which investment expands sharply, typically financed by large capital inflows. Output and consumption growth accelerate. Some of the new investments might seem unproductive, especially if they are made in low productivity sectors such as real estate. But this is not perceived to be a major problem contemporaneously. After all, the population enjoys a high level of consumption and well-being.

Eventually, again for no really good reason, asset values and credit drop, often quite dramatically. This leads to a sudden collapse in investment and a reversal of capital flows. Output and consumption growth stop abruptly and might even turn negative. Some of the investments made during the expansionary phase turn out to have little value, and they might even be abandoned or

dismantled. The population now suffers a low level of consumption and well-being.

This is the stylized view of bubbly boom-bust cycles held by many economic analysts and policymakers around the world, and it is based on experiences such as those shown in Figures 1-4. Perhaps the most defining aspect of this view is that movements in asset values and credit do not seem to be justified by major changes in economic conditions. Instead, they seem to be driven by random and capricious shifts in market psychology. Another important aspect of this view is that even in those cases in which investments are mostly unproductive or even useless, they still seem to create value and raise wealth during the expansionary period. These aspects of bubbly episodes are hard to generate in conventional macroeconomic models. But they are a central feature of models of rational bubbles. Thus, a major selling point of the theory is that it can formalize this popular view and make sense of it.

There is much more to the theory of rational bubbles than its ability to formalize this view, of course. We shall show this amply here. But we have to start somewhere, and this constitutes an excellent entry point to the macroeconomics of rational bubbles. Next, we construct this popular view step by step, showing how each of the elements fits into a progressively more sophisticated and nuanced story. To do this, we begin with a simple "lab economy" that provides a useful starting point to explore the macroeconomic role of bubbles.

1.1 A simple lab economy

Imagine an economy that is only a very small part of a large world. This economy contains equal-sized overlapping generations that live for two periods. All generations are endowed with one unit of labor when young and their goal is to maximize expected consumption when old.[1] Domestic residents and foreigners interact in the credit market, where they exchange consumption goods today for promises to deliver consumption goods in the future. Foreigners are willing to buy or sell any credit contract that offers a gross expected return equal to R. We refer to R as the world interest rate.

Production of consumption goods requires capital and labor, using a standard Cobb-Douglas technology: $Y_t = A \cdot K_t^\alpha \cdot \left(\gamma^t \cdot L_t\right)^{1-\alpha}$; with $A > 0$, $\gamma > 0$ and $\alpha \in (0,1)$; where Y_t, K_t and L_t denote output, the capital stock and the labor force, respectively. To produce one unit of capital for period $t+1$, one unit of the final good is needed in period t. Capital depreciates at rate $\delta \in (0,1)$,

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[1]This assumption simplifies the analysis substantially since it implies that the young save all their income and use it to construct portfolios with the highest possible expected return.

and it is reversible. The labor force is constant and equal to one. But labor productivity grows at the rate $\gamma \geq 1$. As usual we work with quantity variables expressed in efficiency units and denote them with lowercase letters. For instance, we refer to k_t as the capital stock, and we define it as $k_t \equiv \gamma^{-t} \cdot K_t$.

Factor markets are competitive and factors earn their marginal product:

$$w_t = (1 - \alpha) \cdot A \cdot k_t^{\alpha} \tag{1}$$

$$r_t = \alpha \cdot A \cdot k_t^{\alpha-1} \tag{2}$$

where w_t and r_t are the wage per effective worker and the rental per effective unit of capital, respectively.

We make two assumptions throughout. All capital is owned by domestic firms, which receive the capital income. All domestic firms are owned by domestic residents, who buy old firms and create new ones at zero cost.[2] Let v_t be the market value of all firms after the rental has been distributed to firm owners, but before new investments have been made. These firms own all the undepreciated capital left after production, i.e. $(1 - \delta) \cdot k_t$. We think of v_t as the value of all assets contained in the country, that is, the theoretical counterpart of the net worth data shown in Figures 1-4.

Domestic residents may want to borrow from foreigners to purchase firms and to invest. We introduce a friction that limits borrowing, though. Domestic courts can only seize the assets of domestic residents, i.e., their firms, but not the rental income of capital. One interpretation of this assumption is that the rental income can be "hidden" by domestic residents from the courts. As a result, the young can only promise a payment of v_{t+1} to their creditors. Let f_t be borrowing or credit. Since contingent contracts are possible, young firm owners face the following borrowing limit:[3]

$$R \cdot f_t \leq \gamma \cdot E_t v_{t+1} \tag{3}$$

where E denotes the expectation operator and we include it here because, as we shall see, v can be stochastic. This borrowing limit links credit and asset values, and it is a key element of many conventional macroeconomic models nowadays.

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[2]This is an assumption about effective or ultimate control of assets. It does not preclude domestic firms to issue and sell contingent credit contracts such as equity.

[3]To implement this borrowing limit, entrepreneurs sell credit contracts that promise a contingent return equal to $R_{t+1} = \frac{v_{t+1}}{E_t v_{t+1}} \cdot R$. This contract maximizes promised payments in all possible histories.

Some readers might be wondering why our notation distinguishes between the price of firms and the value of their capital. The reason, of course, is that bubbles create a wedge between these two concepts, as we shall see shortly. But the overwhelming choice in current macroeconomic research is to disregard this wedge and focus on equilibria in which these two quantities coincide:

$$v_t = (1 - \delta) \cdot k_t \tag{4}$$

If Equation (4) holds, we can write the maximization problem of the young as follows:

$$\max \gamma \cdot E_t c_{t+1} = (r_{t+1} + 1 - \delta) \cdot \gamma \cdot k_{t+1} - R \cdot f_t \tag{5}$$

$$\text{s. t.} \qquad \gamma \cdot k_{t+1} = w_t + f_t$$

$$R \cdot f_t \leq (1 - \delta) \cdot \gamma \cdot k_{t+1}$$

The young take the wage and rental as given and maximize expected old age consumption. The latter equals the rental, i.e. $r_{t+1} \cdot k_{t+1}$; plus the price obtained by selling their firms, i.e. $(1 - \delta) \cdot k_{t+1}$; minus interest payments. This maximization is subject to two constraints. The first one is the budget constraint, and it says that investment, i.e. $\gamma \cdot k_{t+1} - (1 - \delta) \cdot k_t$; plus the purchase of old firms, i.e. $(1 - \delta) \cdot k_t$; must equal labor income plus borrowing. The second constraint is the borrowing limit and it says that interest payments cannot exceed the value of firms, i.e. $(1 - \delta) \cdot \gamma \cdot k_{t+1}$.

The choice variables in the maximization problem (5) are borrowing (f_t) and the capital stock (k_{t+1}). If firms can costlessly merge or separate, or if they can buy and sell used capital, the concept of firm becomes a veil. Choosing which specific firms to buy and create does not really matter. The only thing that matters is how much capital is ultimately being held.

Maximization and market clearing imply that:

$$\gamma \cdot k_{t+1} = \min \left\{ \frac{R}{R + \delta - 1} \cdot (1 - \alpha) \cdot A \cdot k_t^\alpha, \gamma \cdot \left(\frac{\alpha \cdot A}{R + \delta - 1} \right)^{\frac{1}{1-\alpha}} \right\} \tag{6}$$

Equation (6) is the law of motion of the capital stock and it contains two distinct regions. The key variable that determines these regions is wealth, which here equals the wage of the young. If k_t weakly exceeds a threshold $\bar{k}$, wealth is high enough to ensure that the borrowing limit is not binding.[4] The young invest up to the point at which the return to capital equals the world interest rate: $r_{t+1} + 1 - \delta = R$. If $k_t < \bar{k}$, wealth is too low and the borrowing limit is binding. The return to capital exceeds the world interest rate: $r_{t+1} + 1 - \delta > R$. As it is customary in models of the

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[4]Formally, $\bar{k} = \left[\left(\frac{\alpha \cdot A}{R + \delta - 1} \right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{\gamma}{R} \cdot \frac{\alpha}{1 - \alpha} \right]^{\frac{1}{\alpha}}$.

financial accelerator type (of which this one is an example), when the borrowing limit binds the capital stock is a multiple of wealth. This multiple is known as the financial multiplier since it measures how many units of capital can be purchased for each unit of wealth. The intuition behind this multiplier is well known. One additional unit of wealth allows the young to purchase one unit of capital. This allows the young to borrow and raise the capital stock by $\dfrac{1-\delta}{R}$ additional units. And this allows them to borrow and raise the capital stock by $\left(\dfrac{1-\delta}{R}\right)^2$ additional units. And so on. Thus, one unit of wealth allows the young to purchase $1 + \dfrac{1-\delta}{R} + \left(\dfrac{1-\delta}{R}\right)^2 + ... = \dfrac{R}{R+\delta-1}$ units of capital.

From any initial condition the capital stock monotonically converges to a unique steady state k_F. This convergence is fast if $k_t > \bar{k}$, but slow if $k_t < \bar{k}$. The borrowing limit is binding in the steady state if the interest rate is low enough.[5] These are definitely quiet dynamics. It might seem that we have chosen the wrong economy to study the sort of messy and often dramatic events that are associated with the popular view of bubbles. But events of this sort are lurking in the background. To bring them to the fore, we just need to relax the assumption that firms are worth the capital they own. There is no theoretical reason to keep this assumption, and there is much to gain from relaxing it.

1.2   Building a theory of bubbly episodes

The theory of rational bubbles expands the set of equilibria under consideration. In particular, it also considers equilibria in which the price of firms does not coincide with that of their capital:

$$v_t = (1-\delta) \cdot k_t + b_t \tag{7}$$

where b_t is the bubble or bubble component of firm values. It is useful to think about this bubble as the sum of bubbles attached to specific firms in the economy. Thus, the bubble has two sources of dynamics: the growth of bubbles attached to old firms and the creation of new bubbles attached to new firms. We can express this idea with the following notation:

$$\gamma \cdot b_{t+1} = g_{t+1} \cdot (b_t + n_t) \tag{8}$$

Each period, the economy arrives with old bubbles b_t attached to old firms. New firms are created with new bubbles n_t attached to them. Equation (8) recognizes that new bubbles in period t are

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[5]In particular, if $R < \gamma \cdot \dfrac{\alpha}{1-\alpha}$.

already old bubbles by period $t+1$, and it implicitly defines g_{t+1} as the growth rate of bubbles (old and new) from period t to period $t+1$.

It is almost universally assumed that bubbles cannot be negative. This restriction is motivated by appealing to some notion of free disposal. If a firm were to contain a negative bubble, the argument goes, the owner could always start a new firm without bubble, transfer all the capital from the old firm to it, and close the old firm. Naturally, there might be costs of opening/closing firms and transferring capital among them. Or it might not be possible to start a new firm without a bubble. We abstract from these complications and follow standard practice by assuming free disposal of bubbles, i.e. $b_t \geq 0$ and $n_t \geq 0$.

Why do new bubbles pop up only in new firms? Is it possible that new bubbles pop up also in old firms? Diba and Grossman (1988) argued that, if a firm contains a bubble, this bubble must have started on the first date in which the firm was traded. Bubble creation after the first date of trading would involve an innovation in the firm price. If markets are efficient, this innovation must have had a zero expected value on the first date of trading. If there is free disposal of bubbles, this innovation must be non-negative. Combining these two observations we reach the conclusion that bubble creation must be exactly zero after the first date of trading. The Diba-Grossman argument is sometimes invoked as a "proof" that bubbles cannot be created. This is obviously misleading, since their argument does not impose any restriction on the size of new bubbles attached to new firms. Moreover, one can also relax the assumptions of market efficiency and/or free disposal to make it possible for new bubbles to pop up in old firms. To simplify the exposition, though, we keep these two assumptions here.

As this discussion highlights, a key concept of the theory of rational bubbles is that of market psychology. By this we mean a set of assumptions that define the bubble and its evolution. The theory of rational bubbles is interested in the set of market psychologies that are consistent with maximization and market clearing. Indeed, it is precisely the focus on this particular set that gives the name to the theory. Sometimes, this set contains a unique market psychology.[6] But usually the set contains many market psychologies, and the modeler is forced to make a choice. This is the case here. By choosing the specific market psychology that rules out bubbles, we obtained Equation (6) and the quiet dynamics associated with it. What happens if we make another choice?

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[6]If there is a unique rational market psychology, it is typically the one that rules out bubbles. But we know since Tirole (1985) that there exist environments in which the unique rational market psychology must feature a bubble. This case has been used recently by Allen, Barlevy and Gale (2017).

1.3 The wealth effect of new bubbles

We begin by considering a market psychology according to which the economy transits between two states: $z_t \in \{B, F\}$. During the bubbly state, new bubbles pop up with a combined size $\eta > 0$. During the fundamental state, no new bubbles pop up. Let φ and ψ be the transition probabilities from the bubbly to the fundamental state and vice-versa, respectively. With these assumptions, we have that:

$$n_t = \begin{cases} \eta & \text{if } z_t = B \\ 0 & \text{if } z_t = F \end{cases} \tag{9}$$

This is a stylized model of market psychology. Some periods investor sentiment is such that markets are willing to finance investment on the basis of new bubbles. Some other periods, investor sentiment is such that markets are unwilling to do so. The transition between these states is random and unrelated to economic conditions.

