The Dynamic Provision of Product Diversity under Duopoly

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April 2019

Abstract

This paper builds a dynamic duopoly model to examine the provision of new varieties over time. Consumers experience temporary satiation, and hence higher consumption of the current variety lowers demand for future varieties. The equilibrium can be characterized by a combination of monopolistic pricing and nearly zero profits (competitive timing). In particular, if the cost of producing a new variety is not too low then firms tend to avoid head-to-head competition and set the short-run profit maximizing price. However, firms tend to introduce new varieties as soon as demand has grown sufficiently to cover costs. From a second best perspective, the equilibrium may exhibit excessive product diversity. However, if firms coordinate their frequency of new product introductions, then consumers are likely to be harmed. It is also shown that equilibrium prices are moderated by two factors. First, consumers' option value of waiting reduces their willingness to pay. Second, competition reduces firms' incentives to engage in intertemporal price discrimination.

Key words: temporary satiation, product diversity, dynamic duopoly, repeat purchases, endogenous timing.

JEL codes: L13

1   Introduction

There is an extensive literature on the effect of the market structure on the provision of product diversity. Most of these studies are built around the idea that a larger number of varieties available at a point in time allows for a better match with the preferences of heterogeneous consumers.[1] However, much less attention has been devoted to the temporal dimension of variety. In particular, in many product categories individual consumers have a taste for variety that can only be satisfied over time. This is the case, for example, of leisure goods -such as films, books, music recordings, plays, live musical performances, computer games, etc. In these markets, a consumer tends to purchase only one unit of a particular variety at a certain date, but engage in repeat purchases in the same product category as new varieties become available (*dynamic product diversity*). These markets usually exhibit two important characteristics. First, the rate at which consumers can absorb new varieties is clearly limited, and closely related to the amount of time they need to recover from a previous consumption episode (*temporary satiation*). Second, there is a high degree of synchronization between commercialization and consumption, as most purchases are typically made immediately following the release of a new variety.[2]

Temporary satiation is relevant in a broad range of markets that includes, but is not restricted to, the leisure goods mentioned above. The rate of recovery may vary quite markedly across consumers. Some people may be willing to attend a live performance of a rock band or go see a romantic comedy film every week, whereas others may prefer to wait much longer until they feel ready for a new experience of that sort. Indeed, observed differences in the frequencies of repeat purchases across consumers tend to reflect a heterogeneous degree of exposure to temporary satiation (See, for instance, Hartmann and Viard, 2008).[3] Such *durability of consumption* has been shown to generate elasticity patterns similar to those observed in storable and durable goods, as higher current consumption implies lower future demand (Hartmann, 2006). Unfortunately, hard empirical evidence on the effect of temporary satiation in the markets of interest (in which new vari-

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[1] This principle is clearly reflected in some of the most popular "spacial" models; including Hotelling (1929), Salop (1979) or Chen and Riordan (2007). Other models, like Spence (1976) or Dixit-Stiglitz (1977), postulate a "representative consumer" with a preference for diversity. However, this is interpreted as an aggregation device rather than as a literal representation of individual behavior.

[2] The marketing literature calls the products exhibiting such synchronization "short life-cycle" products (See, for instance, Calantone et al.(2010))

[3] This paper shows that low frequency consumers can hardly take advantage of loyalty programs, because of the existing deadlines to redeem a reward. This indicates that the timing of purchases is largely determined by temporary satiation.

eties of a leisure good are introduced over time) does not abound. One exception is Einav (2010), who shows that box office revenues in the US would increase if film distributors did not cluster their releases so much. Thus, in a market with essentially fixed consumer prices, such a relationship between aggregate revenues and the timing of releases clearly manifests the presence of temporary satiation.

The high degree of synchronization between commercialization and consumption can also be easily illustrated. For example, many artists and performers often present their new work in a specific location in a single event, or in several events taking place on consecutive days. Thus, consumption happens at the same time the new variety is released. Of course, this is an extreme example. In most industries, such synchronization is not perfect, but still very high. In the film industry, between 40 and 50 per cent of US box-office revenues are taken during a movie's first week and very few movies generate significant revenue beyond the sixth week.[4,5] In a similar vein, two thirds of the purchases of video games are made during the first three months after release.[6]

A natural question is whether different market structures provide a socially optimal level of dynamic product diversity at reasonable prices. In Caminal (2016) I analyzed a model in which a monopolist provides different varieties of a good over time to a customer base whose preferences are subject to temporary satiation. The welfare results depend on the balance between two opposing effects. On the one hand, if varieties are introduced very frequently then they become imperfect substitutes and the firm has incentives to engage in intertemporal price discrimination: raise prices above the short-run profit maximizing level, and sell each variety only to consumers with very high valuations (better preference matching). On the other hand, higher frequency also generates market expansion. I showed that under strong temporary satiation, better preference matching dominates and the equilibrium frequency of new product introductions is socially excessive.

Clearly, most of the markets for leisure goods, no matter how narrowly the product category is defined, are characterized by intense competition. Indeed, given the high degree of synchronization between commercialization and consumption and the relevance of temporary satiation, firms use the timing of their new product introductions as a crucial strategic variable. It has been observed by the popular media that large Hollywood studios

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[4]www.boxofficemojo.com

[5]See also Corts [2001], Krider and Weinberg [1998], and Einav [2007] for stylized facts and common practices in the motion picture industry.

[6]This is a summary statistic privately provided by Ricard Gil, from the dataset used in (Gil and Warzynski, 2014).

and publishing houses often play around with the timing of their releases, as a response to new information about rivals' moves.

In the current paper I study the effect of competition on both prices and the timing of new product introductions in markets characterized by temporary satiation and perfect synchronization between commercialization and consumption.[7] The model aims at capturing the main features of a specific segment of a leisure good; like horror movies, historical novels, classical music concerts, etc.[8]

More specifically, I consider a dynamic duopoly model in which two symmetric firms sequentially introduce new varieties of a non-durable good. After a consumption episode individual consumers stay out of the market for a random number of periods until they become active again. Hence, current demand depends negatively on past consumption. The model considers both dimensions of product differentiation, static and dynamic. The static dimension is represented by Hotelling's linear city model. From a dynamic point of view, and because of temporary satiation, two consecutive varieties become imperfect substitutes.

Two important features significantly contribute to the tractability of the model. First, consumers are ex-ante identical but they differ in their valuations of specific varieties. In such a framework, we can study the effect of competition under static product differentiation and intertemporal substitutability, and yet avoid Coasian price dynamics. Second, I focus on parameter values for which all active consumers purchase one unit of the good (covered market). This feature places some limits on the extent of intertemporal price discrimination along the equilibrium path, but nevertheless it is still possible to shed some light on how pricing incentives vary across alternative market structures (monopoly versus duopoly).

As expected, the frequency of new product introductions decreases as the ratio of the rate of demand recovery per period to the fixed cost per variety falls. The value of this ratio is closely related to the length of the period. As time periods get shorter we should expect lower rates of demand recovery per period, as well as a higher discount factor. In one extreme, if this ratio is sufficiently high (the fixed cost is sufficiently low) then both firms may choose to introduce one variety every period, and thus compete head-to-head.

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[7]Thus, firms do not need to worry about the potential cannibalization of existing varieties.

[8]As far as I know, the only paper with a similar goal is Krider and Weinberg (1998). They study the timing of movie releases in a duopoly model in which aggregate demand changes deterministically (there is a peak season) and individual demands decay after the release. It is a one-shot timing game, as each firm only introduces a single variety.

If this ratio is not too high then firms will tend to stagger their new product introductions and behave as temporary monopolists.[9]

Because of temporary satiation, demand evolves deterministically over time. As a result, firms use the timing of new product introductions as an additional strategic variable. In fact, when the rate of demand recovery is not too high relative to the fixed cost per variety, firms have incentives to undercut each other in terms of the timing of launching a new variety. As a result, a firm may always be ready to introduce a new product as soon as demand has grown enough to cover costs. Such a behavior is fuelled by the rival firm's threat to respond immediately to the failure to introduce a new variety by introducing its own in the following period, a magnified business-stealing effect. Thus, in the extreme case that this ratio is sufficiently low (which occurs when time periods are arbitrarily short), profits will be driven down to almost zero. In this case, every cycle looks like a race in which the winner takes (almost) nothing.

Pricing policies crucially depend on whether or not firms can avoid head-to-head competition. If the rate of demand recovery is sufficiently high relative to the fixed cost, then both firms may introduce new varieties every period and prices are equal to those prevailing in the static Hotelling model. In this case, the static competition effect dominates intertemporal substitution. However, even when fixed costs are very low, there may exist another equilibrium in which firms stagger their new product introductions, and behave as temporary monopolists setting the price that maximizes short-run profits. Thus, avoiding head-to-head competition may simply be a coordination problem for firms.

Thus, back to the case of a sufficiently low ratio of the rate of demand recovery to the fixed cost, temporary monopolists set the short-run profit maximizing price, and nevertheless firms intensively compete in the timing of launching new varieties. Thus, the equilibrium outcome may be characterized by the unusual combination of monopolistic prices and (nearly) zero profits. Abusing the language, it could perhaps be argued that the Bertrand and Diamond paradoxes can be reconciled in this type of market. On the one hand, two is enough for competition (zero profits), as in the Bertrand paradox. On the other hand, competition between two (ex-ante) identical firms with constant marginal cost results in monopoly prices, as in the Diamond paradox.

Even when firms are temporary monopolists, their ability to extract surplus from consumers is limited by two factors. First, forward-looking consumers anticipate that current

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[9]As the length of time periods decreases the number of periods between two consecutive new product introductions will increase.

consumption reduces their expected future surplus, since they might be out of the market when future varieties are introduced. Hence, they require a sufficiently high current surplus (a sufficiently low price) in order to compensate for the option value of waiting. More specifically, the short-run profit maximizing price decreases with the intensity of temporary satiation and with the discount factor. Second, competition reduces firms' incentives to engage in intertemporal price discrimination. As mentioned above, a monopolist may have incentives to raise the price above the short-run profit maximizing level, in order to boost future demand and sell the next variety to these marginal consumers at higher prices. Under duopoly, and provided that firms alternate, it would be the rival firm who would mostly benefit from such an increase in future demand, and hence firms tend to stick to the short-run profit maximizing price.

