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Abstract

Pierre Daunou, a contemporary of Borda and Condorcet during the era of the French Revolution and active debates on alternative voting rules, proposed a rule that chooses the strong Condorcet winner if there is one, otherwise eliminates Condorcet losers and uses plurality voting on the remaining candidates. We characterize his rule which combines potentially conflicting desiderata of majoritarianism by ordering them lexicographically. This contribution serves not just to remind ourselves that a 19^{th}-century vintage may still retain excellent aroma and taste, but also to promote a promising general approach to reconcile potentially conflicting desiderata by accommodating them lexicographically.

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A study of the history of opinion is a necessary preliminary to the emancipation of the mind. I do not know which makes a man more conservative—to know nothing but the present, or nothing but the past.

John Maynard Keynes, *The End of Laissez-Faire*, 1926.

1 Introduction

This paper provides a characterization of a voting rule proposed by Daunou in his 1803 Paper on Elections by Ballot. An interesting feature of his rule is that it combines two different (possibly conflicting) desiderata based on Condorcet's (1785) proposals and the plurality rule. Daunou gives priority to the Condorcet winner criterion and the Condorcet loser criterion and then uses the plurality rule in situations where these two criteria do not apply. Our characterization of Daunou's voting rule brings the nature of his proposal into clearer relief, arguing that none of the excellent aroma and flavor of his 19^{th}-century vintage has been lost over the last two centuries. In addition, the technique presented here promises to render some service that goes well beyond an appreciation of a historically highly significant proposal. By combining potentially incompatible desiderata lexicographically, we succeed in avoiding impossibilities and come up with a reasonable compromise. This variant of the axiomatic method has the potential of playing a useful and fundamental role in many different contexts within economic and political theory.

Daunou's contribution dates from a period of deep changes in Europe that were accompanied by active debates regarding all aspects of political philosophy and, in particular, the design of new voting rules. The intellectual atmosphere during the last part of the 18^{th} and the early 19^{th} centuries not only provided the theoretical foundations for political change, but also received feedback from it, in the form of opportunities to experiment with new rules to express the collective views of society. Great and well-known contributors to such debates were Rousseau and Montesquieu, on the philosophical side, while Borda (1781) and Condorcet (1785), two achieved thinkers and mathematicians, were important proposers of voting rules that, in spite of some precedents, were perceived not only as being new, but also opposed to each other.

Condorcet proposed the choice of the candidate who would defeat all others in pairwise voting whenever there is such a candidate (this requirement is known as the Condorcet winner criterion) and completed the description of his rule by a hard-to-interpret proposal for the case where such a winner does not exist. In contrast, Borda suggested to choose those candidates who would accumulate more individual wins over other candidates, added across all voters; this rule can alternatively be expressed as the maximization of the points assigned to each candidate on the basis of its position in the rankings of the different voters.

The two proposals fundamentally diverge. The rule advocated by Daunou, that we shall examine here, partially deviates from each of these two. Yet, all three start from a shared agreement about the shortcomings of the plurality rule—the prevailing voting rule at the time. One of their most profound criticisms regarding plurality voting was that it did not always appropriately respond to the majoritarian principle when more than two candidates were at stake and no candidate obtained an absolute plurality. Indeed, choosing a candidate that merely has a relative plurality may lead to numerous anomalies. Avoiding two of these anomalous consequences of the plurality rule is the clear objective of Daunou's proposal. Yet, his approach was less radical than Borda's, who proposed a completely different rule. Daunou's rule corrects the major shortcomings of plurality but then reverts again to this traditional rule in those cases where its flaws are taken care of. Because of that, our characterization suggests a new way to approach the study of different voting rules in modern terms. Unlike most characterizations that require each axiom to hold on the whole domain of the rule, we follow Daunou's suit and propose to classify the axiomatic requirements lexicographically, in terms of importance, and then allow for other axioms to be added to the fundamental ones in those cases where the major requirements are no longer operational.

Condorcet, Borda and Daunou were members of the French National Institute of Sciences and Arts, an institution that became a living laboratory where to test, in the small, the virtues and the vices of different voting rules. Actually, the Borda rule was adopted for the election of new members in 1796 in response to Daunou's proposal to that effect; the Institute terminated the use of the Borda rule in 1804 as a response to intense pressure that it was subjected to by Napoleon Bonaparte. However, Daunou later became a strong critic of the rule, as is apparent in his 1803 text. There he attacked that rule on different grounds, both directly and by criticizing the arguments put forward in its defense by Morales, the author of a Mathematical Memoir on the Calculation of Opinion in Elections (1797) that defended the use of Borda's rule and was well appreciated by different members of the Institute, including Borda.