Replacing Equation (4) with Equations (7) and (8), we can write the maximization problem of the young as follows:

$$\max \gamma \cdot E_t c_{t+1} = (r_{t+1} + 1 - \delta) \cdot \gamma \cdot k_{t+1} + E_t g_{t+1} \cdot (x_t \cdot b_t + n_t) - R \cdot f_t \tag{10}$$

$$\text{s. t.} \quad \gamma \cdot k_{t+1} + x_t \cdot b_t = w_t + f_t$$

$$R \cdot f_t \leq (1 - \delta) \cdot \gamma \cdot k_{t+1} + E_t g_{t+1} \cdot (x_t \cdot b_t + n_t),$$

where x_t denotes bubble demand by the young as a share of the aggregate bubble. Now the choice variables are borrowing (f_t), the capital stock (k_{t+1}) and the bubble demanded (x_t). Bubble creation n_t is not a choice variable because, according to the market psychology of Equation (9), it is exogenous. Once again, if firms can costlessly merge or separate, or if they can buy and sell used capital, the concept of a firm becomes a veil. Choosing which specific firms to buy and create does not really matter. The only thing that matters is how much capital and bubble are ultimately being held.[7]

Individual maximization plus market clearing $(x_t = 1)$ implies,

$$E_t g_{t+1} = R, \tag{11}$$

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[7]We are assuming here that mergers/separations and purchases/sales of capital do no affect the bubble component. If they do, then firms are no longer a veil and we cannot take this shortcut. Note that the same assumption applies to the capital stock. If its value or productivity is affected by mergers/acquisitions and purchases/sales, firms are no longer a veil either.

i.e., expected bubble growth equals the world interest rate. The return to holding a bubble is its growth since the bubble does not produce a rental or dividend. If expected bubble growth exceeded the interest rate, the young would make a riskless profit by purchasing bubbles and borrowing to finance these purchases.[8] The demand for bubbles would be unbounded (i.e., $x_t \to \infty$). If expected bubble growth fell short of the interest rate, the young would make a riskless profit by lending and shortselling bubbles to finance this lending.[9] The demand for bubbles would be negative (i.e., $x_t \to -\infty$). Thus, expected bubble growth must equal the interest rate.[10]

One of the most attractive features of the theory of rational bubbles is that market psychology affects capital accumulation and growth, which are now given by:

$$\gamma \cdot k_{t+1} = \min \left\{ \frac{R}{R + \delta - 1} \cdot [(1 - \alpha) \cdot A \cdot k_t^\alpha + n_t], \gamma \cdot \left(\frac{\alpha \cdot A}{R + \delta - 1} \right)^{\frac{1}{1-\alpha}} \right\} \tag{12}$$

Equation (12) is the law of motion of the capital stock under the new market psychology. Its shape depends on the size of new bubbles, but it does not depend on the size of old bubbles. In the fundamental state, $n_t = 0$ and the law of motion is the same as before. In the bubbly state, $n_t = \eta$ and the law of motion is shifted up for values of $k_t \leq \bar{k}$. This means that, if the borrowing limit is binding, bubbly episodes foster capital accumulation and growth.

Why do new bubbles foster capital accumulation? Why do old bubbles have no effect on capital accumulation? The key difference is that new bubbles are free, whereas old bubbles need to be paid for. When the young create firms with new bubbles, they anticipate that they will be wealthier during old age when they sell these firms. Since they are constrained, they borrow against this future wealth and use the funds to invest; i.e. $\frac{E_t g_{t+1} \cdot n_t}{R} = n_t$. Thus, new bubbles raise wealth, and each additional unit of wealth can be leveraged to invest $\frac{R}{R + \delta - 1}$ units of capital. This is the wealth effect of new bubbles. When the young purchase firms with old bubbles, they borrow against these bubbles just enough to finance their purchase: $\frac{E_t g_{t+1} \cdot b_t}{R} = b_t$. Thus, old bubbles affect neither wealth nor the capital stock.

Bubbles make the dynamics of our lab economy more interesting. From any initial condition, the capital stock converges to the interval $[k_F, k_B]$. Once the economy has reached this interval,

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[8]For instance, the young could sell credit contracts that offer a return $R_{t+1} = g_{t+1} + R - E_t g_{t+1}$, and use the proceeds to purchase bubbles. This would yield a riskless profit equal to $E_t g_{t+1} - R$ per unit of credit contract sold.

[9]In this case, the young would like to shortsell bubbles and use the proceeds to purchase the credit contracts described in the previous note.

[10]As for unexpected bubble growth, i.e. $g_{t+1} - E_t g_{t+1}$, we leave it unspecified for now. Our justification is that this growth does not play a role until section 1.5, and then we will be forced to make additional assumptions.

it fluctuates within it forever. We refer to the invariant or steady-state distribution of k as the "steady state". In this steady state, the economy perpetually transits between the bubbly and fundamental states. In the bubbly state, asset values, foreign borrowing and investment are high and the economy grows towards k_B. Consumption and welfare also grow. In the fundamental state, asset values, foreign borrowing and investment are low and the economy shrinks towards k_F. Consumption and welfare also shrink. Transitions between states are random and capricious, without any really good reason. Figure 5 shows a simulated boom-bust cycle in our lab economy. As the figure illustrates, these bubbly fluctuations capture quite closely the type of real-world episodes shown in Figures 1-4.

A bubble shock is quite similar to a natural resource shock. To see this, imagine that oil or some other natural resource were suddenly discovered, extracted and exported abroad. This export revenue (n_t) constitutes a windfall or wealth shock to domestic residents. If the borrowing limit is not binding, this wealth shock does not affect capital accumulation. But if the borrowing limit is binding, this shock leads to an increase in the capital stock and a surge in capital inflows. Net worth increases more than proportionally with the change in the capital stock, to reflect the value of the natural resource discovered. If eventually the natural resource is exhausted or a better synthetic substitute is invented, export revenue stops and the wealth effect vanishes. All the effects of the discovery are reversed, and the economy returns to its initial situation.

This comparison of shocks provides a clean intuition for what is going on in the bubbly economy. Instead of exporting natural resources, domestic residents are exporting bubbles to the rest of the world. Obviously, a difference between natural resources and bubbles is the source of their value, that is, the reason why the rest of the world demands them. A natural resource such as oil derives its value from its use in production, and its demand depends on the specific technologies that are used. A bubble derives its value from its use as an asset or store of value, and its demand depends on the specific market psychology that prevails. Whatever the source of value, though, natural resources and bubbles raise wealth. If the borrowing limit is binding, wealth raises investment and foreign borrowing.

1.4 The subsidy effect of new bubbles

We have just seen that our lab economy can experience bubbly boom-bust cycles in investment. But many observers have emphasized that it is not only the size of investment that fluctuates sharply

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during boom-bust cycles, but also its quality.[11] The expansionary phase is often characterized by low-quality investments, most notably in real estate, that seem to be chasing the bubble with little or no concern for productive efficiency. These low-quality investments are often abandoned or dismantled during the recessionary phase. Interestingly, this is exactly what happens in our lab economy if we introduce low-quality investments and then add a small wrinkle to our model of market psychology.

Let us assume that domestic residents can now invest in a low-quality capital h_t that produces output with a linear technology: $y_t = \rho \cdot h_t$. This type of capital produces a rental equal to ρ. Other than this, low-quality capital h_t is similar to high-quality capital k_t. To produce one unit of any capital for period $t+1$, one unit of the final good is needed in period t; both capitals depreciate at rate δ; both capitals are reversible, and their rental incomes cannot be seized by domestic courts and are therefore not pledgeable to creditors. We say that h_t is "low quality" because it delivers a return that is below the world interest rate, i.e. $\rho + 1 - \delta < R$.

Besides low-quality capital, we also add a new and realistic element to our model of market psychology. Up to now, we have assumed that new bubbles are independent of the size and type of investment undertaken by new firms. But often new bubbles seem to be associated or attached to new firms in specific sectors or technologies, such as housing or high-tech industries. The larger is the investment that goes to these sectors or technologies, the larger is the size of the new bubble. To capture this feature of real-world market psychology, we now replace Equation (9) by the following one:

$$n_t = \begin{cases} \eta + \sigma \cdot \dfrac{h_{t+1}^{1-\theta}}{1-\theta} & \text{if } z_t = B \\ 0 & \text{if } z_t = F \end{cases} \tag{13}$$

where $\sigma > 0$ and $\theta \in (0,1)$. Equation (13) says that a fraction of bubble creation is attached to low-quality capital. The more the representative young individual invests in this type of capital, the larger is the new bubble that she receives. We keep all the other assumptions as in section 1.3.

With this additional investment option, the maximization problem of the young becomes:

$$\max \gamma \cdot E_t c_{t+1} = (r_{t+1} + 1 - \delta) \cdot \gamma \cdot k_{t+1} + (\rho + 1 - \delta) \cdot \gamma \cdot h_{t+1} + E_t g_{t+1} \cdot (x_t \cdot b_t + n_t) - R \cdot f_t \tag{14}$$

$$\text{s. t.} \qquad \gamma \cdot (k_{t+1} + h_{t+1}) + x_t \cdot b_t = w_t + f_t$$

$$R \cdot f_t \leq (1 - \delta) \cdot \gamma \cdot (k_{t+1} + h_{t+1}) + E_t g_{t+1} \cdot (x_t \cdot b_t + n_t),$$

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[11]Gopinath et al. (2017), for instance, have recently argued that the Spanish boom was accompanied by a declining efficiency in the allocation of resources.

There is now an additional choice variable: the amount of low-quality capital (h_{t+1}). Adding this choice does not affect the equilibria analyzed in sections 1.1 and 1.3, because there is no reason to invest in this type of capital in those environments. Since $\rho + 1 - \delta < R \leq r_{t+1} + 1 - \delta$, it does not pay to produce low-quality capital when it is possible to lend abroad or, even better, produce high-quality capital.

Maximization together with clearing in the market for bubbles (i.e., $x_t = 1$) implies that Equation (11) still holds. Under the market psychology of Equation (13), however, young individuals have incentives to invest in low-quality capital to increase bubble creation. In this case, the demand for high-quality capital becomes:

$$\gamma \cdot k_{t+1} = \min \left\{ \frac{R}{R + \delta - 1} \cdot [(1 - \alpha) \cdot A \cdot k_t^\alpha + n_t] - \gamma \cdot h_{t+1}, \gamma \cdot \left(\frac{\alpha \cdot A}{R + \delta - 1} \right)^{\frac{1}{1-\alpha}} \right\} \quad (15)$$

Equation (15) is a natural generalization of Equation (12). If the borrowing limit is not binding, low-quality capital does not affect the demand for high-quality capital. If the borrowing limit is binding, though, low-quality capital affects the demand for high-quality capital. For a given amount of wealth, an increase in low-quality capital lowers the resources available for high-quality capital one-for-one. In the bubbly state, though, low-quality capital "produces" new bubbles (see Equation (13)) and this raises wealth and the resources available for overall investment. To determine the net effect of these two forces, we need to know the equilibrium mix of capitals.

In the bubbly state, this mix is determined as follows:

$$\rho + \frac{\sigma}{\gamma} \cdot h_{t+1}^{-\theta} \cdot \frac{R}{R + \delta - 1} \cdot \alpha \cdot A \cdot k_{t+1}^{\alpha-1} = \alpha \cdot A \cdot k_{t+1}^{\alpha-1} \quad \text{if} \quad z_t = B \quad (16)$$

Equation (16) says that investments in both types of capital are such that their marginal returns are equalized. The marginal return to high-quality capital is its rental, i.e. $\alpha \cdot A \cdot k_{t+1}^{\alpha-1}$. The marginal return to the low-quality capital has two components. The first one is also its rental, i.e. ρ. The second one is the *subsidy effect of new bubbles*, and one can think of it as the return to producing bubbles. At the margin, producing an additional unit of low-quality capital produces $\sigma \cdot h_{t+1}^{-\theta}$ worth of new bubbles. One unit of new bubbles allows the young to purchase $\frac{R}{R + \delta - 1}$ units of high-quality capital, and each of these delivers a rental equal to $\alpha \cdot A \cdot k_{t+1}^{\alpha-1}$.