From a welfare point of view, the timing of new product introductions may or may not be efficient. The welfare analysis of the speed of new product introductions is more transparent if the rate of demand recovery is low relative to the fixed cost, and thus several periods separate two consecutive varieties. In this case, it is easy to find conditions under which a social planner that can dictate the timing of new product introductions as well as prices, under the constraint that firms make non-negative profits (and consumers maximize their utility), can raise total welfare by slowing down the introduction of new products, relative to the equilibrium frequency. In this case, there will be more consumption per variety (more time for demand to recover), but lower number of varieties per unit of time. If the discount rate is sufficiently low relative to the rate of demand recovery, then the first effect dominates and a reduction in the frequency of new product introductions raises total welfare. That is, in equilibrium firms introduce new varieties too quickly compared to this second best benchmark.[10]

From a policy perspective, it is also interesting to examine firms' incentives to coordinate the timing of their new product introductions, and their effect on consumer and social welfare. If the rate of demand recovery per period is sufficiently low relative to the fixed cost (so that in equilibrium two consecutive new product introductions are separated by more than one period), then firms have incentives to collude and slow down their new product introductions. Such coordination efforts are likely to hurt consumers and they may even reduce total welfare. By increasing the distance between two consecutive varieties, firms can raise consumers' willingness to pay (by lowering their option

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[10]Note that in the current framework the monopolistic equilibrium prices are not distortionary, since all active consumers purchase the new variety.

value of waiting), as well as the fraction of active consumers who are ready to purchase each variety. Thus, firms would set higher prices and sell larger quantities.[11] However, consumers are likely to be harmed. First, total welfare may decrease, and hence consumer surplus would fall even further (as profits increase). Second, even if total surplus increases as a result of less frequent new product introductions, consumers can still lose because of the higher prices (profits may increase more than total welfare). Thus, private and social interests are not necessarily aligned, and hence competition authorities should be concerned about any attempt by rival firms to coordinate the timing of their releases; even more so if the authorities put a higher weight on consumer surplus.[12]

As mentioned above, the present model captures some of the characteristics attributable to durable goods. It would not be hard to rephrase some of the elements of the model and describe every purchase as an investment in a stock that provides a flow of services that decreases over time. However, several features of the model seem more suitable for representing leisure goods rather than standard durable goods. In particular, different varieties introduced over time have independent characteristics in the present model, whereas new varieties of durable goods typically introduce quality improvements. Additionally, in the present model consumers have idiosyncratic preferences over individual varieties whereas consumers' willingness to pay for quality improvements are likely to be highly correlated over time. Indeed, the literature on durable goods has emphasized how the cumulative nature of innovation affects the frequency of new product introductions (Fishman and Rob, 2000), underinvestment in the durability of the good (Waldman, 1993), compatibility between old and new models (Ellison and Fudenberg, 2000) and, of course, pricing policies in the absence of commitment (Bulow, 1982; Stokey, 1982). In contrast, the present paper focuses on horizontal rather than vertical product differentiation and abstracts from Coasian price dynamics.

The next section presents the model. Section 3 examines the case of a sufficiently low ratio of the rate of demand recovery per period to the fixed cost per variety. The focus is on equilibrium configurations in which both firms introduce a variety every period, firms compete head-to-head and prices are driven by the static competition effect. Section 4

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[11]A lower frequency of new product introductions, ceteris paribus, has a negative effect on the present value of profits. However, this effect is dominated by the other two when profits per variety are sufficiently low.

[12]In some countries, including the UK, competition authorities have been concerned about film distributors coordinating the timing of their releases. As far as I know, none of these attempts have been taken to courts, except in Spain. In 2006 five major distributors were fined for their attempts at coordinating the timing of movie releases.

characterizes equilibria in which one variety is introduced every period and firms alternate in its provision. Thus, firms are temporary monopolist that set the short-run profit maximizing price. In this case firms' incentives to undercut the rival's timing of new product introductions are very mild because by accelerating the frequency of new product introduction the firm would change the competitive regime by inducing head-to-head competition. Section 5 generalizes section 4 by building an equilibrium in which firms alternate in the provision of new varieties, but the number of periods between two consecutive new product introductions is higher than one. In this case, firms have a strong incentive to the undercut the rival's timing of new product introductions. Finally, Section 6 discusses alternative equilibrium configurations.

2   The model

Consider a market in which two firms sequentially provide different varieties of a non-durable good to a mass one of consumers. Time is discrete, and indexed by $t, t = 0, 1, 2, ...,$ and horizon is infinite. All agents are forward looking and discount the future using the same discount factor, $\delta \in (0, 1)$.

In each period the two firms, A and B, choose whether or not to introduce a new variety. If they do, they incur a fixed cost γ and can produce any arbitrary amount of the variety at zero variable cost. In period t innovating firms announce their price p_t^j, $j = A, B$. I assume perfect synchronization between commercialization and consumption; that is, the new variety is only available during the introductory period and is immediately consumed. In the introduction I mentioned some real world examples that motivate such an assumption.

Consumers are ex ante identical but they differ in their valuation of specific varieties. They are also subject to temporary satiation. More specifically, a consumer can be either active or inactive in a particular period. An inactive consumer does not have a taste for the new variety(ies) and hence does not participate in the market. In contrast, if consumer i is active in a period t in which either one or two varieties are introduced, then she learns her valuation of the new variety(ies) $v_{it}^j, j = A, B$. If two varieties are introduced then consumer valuations are negative correlated: $v_{it}^A = R + 1 - z_{it}$, and $v_{it}^B = R + z_{it}$, where R is an exogenous parameter, and z_{it} is an independent realization of a random variable, uniformly distributed on $[0, 1]$. A consumer purchases at most one unit of a single variety. If only firm j introduces a variety in period t, then consumer valuations are still given by

v_{it}^j as specified above. Thus, the preference representation corresponds to the standard Hotelling model, with the unit transportation cost normalized to 1, and firms located at the two extremes of the $[0, 1]$ segment. If consumer i purchases one unit of the variety introduced by firm j then she obtains a net payoff equal to $v_{it}^j - p_t^j$, and zero if she does not make any purchase.

Those consumers who are active in period t will remain active in $t + 1$, unless they consume in t, in which case they will be active again in period $t+1$ with probability μ (and inactive with the complementary probability). Consumers who are inactive in period t will become active in $t + 1$ with the same probability μ.[13] Thus, consumers are heterogeneous with respect to the current varieties, although the future looks exactly the same for all active consumers, on the one hand, and for all inactive consumers, on the other.[14]

The timing of decisions is the following. At the beginning of period t, with full knowledge of the history of purchases, firms simultaneously choose whether or not to introduce a new variety. Next, innovating firms, with perfect information on the rival's decision, simultaneously quote prices. Finally, consumers observe these prices and whether or not they are active. If they are, then they learn their valuations and choose which variety to purchase, if any.

The intensity of temporary satiation is reflected in parameter μ, with a lower value of μ indicating more persistent satiation. Obviously, the impact of temporary satiation depends on how much agents discount the future, δ. The remaining parameters of the model are R (that measures consumers' willingness to pay), γ (the fixed cost of introducing a new variety), and x_0, the fraction of active consumers in the initial period ($t = 0$).

Throughout the paper, I focus on equilibria in which the market is covered (all active consumers purchase one unit). In this case, according to the specification of temporary satiation laid out above, if at least one variety was introduced in period t then the fraction of active consumers in $t + 1$, x_{t+1}, will be equal to μ, independently of x_t. As long as no new variety is introduced in $t + 1$ then $x_{t+2} = \mu + (1 - \mu)\mu$. By iteration, the fraction of active consumers T periods after the last product introduction is

$$\alpha_T = 1 - (1 - \mu)^T. \tag{1}$$

Hence, the initial condition, x_0, will only affect the equilibrium outcome during the

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[13]The working paper version (Caminal, 2018) also considered alternative specifications of temporary satiation.

[14]Thus, the model overlooks brand-specific preferences.

first new product introduction. Thus, we can simplify the presentation by assuming that $x_0 = \mu$. Furthermore, note that α_T increases with T and converges to 1 as T goes to infinity. That is, demand recovers over time but it does not fully recover in finite time. Indeed, α_T can also be interpreted as the probability that a consumer who purchased the last product introduction is active again T periods later.

Notice that the state variable, x_t, can never fall below μ. If all active consumers purchased in $t-1$ then $x_t = \mu$, and if only a fraction $q_{t-1} < 1$ of active consumers purchased the good (which will only happen out of the equilibrium path) then $x_t = \mu + (1-\mu)(1-q_{t-1}) x_{t-1} > \mu$.

The goal of this paper is to study the effect of competition in a fully dynamic framework. Hence, we should ignore any form of tacit collusion. Thus, focusing on Markov strategies (prices and production decisions as function of the state variable) looks like a reasonable choice. However, it is also reasonable to study equilibria in which firms alternate in the introduction of new products. Therefore, I will allow firms to use strategies that depend on the state variable as well as on the identity of the firm that introduced the last variety.

The assumption about the perfect synchronization between commercialization and consumption implies that the likelihood of head-to-head competition (two products being available simultaneously) depends on the length of the period. In this context, it is natural to interpret the length of the period as the amount of time that suppliers need to take the new variety to the hands of the potential customers. Such an amount of time is likely to vary across industries.[15] In the remaining sections I study how the equilibrium pattern changes with $\frac{\mu}{\gamma}$. Since μ is the rate of demand recovery per period, as time periods get shorter then we should expect lower values of $\frac{\mu}{\gamma}$ (and higher values of δ). If $\frac{\mu}{\gamma}$ is sufficiently high, then the frequency of new product introductions will mostly be driven by the incentives to avoid head-to-head competition. Except for that, the frequency of new product introductions will be at its physical maximum. In this sense, there is a large discrete-time friction that blurs the intuition about the timing of launching new varieties. Alternatively, if $\frac{\mu}{\gamma}$ is low (short time periods) then firms can use an additional strategic variable: the distance between two consecutive varieties. In this case, the discrete-time friction is much less relevant and the determinants of the frequency

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[15]The evidence discussed in the introduction suggests that the length of a period may be about three months in the case of videogames, six weeks in the case of films, and probably one or two days in the case of live performances of artists in a particular city.

of new product introductions can be more clearly identified. In spite of the fact that $\frac{\mu}{\gamma}$ is also affected by the length of the time period, for convenience, I will refer to the patterns arising under high and low $\frac{\mu}{\gamma}$ as "frequent", and infrequent introduction of new varieties, respectively. Hence, "frequency" will not necessarily refer to the amount of time elapsed between two consecutive new product introductions, but to the inverse of the number of periods in between the launching of two varieties.