Daunou introduces his proposal both in negative and positive terms. He first postulates a number of maxims about the problems that should be avoided when proposing a voting method, and states that (Daunou, 1803; 1995, p. 251) "... we shall declare faulty all election methods

> [(i)]—which have a tendency to correct the election results rather than just counting them,
>
> [(ii)]—or which take into account the supposed intensity of the votes, instead of taking them all to be equal and simply counting them,
>
> [(iii)]—or which allow the election of a candidate decisively rejected by an absolute majority of the voters,
>
> [(iv)]—or which make no distinction between cases in which there is an absolute majority and cases in which there is not,
>
> [(v)]—or finally which allow or facilitate the victory of a minority candidate over one preferred by an absolute majority to all others, taken together or individually."

Maxims (i) and (ii) are the most obscure for the modern reader, and we take them to reflect several of Daunou's concerns. One of these concerns is to guarantee the simplicity of the method. In that sense, the first sentences of his positive proposal, where he describes the ballot as the one according to Borda's rule, can just be translated in modern terms as requiring the voters to express their ranking of the candidates. In the same spirit, also reflected in his proposal, we shall translate these concerns by concentrating on anonymous social choice correspondences. In certain parts of Daunou's work he criticizes Borda's rule, *inter alia,* for providing voters with ample possibilities to distort the results of the election by manipulating the voting scheme via the strategic use of their votes. In fact, this strategic manipulation was one of the reasons why Daunou was dismayed with the performance of the rule that he had recommended to the Institute in the first place. Thus, one of his reasons for expressing (ii) might also be his association between the possibility of manipulation and the use of intensities.

Maxim (iii) is intended to rule out the election of a strong Condorcet loser. Therefore, the Condorcet loser criterion is the natural axiom to address this. Likewise, maxims (iv) and (v) ensure that if a strong Condorcet winner exists, then this candidate—and only this candidate—should be elected. As Daunou (1803; 1995, p. 244) writes,

> "If there is an absolute majority which ranks one of the candidates in first place, then that candidate is elected and there is no problem.
>
> If there is an absolute majority which, while not directly ranking one of the candidates in first place, prefers him positively—and not just as an indirect consequence—to each of the others taken individually, then this candidate must be considered elected. The difficulty here was only apparent."

Thus, the Condorcet winner criterion takes care of maxims (iv) and (v).

It appears that Daunou's (1803) maxims are fully described by the conjunction of anonymity, the Condorcet loser criterion and the Condorcet winner criterion. Daunou (1803; 1995, p. 273) makes a distinction between elections with a large number of voters and candidates and those for which these numbers are relatively small. The rule that he proposed in the preceding text and that we shall characterize is the one corresponding to the small-numbers case. He is much less explicit about his proposal for the case of large numbers. In an admittedly arbitrary statement, Daunou distinguishes between those elections in which there are both more than seven candidates and more than fifty voters, and those in which there are either not more than fifty voters or not more than seven candidates. We take it as a way to illustrate by means of example the increasing complexity of problems related to the filling of ballots and their scrutiny. Indeed, the latter may not be a problem nowadays, but it remains true that asking voters to rank an excessive number of candidates is still very demanding.

Later on (Daunou, 1803; 1995, p. 274), he makes a positive proposal for his voting rule (the rule that we shall characterize after some comments and clarifications) in the following terms:

> "Each voter has only to draw up one ballot paper according to Borda's method. Everything else is done by the scrutineers.
>
> They first of all check whether any candidate has obtained an absolute majority of first place votes. If so, they declare him elected.
>
> If not, they check how many times each candidate is preferred to each of his opponents, and if an absolute majority prefers one candidate to each of the others taken individually, that candidate is elected.

> If there is still no result, they examine whether any candidate is ranked below all of the others either collectively or individually, on an absolute majority of ballot papers. If so, they eliminate him.
>
> Then, they work out which of the candidates who have not been eliminated obtained the relative plurality of first votes, and this candidate is elected if it is absolutely necessary to elect someone."