Somewhat paradoxically, the worse low-quality investments are, the more they facilitate high-quality investments. The latter are maximized if $\rho = 0$. In this case, the only reason to invest

in low-quality capital is to produce new bubbles to finance high-quality investments.[12] If $\rho > 0$, investment in low-quality capital also yields a rental and this induces the young to invest beyond the point that maximizes the resources available for high-quality investments. The larger is ρ, the larger is this incentive. If $\rho < \theta \cdot r_{t+1}$, low-quality investments produce more bubbles than needed to finance themselves, and this facilitates or crowds in high-quality investments. If $\rho > \theta \cdot r_{t+1}$, low-quality investments do not produce enough bubbles to finance themselves, and this obstructs or crowds out high-quality ones.[13]

In the fundamental state, low-quality capital is neither produced nor used:

$$h_{t+1} = 0 \quad \text{if} \ \ z_t = F \tag{17}$$

Once low-quality capital is unable to "produce" bubbles, the subsidy effect disappears and its return falls below the world interest rate. If there is any low-quality capital when the economy transitions to the fundamental state, it is always preferable to dismantle it, convert it back into goods and use these goods to lend abroad or to produce high-quality capital.[14]

The dynamics of our lab economy are still very much the same as those described in the previous section. The only novelty is that, during expansions, young agents devote resources to the production of low-quality capital in order to chase the bubble. Once the economy enters a recession there is no longer a bubble to chase, these investments stop and existing low-quality capital is dismantled. These low-quality investments might appear wasteful if one looks exclusively

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[12]Equations (13) and (15) can be used to show that the level of h_{t+1} that maximizes k_{t+1} is defined as follows:

$$\frac{\sigma}{\gamma} \cdot h_{t+1}^{-\theta} \cdot \frac{R}{R + \delta - 1} = 1$$

Equation (16) shows that this exactly defines the equilibrium level of h_{t+1} if $\rho = 0$.

[13]To see this, combine Equations (13) and (15) to determine that the presence of h_{t+1} raiseses k_{t+1} if and only if $\frac{R}{R+\delta-1} \cdot \frac{\sigma}{\gamma} \cdot \frac{h_{t+1}^{1-\theta}}{1-\theta} - h_{t+1}^{1-\theta} > 0$, that is, if and only if

$$h_{t+1} < \left(\frac{R}{R + \delta - 1} \cdot \frac{\sigma}{\gamma} \cdot \frac{1}{1 - \theta} \right)^{\frac{1}{\theta}}$$

But we know from Equation (16) that, in equilibrium,

$$h_{t+1} = \left(\frac{R}{R + \delta - 1} \cdot \frac{\sigma}{\gamma} \cdot \frac{r_{t+1}}{r_{t+1} - \rho} \right)^{\frac{1}{\theta}}$$

The result follows from these two observations.

[14]Here the assumption that capital is reversible simplifies the discussion without affecting our arguments. If capital were irreversible, it would not be possible to dismantle it and, instead, its price would drop below one and remain low throughout the fundamental state. Investment in low-quality investment would be zero, and its stock would decline at the rate of depreciation.

at their rental income and neglects the subsidy effect. But this would be misleading, because low-quality investments produce valuable bubbles.

Stressing again the formal similarity between bubble and natural resource shocks, we note that the formation of bubbles might have effects similar to those of the Dutch disease. To show this, one simply needs to follow well-trodden paths and assume that the social return to high-quality capital exceeds its private return due to external learning-by-doing and/or spillovers in knowledge production. If low-quality capital crowds out high-quality capital during the expansionary phase of the boom-bust cycle, this might inefficiently reduce growth in the long run. Knowing that there is this option is useful for the modeler, since Dutch-disease effects are likely to be relevant in applications. But we shall not pursue this thought further here.[15] Instead, we turn our attention to the old bubbles that have been conspicuously absent from the discussion so far.

1.5   The overhang effect of old bubbles

Up to now we have focused exclusively on the effects of new bubbles. Indeed, we have not even specified how old bubbles behave, except for showing that their expected growth equals the world interest rate in all periods. This may not seem like a realistic model of market psychology since, as the episodes outlined in the introduction suggest, real-world bubbles appear to alternate between periods of rapid growth and crashes.

Let us then refine our model of market psychology by assuming that a fraction μ of the old bubbles bursts in the transition from the bubbly to the fundamental state. Expected bubble growth is still given by Equation (11), but now realized bubble growth during the bubbly state is given by:

$$
g_{t+1} = \begin{cases} \dfrac{1}{1-\varphi \cdot \mu} \cdot E_t g_{t+1} & \text{if } z_{t+1} = B \\[2mm] \dfrac{1-\mu}{1-\varphi \cdot \mu} \cdot E_t g_{t+1} & \text{if } z_{t+1} = F \end{cases} \quad \text{if } z_t = B \tag{18}
$$

while in the fundamental state this realized growth is given by:

$$
g_{t+1} = E_t g_{t+1} \quad \text{if } z_t = F \tag{19}
$$

In both states, expected bubble growth equals the world interest rate. In the bubbly state, holding bubbles is risky. If the economy remains in the bubbly state, the return to the bubble is above the

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[15]Our lab economy offers an interesting insight that is reminiscent of the Dutch disease result. If low-quality investments crowd out high-quality ones, the wage falls and so does savings. As a result, growth might slow down in the future. Obviously, this result depends on our assumption that the low-quality sector is the capital-intensive one, and it would be reversed if we were to assume that the low-quality sector is the labor-intensive one.

world interest rate. But this just compensates bubble owners for the loss of a fraction μ of their bubbles when the economy transitions to the fundamental state. Once there, holding bubbles is safe. As a result, bubble growth equals the world interest rate.

With this market psychology, owners of old bubbles also experience wealth shocks. The size and time distribution of these shocks depends on our assumptions. If φ is small and μ is large, for instance, we have that bubble owners receive (on average) a large sequence of small positive wealth shocks during a bubbly episode. The episode ends however with a single very large negative wealth shock. This does not seem too unrealistic a model of market psychology, by the way.

Owners of old bubbles must come up with the resources to purchase them, and they receive the wealth shocks associated with them. Where do these resources come from? How are they affected by these shocks? Up to now, the owners of old bubbles have been the foreigners.[16] Since we have not modeled the rest of the world yet, we cannot really say where do foreigners find the resources to purchase bubbles, or what do they do with the wealth shocks associated with them. A full and satisfactory answer to these questions must wait until the next section, when we take a look at the rest of the world and examine the general equilibrium implications of bubbles.

We can obtain some preliminary answers, though, by "forcing" domestic residents to hold some old bubbles. To do this, we add another small wrinkle to our model of market psychology.[17] Whenever a firm owner defaults on her credit contracts and is taken to court, all of the bubbles attached to her firms burst. It is well known that litigation is costly, but here it neither uses resources nor distorts incentives. Instead, it is the market that punishes litigation with the loss of the bubble. This does not seem too unrealistic and it shows that not even the most efficient courts might be able to eliminate all litigation costs. We then make two assumptions about the interaction between debtors and creditors. Ex-post bargaining is efficient and litigation never takes place in equilibrium. Ex-post bargaining is such that creditors obtain a fraction ϕ of the surplus, which in this case is the size of the bubble. Under these assumptions, the borrowing limit is now given by:

$$R \cdot f_t \leq (1 - \delta) \cdot \gamma \cdot (k_{t+1} + h_{t+1}) + \phi \cdot E_t g_{t+1} \cdot (b_t + n_t) \qquad (20)$$

---

[16]Some readers might find this terminology a bit puzzling. Are bubbles not a component of firm prices? And are these firms not owned by domestic residents? What we mean, of course, is that credit contracts are such that all the risk associated with changes in the value of old bubbles are held by foreigners. Since these contracts are rolled over forever, one can say that foreigners effectively own the bubbles.

[17]Another way to do this would be to limit the set of contracts that are available. For instance, we could impose the popular but ad-hoc restriction that credit contracts cannot be contingent.

Equation (20) simply says that the young cannot borrow against the whole value of their firm since creditors know that during old age they would have to agree to a reduction in debt equal to a fraction $1 - \phi$ of the bubble to avoid litigation. Thus, the borrowing limit is the value of the firm, i.e. $\gamma \cdot E_t v_{t+1}$ minus a fraction of the bubble, i.e. $(1 - \phi) \cdot \gamma \cdot E_t b_{t+1}$. This forces the young to effectively hold a fraction $1 - \phi$ of the bubble.

Expected bubble growth must now equal:

$$E_t g_{t+1} = \frac{R \cdot \alpha \cdot A \cdot k_{t+1}^{\alpha-1}}{\phi \cdot \alpha \cdot A \cdot k_{t+1}^{\alpha-1} + (1 - \phi) \cdot (R + \delta - 1)} \tag{21}$$

The best intuition for this result comes from examining the limiting cases. As $\phi \to 1$, we have that $E_t g_{t+1} \to R$. Domestic residents can borrow to finance the bubble and the cost of this is the world interest rate. As $\phi \to 0$, we have that $E_t g_{t+1} = \frac{R}{R + \delta - 1} \cdot \alpha \cdot A \cdot k_{t+1}^{\alpha-1}$. Domestic residents must reduce their holdings of capital to finance the bubble and the cost of this is the return to capital. For intermediate values of ϕ, domestic residents can borrow part of the cost of the bubble, but must finance the rest by reducing their holdings of capital. Thus, the return to the bubble is somewhere in between the world interest rate and the return to capital. Whatever the case, though, realized bubble growth is still given by Equations (18) and (19).

We must replace Equation (15) by the following generalization:[18]

$$\gamma \cdot k_{t+1} = \min \left\{ \frac{R}{R + \delta - 1} \cdot \left[(1 - \alpha) \cdot A \cdot k_t^\alpha + \frac{\phi \cdot E_t g_{t+1}}{R} \cdot (b_t + n_t) - b_t \right] - h_{t+1}, \gamma \cdot \left(\frac{\alpha \cdot A}{R + \delta - 1} \right)^{\frac{1}{1-\alpha}} \right\} \tag{22}$$

Since $\phi \cdot E_t g_{t+1} \leq R$, old bubbles reduce the capital stock. The reason is that the young can no longer pledge them fully to foreigners, so that they need to use some of their own resources to purchase them. They do so expecting that the next generation of young entrepreneurs does the same. And future generations keep doing so following this logic. Thus, the bubble is like a debt that is passed across generations and absorbs part of the resources that would have been used to invest in capital. This is the *overhang effect of old bubbles*, and it always crowds investment out, reducing the capital stock.

The overhang effect has important implications for the dynamics of our lab economy. Bubbly episodes, for instance, need not be expansionary. To see this, note that wealth effect of new bubbles is smaller if borrowing against them is restricted. As $\phi \to 0$, the wealth effect of new bubbles vanishes and the overhang effect of old bubbles is maximized. In this limit, the shape of

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[18]Equations (16)-(17) still hold. Whether bubbles are pledgeable or not, the marginal returns to both types of capital must be equalized in equilibrium.

the law of motion depends on old bubbles, but not on new bubbles. In this extreme case, bubbly episodes are contractionary and reduce the capital stock. This case is just the opposite of the limiting case $\phi \to 1$ that we have been focusing upon until now.

New bubbles raise wealth and provide resources for investment and growth, but they eventually turn into old bubbles that take away resources from investment and growth. The dynamic balance of these effects is complex and can go either way. One can generate scenarios in which bubbly episodes lead to a strong expansion initially, when the ratio of new to old bubbles is large. Over time, the expansion weakens, as the ratio of new to old bubbles declines. It might be even possible that the expansion turns into a contraction before the economy transits to the fundamental state. Interestingly, the larger is the fraction of bubbles that burst when the latter happens, the milder is the recession that follows. When bubbles burst, the overhang effect is dampened and this liberates resources that can be used to invest and grow.

To conclude our analysis of bubbly boom-bust cycles, we ask whether our model of market psychology is indeed rational as claimed. To check this, we must ensure that two conditions are met. The first one is that the bubble grows fast enough to be attractive to those that hold it. Equation (21) already imposes this condition. The second condition is that this growth does not lead the supply of bubbles to outgrow their demand. Up to now, we avoided this issue by assuming that foreigners have unbounded resources. But this is not enough now, since part of the bubble is held by domestic residents. How do we know that the young have enough resources to purchase their part of the bubble? To ensure that our model of market psychology is rational we must now check that it generates an equilibrium in which

$$(1 - \alpha) \cdot A \cdot k_t^\alpha + n_t > \left(1 - \frac{\phi \cdot E_t g_{t+1}}{R}\right) \cdot (b_t + n_t) \tag{23}$$

in all possible periods and histories. Condition (23) says that the wealth of the young, wage plus new bubbles, must be large enough to purchase the fraction of the bubbles that cannot be sold to foreigners.

All models of rational bubbles must satisfy a feasibility condition like this one.[19] Although the details are specific to each environment, it always involves some sort of comparison between the interest rate (or return to the relevant assets) and the growth rate of the economy. For instance, assume that $\varphi = 0$ so that the bubbly state is absorbing, and let the economy start in it. If the

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[19]We did not check explicitly this condition before since it is always satisfied in the limit $\phi \to 1$ (recall that in this limit $E_t g_{t+1} = R$). But the condition was indeed there!

borrowing limit is not binding, the bubble grows at the world interest rate, so that:

$$\lim_{t \to \infty} b = \begin{cases} \dfrac{R}{\gamma - R} \cdot \eta & \text{if } R < \gamma \\ \infty & \text{if } R \geq \gamma \end{cases} \tag{24}$$

If $R \geq \gamma$, the bubble grows without bound and eventually exceeds the wealth of the young. Standard backward induction arguments rule this out, and this leads us to conclude that our assumed market psychology is not rational in this case. If $R < \gamma$, the bubble converges to a finite value and Condition (23) is satisfied if η is not too large.[20] In this case, we conclude that our assumed market psychology is rational. If the borrowing limit were not binding and/or the transition probability φ were positive, the calculations would be more involved but the idea would be pretty much the same. The interest rate must be low enough to ensure that the bubble does not outgrow the wealth of the young.