3   Head-to-head competition

In this section I provide conditions for the existence of an equilibrium in which the two firms introduce a new product every period, charge prices $p_t^A = p_t^B = 1$, and all active consumers always make a purchase. Given that new products are introduced every period and the market is covered, the fraction of active consumers every period is μ. Also, the equilibrium is symmetric and hence firms split demand equally. Therefore, individual profits per period are $\pi_t^j = \frac{\mu}{2} - \gamma$. Clearly, a necessary condition for the existence of such an equilibrium is the following assumption:

Assumption $1 : \gamma \leq \frac{\mu}{2}$.

That is, head-to-head competition is only possible if $\frac{\mu}{\gamma}$ is sufficiently high.

Consumer choices. Consumers are subject to temporary satiation. Hence, they anticipate that if they purchase one of the current varieties their continuation value will fall, since all future purchases will in expectation be delayed. That is, there is a chance that, after a consumption episode, they may not have recovered and are thus unable to enjoy some of the future varieties. In order to measure consumers' option value of waiting, I start by characterizing consumers' continuation value, conditional on being active. Consumers expect that, in any future period, if they are active, they will purchase one unit every period at a price equal to 1. Hence, the continuation value of an active consumer at the beginning of period t, before learning the realization of z_i, is equal to $U^* = R - \frac{1}{4} + \mu\delta U^* + (1-\mu)\mu\delta^2 U^* + ... = R - \frac{1}{4} + \frac{\mu\delta}{1-(1-\mu)\delta}U^*$. Since average transportation costs are equal to $\frac{1}{4}$ (transportation costs are uniformly distributed over the interval $\left[0, \frac{1}{2}\right]$), then the current expected surplus is $R + 1 - p - \frac{1}{4} = R - \frac{1}{4}$. Additionally, with probability μ the consumer will be active again in period $t+1$ and hence will enjoy a discounted continuation value of δU^*, with probability $(1-\mu)\mu$ will be active again in period $t+2$ and thus obtain a discounted continuation value of $\delta^2 U^*$, and so on. Solving

for U^*:
$$U^* = \frac{[1 - (1 - \mu)\,\delta]\left(R - \frac{1}{4}\right)}{1 - \delta} \tag{2}$$

If $\mu = 1$ (no satiation) consumers obtain an expected surplus of $R - \frac{1}{4}$ every period, and hence the continuation value is $\frac{R - \frac{1}{4}}{1 - \delta}$. However, if there is a probability $1 - \mu$ that the consumer stays out of the market the period after consumption, then the continuation value per period is reduced by a factor $(1 - \mu)\,\delta$, which can be interpreted as the value of temporary satiation.

In order to assess consumers' reaction to firms' deviations, we need to consider the case in which only one variety is available in the current period, sold at the price p. A consumer located at a distance $q \in [0, 1]$ of the temporary monopolist will be indifferent between purchasing and not purchasing if $R + 1 - q - p + \mu\delta U^* + (1 - \mu)\,\mu\delta^2 U^* + \ldots = \delta U^*$. That is,
$$R + 1 - q - p = \frac{\delta\,(1 - \delta)\,(1 - \mu)}{1 - (1 - \mu)\,\delta}U^* \tag{3}$$

The left hand side is the current surplus, and the right hand side is the option value of waiting; that is, the negative effect of a current purchase on future utility. Taking into account equation (2), then condition (3) indicates that all active consumers will purchase the good ($q = 1$) if and only if $p \leq \overline{p}$, where $\overline{p}$ is given by:

$$\overline{p} = R\left[1 - (1 - \mu)\,\delta\right] + \frac{\delta\,(1 - \mu)}{4} \tag{4}$$

In other words, $\overline{p}$ is the highest price compatible with a covered market. Note that $\overline{p}$ decreases with the value of temporary satiation, $\delta\,(1 - \mu)$. If consumers were myopic $(\delta = 0)$, or in the absence of temporary satiation $(\mu = 1)$, then the most distant consumer would be willing to purchase the good if the price was below or equal to her willingness to pay, R. Forward-looking consumers, subject to temporary satiation, require a positive current surplus in order to compensate for the option value of waiting. Thus, the maximum price that the most distant consumer is willing to pay, $\overline{p}$, is lower than R.

If the firm deviates in the current period and sets $p \geq \overline{p}$, then the fraction of active consumers that choose to purchase the variety, $q(p)$, is implicitly given by equation (3), which implies that $q'(p) = -1$.

Notice also that if $\overline{p} \geq 1$, and both firms introduce a new product, but one firm sets an arbitrarily high price, then the most distant consumer will still purchase the good at $p = 1$.

Firms' strategies. For all $x_t \geq \mu$, each firm introduces a new product in period t and charges a price equal to one. Thus, firms use symmetric Markov strategies. We need to determine under which conditions these strategies conform a subgame perfect Nash equilibrium. I first check that, under certain conditions, the market is covered, not only on-the-equilibrium path, but also after some deviations. First, suppose that in period t firm i deviates and does not supply a new variety. Then firm j, $j \neq i$, becomes a temporary monopolist. Firm j will never find it profitable to set a price below $\bar{p}$, since such a price induces the same current and future demand, but it might consider setting a price above $\bar{p}$, trading off lower current demand for higher future demand. I next establish conditions that rules this out. If $p \geq \bar{p}$, then the fraction of active consumers in period $t+1$, x_{t+1}, is given by $x_{t+1} = \mu + x_t (1 - \mu) [1 - q(p)]$, where $q(p)$ is implicitly defined by equation (3). Since the market will be covered in period $t+1$, firm j's deviation only affect profits in periods t and $t+1$, which can be written as $x_t p q(p) - \gamma + \delta \{\mu + x_t (1 - \mu) [1 - q(p)]\} \frac{1}{2} - \delta\gamma$. Hence, from the first order condition, firm j will optimally choose $p = \bar{p}$, for all $x_t \geq \mu$, provided $\bar{p} \geq 1 + \frac{1}{2}\delta(1 - \mu) > 1$. Thus, the price that maximizes the short-run profits, $x_t p q(p)$, is $\bar{p}$, provided that $\bar{p} \geq 1$. However, because of the intertemporal price discrimination effect the lower bound on $\bar{p}$ is strictly higher than one. To see why suppose that $\bar{p} = 1$. Then the firm has incentives to set the current price above $\bar{p}$: a small deviation would cause a second order loss in current profits, but a first order gain in future profits. Hence, in order to eliminate the incentives to engage in intertemporal price discrimination we need the above condition on $\bar{p}$.[16] Given equation (4), this condition is equivalent to the following assumption:[17]

Assumption 2 : $R \geq \frac{4 - \delta(1-\mu)}{4[1 - (1-\mu)\delta]} > 1$.

Second, if both firms introduce a new product in period t, and since (under Assumption 2) $\bar{p} > 1$, then if one of the firms deviates and sets an arbitrarily high price, still all active consumers will purchase from the the rival firm who sets at a price equal to 1. Hence, the market will also be covered.

Consequently, if both firms introduce a new variety in period t, and firm i sets $p_t^i = 1$ then $x_{t+1} = \mu$ independently of p_t^j. Hence, firm j's optimization problem becomes static, and the best response to $p_t^i = 1$ is $p_t^j = 1$.

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[16] The size of such intertemporal substitution effect is limited by two factors: the increase in future demand is shared with the rival firm and future prices are moderated by the head-to-head competition.

[17] In a static Hotelling model the market is covered (all consumers purchase) if and only if $R \geq 1$. In our case, consumers demand a strictly positive current surplus and thus the lowest value of R compatible with a covered market is higher than 1.

Also, in the first stage, under Assumption 2, if firm i introduces a new variety, then firm j cannot affect x_{t+1}. Hence, under Assumption 1, the best response is also to introduce a new variety and make a positive current profit.

Equilibrium. This discussion can be summarized as follows.

Proposition 1 *Under Assumptions 1 and 2, there exists a Markov perfect equilibrium in which, for all $t > 0$, both firms introduce a new product every period and charge prices equal to 1. All active consumers purchase one of the new varieties and, as a result, the fraction of active consumers is equal to μ, for all t.*

Thus, if the rate of demand recovery is sufficiently high with respect to the fixed cost, and consumers' willingness to pay is sufficiently high, there is an equilibrium in which both firms introduce a new variety every period, and hence they compete head-to-head. In this case, the "static" competition effect dominates the intertemporal substitution effect. As a result, temporary satiation has no effect on equilibrium prices, but it does drive aggregate consumption.

4   Staggered and frequent new product introductions

In this section I focus on a stationary equilibrium configuration in which one variety is introduced every period, firms alternate in its provision, and charge a constant price, p^m. Like in the previous section, I will restrict attention to those parameter values for which firms find it optimal to sell to all active consumers.

Consumer choices. I first characterize consumers' option value of waiting. The consumer's continuation value, contingent on being active, can be computed analogously to equation (2) in the previous section. The only difference is that the average transportation cost is now equal to $\frac{1}{2}$ instead of $\frac{1}{4}$, and the price is p^m (to be specified below) instead of 1. Thus, the continuation value of an active consumer at the beginning of a period, before learning the realization of z_i, can be written as :

$$U^* = \frac{\left[1 - (1 - \mu)\,\delta\right]\left(R + \frac{1}{2} - p^m\right)}{1 - \delta} \tag{5}$$

Once again, we need to pay attention to consumers' reaction to deviations from equilibrium prices. Thus, in any arbitrary period a consumer located at a distance $q \in [0, 1]$ of the temporary monopolist will be indifferent between purchasing and not purchasing

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if equation (3) holds. The only difference is that in this case U^* is given by equation (5) instead of (2). Once again, $q'(p) = -1$. Also, in case the firm finds it optimal to serve all active consumers (I will provide the precise condition below), then the equilibrium price, p^m (analogous to $\bar{p}$, in the previous section) can be found by setting $q = 1, p = p^m$, in the above equation. If we solve for p^m, using equation (5), we obtain:

$$p^m = R - \frac{\delta (1 - \mu)}{2 [1 - (1 - \mu) \delta]} \tag{6}$$

Like in the case of $\bar{p}$ in the previous section, p^m decreases with the value of temporary satiation, $\delta (1 - \mu)$. In fact, the only difference between $\bar{p}$ and p^m is that in the latter case consumers' option value of waiting is lower, since they face higher expected transportation costs.[18] As a result, $p^m > \bar{p}$.