The last step in Daunou's proposal reverts to the use of plurality, when all other criteria do not reach a definite choice. He explicitly justifies and explains his proposal on pages 268–269 of Daunou (1803; 1995) as follows:

> "... [when] there is no general will and no majority preference for one of the candidates over the others ... [and] it is absolutely necessary to elect someone ... I consider it best to elect the candidate with the most first place votes: for as soon as we have ascertained that there is no absolute majority, whether clear or hidden, in favor of any of the other candidates, it seems perfectly just to give the authority of this majority to the simplest, most direct, and clearest relative plurality."

Daunou's ambiguous use of the singular form in this quotation needs to be clarified—clearly, there may be multiple plurality winners. We interpret his statement to mean that all of them are to be elected.

Another potential problem of interpretation has to do with whether we require strong or weak dominance to define Condorcet winners and losers. We observe that if there exists a candidate who has obtained an absolute majority of first place votes, this candidate is a strong Condorcet winner who, by definition, is the unique candidate who is preferred to each of the others taken individually by an absolute majority. Thus, the first step to be performed by the scrutineers according to Daunou's voting rule consists of checking whether there is a strong Condorcet winner and, if yes, to declare this candidate to be elected. We thus favor the interpretation that strong dominance is required in all statements regarding the Condorcet principle. Moreover, we observe that the notion of weak Condorcet winners and weak Condorcet losers is fundamentally problematic because it is possible for a candidate to be a weak Condorcet winner and a weak Condorcet loser at the same time, rendering the simultaneous choice and rejection of such a candidate a meaningless concept. We illustrate this possibility via an explicit example once the requisite definitions have been introduced.

In general, we cannot but claim some interpretative license in passing from this descriptive section to the formal analysis that follows. Similar results could be obtained if these interpretations were modified, and we shall comment on these observations as we go along. But, in essence, this is the spirit that is distilled from our reading of Daunou's work: being aware of two major shortcomings of the plurality rule, he proposes the predominance of the Condorcet criterion, as a principle to choose a winner and also to discard some definite losers, and then reverts to plurality as a lesser evil when these major drawbacks are not a threat. In Daunou's own words (Daunou, 1803; 1995, pp. 274–275),

> "This method ... guarantees both the negative and positive rights of the absolute majority and only takes the relative plurality into account after having ensured that there is no absolute majority, whether positive or negative, clear or hidden."

To begin our formal analysis, we define voting rules (Section 2) and the axioms that we employ (Section 3). We then proceed to a characterization of Daunou's rule (Section 4). The characterization clearly displays a combination of the plurality rule, in cases where two basic criticisms of it do not apply, along with solutions to those criticisms, when Daunou deems them relevant, through the use of Condorcet's ideas. To accommodate Daunou's modifications, some of the axioms in our characterization are stated conditional on the absence of strong Condorcet winners and strong Condorcet losers. If these axioms are amended by making them apply unconditionally, an alternative characterization of the plurality rule emerges.

We note that characterizing Daunou's rule represents a challenge because the classical characterizations of the plurality rule such as that provided by Ching (1996) cannot be taken as a starting point for our approach; this is because Ching's proof method involves profiles for which a strong Condorcet loser may be chosen, a feature that is explicitly ruled out according to Daunou's rule. Moreover, an alternative characterization of the plurality rule established by Goodin and List (2006) cannot be adapted to our setting either. In essence, Goodin and List assume that each voter can either submit one candidate or abstain—that is, submit an empty ballot. This domain is perfectly suitable when dealing with the plurality rule but it fails to be appropriate in our setting where the voters' full preference orderings are needed in order to determine whether there are strong Condorcet winners or strong Condorcet losers. We provide further details on the comparison of our result and these two earlier contributions after stating and proving our characterization theorem.

We view Daunou's as well as our approach as an important instance of a general systematic method to accommodate potentially conflicting desiderata lexicographically. Section 5 puts this aspect of our work into perspective by linking it to some related contributions.

The Appendix contains a discussion of the insufficiency of a weak axiom related to the exclusion of strong Condorcet losers (Part A), a collection of examples that prove the independence of the axioms we use in our result (Part B), and an illustration of the proof of our theorem (Part C).

2 Voting rules

For any two sets A and B, we use the notation $A \subseteq B$ to indicate that A is a subset of B, and $A \subsetneq B$ means that A is a *strict* subset of B. There is a non-empty and finite set of candidates X and we use $\mathcal{X}$ to denote the set of all non-empty subsets of X. The set of potential voters is the set $\mathbb{N}$ of positive integers. The set of possible societies under consideration is the set $\mathcal{N}$ of all non-empty and finite subsets of $\mathbb{N}$.