The bottom line of this discussion is that, for bubbly boom-bust cycles to happen, our lab economy must be inserted into a world economy that is capable of supplying plenty of financing (so that a large part of the bubble is exported) at low interest rates (so that the part of the bubble that remains at home does not grow too fast). Is this a plausible description of the world economy? Can such an environment explain the type of boom-bust episodes discussed in the introduction? To answer these questions, we need to move beyond the borders of our lab economy and explore the rest of the world.

2   The bubbly world economy

Many observers refer to the last 45 years as the era (or new era) of financial globalization.[21] It all started in the early 1970s in industrial countries, with the abandonment of the Bretton Woods system and the removal of capital controls and many other restrictions to cross-border transactions. The effects of this policy reversal were amplified by new trends in the 1980s. Industrial countries deregulated their financial markets, and new technologies facilitated the development of sophisticated financial products. But the major impulse to financial globalization came in a second wave during the early 1990s, when many emerging markets joined the world financial system. Up to then, the private sectors in these developing economies had been prevented from participating in global markets, which was a privilege retained by their sovereigns. The painful sovereign debt crisis of the

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[20]If $R \leq \phi \cdot \gamma$, Condition (23) is satisfied for any value of η. If $\phi \cdot \gamma < R < \gamma$, Condition (23) is satisfied if η is not too large.

[21]See, for instance Eichengreen and Bordo (2002), or Beck, Claessens and Schmukler (2013)

1980s uncovered the weakness of this model and led to its downfall. Capital controls were removed and market-friendly policies were adopted throughout the emerging world.

The entry of emerging markets into the world financial system has coincided with profound changes in the world economic environment. The first of these is cheap credit. Interest rates have declined steadily since the early 1990s, reaching zero or even turning negative. Low interest rates are a feature of systems with financial repression where funds are limited and rationed. But the world financial system could not be farther away from such systems. If anything, financial markets have incorporated large pools of savings from the emerging world, which move rapidly around the globe in search of assets or stores of value.[22] This is what Ben Bernanke famously described as a global savings glut. As our lab economy showed us, low interest rates and plenty of financing create the sort of environment that is conducive to bubbly boom-bust cycles. It is therefore not surprising to find that financial integration with the emerging world has also been accompanied by a marked increase in the frequency of credit booms and busts. More surprising, though, are the so-called global imbalances, which refer to large capital flows from emerging economies with fast productivity growth like China to advanced economies with slower productivity growth like the United States. Most observers expected financial integration with emerging markets to be followed by large capital flows in the opposite direction.

In this section, we use the theory of rational bubbles to explore the relationship between financial globalization and bubbles. This relationship is complex and goes both ways, as we argue. Financial globalization with emerging markets may well have created a bubble-friendly environment, but bubbles have played a critical role shaping the effects of globalization as well. To show this, we proceed again step by step, explaining first how to create bubbly environments, deriving then a couple of important additional effects of old bubbles, and finally mixing all these ingredients to develop a view of financial globalization with bubbles. We conclude the section by exploring the implications for welfare and policy.

2.1 Creating bubbly environments

How do we create a bubbly environment? In the lab economy, it was just enough to assume that the world interest rate, R, lies below the long-run growth rate, γ. Once we adopt a global perspective, the world interest rate becomes an endogenous variable and it cannot be treated parametrically. A bubbly environment is still a low interest rate environment. But to create one, we need to take a

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[22]See, for instance, Caballero et al. (2008) and Coeurdacier et al. (2015).

detailed look at the determinants of the world interest rate.

Let us consider a world economy with many countries. All countries have the same population, but they differ from the lab economy of the previous section in three respects. First, factor markets are global and the wage and rental therefore depend on the world capital stock and not the country's capital stock. Thus, Equations (1)-(2) still apply, but now k_t must be interpreted as the world capital stock, and w_t and r_t as the common wage and rental.[23] Second, only a fraction ε of each country's residents can manage and own capital. We refer to these individuals as entrepreneurs. The rest cannot do so, and we refer to them as savers.[24] Third, consumption goods are not perishable and can be stored from one period to the next one. A literal way to think about storage is as inventory accumulation. But there are other interpretations, as we shall show later. The rest of our assumptions regarding preferences, technology, demography and domestic courts remain the same as before.

As is customary by now, we must specify a market psychology to complete the model. We adopt again the familiar market psychology of section 1.3. Recall that in this market psychology there is no subsidy effect of bubbles, so that Equation (9) holds, and bubbles are fully pledgeable, so that Equation (11) also applies. What kind of environments make such a market psychology rational? What does the modeler have to do to create them?

The steady-state dynamics of the world capital stock are as follows:[25]

$$\gamma \cdot k_{t+1} = \min\left\{\frac{1}{\delta} \cdot \left[\varepsilon \cdot (1-\alpha) \cdot A \cdot k_t^\alpha + n_t\right], (1-\alpha) \cdot A \cdot k_t^\alpha - b_t\right\} \tag{25}$$

The aggregate resource constraint of the economy says that the total wealth of the economy, which consists of the wage, must be allocated to the three available assets: capital, bubbles, and storage.

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[23] Since moving capital and labor physically is costly, the assumption of global factor markets can only be justified if technologies allow factors of production located in different countries to embed their contributions to production in specialized intermediate inputs. Trading these inputs would then lead to the equalization of factor prices. Technologies have certainly evolved in this direction, and trade in intermediates has exploded in the last couple of decades. But we are still far away from having factor markets that are truly global. We adopt this assumption because it helps tremendously to provide clean and instructive derivations of the theoretical results we are after. But it is admittedly unrealistic, and we shall remove it in section 2.3.

[24] Since not all countries need to have the same proportion of savers, one could think of our lab economy as a country without savers. But nothing of substance would really change from our analysis in section 1 if we added savers to the lab economy. The only change would be that, when we referred to wealth, we would now refer to "entrepreneurial" wealth. And when we referred to borrowing, we would now refer to "entrepreneurial" borrowing.

[25] Here we are using the assumption that $\frac{\alpha \cdot \gamma}{1-\alpha} + 1 - \delta > 1$. This condition implies that, in any steady state, the return to investment is higher than the return to storage.

Equation (25) says that there are two regimes. In one regime, the interest rate equals the return to storage, i.e., $R_{t+1} = 1$, and the three assets are used in equilibrium. We refer to this case as the "partial intermediation" regime. The borrowing limit is binding and some savings remain in the hands of savers, who store them. Investment is limited or determined by entrepreneurial wealth. In the second regime, the interest rate is higher than the return to storage:

$$R_{t+1} = \min \left\{ \frac{\gamma \cdot [(1-\delta) \cdot k_{t+1} + E_t b_{t+1}]}{(1-\varepsilon) \cdot (1-\alpha) \cdot A \cdot k_t^\alpha}, \alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta \right\} \tag{26}$$

and only capital and bubbles are held in equilibrium. We refer to this case as the "full intermediation" regime. The borrowing limit might be binding or not, but all savings end up in the hands of entrepreneurs. If the borrowing limit is binding, the interest rate is determined by "cash-in-the-market", i.e. the ratio of entrepreneurial collateral to savings of the savers. If the borrowing limit is not binding, the interest rate equals the return to investment.

The effects of bubbles depend on the regime. In the partial intermediation regime, the wealth effect of new bubbles raises the capital stock at the expense of storage, with a constant interest rate. Old bubbles have no effect on the capital stock. Thus, new and old bubbles have the same effect as in the lab economy! But old bubbles are no longer held by foreigners with unbounded resources. Instead, they are held by savers who reduce their holdings of stored goods. Interestingly, this places a general-equilibrium limit to the effects of bubble creation: the complete elimination of storage.

In the full intermediation regime, this limit has been reached and new bubbles no longer affect capital accumulation. Interestingly, old bubbles have an overhang effect now. New and old bubbles have the same effect as they did in the lab economy when they were not pledgeable, i.e. $\phi = 0$. But here we have assumed that they are pledgeable, i.e. $\phi = 1$. What is going on? In the economy of section 1.5, the overhang effect was a partial-equilibrium effect. Our assumption that the bubble was not pledgeable forced entrepreneurs to hold part of it. The remaining part was held by foreigners, and the latter had unbounded resources. Now, the overhang effect is a general-equilibrium effect. Entrepreneurs need not hold any part of the bubble. But the rest of the world no longer has unbounded resources. The larger is the bubble, the smaller are the resources that are available for capital accumulation.

To create bubbly environments, note first that the equilibrium interest rate is (weakly) increasing in the bubble. Calculate the steady-state interest rate in the absence of bubbles, i.e., in the history

21

in which the economy remains always in the fundamental state:

$$
R_\infty = \begin{cases} 1 & \text{if } \varepsilon < \delta \\ \min\left\{\dfrac{1-\delta}{1-\varepsilon}, \dfrac{\alpha \cdot \gamma}{1-\alpha} + 1 - \delta\right\} & \text{if } \varepsilon \geq \delta \end{cases}
\tag{27}
$$

Since the interest rate is weakly increasing in the bubble, any equilibrium bubble must grow at least at rate R_∞. Thus, we need $R_\infty < \gamma$ for bubbles to be possible. Otherwise, any bubble would eventually exceed the wealth of the economy. If $R_\infty < \gamma$, a bubble can always exist and its size will be limited by the need to keep the interest rate below the growth rate.[26]

The traditional literature on rational bubbles considers economies in which the borrowing limit is not binding and R_∞ equals the return to investment. This approach has been criticized on two grounds however. Under these conditions, bubbles are only possible if additional investments are dynamically inefficient and reduce steady state consumption. Abel et al. (1989) argued that this condition was not met in the data, and even though Geerolf (2013) has recently challenged this view, it is fair to say that most macroeconomists still believe that investment is dynamically efficient. The second critique is that, if the borrowing limit is not binding, bubbles are contractionary and should be associated with reductions in the capital stock and output. This seems to be contrary to empirical evidence that shows that asset prices tend to be pro-cyclical. Mostly for these two reasons, recent research on rational bubbles has focused instead on environments in which the borrowing limit is binding and R_∞ is lower than the return to investment. In this case, bubbles exist in environments in which average investment is dynamically efficient and bubbles are expansionary. We shall also follow this route here and, from now on, we focus exclusively on environments in which the borrowing limit is binding.

We conclude this section with a comment on the concept of storage. A literal interpretation of storage is inventory accumulation. But one could also interpret storage as low-quality capital, as in section 1.4. In this case the return to "storage" would be $\rho + 1 - \delta$ instead of one. Thus, a reduction in storage could also be interpreted as a reduction in inefficient investments. A simple variation on the model shows that there is also a third possibility. Assume now that the young attach a positive value to consumption during youth. In particular, young savers in generation t maximize,

$$
U_t = c_{t,t} + \beta \cdot E_t c_{t,t+1}
\tag{28}
$$

_____

[26]In some environments, bubbles lower the interest rate. In this case, they can exist even if $R_\infty > \gamma$. Interestingly, in this case small bubbles do not exist, but large enough bubbles that lower the interest below the growth rate do exist. See Martin and Ventura (2012) for an example.

where β is the rate of time preference. Now, the return to "storage" would be β^{-1} instead of one. Thus, a reduction in storage could also be interpreted as a reduction in early consumption. The appropriate interpretation of "storage" depends on the context in which the theory is applied, and the modeler has at least these three choices.

2.2 Are old bubbles always contractionary?

In the models we have developed so far new bubbles can only be expansionary while old bubbles can only be contractionary. It seems to us that, under reasonable assumptions, the notion that new bubbles are expansionary should be quite robust. But we know that the conclusion that old bubbles are contractionary is not robust. To show this, we use now a popular market psychology according to which no new bubbles are created, and either there is no bubble (the bubbleless equilibrium), or the bubble has existed forever and it is stationary (the bubbly equilibrium). This market psychology generates two possible steady-state situations with a constant bubble. A bubbleless steady state in which the interest rate equals R_∞, and a bubbly steady state in which the interest rate equals γ.[27]

Let us apply this market psychology first to the world economy of the previous section. The law of motion of the capital stock is (recall that we are assuming from now on that the borrowing limit is binding):

$$\gamma \cdot k_{t+1} = \frac{R_{t+1}}{R_{t+1} + \delta - 1} \cdot \varepsilon \cdot (1 - \alpha) \cdot A \cdot k_t^\alpha. \tag{29}$$

Thus, in any steady state we have that:

$$k = \left[\frac{R}{R + \delta - 1} \cdot \varepsilon \cdot \frac{(1 - \alpha) \cdot A}{\gamma} \right]^{\frac{1}{1-\alpha}} \tag{30}$$

In the bubbleless equilibrium $R = R_\infty$, while in the bubbly equilibrium $R = \gamma$. Since the capital stock declines with R, we just confirm our earlier finding that if bubbles exist, i.e. $R_\infty < \gamma$; they raise the interest rate and are therefore contractionary.