Firms' strategies. Since we are building an equilibrium in which firms alternate in their launching of new varieties, their strategies must depend not only on the state variable, x_t, but also on some elements of the history of new product introductions. We must also consider the possibility that, off-the-equilibrium path, both firms introduce a new variety in the same period. In particular, let $\bar{x}^d \in (\mu, 1]$ be a threshold value of the fraction of active consumers, which will be characterized below. Firms' strategies are the following. If $x_t \geq \bar{x}^d$ then, for all histories, both firms are expected to introduce a new variety in period t. Suppose that j was the last firm who was a temporary monopolist in the past. Let n be the number of periods with no product introduction since then. If $x_t < \bar{x}^d$, and if n is an odd number ($n = 0, 2, 4, ...$), then it is firm $i's$ turn to introduce a new variety (firm i is the designated firm), but if n is an even number then it is firm $j's$ turn again (firm j is the designated firm). If two varieties were simultaneously introduced after the last period with a temporary monopolist, then the designated firm still depends in the same fashion on the number of periods with no product introduction, n. Finally, and without loss of generality, I assume that firm A is the designated firm at $t = 0$. Thus, if firm i was the designated firm in period t, and i chooses not to launch a new variety, then firm j will be the designated firm in $t + 1$ (firm i losses its turn). Similarly, if x_t grows above $\bar{x}^d$, then the simultaneous introduction of varieties in that particular period does not affect the order of the future designated firms.

As far as prices are concerned, strategies specify that firms set a price equal to p^m if $x_t \in [\mu, \bar{x}^d)$ and equal to 1, if $x_t \geq \bar{x}^d$. Thus, along the equilibrium path firm A will

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[18]Such an effect is reinforced by the expectation of higher future prices.

introduce a new product in odd periods and firm B in even periods, and the fraction of active consumers will be constant over time and equal to μ. Hence, equilibrium profits per variety are equal to $\mu p^m - \gamma$. Consequently, a necessary condition for this equilibrium configuration is the following assumption:

Assumption 3 : $\gamma \leq \mu p^m$.

Below I will impose conditions on R that will imply that $p^m > 1$. Hence, p^m will be the short-run profit maximizing price. Moreover, if $\gamma \in \left[\frac{\mu}{2}, \mu p^m\right]$ Assumption 1 is violated and Assumption 3 is satisfied.

We can now characterize the value of $\overline{x}^d$. Suppose that $x_t \geq \overline{x}^d$, and j was the last temporary monopolist. Hence, firm i is the designated monopolist in period $t+1$ (provided $x_{t+1} < \overline{x}^d$). If firm i deviates and does not supply a new product in period t, then future profits will remain constant, since all active consumers will still buy firm j's product, and the deviation will not alter the order of moves. Hence, firm i is willing to introduce a new variety if and only if $\frac{x_t}{2} - \gamma \geq 0$. In contrast, if firm j abstains in period t then all future new product introductions will take place one period in advance. Hence, firm j will find it optimal to introduce a new variety if: $\frac{x_t}{2} - \gamma + \frac{\delta^2}{1-\delta^2}\left(\mu p^m - \gamma\right) \geq \frac{\delta}{1-\delta^2}\left(\mu p^m - \gamma\right)$. Hence, the lowest value of x_t compatible with two new varieties being introduced is given by $\overline{x}^d = 2\gamma + 2\frac{\delta}{1+\delta}\left(\mu p^m - \gamma\right)$. Notice that if $\mu p^m < \frac{1}{2}$, then for all values of γ that satisfy Assumption 3, $\overline{x}^d < 1$. Furthermore, a necessary condition for such an equilibrium configuration is $\overline{x}^d > \mu$ (otherwise, an equilibrium with alternating firms is not feasible), which holds if and only if $\gamma > \frac{\mu}{2}\left[1 - \delta\left(2p^m - 1\right)\right]$. Notice that if $p^m \geq \frac{1+\delta}{2\delta}$ (which may or may not hold under the assumption on R that I impose below) then the right hand side of this inequality is negative. Hence, if we let $\underline{\gamma} = \max\left\{0, \frac{\mu}{2}\left[1 - \delta\left(2p^m - 1\right)\right]\right\}$, a staggered equilibrium requires the following condition:

Assumption 4: $\gamma \geq \underline{\gamma}$.

Moreover, since $p^m > \frac{1}{2}, \underline{\gamma} < \frac{\mu}{2}$. Therefore, if $\gamma \in \left[\underline{\gamma}, \frac{\mu}{2}\right]$ there will be multiple equilibria. Below I compare the welfare properties of these two equilibrium configurations.

Let us now consider the case $x_t < \overline{x}^d$ and let firm i be the designated monopolist in period t. If firm i deviates and abstains from providing a new variety, then current profits, $x_t p^m - \gamma > 0$, would be forgone. Moreover, future profits cannot increase either. In particular, if $x_{t+1} < \overline{x}^d$ then firm j will introduce a new variety in period $t + 1$ and hence future profits will be the same as in the equilibrium path. Finally, if $x_{t+1} \geq \overline{x}^d$ then firm i's net gain from deviating will be $\delta\left(\frac{x_{t+1}}{2} - \gamma\right) + \frac{\delta^3}{1-\delta^2}\left(\mu p^m - \gamma\right) - \left(x_t p^m - \gamma\right) -$

$\frac{\delta^2}{1-\delta^2}\left(\mu p^m - \gamma\right)$. Since $\frac{x_{t+1}}{2} = \frac{\mu + (1-\mu)x_t}{2} < x_t$, for all $x_t \geq \mu$, and since $p^m \geq 1$, then this net gain is negative.

Pricing policies. The next step is to check that, if $x_t \in \left[\mu, \overline{x}^d\right)$, a temporary monopolist does not have incentives to deviate and set a price different from p^m. Clearly, setting a price $p < p^m$ is never profitable since such a deviation would not affect neither current nor future demand. If firm i is the designated firm in period t, and sets a price $p > p^m$, then current demand would be lower than at p^m, but demand in $t + 1$ would be higher. What happens in period $t + 1$ depends on whether x_{t+1} is higher or lower than $\overline{x}^d$. If $x_{t+1} < \overline{x}^d$, then firm j is expected to introduce a new product, the market will be covered and hence, such a deviation in period t would not affect firm i's future profits. In other words, the potential increase in future demand can only benefit the rival firm. In this case, since $p^m \geq 1$, firm i will find it optimal to set p^m, the short-run profit maximizing price. Alternatively, if $x_{t+1} \geq \overline{x}^d$, then two new varieties are expected to be introduced in $t + 1$. In this case, firm i would share the benefits of higher future demand. However, it would also delay all other future product introductions. In this case the net gains from the deviation would be $x_t p q\left(p\right) - \gamma + \delta\left\{\mu + (1 - \mu)\left[1 - q\left(p\right)\right]x_t\right\}\frac{1}{2}$ $-\delta\gamma - \frac{\delta^2(1-\delta)}{1-\delta^2}\left(\mu p^m - \gamma\right)$. A sufficient condition for p^m being the optimal price will be given by the first order condition: $p^m \geq 1 + \frac{\delta(1-\mu)}{2}$. Given equation (6), this condition holds if and only if the following assumption holds:

Assumption 5 : $R \geq \frac{2 - \delta^2(1-\mu)^2}{2[1 - (1-\mu)\delta]}$.

Finally, since $p^m > 1$, then if $x_t \geq \overline{x}^d$, both firms introduce a new variety, and firm i sets $p_t^i = 1$, then future demand and profits are independent of p_t^j, which implies that the best reply is also $p_t^j = 1$.

Equilibrium. Summarizing:

Proposition 2 *Under Assumptions 3 to 5, there exists a subgame perfect equilibrium in which, for all t, firms alternate in the introduction of new varieties, and charge the short-run profit maximizing price, p^m. All active consumers purchase the new variety, and as a result the fraction of active consumers is equal to μ, for all $t \geq 0$.*

Thus, if the fixed cost is not too low then firms can avoid head-to-head competition and become temporary monopolists. However, their ability to extract surplus from consumers is limited by both consumers' and firms' intertemporal substitution. First, if consumers were myopic the optimal price would be R, and firms would still sell to all

active consumers. In contrast, when firms face forward-looking consumers the optimal price is $p^m < R$ because consumers require a strictly positive current surplus in order to compensate for the option value of waiting. Second, given that current prices can affect future demand, and since alternating firms do not internalize the effect of current prices on their rivals' demand, prices tend to be lower than in the case where all varieties are provided by the same firm. Next, I elaborate on this second point.

In order to underline the different pricing incentives under monopoly and alternating duopoly, I examine the conditions under which a single supplier optimally sets a price equal to p^m (given by equation (6)) and sells new varieties to all active consumers.[19] Then, I will compare it with Assumption 5. In the monopoly case, the continuation value of the firm at the beginning of period $t+1$ can be written as: $\Pi(x_{t+1}) = x_{t+1} p^m - \gamma + \frac{\delta(p^m \mu - \gamma)}{1-\delta}$. Also, since $q'(p) = -1$, and $x_{t+1} = \mu + (1-\mu)[1 - q(p)] x_t$, a price above p^m reduces current demand but increases future demand and profits.[20] Hence, p^m will be the optimal price if it maximizes $\{x_t p_t q(p_t) - \gamma + \delta \Pi(x_{t+1})\}$. The solution is $p = p^m$ and $q = 1$ provided:

$$p^m \geq \frac{1}{1 - \delta(1-\mu)}. \tag{7}$$

Note that the lower bound on p^m is higher than in the case of alternating firms, $\frac{1}{1-\delta(1-\mu)} > 1 + \frac{\delta(1-\mu)}{2}$. Thus, if R is such that $p^m \in \left[1 + \frac{\delta(1-\mu)}{2}, \frac{1}{1-\delta(1-\mu)}\right]$ then a monopolist would choose a price higher than the one prevailing in the alternating duopoly equilibrium. In other words, in a duopoly market with staggered provision of new varieties, firms have less incentives to engage in intertemporal price discrimination. Thus, even though firms do not compete head-to-head there exists some kind of "intertemporal" price competition.