An *ordering* is a reflexive, complete and transitive relation on the set X. For all $i \in \mathbb{N}$, $\mathcal{R}_i$ is the set of all antisymmetric orderings on X with typical element R_i. For $i \in \mathbb{N}$, $R_i \in \mathcal{R}_i$, $S \in \mathcal{X}$ and $x \in S$, x is i's top candidate for R_i in S if xR_iy for all $y \in S$.

This top candidate is denoted by $t(R_i, S)$. For all $N \in \mathcal{N}$, $\mathcal{R}_N = \prod_{i \in N} \mathcal{R}_i$ is the set of all profiles of antisymmetric orderings for the population N with typical element $R_N = (R_i)_{i \in N}$. Furthermore, we define $\mathcal{R} = \cup_{N \in \mathcal{N}} \mathcal{R}_N$.

Let $S \in \mathcal{X}$ and $N \in \mathcal{N}$ be such that $|N| = |S|$; that is, the number of voters is equal to the number of candidates. A profile $R_N \in \mathcal{R}_N$ is a *completely symmetric profile* if each candidate is placed in each possible position exactly once in the profile R_N. For instance, if $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$, the profile R_N given by

$$xR_1yR_1z,$$
$$yR_2zR_2x,$$
$$zR_3xR_3y$$

is completely symmetric but the profile R'_N defined by

$$xR'_1yR'_1z,$$
$$yR'_2xR'_2z,$$
$$zR'_3xR'_3y$$

is not because x appears twice in the second position and not at all in the third (and z appears twice in the third position and not at all in the second). The notion of a completely symmetric profile also appears in Ching (1996, p. 300).

If $|N| = m|S|$ for some $m \in \mathbb{N}$ (that is, if the number of voters is a multiple of the number of candidates), a profile $R_N \in \mathcal{R}_N$ is an *m-fold replica of a completely symmetric profile* if each candidate in S appears exactly m times in each position in the profile R_N. Profiles of this nature are of importance because they allow us to invoke axioms that ensure the impartial treatment of all voters and all candidates; see the axioms of anonymity and neutrality introduced in the following section.

A *voting rule* is a social choice correspondence, that is, a mapping $f\colon \mathcal{R} \times \mathcal{X} \to \mathcal{X}$ such that, for all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$, $f(R_N, S) \subseteq S$.

Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. A candidate $x \in S$ is a *strong Condorcet winner for R_N in S* if

$$|\{i \in N \mid xR_iy\}| > \frac{|N|}{2} \text{ for all } y \in S \setminus \{x\}$$

and $x \in S$ is a *strong Condorcet loser for R_N in S* if

$$|\{i \in N \mid yR_ix\}| > \frac{|N|}{2} \text{ for all } y \in S \setminus \{x\}.$$

Clearly, for any profile R_N and for any feasible set S, there is at most one strong Condorcet winner for R_N in S and at most one strong Condorcet loser for R_N in S. We denote the set of strong Condorcet winners for R_N in S by $CW(R_N, S)$ and the set of strong Condorcet losers for R_N in S by $CL(R_N, S)$. Each of these sets is either empty or a singleton.

We focus on strong Condorcet winners and strong Condorcet losers, as we believe is in the spirit of Daunou. In addition, as alluded to in the introduction, the notions of weak Condorcet winners and weak Condorcet losers (which are obtained by replacing the strict inequalities in the above definitions by weak inequalities) can be seen as fundamentally flawed because it is possible for a candidate to be a weak Condorcet winner and a weak Condorcet loser at the same time. To see that this is the case, consider the set of candidates $S = \{x, y, z, w\}$ and the set of voters $N = \{1, 2, 3, 4, 5, 6\}$. Now define the profile R_N by

$$\begin{aligned}
&yR_1xR_1zR_1w,\\
&yR_2zR_2xR_2w,\\
&zR_3xR_3wR_3y,\\
&zR_4wR_4xR_4y,\\
&wR_5xR_5yR_5z,\\
&wR_6yR_6xR_6z.
\end{aligned}$$

Candidate x is the unique weak Condorcet winner and the unique weak Condorcet loser for R_N in S because x weakly beats and is weakly beaten by every other candidate with a score of 3 to 3. It is straightforward to verify that none of the other candidates can be a weak Condorcet winner or a weak Condorcet loser.

A candidate $x \in S$ is a *plurality winner for R_N in S* if

$$|\{i \in N \mid t(R_i, S) = x\}| \geq |\{i \in N \mid t(R_i, S) = y\}| \text{ for all } y \in S.$$

The set of plurality winners for R_N in S is non-empty and may have any number of elements between one and $|S|$. We denote this set by $PW(R_N, S)$. The plurality rule f^p is defined by letting $f^p(R_N, S) = PW(R_N, S)$ for all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$.