Let us now consider a simple modification of the model in which individuals live for three periods: youth, middle age and old age. The young are endowed with $1 - \varepsilon$ units of labor, while the middle-aged are endowed with ε units. Assume also that young individuals are savers, while middle-aged individuals are entrepreneurs. Superficially, this model looks almost identical to the one of the previous section. There are gains from intermediating resources between savers and entrepreneurs, but financial frictions – and the wealth of entrepreneurs – limits the extent to which

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[27]This type of modeling is quite restrictive, and we do not recommend it in applications. But it does simplify the analysis dramatically, and this makes it convenient for theoretical explorations of specific mechanisms.

this can be done. But the life-cycle structure of this model introduces a crucial innovation: now, the entrepreneurs of period t are the savers of period $t-1$, so that the wealth of the former depends on the return to the savings of the latter.

The law of motion of the capital stock in this modified economy is given by:

$$\gamma \cdot k_{t+1} = \frac{R_{t+1}}{R_{t+1} + \delta - 1} \cdot \left[\varepsilon \cdot (1 - \alpha) \cdot A \cdot k_t^\alpha + \frac{R_t}{\gamma} \cdot (1 - \varepsilon) \cdot (1 - \alpha) \cdot A \cdot k_{t-1}^\alpha \right]. \tag{31}$$

The key novelty here is that entrepreneurial wealth has two components: wages plus the return to savings. In any steady state, we now have that:

$$k = \left[\frac{R}{R + \delta - 1} \cdot \left(\varepsilon + \frac{1 - \varepsilon}{\gamma} \cdot R \right) \cdot \frac{(1 - \alpha) \cdot A}{\gamma} \right]^{\frac{1}{1-\alpha}} \tag{32}$$

It is not clear now whether the steady state capital stock is decreasing or increasing with R. On the one hand, a high interest rate lowers the multiplier and reduces entrepreneurial leverage. This is the familiar bubble overhang effect of old bubbles that lowers capital accumulation. On the other hand, a high interest rate raises entrepreneurial wealth and this raises capital accumulation. This is the *liquidity effect of old bubbles* since, by raising the interest rate, old bubbles make it cheaper to carry funds across periods. If bubbles exist, i.e. if $R_\infty < \gamma$, they still raise the interest rate. But this need not be contractionary.[28] To see this, consider the case in which $\delta = 1$, so that the financial multiplier equals one. The overhang effect vanishes, only the liquidity effect operates, and old bubbles raise capital accumulation.

2.3   Financial globalization with bubbles

Let us now bring our focus back to financial globalization. To do this, we consider a world with local factor markets and countries that differ in their level of financial development. In a subset of countries $j \in C$, which we call Core, domestic courts can seize the revenues obtained through the sale of firms so that their entrepreneurs face the familiar borrowing limit in Equation (3). In the remaining set of countries $j \in P$, which we call Periphery, domestic courts are unable to seize any income and their entrepreneurs cannot borrow. Let π be the share of the world population that lives in Periphery countries. Throughout, we consider two possible market psychologies for each country j. The first one is the fundamental one in which there are no bubbles $n_{j,t} = 0$. The second one is the bubbly market psychology we used in section 1.3 in which $n_{j,t}$ is given by Equation (9).

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[28]In this modified model $R_\infty = \max \left\{ 1, \frac{1 - \delta}{1 - \varepsilon} \cdot \frac{\gamma}{\gamma + \delta - 1} \right\} < \gamma.$

Consider an initial situation in which Core countries participate in a world credit market, but Periphery countries do not (1970s and 1980s). The capital stocks of the different countries evolve as follows

$$\gamma \cdot k_{j,t+1} = \frac{R_{t+1}}{R_{t+1} + \delta - 1} \cdot \left[\varepsilon \cdot (1 - \alpha) \cdot A \cdot k_{j,t}^{\alpha} + n_{j,t}\right], \text{ for } j \in C, \tag{33}$$

$$\gamma \cdot k_{j,t+1} = \varepsilon \cdot (1 - \alpha) \cdot A \cdot k_{j,t}^{\alpha} + n_{j,t}, \text{ for } j \in P. \tag{34}$$

The main difference between capital accumulation in Core and Periphery countries is the size of the financial multiplier. This multiplier is above one in Core countries, since their entrepreneurs are subject to the borrowing limit in Equation (3). The financial multiplier is one in Periphery countries, since their entrepreneurs cannot borrow. The interest rate in Core countries is given by:

$$R_{t+1} = \max \left\{ \frac{\gamma \cdot \sum\limits_{j \in C} [(1 - \delta) \cdot k_{j,t+1} + E_t b_{j,t+1}]}{(1 - \varepsilon) \cdot (1 - \alpha) \cdot A \cdot \sum\limits_{j \in C} k_{j,t}^{\alpha}}, 1 \right\}. \tag{35}$$

If their combined entrepreneurial collateral is large enough relative to the wealth of their savers, the interest rate in Core countries is determined by "cash-in-the-market" and storage is not used in equilibrium. Otherwise, the interest rate equals one and storage is used in equilibrium. In Periphery countries, the interest rate equals one so that savers are indifferent between storage and credit. However, this interest rate is only notional since the equilibrium amount of credit is zero. Entrepreneurs would like to borrow at this rate, but they have no collateral to offer. Periphery savers are forced to use storage.

Let us assume that:

$$\gamma < \frac{1 - \delta}{1 - \varepsilon} < \varepsilon^{-1} \cdot \frac{\alpha \cdot \gamma}{1 - \alpha} + 1 - \delta. \tag{36}$$

In Core countries, bubbles are fully pledgeable and their expected growth must equal the world interest rate. But the first inequality in assumption (36) says that this interest rate is above the growth rate. In Periphery countries, bubbles are not pledgeable and their expected growth in equilibrium must equal the return to investment. But the second inequality in assumption (36) says that this return to investment is above the growth rate. Thus, bubbles are not possible in any country. The only feasible market psychology is the fundamental one and the Core-Periphery world exhibits quiet dynamics. There are no shocks and all countries monotonically converge to their respective steady states. Given that countries differ only in their level of financial development, there is full convergence within regions but not across regions. Core countries converge to a steady state with a higher capital stock and a higher interest rate than Periphery countries.

Let us consider now what happens if Periphery countries join the world credit market (1990s-today). The world interest rate, which now applies to all countries, becomes:

$$R_{t+1} = \max\left\{ \frac{\gamma \cdot \sum\limits_{j \in C} [(1-\delta) \cdot k_{j,t+1} + E_t b_{j,t+1}]}{(1-\varepsilon) \cdot (1-\alpha) \cdot A \cdot \sum\limits_{j \in C \cup P} k_{j,t}^{\alpha}}, 1 \right\}. \tag{37}$$

The entry of Periphery countries into the world financial market does not raise entrepreneurial collateral, because Periphery entrepreneurs are still unable to borrow due to the inefficiency of their domestic courts. But Periphery savers can now lend to Core entrepreneurs, and this raises the wealth of savers in the market. Thus, the world interest rate drops and capital flows from Periphery to Core. These capital flows raise world efficiency because they convert Periphery storage into Core capital.

If the collective size of Periphery countries is sufficiently large relative to that of Core countries, capital flows are not large enough to eliminate all Periphery storage and the world interest rate drops to one.[29] The entry of Periphery countries into the world financial system has created a bubbly environment! How do bubbles affect financial globalization? To answer this question, we simulate a six-country version of the Core-Periphery world, two of which are in Core. Figure 6 plots the evolution of the aggregate capital stock, bubble, and current account deficit for each of the countries in Core.[30] The figure assumes that all countries are in the steady state when Periphery joins the world financial system. The dashed lines depict the benchmark dynamics with the fundamental market psychology, whereas the solid lines depict the evolution of these same variables for a simulated realization of the bubbly market psychology.

There are three main takeaways from Figure 6. First, bubbles reinforce the effects of financial globalization. By creating collateral in Core, bubbles sustain additional capital flows and deepen financial integration between Core and Periphery. In the simulation, both the capital stock and the current account deficit in each Core country rises and falls alongside its bubble. From the perspective of an outside observer, bubbly episodes are accompanied by a high degree of *de facto*

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[29]In particular, this requires that $\pi > \dfrac{\delta^{\frac{\alpha}{1-\alpha}} \cdot (1-\delta)}{1 - \varepsilon + \delta^{\frac{\alpha}{1-\alpha}} \cdot (1-\delta)}$. Note that this is a sufficient but not necessary condition for creating a bubbly environment. Even if all storage is eliminated, the interest rate can fall below the growth rate.

[30]In this version of the model, the capital stocks of Periphery countries are not affected. The reason is that entrepreneurs cannot borrow. But this need not be the case if entrepreneurs also save. In a multi-country version of the life-cycle model of Section 2.2., for instance, bubbles would be a source of liquidity regardless of the country in which they are located, so that bubbles in Core could lead to an expansion in both, Core and Periphery.

financial integration, whereas reversals to the fundamental state are accompanied by retrenchment. This leads to the second point, which is that the capital flows sustained by bubbles are volatile. Indeed, as the figure shows, Core countries experience surges in inflows and sudden stops driven solely by market psychology. Third, because they are country-specific, bubbles lead to dispersion within Core. Although both Core countries are fundamentally identical to one another, market psychology can favor any one of them over the other. The general insight here is that the global allocation of savings will be determined both by bubbles and by productivity and, in principle, there is no reason for both forces to coincide.

2.4 Managing bubbles

Up to now, we have focused exclusively on the positive aspects of bubbles, namely when they can exist and how they affect macroeconomic dynamics. We have shown the reader how the theory of rational bubbles can help us interpret the boom-bust episodes of recent years. We now turn to some normative issues and explore both the desirability of bubbles and the role, if any, of policy in managing them. Both questions have been the object of a lively, if often unstructured, debate in the academic and policy communities. But the theory of rational bubbles has much to offer to this debate.

In the bubbly economy, market psychology is an essential component of equilibrium. This raises two central questions. Which is the most desirable market psychology? Can the policy maker implement it? Much has been written about these questions, and a thorough treatment of them would require more space than we have at our disposal. We therefore address them within the context of a particular example, the Core-Periphery world of the previous section. Although the setting is specific, the insights that it delivers are easily generalizable to other environments.

2.4.1 What should governments do?

We return to the Core-Periphery world and assume that there is a global planner with the ability to coordinate the market on its preferred psychology. Which one would she select? Answering this question requires defining the objective function of the planner. This is not trivial because bubbles entail a complex web of intra- and inter-generational transfers. One common approach, which we adopt here, is to assume that the planner's goal is to maximize some measure of welfare in the steady state. Since the only source of uncertainty in this world is market psychology itself, the planner selects a "deterministic" market psychology with a constant rate of bubble creation , i.e.,

$n_{j,t} = \eta_j$, and no shocks to the return of old bubbles, i.e., $g_{j,t+1} = R_{t+1}$, for all t and $j \in C$. Note that we need only specify the market psychology for Core, as we know that in Periphery the only admissible psychology is $n_{j,t} = b_{j,t} = 0$, for all t and $j \in P$.

All market psychologies of this type yield a deterministic steady state, which is characterized by the following set of equations:

$$k_j = \frac{g}{g + \delta - 1} \cdot \left[\varepsilon \cdot A \cdot (1 - \alpha) \cdot k_j^\alpha + \eta_j \right] \quad \text{for } j \in C, \tag{38}$$

$$k_j = \left[\frac{\varepsilon \cdot A \cdot (1 - \alpha)}{\gamma} \right]^{\frac{1}{1-\alpha}} \quad \text{for } j \in P, \tag{39}$$

$$g = \max \left\{ \frac{\gamma \cdot \sum_{j \in C} \left[(1 - \delta) \cdot k_j + b_j \right]}{(1 - \varepsilon) \cdot (1 - \alpha) \cdot A \cdot \sum_{j \in C \cup P} k_j^\alpha}, 1 \right\}. \tag{40}$$

where g denotes the world interest rate, which must equal the growth rate of bubbles in all countries. Equation (38) implicitly defines the steady-state capital stock in Core countries solely as a function of g and η_j. Equation (39) instead shows that, because its entrepreneurs are cut-off from the credit market, the capital stock in Periphery is independent of bubbles and the world interest rate. Equation (40), in turn, characterizes the world interest rate. Finally, note that there are two market psychologies under which $b_j > 0$: if $g = \gamma$, it must be that $\eta_j = 0$, or; if $g < \gamma$, it must be that $\eta_j > 0$ and $b_j = \eta_j \cdot g \cdot (\gamma - g)^{-1}$.[31]

Which psychology would a global planner choose? To answer this question, let us first characterize the set of Pareto-optimal allocations. This may seem like a complicated task, given that this world is populated by savers and entrepreneurs in many different countries. But it is actually quite simple.[32] An allocation can only be Pareto-optimal if storage is not used. The intuition is simple: if storage is being used, it is always possible to increase bubble creation – and thus investment – in some country without reducing investment (and therefore welfare) anywhere else. Once

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[31]Indeed, by evaluating Equation (8) in steady state and setting $n_{j,t} = \eta_j$ for all t, we obtain,

$$\gamma \cdot b_j = g \cdot (b_j + \eta_j), \quad \text{for } j \in C,$$

so that $b_j > 0$ only if $\gamma = g$ and $\eta_j = 0$, or if $\gamma > g$ in which case,

$$b_j = \frac{g}{\gamma - g} \cdot \eta_j.$$

[32]In Core, the welfare of entrepreneurs equals only the rental income of capital because undepreciated capital has been fully pledged to creditors. In Periphery, instead, the welfare of entrepreneurs equals the entire capital income. The steady-state welfare of savers, in turn, equals their labor income times the market interest rate.

storage is completely eliminated, any additional increase in bubble creation comes at the expense of investment somewhere else and cannot be Pareto-improving.