Welfare. As mentioned above, if $\gamma \in \left[\underline{\gamma}, \frac{\mu}{2}\right]$ (and provided Assumptions 2 and 5 hold), then there are multiple equilibria: one with head-to-head competition and the other with staggered new product introductions. Thus, if firms expect that two products will be introduced in the future, then no firm wishes to deviate and miss the current profits. However, if firms expect staggered product introduction, then a firm that deviates and introduces a second variety can reasonably expect that all future introductions will

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[19] To focus exclusively on price differences I fix the frequency of new product introductions and hence ignore the monopolist's incentives to introduce a new product every other period.

[20] Such intertemporal effect on monopoly prices was studied in detail in Caminal (2016). In particular, in a set up with a smooth demand function, it is shown that a monopoly firm finds it optimal to set prices above the short-run profit maximizing level. A small increase with respect to the short-run profit maximizing level generates second order losses. However, consumers excluded by the higher current price are more likely to be willing to pay a higher price for future varieties, which implies a first order gain.

be delayed by one period, since the rival firm would still be "entitled" to be the next temporary monopolist. Hence, different beliefs about the future can be self-fulfilling.

It is natural to ask which equilibrium provides the highest total welfare. Since prices are not distortionary (all active consumers purchase a new variety) the trade-off is simple: if we move from an equilibrium with head-to-head competition to the one with staggered new product introductions, then total transportation costs per period increase by $\frac{1}{4}$ (worse preference matching) but fixed costs per period decrease by γ. This is exactly the trade-off in the static Hotelling model with free entry, in which case two varieties can be sustained in equilibrium if and only if $\gamma < \frac{1}{2}$. As a result, if $\gamma \in \left[\frac{1}{4}, \frac{1}{2}\right]$ then the equilibrium of the static game exhibit too many varieties with respect to the total welfare maximizing benchmark.[21]

Back to the current dynamic model, if we still focus on the range of parameter values for which multiple equilibria exists, then the welfare comparisons are not so simple. If $p^m > \frac{1+2\delta}{4\delta}$ then $\underline{\gamma} < \frac{\mu}{4}$. Hence, if $\gamma \in \left[\underline{\gamma}, \frac{\mu}{4}\right]$ then the equilibrium with head-to-head competition generates higher total welfare. However, if $\gamma > \left[\frac{\mu}{4}, \frac{\mu}{2}\right]$, analogously to the static model, the social planner would prefer the equilibrium with one new product per period. Consequently, the equilibrium frequency of new product introductions can be excessive or insufficient, not only with respect to the total welfare benchmark (which involves some sort of public intervention), but compared with the alternative equilibrium.

The distributional consequences of these alternative equilibria are more straightforward. Consumers will always prefer the equilibrium with head-to-head competition: lower transportation costs and lower prices. In contrast, firms always prefer the equilibrium with staggered new product introductions: since $p^m > 1$, $\frac{p^m \mu - \gamma}{1-\delta^2} > \frac{\frac{\mu}{2} - \gamma}{1-\delta}$. Thus, it is important to note that firms have incentives to coordinate on the more profitable equilibrium, a move that would hurt consumers, but that may or may not decrease total welfare.[22]

5   Less frequent new product introductions

In this section I generalize the results of Section 4 by considering arbitrary frequencies of new product introductions. More specifically, I focus on equilibrium configurations in which a new variety is introduced every T periods, $T > 1$, firms alternate in the provision

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[21]Notice that the implementation of the social optimum in the static model requires an explicit policy intervention restricting entry.

[22]Firms could also find it profitable to collude and introduce new products every other period. These incentives will be examined in the next section.

of new varieties, and sell each one to all active consumers at the price that maximize short-run profits. Thus, the fraction of active consumers that buy a particular variety is α_T (given by equation (2)). In this case the timing of new product introductions becomes a more powerful strategic tool. In particular, firms now have incentives to undercut each other's timing of new releases; they can raise the frequency of new product introductions without risking a change in the competitive regime (without inducing head-to-head competition). As a result, we can build equilibria in which a new product is introduced as soon as demand has grown sufficiently to cover costs.

Consumer choices. The characterization of consumer behavior is a straightforward generalization of the results of the previous sections. More specifically, if consumers expect a new variety to be introduced every T periods at a price $p^m(T)$, then the continuation value of an active consumer at the beginning of a period in which a new variety is introduced (before learning the realization of z) is $U^* = R + \frac{1}{2} - p^m + \alpha_T \delta^T U^* + (1 - \alpha_T) \alpha_T \delta^{2T} U^* + (1 - \alpha_T)^2 \alpha_T \delta^{3T} U^* .....$That is,

$$U^* = \frac{1 - (1-\mu)^T \delta^T}{1 - \delta^T} \left(R + \frac{1}{2} - p^m \right) \tag{8}$$

Equation (8) generalizes equation (5) for any arbitrary frequency, T. Thus, the effect of the various parameters on U^* have the usual sign.

Consumers' option value of waiting and the highest price compatible with a covered market also generalize the expressions obtained in the previous section. In particular, in a period in which a new variety is introduced, a consumer located at a distance q of the temporary monopolist will be indifferent between purchasing and not purchasing the new variety if $R + 1 - q - p + \alpha_T \delta^T U^* + (1 - \alpha_T) \alpha_T \delta^{2T} U^* + (1 - \alpha_T)^2 \alpha_T \delta^{3T} U^* + ... = \delta U^*$, which is again analogous to equation (3). Such a condition can be written as:

$$R + 1 - q - p = \frac{(1-\mu)^T \left(1 - \delta^T\right)}{1 - (1-\mu)^T \delta^T} \delta^T U^* \tag{9}$$

Thus, the right hand side of (9) represents the option value of waiting, and the left hand side the short-run surplus. If $q \in [0,1]$, and taking into account equation (8), then equation (9) determines the fraction of active consumers willing to purchase the good, $q(p)$. As usual $q'(p) = -1$.

If firms set the maximum price that induces all active consumers to purchase the new variety, then such a price can be found by evaluating equation (9) at $q = 1$ and $p = p^m(T)$. Taking into account equation (8) and solving for $p^m(T)$:

$$p^m(T) = R - \frac{\delta^T (1 - \alpha_T)}{2 \left[1 - (1 - \alpha_T) \delta^T \right]} \tag{10}$$

Note that $p^m(1)$ coincides with equation (6). More generally, $p^m(T)$ increases with T, and converges to R as T goes to infinity. I will argue below that, under certain conditions, $p^m(T)$ is indeed the equilibrium price.

The timing of new product introductions. If firms charge a price $p^m(T)$ for every new variety, then we can denote the fraction of active consumers that involves zero profits by $\overline{x}^m(T)$, which is given by

$$\overline{x}^m(T) = \frac{\gamma}{p^m(T)}. \tag{11}$$

Since the market is covered then, after a new product introduction, the fraction of active consumers will evolve according to α_T, which is given by equation (2). Thus, a new product introduction after T periods will be profitable if and only if α_T is higher or equal to $\overline{x}^m(T)$.

In order to specify firms' strategies, it will be useful to introduce the following sequence: $y_{T+n} = \mu + (1 - \mu) y_{T+n-1}, n > 0, y_T = \overline{x}^m(T)$. That is, if no firm introduces a new variety when the fraction of active consumers is equal to $\overline{x}^m(T)$ then, in the next period, such state variable will be equal to $y_{T+1} = \mu + (1 - \mu) \overline{x}^m(T)$. Other values of y_{T+n} are constructed following the same logic. Using this notation I can now define the following intervals: $I_{T+n} = [y_{T+n}, y_{T+n+1}), n \geq 0$. Note that any value of $x \in [\overline{x}(T), 1)$ belongs to one and only one interval $I_{T+n}, n \geq 0$. In other words, intervals I_{T+n}, for all $n \geq 0$ are a partition of $[\overline{x}(T), 1)$, with the length of these intervals decreasing in n and converging to zero as n goes to infinity. Moreover, the length of any interval I_{T+n} increases with μ and converges to 0 as μ goes to 0.

In this section I characterize the most "competitive" equilibrium with alternating firms. In particular, if j was the firm who introduced the last variety, then firm $i, i \neq j$, is expected to introduce a new product as soon as instantaneous profits turn non-negative. Thus, after every introduction firms switch roles. The equilibrium number of periods is the value of T such that $\alpha_T \in I_T$. Notice that α_T increases with T and takes values between μ and 1. In addition, $\overline{x}^m(T)$ decreases with T and take values between $\frac{\gamma}{p^m(1)}$ and $\frac{\gamma}{R}$. Hence, if the following assumption holds there exists a unique value of T such that $\alpha_T \in I_T$.

Assumption 6 : $\gamma < R$.

Moreover, if Assumption 3 fails ($\gamma > \mu p^m(1)$) then $T > 1$.

Thus, instantaneous profits, $\overline{\pi}$, are non-negative and given by $\overline{\pi} = p^m \alpha_T - \gamma = p^m [\alpha_T - \overline{x}^m(T)]$ and lie in the interval $[0, \mu p^m 1 - \overline{x}^m(T))$. Note that the upper bound of profits increases with μ and goes to zero as μ goes to zero. Thus, in the extreme case of arbitrarily short time periods both μ and $1 - \delta$ are arbitrarily low, which implies that equilibrium profits will be negligible.

Out of the equilibrium path, if x_t is sufficiently high then there may be room for two varieties being introduced in the same period. Analogously to the previous section, there is a threshold value, $\overline{x}^d(T) = 2\gamma + 2\frac{\delta^T (1-\delta^T)}{1-\delta^{2T}}\overline{\pi}$, such that if $x_t \geq \overline{x}^d(T)$ then both firms are expected to introduce a new variety in the same period. Notice if $\gamma < \frac{1}{2}$ and $\overline{\pi}$ is low, then $\overline{x}^d(T) < 1$. In order to simplify the presentation, in the main text I restrict attention to the case that the fixed cost is sufficiently high so that $\overline{x}^d(T) \geq 1$:

Assumption 7: $\gamma \geq \frac{1}{2}$.