A new strong Condorcet loser may appear after the strong Condorcet loser for a profile has been removed. It seems to us that Daunou would have objected to the choice of such a candidate and, therefore, we may want to eliminate strong Condorcet losers in a cumulative fashion—that is, after each step in the elimination process, we determine whether there is a candidate that has become a strong Condorcet loser as a consequence of removing those that were disqualified in earlier steps. The procedure can be continued until a set is reached that contains no strong Condorcet loser. This elimination process cannot generate strong Condorcet winners in the reduced sets so that an analogous iterative procedure is not required for strong Condorcet winners.

To show that there are societies $N \in \mathcal{N}$, profiles $R_N \in \mathcal{R}_N$ and feasible sets $S \in \mathcal{X}$ such that a candidate who is not a strong Condorcet loser for R_N in S is both a strong Condorcet loser and a plurality winner for R_N in $S \setminus CL(R_N, S)$, let $N = \{1, 2, 3\}$, $S = X = \{x, y, z, w, v\}$ and define the profile R_N by

$$\begin{aligned}
&xR_1yR_1zR_1wR_1v,\\
&yR_2zR_2xR_2wR_2v,\\
&wR_3zR_3xR_3yR_3v.
\end{aligned}$$

It follows that $CW(R_N, S) = \emptyset$, $CL(R_N, S) = \{v\}$, $CL(R_N, S \setminus \{v\}) = \{w\}$ and $w \in PW(R_N, S \setminus \{v\})$. As this example illustrates, it is possible that a new (unique) strong Condorcet loser emerges (w in the example) once the (unique) strong Condorcet loser (in the example, candidate v) is eliminated from a feasible set. It seems to be in the spirit of Daunou's proposal that this new Condorcet loser be removed as well. Of course, this elimination may yield yet another strong Condorcet loser which then is to be removed in another step, and so on.

More formally, we define $CCL(R_N, S)$, the *cumulative set of strong Condorcet losers for R_N in S,* by means of the following iterative method. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. If $CL(R_N, S) = \emptyset$, we (trivially) obtain $CCL(R_N, S) = \emptyset$. If $CL(R_N, S) \neq \emptyset$, we define the set $S^1 = S \setminus CL(R_N, S)$ in the first step of the successive elimination process. If $CL(R_N, S^1) \neq \emptyset$, we define $S^2 = S^1 \setminus CL(R_N, S^1)$ and so on until we reach a step K such that no strong Condorcet loser remains—that is, $CL(R_N, S^K) = \emptyset$. Because S is finite, such a step K must exist. The cumulative set of strong Condorcet losers for R_N in S is given by

$$CCL(R_N, S) = CL(R_N, S) \cup CL(R_N, S^1) \cup \ldots \cup CL(R_N, S^{K-1})$$

and, because the last iteratively eliminated strong Condorcet loser must be dominated by *some* candidate(s) in the sense of Condorcet, the set $S \setminus CCL(R_N, S)$ of candidates that remain must be non-empty.

To illustrate, let us return to the above example. Candidate v is the strong Condorcet loser for R_N in $S = \{x, y, z, w, v\}$ so that $CL(R_N, S) = \{v\}$. Thus, v is eliminated and we arrive at the remaining set $S^1 = S \setminus CL(R_N, S) = \{x, y, z, w\}$. In this reduced set, $\{w\} = CL(R_N, S^1) = CL(R_N, \{x, y, z, w\})$ so that w is the strong Condorcet loser for R_N in $\{x, y, z, w\}$. After removing w in this step, the set that remains is $S^2 = S^1 \setminus CL(R_N, S^1) = \{x, y, z\}$. There is no strong Condorcet loser for R_N in $\{x, y, z\}$ so that $CL(R_N, S^2) = \emptyset$ and we obtain the cumulative set of strong Condorcet losers $CCL(R_N, S) = \{w, v\}$. Thus, the set of remaining candidates is given by $S \setminus CCL(R_N, S) = \{x, y, z\}$.