Indeed, there is a continuum of Pareto-optimal allocations in the Core-Periphery world, each of them sustained by a different market psychology. Although the use of storage is eliminated in all of them, these allocations differ in their bubbles, their capital stocks, and their interest rates, which can lie anywhere between 1 and γ. Whether one market psychology is preferred over another depends on the relative weights that the global planner assigns to the different type of individuals. If the planner favors entrepreneurs, she will prefer allocations with the lowest interest rate to maximize investment. If instead the planner favors savers, she may prefer allocations with higher interest rates even if this comes at the expense of investment.

This discussion illustrates some key normative implications of the theory of rational bubbles. Perhaps the main one is that, far from the widespread notion that bubbles are undesirable, the theory suggests that they can be Pareto-improving.[33] Naturally, different bubbles have different effects on global output and its distribution.[34] But characterizing the set of optimal bubbles is only the first step towards a proper analysis of policy. The other is thinking about implementation, which raises further questions of its own. First, the world is not run by a global planner but rather by a collection of individual governments. It is natural to expect each government to favor bubble creation in its own economy even if this comes at the expense of investment elsewhere in the world. As in other areas of international economics, this creates a need for policy coordination on which we shall not elaborate here. We turn instead to a second crucial point: once governments have decided on their preferred market psychology, what can they do to induce it?

2.4.2 What can governments do?

This section considers the problem of governments that, having settled on a preferred market psychology, want to implement the corresponding allocation. One possible approach is to assume that governments can choose this psychology directly, for instance, by forcing individuals to act in a certain way and punishing those that deviate. This is not very interesting, however. An

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[33]This need not be the case, of course, as bubbles may also entail costs. They may redirect investment towards unproductive uses, as we have seen, they may be too volatile, or they may become too large thereby leading to inefficiently severe crises when they burst (see, for instance, Caballero and Krishnamurthy 2006).

[34]These effects of bubbles, moreover, are model-specific. In the Core-Periphery world, for instance, it is the wealth effect of bubble creation that enables entrepreneurs to expand their investment. In the life-cycle model of Section 2.2., instead, the liquidity effect of old bubbles can do the trick.

alternative approach, which we have developed over various papers and follow here, is to endow governments with specific policy tools and study how these tools can be used to re-create the transfers implemented by the preferred bubble, thereby implementing the desired allocation.[35] Since bubbles implement these transfers through the credit market, it is reasonable to begin by looking at credit market interventions.

Let us assume that governments want to implement the Pareto-optimal allocation associated with market psychology $\{\eta_j\}_{j \in C}$. To do so, we assume that governments in Core are able to subsidize and/or tax their entrepreneurs. In particular, we assume that the government of country $j \in C$ promises to give its entrepreneurs of generation t a gross transfer of $s_{j,t+1}$ units of the consumption good during old age. These transfers can be negative and/or contingent on the state of the economy, and we assume that they can be pledged by entrepreneurs to outside creditors. We do not allow for cross-country subsidies, though. Thus, positive transfers in country $j \in C$ are financed through a tax on the country's young entrepreneurs; negative transfers, in turn, generate a revenue that is distributed among young entrepreneurs.

These interventions can be thought of as credit management policies. Given policies $\{s_{j,t}\}_{j \in C}$ for all t, we can define:

$$n_{j,t}^s = \frac{\gamma \cdot E_t s_{j,t+1}}{g} - s_{j,t}. \tag{41}$$

Thus, $n_{j,t}^s$ represents the net transfers to young entrepreneurs in country j, i.e., the difference between the present value of subsidies that they will obtain in old age and the taxes paid in young age. This difference can be positive or negative. The key observation is that there are now three sources of entrepreneurial wealth: wages, new bubbles, and net transfers, i.e., entrepreneurial wealth equals $w_{j,t} + n_{j,t} + n_{j,t}^s$. Moreover, there are now three sources of collateral: undepreciated capital, bubbles, and gross transfers, i.e., collateral equals $(1 - \delta) \cdot k_{j,t+1} + E_t \{b_{j,t+1} + s_{j,t+1}\}$.

As this discussion makes it clear, these interventions replicate the effects of bubbles. For instance, to implement the desired allocation, governments simply need to set $s_{j,t} = b_j - b_{j,t}$, which implies that $n_{j,t}^s = \eta_j - n_{j,t}$ for any market psychology $\{n_{j,t}\}_{j \in C}$. That is, governments can replicate any allocation by appropriately designing state-contingent policies! In our case, this requires "leaning against market psychology". In countries and times where the bubble is low relative to the desired benchmark, the policy requires the corresponding government to subsidize credit. In countries and times where the bubble is high relative to the desired benchmark, the policy requires the corresponding government to tax credit.

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[35]See, for instance, Martin and Ventura (2015, 2016).

A salient feature of these policies is that they are *expectationally robust*, in the sense that they implement the desired allocation regardless of market psychology. That is, they do not target asset prices, but instead insulate the economy from their fluctuations. Note that these policies achieve their objectives even if their adoption changes market psychology.

Bubbles create a web of inter and intra generational transfers. The key insight of this section is that, if governments have the necessary tools to replicate these transfers, they can always replicate the desired bubble. We have focused here on the case of taxes and subsidies on credit, but nothing changes if we allow governments to finance these policies through debt. Moreover, depending on the setup, these interventions can also be interpreted as capital controls or bailouts.

What if governments do not have this full set of tools at their disposal? They may, for instance, have only a limited ability to tax and/or subsidize entrepreneurs. Or, as in some monetary models, they may be able to influence the real interest rate, which in turn determines the equilibrium growth rate of old bubbles. Although such limitations certainly curtail governments' ability to replicate a desired bubble, they do not change the general nature of the problem. At times, the observed bubble is too small relative to the desired one and policy should try to raise its size; at other times, the observed bubble is too large relative to the desired one and policy should try to reduce its size.

3   A guide to the literature

In the previous sections, we have used simple models and examples to convey the main positive and normative insights of the theory of rational bubbles for macroeconomics. We now provide a brief guide to the literature that has developed these insights and built on them to generate additional results.

3.1   The traditional view of bubbles

The seminal papers in the theory of rational bubbles are Samuelson (1958) and Tirole (1985). Samuelson showed how, in an endowment economy with overlapping-generations, rational bubbles could offer a remedy to the problem of dynamic inefficiency. Tirole extended these insights to the classic Diamond (1965) model of capital accumulation, providing the first full treatment of bubbles in a production economy. We describe below his results.

In the absence of frictions, the interest rate in the Diamond (1965) model equals the marginal return to investment, i.e., the return to capital accumulation. It is well-known that the equilibrium

in this model can be dynamically inefficient, which happens whenever the steady-state interest rate lies below the growth rate of the economy. These are situations in which the capital stock is excessive and its return is too low. Tirole (1985) showed that, in such situations, there exists an alternative *bubbly* steady state in which agents are willing to hold rational bubbles. This steady state has two key features relative to the standard *fundamental* one. First, both the capital stock and output are lower, because bubbles divert resources away from capital accumulation. This is nothing but the overhang effect of old bubbles that we have discussed so extensively. Second, welfare is higher, because the capital stock eliminated by the bubble is inefficient to begin with. Both results correspond exactly to our analysis of the world economy of section 2.1, for the case in which the borrowing limit is not binding.

These first findings were derived in highly stylized models and it was not obvious how robust they were to some natural generalizations of the environment. One such generalization is the inclusion of non-reproducible factors of production such as land, which provide rents to their owners. In equilibrium, the price of these factors must equal the discounted value of the stream of rents that they are expected to generate in the future. This is problematic, because dynamic inefficiency – which generates the low interest rates necessary for the existence of bubbles – may imply infeasibly high prices of non-reproducible factors. Tirole (1985) partially explored this connection and showed that sometimes, in the presence of rents, the unique market psychology must feature a bubble! Rhee (1991) focused specifically on land, and showed that its presence is not incompatible with dynamic inefficiency insofar as the land share of output vanishes asymptotically.

Another unappealing feature of this early work is that it focused on deterministic bubbles, which display a very predictable behavior and never burst. The fact that real-world bubbles are anything but predictable prompted the study of stochastic bubbles like the ones we have used throughout this guide. Blanchard (1979) and Blanchard and Watson (1982) used partial-equilibrium asset-pricing models to show that such bubbles can be consistent with rational expectations. They also argued that stochastic bubbles could display a wide range of behavior, and studied conditions under which their presence could be verified econometrically. Weil (1987) studied the existence of stochastic bubbles in general equilibrium, extending Tirole (1985) to the case of bubbles that have a constant, exogenous, probability of collapsing.

This theory of rational bubbles provided an elegant and powerful way to think about real-world bubbles, suggesting that it would be widely used by macroeconomists. But this did not happen. A first drawback of the theory is its prediction that bubbles can only arise under low interest

rates, which – in the basic models discussed above – is equivalent to dynamic inefficiency. As we anticipated in section 2.1, the empirical validity of the latter was questioned in an influential paper by Abel et al. (1989). Although their findings have been recently questioned by Geerolf (2013), the predominant view has been for many years that real economies are dynamically efficient. This appeared to limit the relevance of the theory of rational bubbles, but only if one sticks to the frictionless models in which the theory was first developed. In the presence of frictions, in fact, the interest rate need not accurately reflect the return to capital and bubbles may exist even if the economy is efficient.

One set of papers made this point in the context of endogenous growth. Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993), for instance, extended the analysis of Tirole (1985) to economies with endogenous growth due to externalities in capital accumulation. The key insight of these papers is that, in the presence of such externalities, the market interest rate underestimates the true return to investment. Thus, the interest rate can be low enough for bubbles to exist even if the economy is dynamically efficient. In these models, bubbles reduce capital accumulation exactly as in Tirole (1985). But their welfare implications are different because these economies accumulate *too little*, as opposed to *too much*, capital to begin with. By reducing capital accumulation, bubbles slow down the long-term growth rate of the economy so that there always exists a future generation that is harmed. Note that these insights are closely related to our discussion of the Dutch disease in section 1.4., and they would also arise in our example in the presence of externalities.

Another set of papers showed that bubbles could arise in dynamically efficient economies if there are financial frictions. Woodford (1990), Kocherlakota (1992), Azariadis and Smith (1993), and Woodford and Santos (1997), for instance, showed how frictions such as borrowing constraints and adverse selection could depress equilibrium interest rates and create bubbly environments. The role of various financial frictions in creating this type of environments has since then been analyzed in the literature, from the lack of insurance markets (e.g. Aoki et al. (2014)) to borrowing limits like the ones we have explored here (e.g. Hirano and Yanagawa (2016)).

3.2   A new view of bubbles

Even if one accepts that the existence of bubbles does not require dynamic inefficiency, there is a second implication of the basic models that seemed unpalatable; namely, bubbles crowd out capital accumulation and reduce output. This prediction seemed particularly troubling in the aftermath of

the dot-com boom and bust of the late 1990s, during which the performance of the United States economy closely tracked the growth and subsequent collapse of the stock market. If there was ever a boom-bust episode where the theory of rational bubbles should apply, this was it. And yet, it highlighted the theory's limitations.

To address this issue, Olivier (2000) used an endogenous growth model to show that the effects of bubbles depend on the asset on which they appear. Bubbles on equity, in particular, can raise the market value of firms thereby enhancing firm creation, investment, and growth. Bubbles on unproductive assets, in turn, have the typical crowding-out effect and reduce growth. The key insight of the model, in fact, is that bubbles on equity have similar effects to subsidies on R&D. This is closely related to the subsidy effect of section 1.4., with the difference that Olivier (2000) focuses on the case in which the bubble subsidizes productive investment.

But most of the recent literature on rational bubbles has focused instead on models with financial frictions. This has been partly due to the widespread introduction of financial accelerator mechanisms, like those in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), into macroeconomic models. Financial frictions not only relax the conditions for the existence of bubbles, as we have already mentioned, but they also enable bubbles to expand the capital stock and output. Different strands of literature emphasize different mechanisms for this expansionary effect, most of which have already been discussed throughout this guide. The common theme that underlies all of them is that bubbles enable the transfer of resources, from agents who do not want to invest to those that do.