I discuss the case $\gamma < \frac{1}{2}$ in the Appendix.

Like before, I restrict attention to strategies that depend on the state variable, x_t, and on some elements of the history of new product introductions (basically, the identity of the firm that introduced the last variety). In particular, suppose that firm j was the last temporary monopolist. Then firm $i, i \neq j$, is expected to introduce a new variety if and only if the fraction of active consumers lies in I_{T+n}, where n is an odd number. In addition, firm j introduces a new variety only if the fraction of active consumers lies in I_{T+n}, where n is an even number.[23] Thus, as soon as I_T is reached firm i is expected to introduce a new variety. If it fails to do so, then in the next period interval I_{T+1} is reached, and firm j is expected to introduce a new product. If j fails to do so, then it will be firm i's turn again in the following period, and so on.

We must check that these strategies conform a subgame perfect Nash equilibrium. If firm j was the last temporary monopolist and firm i deviates and introduces a new variety when the fraction of active consumers is $x < \overline{x}(T)$, τ periods before expected, then the payoff is $xp^m - \gamma + \frac{\delta^{2T}}{1-\delta^{2T}}\overline{\pi}$, which is lower than waiting for τ periods, in which case firm i obtains $\frac{\delta^\tau}{1-\delta^{2T}}\overline{\pi}$. Hence, such a deviation is not profitable since it generates negative current profits and causes a delay in future product introductions. Similarly, if firm j introduces a new variety when the fraction of active consumers is $x < \overline{x}(T)$ then

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[23]Like in the previous section, I also need to specify which firm introduces a new variety the first time. Without loss of generality, I assume that the first time I_T is reached, then it will be firm A's turn to launch a new product.

it also generates current negative profits and causes a delay in future gains.

Suppose now that firm i deviates and does not introduce a new variety when the fraction of active consumers belongs to the interval $I_{t+n}, n = 0, 2, 4, ...$ Then, it will be firm j who will introduce a new variety in the next period, and hence firm i's payoff will be $\frac{\delta^{T+1}}{1-\delta^T}\overline{\pi}$, which is lower than in the case where firm i follows the prescription: $p^m x_t - \gamma + \frac{\delta^{2T}}{1-\delta^{2T}}\overline{\pi} > \frac{\delta^{T+1}}{1-\delta^{2T}}\overline{\pi}$. The same argument applies if firm j deviates and abstains from introducing a variety in the intervals $I_{T+n}, n = 1, 3, 5, ...$ Finally, if firm i introduces a second variety when it was firm j's turn, then the net gains from such a deviation are $\left(\frac{x_t}{2} - \gamma\right) + \frac{\delta^T}{1-\delta^{2T}}\overline{\pi} - \frac{\delta^T}{1-\delta^{2T}}\overline{\pi} < 0$, since $\gamma \geq \frac{1}{2}$. If firm j introduces a second variety when it was firm i's turn then the net gains are even more negative since its next product introductions will be delayed.

Optimal pricing. In an equilibrium configuration with more than one period between two consecutive new product introductions, intertemporal pricing incentives are different than in the case of $T = 1$. The temporary monopolist can now be in a position to influence the future timing of new product introductions: a price above $p^m(T)$ can induce the rival firm to introduce the next variety earlier, which would bring forward the launching of all future varieties. However, such a temptation cannot be ruled out by requiring a higher R, but by placing a restriction on the level of static profits.

In any case, in order to induce the firm to set the highest price compatible with a covered market the value of R should satisfy the following assumption:

Assumption 8: $R \geq \frac{2-\delta^T(1-\alpha_T)}{2\left[1-(1-\alpha_T)\delta^T\right]}$.

As expected, the lower bound on R increases with the value of temporary satiation, $\delta^T(1-\mu)^T$.[24] If Assumption 8 holds, then $p^m(T) \geq 1$, which implies that $p^m(T)$ is the short-run profit maximizing price. Turning to the role of static profits, if we let $\Delta \equiv \alpha_T - \overline{x}^m(T)$, then, provided Δ is not too high, firms have no incentives to raise the price above p^m. In the Appendix I characterize the threshold value $\overline{\Delta}$ and prove the following result:

Lemma 3 *Under Assumptions 6 to 8, there exists $\overline{\Delta} > 0$ such that if $\Delta \leq \overline{\Delta}$, then no firm has incentives to deviate from $p^m(T)$.*

The intuition of why Δ cannot be too high is the following. Setting a price above p^m

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[24]Notice that $(1-\mu)^T$ is the probability that a consumer has not recovered from the consumption of the previous variety, and δ^T is the discount factor applied to payoffs obtained T periods later. Thus, Assumption 8 is implied by Assumption 5.

generates costs and benefits to the temporary monopolist. The costs are the reduction in current profits, which continuously change with the size of the deviation, $\varepsilon = p - p^m(T)$. However, the potential benefits are a step function. An arbitrarily small deviation will positively affect the rival's profits, but it will not trigger a change in the timing of future varieties. Hence, the effect on future profits would be null. However, there is a deviation, $\overline{\varepsilon}_1$, such that $T - 1$ periods later the fraction of active consumers will exactly be equal to $\overline{x}^m(T)$. This implies that the rival firm will introduce its variety one period earlier and hence, according to the prescribed strategy, all future product introductions will also be brought forward. Similarly, we can denote by $\overline{\varepsilon}_s$ the minimum deviation necessary to bring forward by s periods the next product introduction, $s < T$. Thus, any $\varepsilon \in [\underline{\varepsilon}_s, \underline{\varepsilon}_{s+1})$ has the same effect on future profits as $\underline{\varepsilon}_s$. Hence, the optimal deviation from p^m is $\overline{\varepsilon}_s$, for some s. If $\Delta = 0$, equilibrium profits are zero ($\alpha_T = \overline{x}^m(T)$) and hence bringing forward future product introductions have no effect on the present value of profits. Moreover, the deviation in the current price required to induce the rival to introduce a new variety, say, one period earlier is relatively high: $(p - p^m(T))\alpha_T = \mu(1 - \overline{x}^m)$. Thus, costs are positive but the benefits are null. In the opposite extreme, if Δ is very close to $\mu(1 - \overline{x}^m)$, the static profits ($\overline{\pi} = \Delta p^m$) are relatively high, and hence the potential benefits of bringing forward by one period future product introductions are potentially large, whereas the distortion in current profits is arbitrarily small ($\underline{\varepsilon}_1$ is close to zero). By continuity, if Δ is not too high then there are no incentives to deviate from the short-run profit maximizing price.[25]

All this discussion can be summarized as follows.

Proposition 4 *If Assumptions 6 to 8 hold, 3 fails, and $\Delta \leq \overline{\Delta}$, then there exists a subgame perfect equilibrium in which a new variety is introduced every T periods, $T > 1$, firms alternate, and all active consumers purchase the new variety at the price that maximizes short-run profits, $p^m(T)$. As a result, the fraction of active consumers that buys every variety is equal to α_T.*

Proposition 4 is analogous to Proposition 2. If $T = 1$ (Section 4) the only effect of competition was reflected in the relatively moderate level of prices relative to the monopoly case (absence of intertemporal price discrimination). However, prices were still fairly high (they maximize short-run profits). Moreover, firms were highly restricted in

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[25]Note that firm i's optimal deviation implies that the rival firm makes zero profits in the next product introduction, but obviously such a negative effect is not internalized by the deviating firm.

their timing game: they could not increase the frequency of new product introductions without switching to a much worse competitive regime: head-to-head competition (lower prices and demand sharing). As a result, depending on parameter values, profits could be substantial. However, in the current equilibrium configuration, firms have another instrument of competition: the timing of new product introductions. Firms have incentives to undercut their rival's timing of new product introduction, and in equilibrium they launch a new variety as soon as demand has grown sufficiently so that revenues can cover the fixed cost. Consequently, profits are low and they become negligible as μ goes to zero. As a result, the equilibrium exhibits a combination of low (perhaps, negligible) profits and high prices (firms charge the short-run profit maximizing price).

Like in Section 4, despite of the fact that firms are temporary monopolists, their ability to capture rents is limited by the same two factors: (i) consumers' temporary satiation, and (ii) firms' lower incentives to engage in intertemporal price discrimination. The first factor is reflected in equation (10): the maximum price compatible with a covered market is reduced by the value of temporary satiation. The second factor is also analogous to the one found in Section 4. If a monopolist is restricted to introduce a new variety every T periods (the equilibrium frequency), then it will set the short-run profit maximizing price, $p^m(T)$, only if $p^m(T) \geq \frac{1}{1-\delta^T(1-\mu)^T} > 1$. Therefore, if R is such that $p^m(T) \in \left[1, \frac{1}{1-\delta^T(1-\mu)^T}\right]$ then alternating firms set $p^m(T)$, whereas a monopolist charges a higher price.