3 Axioms for voting rules

The axiom of *anonymity* guarantees that the rule treats all voters symmetrically, paying no attention to their identities. Let Π be the set of all permutations $\pi \colon \mathbb{N} \to \mathbb{N}$. For all $\pi \in \Pi$ and for all $N \in \mathcal{N}$, let

$$N_\pi = \{j \in \mathbb{N} \mid \exists i \in N \text{ such that } j = \pi(i)\}.$$

Thus, we have

$$R_{N_\pi} = \left(R_{\pi(i)}\right)_{i \in N}$$

for all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $\pi \in \Pi$. Anonymity is now defined as follows.

Anonymity. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$, for all $S \in \mathcal{X}$ and for all $\pi \in \Pi$,

$$f(R_{N_\pi}, S) = f(R_N, S).$$

Our next axiom is *neutrality,* which is the counterpart of anonymity that applies to the equal treatment of the candidates. For a set $S \in \mathcal{X}$, let Σ_S be the set of all permutations

$\sigma\colon X \to X$ such that $\sigma(x) = x$ for all $x \in X \setminus S$. For all $i \in \mathbb{N}$, for all $R_i \in \mathcal{R}_i$, for all $S \in \mathcal{X}$ and for all $\sigma \in \Sigma_S$, define the relation $\sigma_i(R_i)$ by letting

$$\sigma(x)\, \sigma_i(R_i)\, \sigma(y) \;\Leftrightarrow\; xR_iy$$

for all $x, y \in X$. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$, for all $S \in \mathcal{X}$ and for all $\sigma \in \Sigma_S$, let

$$\sigma_N(R_N) = \left(\sigma_i(R_i)\right)_{i \in N}.$$

Neutrality. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$, for all $S \in \mathcal{X}$ and for all $\sigma \in \Sigma_S$,

$$f(\sigma_N(R_N), S) = \sigma(f(R_N, S)).$$

The conjunction of anonymity and neutrality implies that, for any profile R_N and any set of candidates S so that R_N is a replica of a completely symmetric profile, the voting rule must choose all elements in S. This is because, in an m-fold replica of a completely symmetric profile, each candidate appears exactly m times in each position. Thus, the equal treatment of the voters (imposed by anonymity) or the equal treatment of candidates (guaranteed by the neutrality axiom) demands that all candidates in S are chosen: because the set of chosen candidates is non-empty, the absence of one of the candidates in the chosen set would necessarily involve an unequal treatment of the voters or the candidates (or both).

We now address the treatment of strong Condorcet winners and strong Condorcet losers. The *Condorcet winner criterion* guarantees the choice of a strong Condorcet winner whenever such a candidate exists.

Condorcet winner criterion. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$, if $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$.

To deal with strong Condorcet losers, we employ an axiom that differs from the standard Condorcet loser criterion in two respects. As already discussed, we require the conclusion of the axiom to apply to the cumulative set of strong Condorcet losers rather than merely the strong Condorcet loser. In addition, we demand that removing the cumulative set of strong Condorcet losers does not change the set of chosen candidates; this is a strengthening of the *cumulative Condorcet loser criterion* which merely requires that cumulative strong Condorcet losers not be chosen. This results in the following axiom of *cumulative Condorcet loser independence.*

Cumulative Condorcet loser independence. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$, $f(R_N, S) = f(R_N, S \setminus CCL(R_N, S))$.

The cumulative Condorcet loser criterion alluded to above is not sufficient for our purposes—it permits the choice from a feasible set to change if a cumulative strong Condorcet loser is removed from this set, as long as a cumulative strong Condorcet loser is not chosen. To illustrate that such a situation can indeed occur, we provide an example in Part A of the Appendix. We note that this observation also applies if merely the strong Condorcet loser rather than the entire cumulative set of strong Condorcet losers is to be removed. Thus, the full force of cumulative Condorcet loser independence (as opposed to the weaker cumulative Condorcet loser criterion) is required in our characterization result.

The following two axioms are conditional on the absence of strong Condorcet winners and cumulative strong Condorcet losers. The first of these ensures that, conditional on the absence of strong Condorcet winners and cumulative strong Condorcet losers, a tops-only condition is satisfied. This axiom is familiar from the literature on *single-peaked* preferences; see, for instance, Black (1958), Dummett and Farquharson (1961) and Moulin (1980). While the axiom appears to be quite forceful, its presence in a conditional form is not too surprising in view of the nature of Daunou's rule—and a variant of it is also used in Goodin and List's (2006) characterization of the plurality rule.