A first strand of the literature stresses the wealth effect of bubble creation, which we first introduced in Martin and Ventura (2012) and has since then been explored in a series of papers. In these models, newly created bubbles reallocate resources because they are sold by productive to unproductive agents, either directly or indirectly through the credit market. This wealth effect of bubble creation has been a recurrent theme throughout sections 1 and 2, and we shall not dwell on it here. It suffices to say that all the main insights derived in the context of our stylized examples survive in richer environments, with more general preferences and alternative credit market specifications (see Martin and Ventura 2011, 2015 and 2016).

A second strand of literature stresses the role of bubbles as providers of liquidity, as in the model of section 2.2. In these models, bubbles enable agents to transfer resources over time, from periods in which they are unproductive to periods in which they are productive. Thus, rational agents are willing to hold bubbles because they expect to sell them or borrow against them in the

future, when the time comes to invest. Some examples of this line of research include Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2012), Miao and Wang (2017) and Guerron-Quintana et al. (2017).

Finally, a third strand of literature highlights the general equilibrium effects of bubbles, which can induce price changes that enable the transfer of production factors from unproductive to productive agents. Ventura (2012) and Ventura and Voth (2015), for instance, study environments in which unproductive and productive agents compete for existing factors of production like capital and labor. When unproductive agents purchase bubbles, they reduce their production and thus their demand for capital and/or labor. The consequent fall in the price of these factors enable productive but constrained agents to expand their production, thereby leading to a boom.

3.3 Macroeconomic applications

Building on these insights, there has been an explosion of research in recent years trying to explain a wide array of macroeconomic phenomena.

Let us start with financial markets. There is mounting evidence of a link between asset bubbles, credit booms and busts, and financial crises (e.g. Schularik and Taylor (2012), Jordà et al. (2015)). Martin and Ventura (2016) and Miao and Wang (2017) explore this connection in general equilibrium models of credit and bubbles, in many ways similar to the world economy of section 2.1. In these models, the bursting of a bubble pushes the economy into a recession but it does not lead to defaults or unemployment because credit contracts are state-contingent and prices are flexible. In related settings, Ikeda and Phan (2016) and Bengui and Phan (2016) introduce non-contingent contracts to generate defaults, while Miao and Wang (2016) and Hanson and Phan (2017) introduce nominal rigidities to generate unemployment.

Delving deeper into financial markets, various papers have explicitly introduced financial intermediaries to analyze how their presence shapes the effects of bubbly booms and busts. Aoki and Nikolov (2015), Miao and Wang (2015), Martin and Ventura (2016 online appendix), and Freixas and Perez-Reyna (2016) are some examples in this regard. In general, these models are similar to the examples explored throughout this guide, with the difference that both entrepreneurs and financial intermediaries are subject to borrowing limits. Bubbles can relax or tighten these limits for financial intermediaries through the same mechanisms outlined above. The key insight is that it is not just the size of the bubble, but also its location – i.e., whether it is created and/or held by financial intermediaries or entrepreneurs – that matters for economic activity.

The location of a bubble is also important in determining the allocation of resources across firms and sectors. Queirós (2017a) provides empirical evidence in this regard. Miao and Wang (2014), Basco (2016) and Tripathy (2017) develop models to study the role of bubbles in determining the cross-sector allocation of resources. Much in the spirit of the subsidy effect discussed in section 1.4, these papers identify conditions under which bubbles can reduce efficiency (and potentially growth) by reallocating resources towards less productive uses. One specific sector that has received much attention is housing, both because of its size and because it exhibits large fluctuations. Arce and Lopez-Salido (2011), Basco (2014), Zhao (2015), and Huber (2016) develop models of rational bubbles to study the macroeconomic implications of fluctuations in housing. Finally, for all this sector-level research, there has been almost no work on the firm-level implications of bubbles. This is somewhat surprising given the growing interest of macroeconomists on firm dynamics. Two exceptions are Tang (2017) and Queirós (2017b), who study models of heterogeneous firms with entry and exit and explore the effects of bubbles on competition, on average productivity, and on its dispersion across firms.

On the international sphere, bubbles affect the allocation of resources across countries, as we illustrated through the Core-Periphery world. The role of bubbles in shaping the size, direction, and volatility of capital flows has been explored in a number of papers. Kraay and Ventura (2007) and Ikeda and Phan (2015) use models of rational bubbles to account for global imbalances between the United States and the developing world. Basco (2014) and Rondina (2017) are closest to the Core-Periphery example in section 2.3, as they provide formal models in which, due to asymmetric financial development, financial globalization itself creates a bubbly environment. In Martin and Ventura (2015), we construct a multi-country model and explore the positive and normative implications of bubbles for capital flows and for the global allocation of resources.

Finally, there is a large literature on the ability (or not) of macroeconomic policy to manage bubbles and their effects. Here, the desirability and effectiveness of policy depends on the tools at the authority's disposal. In Martin and Ventura (2015, 2016), we provide a formal analysis of the policy results illustrated in section 2.4, which assume that the government has the necessary tools to reproduce the transfers induced by bubbles. In Martin and Ventura (2011, 2016), we also explore whether a government with limited ability to tax can nonetheless manage bubbles by issuing public debt. Arce and Lopez Salido (2011) study policy interventions meant to eliminate bubbles on the "wrong" assets, which generate distortions, and foster them on the "right" assets. Other papers have focused not on the ability of policy to replicate the desired bubble, but rather on its role in

preventing crises or mitigating their effects. These interventions can be both ex ante in the form of macroprudential regulation (e.g. Caballero and Krishnamurthy 2006), or ex post in the form of bailouts (e.g. Hirano et al. 2015).

Most of this literature on policy has a fiscal flavor to it, focusing almost exclusively on the role that taxes and subsidies can play in managing bubbles. The role of monetary policy has been much less explored, although it has featured prominently in the academic and policy debate for a long time (e.g. Bernanke and Gertler 2001). This is now changing. Gali (2014) and Barlevy et al. (2017) study how a monetary authority can, by controlling the interest rate, affect the evolution of bubbles. Asriyan et al. (2016) construct a model of money, credit and bubbles and show that market psychology can itself be a source of nominal rigidity if expectations are set in nominal terms. They also explore the general equilibrium effects of bubbles, and of monetary policy, both outside and inside of the liquidity trap. More recently, Gali (2017), Ikeda (2017) and Dong et al. (2017) explore the interaction of monetary policy and bubbles in models with nominal rigidities and long-lived agents, which renders these models more suitable for quantitative analysis.

The key takeaway from these papers is not that policy should eliminate bubbles, as is commonly argued in informal discussions. Rather, policy should "lean against market psychology", making the bubble larger when it is too small and reducing it when it is too large.

This brings us to our last point on this tour of the literature. Most of the papers surveyed here use OLG structures with short lifespans, which are extremely useful to explore the theoretical mechanisms at hand. These models are hard to solve once individuals are long-lived, however, because they give rise to a large degree of inter-generational heterogeneity. This is one of the reasons for which macroeconomics has adopted models with infinitely-lived agents for quantitative work. These models, however, are known to be inconsistent with rational bubbles. So how do we construct macroeconomic models of rational bubbles for quantitative analysis?

There are currently two approaches being explored in the literature. The first is to develop OLG models with perpetual youth as in Yaari (1965) and Blanchard (1985), in which agents face a certain probability of death: these models have the advantage of preserving the OLG structure, and thus all of the basic results discussed above, while maintaining tractability for quantitative analysis. This is the approach recently adopted by Gali (2017). The second approach, used by Kocherlakota (2009) and Miao and Wang (2017) among others, is based once again on financial frictions. In a nutshell, it consists of assuming that some assets have a special status as collateral. From the perspective of potentially constrained agents, bubbles on these assets have the additional

benefit – over and above their growth rate – of relaxing the borrowing limit. Hence, bubbles need not grow at the rate of interest to be attractive and may exist in infinite horizon models. Although an in-depth discussion of this approach is beyond the scope of this guide, we refer the interested reader to Miao (2014) for a survey.

4   Where do we go from here?

This paper has provided a user's guide to the theory of rational bubbles for macroeconomists. It has sought to introduce macroeconomists to the main positive and normative aspects of the theory, and to illustrate its usefulness in accounting for important macroeconomic phenomena. We would like to conclude by pointing out the two main challenges that we see as the literature moves forward.

The first challenge is empirical. To assess the macroeconomic effects of bubbles, we need to measure them. This is difficult, because the fundamental value of an asset is not observable. Thus, any attempt to compare an asset's market and fundamental values is only as reliable as the underlying model that is used to assess fundamentals. This problem lies at the heart of the large discrepancy of findings in the empirical literature on bubbles.[36] Most of this literature, moreover, tests for the existence of bubbles in a particular asset or asset class. From a macroeconomic standpoint, it seems more relevant to measure the aggregate bubble, i.e. the bubble component of the economy's entire stock of assets.

Measuring this aggregate bubble may seem like wishful thinking. But macroeconomists have experience measuring unobservables, most notably the aggregate productivity of an economy. More than half a century ago, Solow taught the profession how to use aggregate input and output data to derive a model-based measure of aggregate TFP. In much the same vein, macroeconomists could use data on asset prices and aggregate capital income from the national accounts to derive a model-based measure of the economy's aggregate bubble, or an aggregate "bubble residual". In Carvalho et al. (2012), we took a first step in this direction and used national accounts to estimate the fundamental value of household net worth in the United States between 1950 and 2010. We found large discrepancies between this estimated value and the actual net worth as reported in the Federal Reserve's flow of funds data, especially during recent decades. But our exercise merely scratched the surface, and there is much to be done along this front.
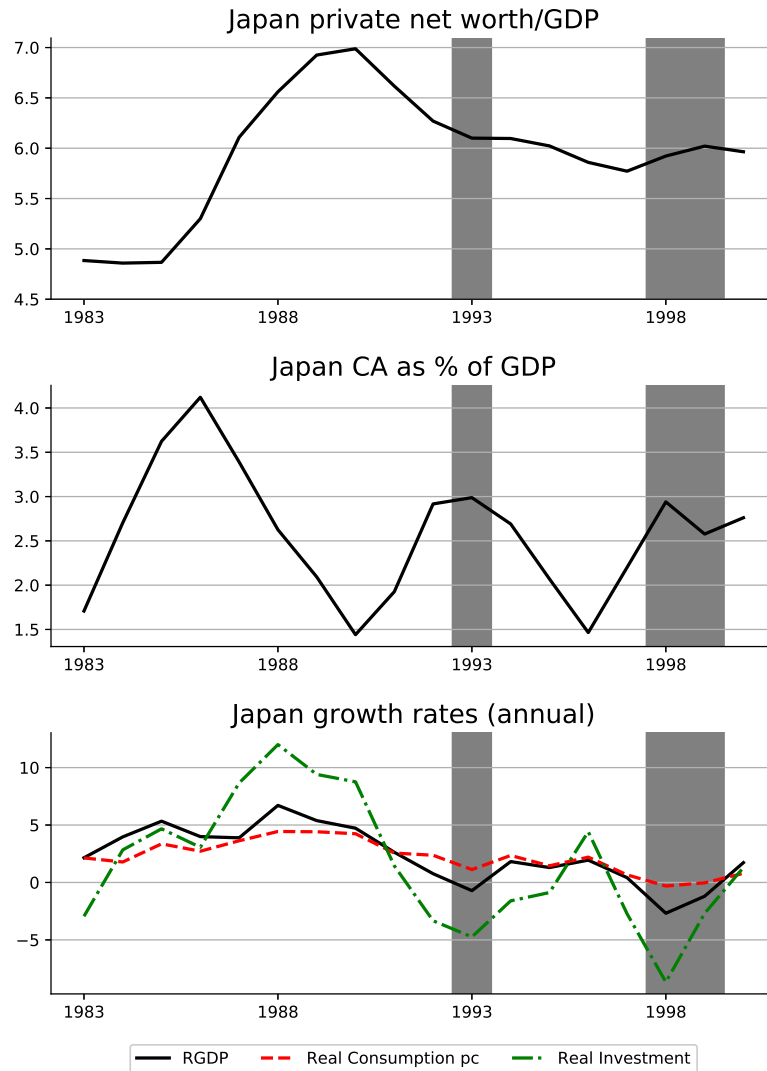
The second challenge is both empirical and conceptual. One of the running themes of this guide

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[36]Two excellent examples are Froot and Obstfeld (1991) and LeRoy (2004). See Gurkaynak (2008) for a survey.