Welfare analysis. Consider the following second best benchmark: the social planner chooses the number of periods between two consecutive varieties, τ, and the price, p, in order to maximize the total surplus under the constraint that firms cannot make negative profits (and consumers maximize their utility). In order to simplify the presentation, suppose that the social planner is also restricted to set prices that induce all active consumers to purchase the new variety, $p \leq p^m(\tau)$.[26] Clearly, since both $p^m(\tau)$ and α_τ increase with τ, the social planner can only choose $\tau \geq T$, since firms would make negative profits if $\tau < T$. If $\tau \geq T$ the present value of total surplus, $W(\tau)$, can be written as:

$$W(\tau) = \frac{\alpha_\tau \left(R + \frac{1}{2}\right) - \gamma}{1 - \delta^\tau}$$

Thus, by increasing τ, $W(\tau)$ raises through α_τ but decreases through δ^τ. In other words, raising the distance between two consecutive varieties generates two countervailing

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[26]See footnote 30 on the consequences of relaxing such a restriction.

effects on total surplus. On the one hand, the fraction of consumers that enjoy each variety increases, as consumers have more time to recover from the previous consumption episode. On other hand, the frequency of new product introductions decreases, and hence these higher surpluses are enjoyed less often. Once again, since prices are compatible with a covered market then they do not affect total welfare, but of course they affect the distribution of surplus between consumers and firms. If we treat τ as a continuos variable, then

$$\frac{dW(\tau)}{d\tau} = \frac{1}{1-\delta^{\tau}} \left[-\left(R + \frac{1}{2}\right)(1-\mu)^{\tau} \ln(1-\mu) + W(\tau)\delta^{\tau} \ln \delta \right]$$

The first term within the square brackets is positive and represents the gains derived from higher consumption of each variety. The second term is negative and represents the losses generated by less frequent new product introductions. Notice that as μ goes to zero, the first term goes to zero faster than the second ($\frac{dW(\tau)}{d\tau}$ is negative for μ arbitrarily small).[27] Similarly, as δ goes to 1 the second term goes to zero ($\frac{dW(\tau)}{d\tau}$ is negative for $1-\delta$ arbitrarily small).[28] Thus, if $\frac{1-\delta}{\mu}$ is sufficiently low, in equilibrium firms introduce new varieties too quickly with respect to the second best. In the opposite case (high $\frac{1-\delta}{\mu}$) total welfare increases with a higher frequency of new product introductions. However, if we require prices to be compatible with a covered market, and firms cannot make negative profits, then more frequent new product introductions are not feasible and the equilibrium is (second best) efficient.[29]

Colluding on the frequency of new product introductions. We can now examine firms' incentives to collude on the frequency of new product introductions. More specifically, suppose that firms can agree on the number of periods between two consecutive new product introductions, τ, but prices are still set non-cooperatively. From the previous analysis it follows that firms' profits will increase if they agree on τ above T.[30]

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[27] α_{τ} (and hence $W(\tau)$ in the second term) also goes to zero, as μ goes to zero, but more slowly than $\ln(1-\mu)$.

[28] Both $\ln \delta$ and $(1-\delta^{\tau})$ go to zero as δ goes to 1, but the former goes faster.

[29] If we relax the restriction $p \leq p^m(\tau)$, then it might be possible (if R is such that $p^m(T)$ is close to 1) to set $\tau < T$ and still allow firms to make non-negative profits, by setting prices above $p^m(\tau)$ and discouraging a positive fraction of active consumers. Unfortunately, an analytical exploration of such an scenario is not feasible. However, even in the case where non-negative profits were compatible with $\tau < T$, it would be unlikely that setting $p > p^m(\tau)$ might generate a total surplus higher than in equilibrium. The reason is that these high prices imply that a fraction of active consumers abstain from making a purchase, and hence the total surplus per variety is further reduced. But if my conjecture is wrong, then the equilibrium frequency of new product introduction could be also inefficiently low with respect to this more flexible second best. benchmark.

[30] If they agree on $\tau < T$, then profits can only decrease, since demand has less time to recover and consumers' willingness to pay is lower (higher option value of waiting)

First, they will be able to raise prices, since $p^m(\tau)$ increases with τ, as consumers' option value of waiting shrinks. Second, the fraction of active consumers when a new variety is introduced will also be higher (α_τ increases with τ). Hence, profits per variety will increase. However, there is a countervailing effect: these higher profits per variety will be enjoyed less frequently. Such a negative effect is likely to be dominated by the other two. The reason is that equilibrium profits are low, and even negligible if time periods are arbitrarily short. Hence, slowing down the collection of small payoffs will most likely be dominated by the two other effects. Summarizing, starting at $\tau = T$, if equilibrium profits are sufficiently low there exists a value of $\tau > T$ such that firms make higher profits.

However, consumers are less likely to benefit from a reduction in the frequency of launching new varieties. First, consumers will enjoy their consumption episodes less frequently (unlike profits, the equilibrium level of consumer surplus per variety is large). Second, new varieties will be sold at higher prices. These two effects have a negative impact on consumer welfare. However, the third effect has a positive sign: consumers will have a higher chance of being active when new products are introduced. It is important to emphasize that the differential impact of less frequent varieties on firms and consumers lies on the fact that prices increase with τ, which favors firms and hurts consumers. Clearly, in the case total surplus decreases with τ (high $\frac{1-\delta}{\mu}$), and since profits increase, then consumer surplus must decrease even further. If total surplus increases moderately with τ, the increase in profits will exceed the expansion of total surplus, and hence the reduction in the frequency of new product introductions will still imply a reduction in consumer surplus. Only if total efficiency gains are high enough (low $\frac{1-\delta}{\mu}$), then consumers have a chance of capturing some of these extra rents. Thus, in the absence of price controls, the distributional consequences of a reduction in the frequency of new product introductions are biased against consumers.

Summarizing, authorities concerned about total welfare should pay close attention to firms' attempts at coordinating the timing of releases. The interest of firms and society are not necessarily aligned. If authorities' objective function puts more weight on consumer surplus than in profits, then they should scrutinize such coordination efforts even more closely.

6   Discussion

Figure 1 depicts the values of γ for which the three equilibrium types examined above exist. Thus, our previous analysis covers all possible values of γ compatible with a positive surplus. Also, as discussed in Section 4, if $\gamma \in \left[\underline{\gamma}, \frac{\mu}{2}\right]$ staggered and synchronized equilibria coexist. The main reason behind the multiplicity of equilibria is that we cannot restrict attention to Markov strategies (actions depending exclusively on the state variable). Since it is very natural to consider equilibrium paths in which firms alternate in their provision of new varieties, we must let strategies also depend to some extent on the history of the game; in particular, on the identity of the firm that introduced the last variety. As a result, players can hold alternative beliefs about the future that can be self-fulfilling. Indeed, multiple stationary equilibria may also exist in the case $\gamma > \mu p^m$ (infrequent new product introductions). However, I will argue that the flavor of these alternative equilibria is similar to the ones analyzed above.

In the last section I focused on the most "competitive" equilibrium, the one in which a new variety is introduced as soon as demand has grown sufficiently to cover costs. However, less competitive equilibria are also likely to exist, even if we stick to the same class of strategies. In order to illustrate this observation, let T be the lowest number of periods between two consecutive new product introductions that is compatible with non-negative profits, and once again restrict attention to equilibria with the market covered. We can now ask whether a stationary equilibrium in which a new variety is introduced every $T+1$ exist. Along the equilibrium path, static profits would be $\overline{\pi} = \alpha_{T+1} p^m (T+1) - \gamma$. If firm i introduced the last variety and chooses to deviate and launch a new variety again T periods later, the net gains from such a deviation will be $\alpha_T p^m (T+1) - \gamma + \frac{\delta^{2(T+1)}}{1-\delta^{2(T+1)}}\overline{\pi}$ $-\frac{\delta^{T+2}}{1-\delta^{2(T+1)}}\overline{\pi}$. Notice that short-run gains, $\alpha_T p^m (T+1) - \gamma$, are positive, but they must be compared against the losses from delaying future gains. If $\alpha_T p^m (T+1) - \gamma$ is sufficiently low, then the short-run gains will be dominated by the losses associated to delaying all future new product introductions.[31] Hence, undercutting the rival's timing is not profitable. Notice that if it is j the firm that deviates and introduces a new variety one period in advance then the losses associated to delaying all future new product introductions are even larger. Hence, at least for certain values of parameters, less competitive equilibria are likely to exist.

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[31]As α_T goes to $\overline{x}^m (T)$, then the net gains go to $-\frac{\delta^{T+2}}{1-\delta^{2(T+1)}}\mu (1 - \overline{x}^m (T)) p^m$.

An important limitation of the analysis of the previous section is that the existence of the most "competitive" equilibrium with alternating firms requires relatively stringent conditions. Thus, it is natural to inquire about other possible equilibrium configurations. Here I will discuss two alternatives: mixed strategy equilibria, and pure strategy equilibria with one active firm.

It might be possible to construct a symmetric, mixed strategy equilibria in which firms make zero expected profits, analogous to the war of attrition. Consider the case $\gamma \geq \frac{1}{2}$ (that is, if both firms simultaneously introduce a new variety, then they make negative profits, for all x). If we let T be the lowest number of periods between two consecutive varieties that is compatible with non-negative static profits, then the strategy of each firm could consist of introducing a new product with probability $\lambda_s > 0$ in period $T + s, s \geq 0$. Thus, with probability λ_s^2 both firms introduce a new product and make negative profits, with probability $(1 - \lambda_s)^2$ no product is introduced and, with the complementary productivity, one firm introduces a new variety and collect the short-run monopoly profits. The idea is to set λ_s in such a way that firms make zero expected profits, and hence their choices are not affected by any intertemporal consideration. Characterizing such equilibrium configuration is a daunting task since consumers' option value of waiting will vary over time. However, the intuition suggests that such an equilibrium is likely to exist, and it would also exhibit a highly "competitive" nature (in the sense of zero profits), but a random provision of new varieties over time.

A second, much more tractable, alternative is a pure strategy equilibria in which only one firm is active. Define T, $p^m(T)$, and I_{T+n} as in the last section, and assume once again that $\gamma \geq \frac{1}{2}$ (so that $\overline{x}^d \geq 1$). Suppose that firm A introduces a new variety if and only if the fraction of active consumers either lies in I_{T+n}, where n is a non-negative, odd number. Additionally, firm B introduces a new variety only if the fraction of active consumers lies in I_{T+n}, where n is a non-negative, even number. Thus, as soon as I_T is reached firm A is expected to introduce a new variety. If it fails to do so, then interval I_{T+1} is reached, and firm B is expected to introduce a new product, and if it fails to do so, then it is firm $A's$ turn again, and so on. The same arguments used in the last section can be used here to check that these strategies conform a subgame perfect Nash equilibrium. The main difference with respect to the equilibrium configuration studied in the previous section is that firm $A's$ incentives to set a price above p^m are now stronger. The reason is that firm A is not strictly a temporary monopolist any more, in the sense that a reduction of

current consumption may generate to some extent future extra gains. In other words, firm A will have incentives to engage in intertemporal price discrimination. As a result, the lower bound on R necessary to guarantee that firm A will cover the market is higher than in the previous section. It can be shown (details available upon request) that if $p^m(T)$ (and hence R) is high enough, then setting $p^m(T)$ is indeed optimal, but no constraint on Δ is needed. Summarizing, if consumers' willingness to pay is sufficiently high then there exists an equilibrium in which a new product is introduced as soon as demand has grown sufficiently to cover costs, like in the previous section. The characteristics of this equilibrium are reminiscent of contestability. Only one firm is active, but its behavior is tightly constrained by the presence of the rival firm, who can credibly enter the market in case the incumbent deviates.