Conditional tops only. For all $N \in \mathcal{N}$, for all $R_N, R'_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$, if $CW(R_N, S) = CW(R'_N, S) = CCL(R_N, S) = CCL(R'_N, S) = \emptyset$ and $t(R_i, S) = t(R'_i, S)$ for all $i \in N$, then

$$f(R_N, S) = f(R'_N, S).$$

The final axiom in our characterization applies to reductions of a profile that involve the removal of voters whose top candidates were chosen prior to their departure. The axiom only applies if the candidate in question is not the only chosen candidate for the original (pre-reduction) profile; this is essential to ensure that the axiom is well-defined.

Conditional reduction monotonicity. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$, for all $M \subsetneq N$, for all $S \in \mathcal{X}$ and for all $x \in S$, if $CW(R_N, S) = CW(R_{N \setminus M}, S) = CCL(R_N, S) = CCL(R_{N \setminus M}, S) = \emptyset$, $\{x\} \subsetneq f(R_N, S)$ and $t(R_i, S) = x$ for all $i \in M$, then

$$f(R_{N \setminus M}, S) = f(R_N, S) \setminus \{x\}.$$

Because of their focus on top candidates, the two conditional axioms defined above exclude other voting rules that satisfy the Condorcet winner criterion.

4 A characterization of Daunou's voting rule

Daunou's proposal can be formalized by means of the following voting rule f^D. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$,

(i) if $CW(R_N, S) \neq \emptyset$, then $f^D(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$, then $f^D(R_N, S) = PW(R_N, S \setminus CCL(R_N, S))$.

Our main result characterizes this voting rule. The independence of the axioms used in this characterization is established in Part B of the Appendix. To make the argument employed in the proof of the only-if part of this theorem easier to follow, we provide an informal explanation of the proof structure and an illustrative example in Part C.

Theorem 1 *A voting rule f satisfies anonymity, neutrality, the Condorcet winner criterion, cumulative Condorcet loser independence, conditional tops only and conditional reduction monotonicity if and only if $f = f^D$.*

Proof. That f^D satisfies the axioms of the theorem statement is straightforward to verify. Conversely, suppose that f is a voting rule that satisfies the axioms. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. We have to show that $f(R_N, S) = f^D(R_N, S)$. This follows trivially if S is a singleton so we now assume that S contains at least two elements.

If $CW(R_N, S) \neq \emptyset$, it follows that $f(R_N, S) = CW(R_N, S) = f^D(R_N, S)$ by the Condorcet winner criterion.

If $CW(R_N, S) = \emptyset$ we can, without loss of generality, assume that $CCL(R_N, S) = \emptyset$ because f satisfies cumulative Condorcet loser independence. Let

$$n^* = \max_{x \in S}\{|\{i \in N \mid t(R_i, S) = x\}|\}$$

denote the plurality-winner score for R_N in S.

Suppose first that $PW(R_N, S) = S$. Let $\overline{R}_N$ be a profile such that $t(\overline{R}_i, S) = t(R_i, S)$ for all $i \in N$ and, moreover, each candidate in $PW(R_N, S) = S$ appears in each position n^* times in $\overline{R}_N$. Thus, $\overline{R}_N$ is an n^*-fold replica of a completely symmetric profile. By anonymity and neutrality, it follows that $f(\overline{R}_N, S) = S = PW(R_N, S)$. We have that $CW(R_N, S) = CCL(R_N, S) = \emptyset$ by assumption, and $CW(\overline{R}_N, S) = CCL(\overline{R}_N, S) = \emptyset$ follows from the definition of $\overline{R}_N$. We can therefore apply conditional tops only to obtain

$$f(R_N, S) = f(\overline{R}_N, S) = S = PW(R_N, S) = f^D(R_N, S).$$

Finally, suppose that $PW(R_N, S) \subsetneq S$. Without loss of generality, let $PW(R_N, S) = \{x_1, \ldots, x_w\}$ and $S \setminus PW(R_N, S) = \{x_{w+1}, \ldots, x_{w+p}\}$ where $w \in \{1, \ldots, |S| - 1\}$ and $w + p = |S|$. Define, for all $k \in \{1, \ldots, |S|\}$,

$$n_k = |\{i \in N \mid t(R_i, S) = x_k\}|.$$

Clearly, $n_k = n^*$ for all $k \in \{1, \ldots, w\}$ and $n_k < n^*$ for all $k \in \{w + 1, \ldots, w + p\}$.