is that, in standard macroeconomic models, multiplicity may be more prevalent than we typically care to admit. Once we accept that there may be many market psychologies that are consistent with rationality, how do we chose among them? Should we choose the market psychology that, within a set of constraints, best fits the data? This is the approach recently used by Guerron-Quintana et al. (2017), who develop a macroeconomic model of bubbles and use it to estimate the persistence and volatility of productivity and psychology shocks on post-war US data (incidentally, the data is strongly consistent with the presence of bubbles). An alternative and complementary strategy is to appeal to theory and develop new refinement techniques that can be used to reduce the set of admissible market psychologies.
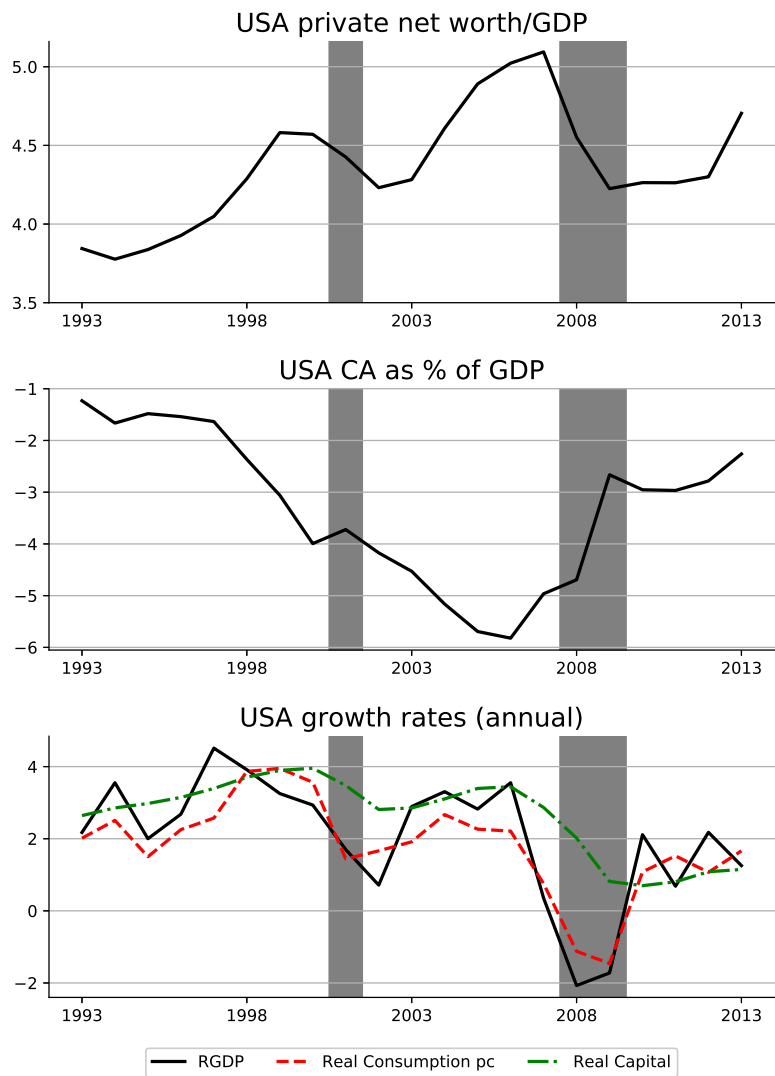
These challenges are far from minor, and they raise difficult questions going forward. But they also point to an exciting path ahead, one that must be tread on if the theory of bubbles is to become an integral part of the macroeconomist's toolkit.
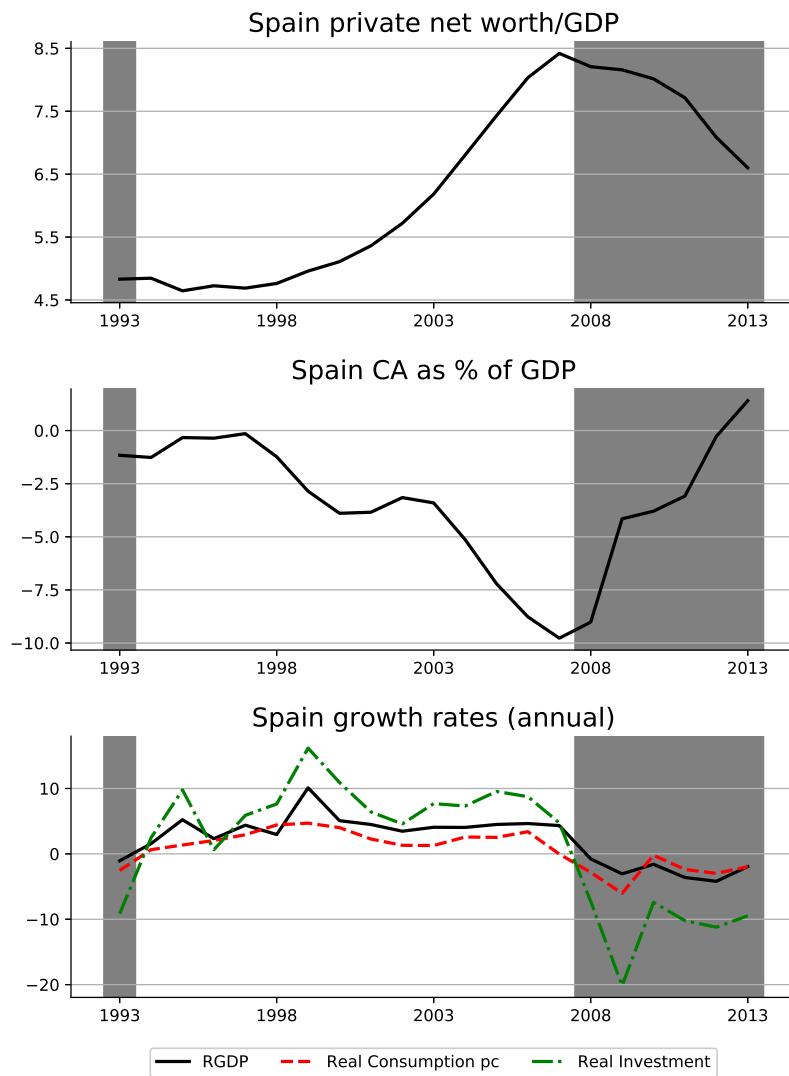
5 Figures

Japan private net worth/GDP

Japan CA as % of GDP

Japan growth rates (annual)

RGDP   Real Consumption pc   Real Investment

Sources: wealth data: world wealth and income database, other data: JST (2017)

Figure 1: Bubbly episode in Japan.

USA private net worth/GDP

USA CA as % of GDP

USA growth rates (annual)

| RGDP | Real Consumption pc | Real Capital |

Sources: wealth data: world wealth and income database, other data: JST (2017)

Figure 2: Dot-com and housing boom in USA.

Spain private net worth/GDP

Spain CA as % of GDP

Spain growth rates (annual)

RGDP — — — Real Consumption pc — · — Real Investment

Sources: wealth data: world wealth and income database, other data: JST (2017)

Figure 3: Housing boom in Spain.
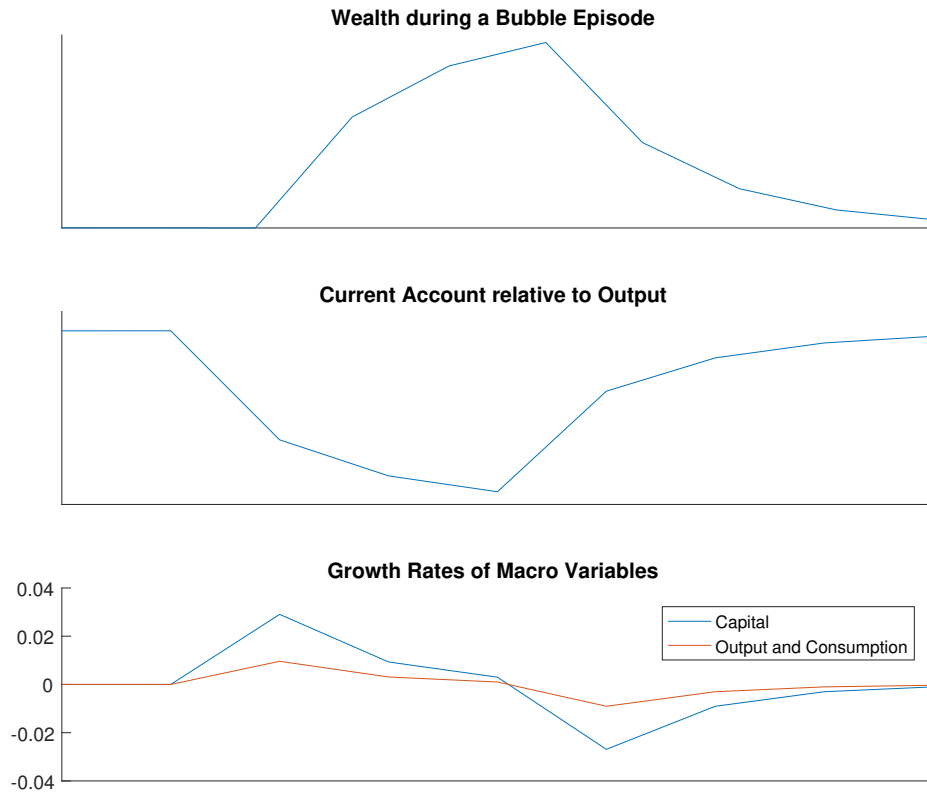
42
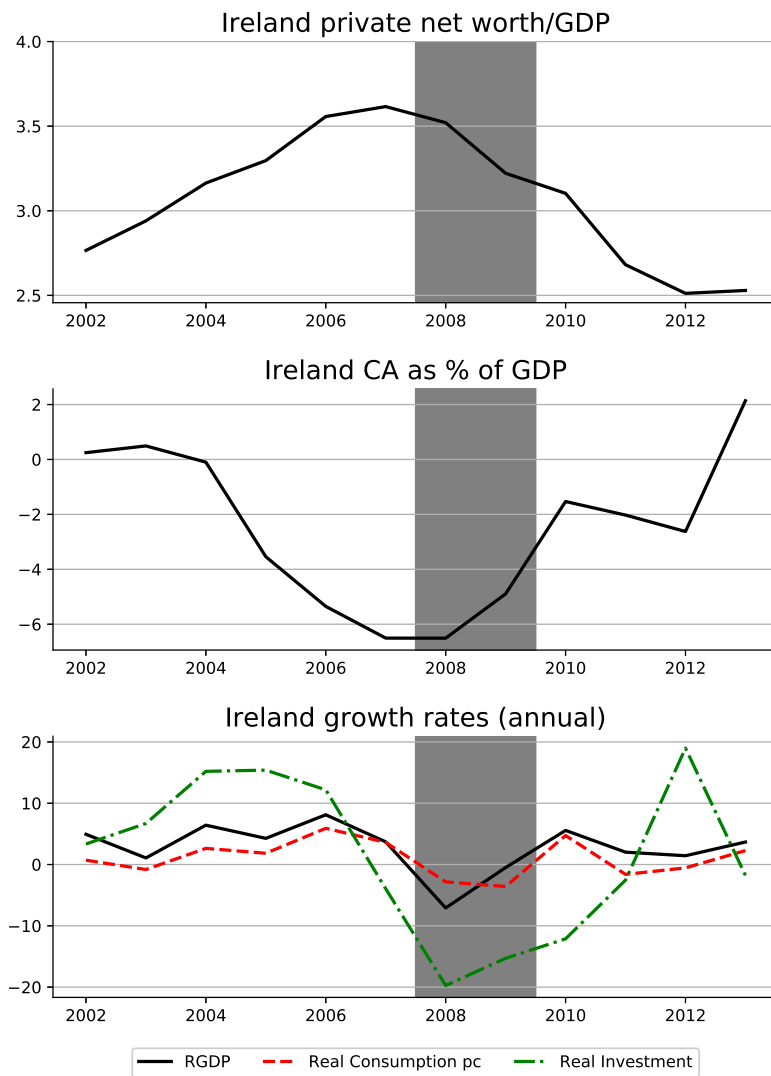
Figure 4: Housing boom in Ireland.
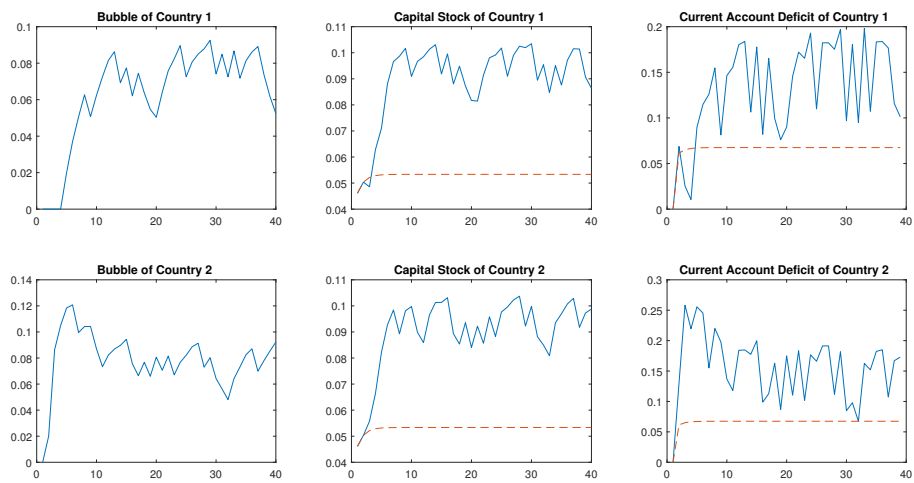
Figure 5: Bubbly episode in the lab economy.

Figure 6: Bubbly episode in the lab economy with two countries.

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