7   References

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8   Appendix

8.1   Infrequent new product introductions in the case $\gamma < \frac{1}{2}$

If $\gamma < \frac{1}{2}$ and δ is sufficiently close to 1, then $\overline{x}^d < 1$. In this case, firms' strategies must a consider a richer set of histories. In particular, let $\overline{n} > 0$, be such that $y_{T+\overline{n}-1} < \overline{x}^d(T) \leq y_{T+\overline{n}}$.

If in the last innovative period firm j introduced a single variety (was a temporary monopolist), then firm i's strategy consists of introducing a new variety only if the fraction of active consumers either lies in I_{T+n}, where n is an odd number lower than $\overline{n}$, or any

number higher or equal than $\overline{n}$. Additionally, firm j introduces a new variety only if the fraction of active consumers either lies in I_{T+n}, where n is an even number lower than $\overline{n}$, or is higher or equal to $\overline{n}$. Thus, as soon as I_T is reached firm i is expected to introduce a new variety. If it fails to do so, then interval I_{T+1} is reached, and firm j is expected to introduce a new product, and if it fails to do so, then it is firm i's turn again, and so on until we reach $I_{T+n}, n \geq \overline{n}$, at which point both firms are expected to introduce a new variety. For convenience, call firm i the "entitled" firm and firm j the "replacement".

Now suppose that firm j was the last temporary monopolist, but in the last innovative period two simultaneous varieties were introduced. If this happened in an interval I_{T+n}, where n is an even number (that is, firm i lost its turn) then firm j is the entitled firm, and firm i the replacement. Finally, if two varieties were introduced in any other interval then their roles are reversed: i is the entitled firm, and j the replacement.[32]

Like in Section 5, no firm has an incentive to introduce a variety if $x_t < \overline{x}^m$. Similarly, no firm has incentives to abstain from being a temporary monopolist when designated as such. However, we now need to check firms' incentives to introduce a second variety.

Suppose $x_t \in I_{T+n}$ where n is an odd number (it is i's turn), and $n < \overline{n}$, then firm j has no incentives to introduce a new variety, since, according to the above strategies, the net gains from deviation will be $\frac{x_t}{2} - \gamma + \frac{\delta^{2T}}{1-\delta^{2T}}\overline{\pi} - \frac{\delta^T}{1-\delta^{2T}}\overline{\pi} < 0$, since $x_t \leq \overline{x}^d(T)$. The argument is symmetric if n is an even number (i missed its turn) and $n < \overline{n}$: firm i has no incentive to introduce a second variety.

Suppose that $x_t \in I_{T+\overline{n}-1}$. If the firm who is supposed to introduce a new variety fails to do so, then in the next period both firms will introduce new varieties. The net gains from such a deviation are $\delta\left(\frac{x_{t+1}}{2} - \gamma\right) + \frac{\delta^{2T+1}}{1-\delta^{2T}}\overline{\pi} - (x_t p^m - \gamma) - \frac{\delta^{2T}}{1-\delta^{2T}}\overline{\pi}$. Since $p^m \geq 1$, $x_{t+1} = \mu + (1 - \mu)x_t$, and $x_t \in \left[\frac{\overline{x}^d(T)-\mu}{1-\mu}, \overline{x}^d(T)\right]$, then these net gains are negative.

Finally, if $x_t \in I_{T+n}$, $n \geq \overline{n}$ then no firm has any incentives to deviate and abstain from introducing a new variety. The arguments are the same as the ones given in Section 5.

In case $\overline{x}^d < 1$, assuming that R is such that $p^m(T) \geq 1$ is not sufficient to guarantee that the market is covered. In particular, if $p^m(T)$ is sufficiently close to 1, and $T = 2$, then (like in Section 4), firms may have incentives to set a price higher than $p^m(T)$. More

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[32]Like in Section 4, I also need to specify which firm introduces a new variety the first time, as well as after some peculiar histories. Without loss of generality, I assume at the beginning of the game, and also in case all previous varieties were simultaneously supplied by both firms, then A will be the entitled firm.

specifically, if x_t is sufficiently close to $\overline{x}^d$, then $p > p^m(T)$ may cause $x_{t+1} \geq \overline{x}^d$. Such a deviation would generate a second order loss in current profits but a first order gain in future profits. Like in Section 4, if $p^m \geq 1 + \frac{\delta(1-\mu)}{2}$ then for all $x_t < \overline{x}^d$ firms have no incentive to engage in intertemporal price discrimination and will choose to stick to the short-run profit maximizing price, $p^m(T)$.

8.2 Proof of Lemma 3

Suppose that firm i introduces a new variety when the fraction of active consumers is $x \in \left[\overline{x}(T), \overline{x}^d\right)$. If the firm deviates and sets $p = p^m(T) + \varepsilon$, instead of $p^m(T)$, then current profits decrease by $\varepsilon(p^m - 1 + \varepsilon)x$. However, future demand increases which may affect both the rival's profits and also the future timing of new product introductions. If ε is arbitrarily small, then the fraction of active consumers will reach the interval I_T T periods later. Hence, the only future effect of such a price deviation is on the rival's profits. Therefore, firm i has no incentives to make a small deviation from $p^m(T)$. However, there is a value of ε, that I denote by $\overline{\varepsilon}_1$, such that $T - 1$ periods later the fraction of active consumers will be $\overline{x}(T)$. $\overline{\varepsilon}_1$ is given by the law of motion: $\overline{x}(T) = \alpha_{T-1} + (1-\mu)^{T-1} x\overline{\varepsilon}_1$. In the main text I defined $\Delta = \alpha_T - \overline{x}(T)$, $\Delta \in \left[0, \mu(1-\mu)^{T-1}\right)$. Hence,

$$x\overline{\varepsilon}_1 = \mu - \frac{\Delta}{(1-\mu)^{T-1}}$$

If firm i sets $\varepsilon = \overline{\varepsilon}_1$ then according to the equilibrium strategies, firm j will introduce a new product $T - 1$ periods later, and hence all future sales will take place one period earlier. Thus, the net benefits from such a deviation will be

$$\Gamma(\overline{\varepsilon}_1, x) = -\overline{\varepsilon}_1(p^m - 1 + \overline{\varepsilon}_1)x + (1-\delta)p^m \Delta \frac{\delta^{2T-1}}{1 - \delta^{2T}}$$

where $x\overline{\varepsilon}_1 = \mu - \frac{\Delta}{(1-\mu)^{T-1}}$. Under Assumption 8, $p^m \geq 1$ and therefore the first term is negative, whereas the second term is positive. If $\Delta = 0$, the second term is zero and hence $\Gamma(\overline{\varepsilon}_1, x) < 0$. In the other extreme, if Δ is arbitrarily close to $\mu(1-\mu)^{T-1}$ and hence $\overline{\varepsilon}_1$ is arbitrarily close to zero. Therefore, $\Gamma(\overline{\varepsilon}_1, x) > 0$. Moreover, $\Gamma(\overline{\varepsilon}_1, x)$ monotonically increases with Δ. Hence, for all $x \in [\overline{x}(T), \mu + (1-\mu)\overline{x}(T))$ there exists $\overline{\Delta}_1(x) \in \left(0, \mu(1-\mu)^{T-1}\right)$ such that if $\Delta \leq \overline{\Delta}_1(x)$ then $\Gamma(\overline{\varepsilon}_1, x) \leq 0$. Notice that $\overline{\Delta}(x)$ increases with x and R, (the latter through p^m).

Similarly, we can define by $\overline{\varepsilon}_s, s < T$, the value of ε such that $T - s$ periods later the

fraction of active consumers is $\bar{x}(T)$. The net gains from setting $\bar{\varepsilon}_s$ are

$$\Gamma(\bar{\varepsilon}_s, x) = -\bar{\varepsilon}_s (p^m - 1 + \bar{\varepsilon}_s) x + (1 - \delta^n) p^m \Delta \frac{\delta^{2T-s}}{1 - \delta^{2T}}$$

where $x\bar{\varepsilon}_s = 1 - (1 - \mu)^s - \frac{\Delta}{(1-\mu)^{T-s}}$. Once again, if $\Delta = 0$ then $\Gamma(\bar{\varepsilon}_s, x) < 0$. However, in this case if Δ is arbitrarily close to $\mu(1 - \mu)^{T-1}$, and $n > 1$, then the sign of $\Gamma(\bar{\varepsilon}_s, x)$ is ambiguous since $\bar{\varepsilon}_s$ is arbitrarily close to $1 - (1 - \mu)^{s-1} > 0$. In either case, $\Gamma(\bar{\varepsilon}_s, x)$ monotonically decreases with Δ. Hence, we can still define by $\overline{\Delta}_s(x)$ as the value of Δ in the interval $\left(0, \mu(1 - \mu)^{T-1}\right)$ such that $\Gamma(\bar{\varepsilon}_s, x) = 0$, if such a value exists. If it does not and $\Gamma(\bar{\varepsilon}_s, x) < 0$ for all $\Delta \in \left(0, \mu(1 - \mu)^{T-1}\right)$ then we let $\overline{\Delta}_s(x) = \mu(1 - \mu)^{T-1}$. Once again, $\overline{\Delta}_s(x)$ weakly increases with x.

Finally, we define $\overline{\Delta}$ as the lowest value of $\overline{\Delta}_s(x = \bar{x}(T))$ for all $s, 0 < s < T$. Clearly, $\overline{\Delta} \in \left(0, \mu(1 - \mu)^{T-1}\right)$ and if $\Delta \leq \overline{\Delta}$ then firms have no incentives to deviate from $p^m(T)$.

In addition, note that a deviating firm that introduces a new product when $x < \bar{x}(T)$ cannot do better by setting a price higher than $p^m(T)$. The reason is that $p^m(T)$ is the short-run profit maximizing price for any x.

Figure 1