In the next step of the proof, we construct a larger profile that is an n^*-fold replica of a completely symmetric profile and invoke the conjunction of anonymity and neutrality to conclude that all candidates in S must be chosen. Clearly, we need to add as many voters as required to arrive at an augmented profile so that each candidate in S appears n^* times in the top position. Because this is already the case for the w candidates in $PW(R_N, S) = \{x_1, \ldots, x_w\}$, we only need to take care of the p candidates in $S \setminus PW(R_N, S) = \{x_{w+1}, \ldots, x_{w+p}\}$. Let the set $N' \subsetneq \mathbb{N} \setminus N$ be composed of $n' = |N'|$ voters, where

$$n' = \sum_{k=w+1}^{w+p} (n^* - n_k).$$

Now consider a profile $\overline{R}_{N \cup N'} = (\overline{R}_N, \overline{R}_{N'})$ such that

$$t(\overline{R}_i, S) = t(R_i, S) \text{ for all } i \in N \tag{1}$$

and

$$|\{i \in N' \mid t(\overline{R}_i, S) = x_k\}| = n^* - n_k \;\text{ for all } k \in \{w+1, \ldots, w+p\} \tag{2}$$

and, moreover, each candidate in S appears n^* times in each position in the profile $\overline{R}_{N \cup N'}$. By (1) and (2), it follows that

$$|\{i \in N \cup N' \mid t(\overline{R}_i, S) = x_k\}| = n^*$$

for all $k \in \{1, \ldots, |S|\} = \{1, \ldots, w+p\}$. By definition, $CW(\overline{R}_{N \cup N'}, S) = CCL(\overline{R}_{N \cup N'}, S) = \emptyset$. The profile $\overline{R}_{N \cup N'}$ is an n^*-fold replica of a completely symmetric profile and, by anonymity and neutrality, it follows that

$$f(\overline{R}_{N \cup N'}, S) = S.$$

Next, we successively reduce the profile $\overline{R}_{N \cup N'} = (\overline{R}_N, \overline{R}_{N'})$ to the profile $\overline{R}_N$ and apply conditional reduction monotonicity in each step. To ensure that the axiom can indeed be applied, we first show that the elimination of any subset of voters in N' does not create a strong Condorcet winner or a strong Condorcet loser (and, therefore, does not create any cumulative strong Condorcet losers). To do so, let M be any non-empty subset of N'. Now observe that, for any $k \in \{1, \ldots, w_{|S|} - 1\}$, there are at least n^* voters corresponding to the profile $\overline{R}_{N \cup (N' \setminus M)}$ who rank x_k above x_{k+1}, and at most n^* voters who rank x_{k+1} above x_k. Therefore, x_k cannot be a (cumulative) strong Condorcet loser and x_{k+1} cannot be a strong Condorcet winner for $\overline{R}_{N \cup (N' \setminus M)}$ in S. Likewise, there are at least n^* voters associated with the profile $\overline{R}_{N \cup (N' \setminus M)}$ who rank $x_{|S|}$ above x_1 so that $x_{|S|}$ cannot be a (cumulative) strong Condorcet loser and x_1 cannot be a strong Condorcet winner. Therefore, all the reduced profiles employed in the subsequent construction are such that there are no strong Condorcet winners and no cumulative strong Condorcet losers so that conditional reduction monotonicity can be applied.

Consider the set $M_1 = \{i \in N' \mid t(R'_i, S) = x_{w+1}\}$. By conditional reduction monotonicity, it follows that

$$f(\overline{R}_{N \cup (N' \setminus M_1)}, S) = f(\overline{R}_{N \cup N'}, S) \setminus \{x_{w+1}\} = S \setminus \{x_{w+1}\}.$$

If $M_1 \subsetneq N'$, this procedure can be used repeatedly where, in each step $q \in \{2, \ldots, p\}$, we define $M_q = \{i \in N' \mid t(R'_i, S) = x_{w+q}\}$ and invoke conditional reduction monotonicity to conclude that

$$f(\overline{R}_{N \cup (N' \setminus (M_1 \cup \ldots \cup M_q))}, S) = f(\overline{R}_{N \cup (N' \setminus (M_1 \cup \ldots \cup M_{q-1}))}, S) \setminus \{x_{w+q}\} = S \setminus \{x_{w+1}, \ldots, x_{w+q-1}, x_{w+q}\}.$$

At the final step $q = p$, we obtain $N' = M_1 \cup \ldots \cup M_p$ and hence

$$\begin{aligned} f(\overline{R}_N, S) &= f(\overline{R}_{N \cup (N' \setminus (M_1 \cup \ldots \cup M_p))}, S) \\ &= f(\overline{R}_{N \cup N'}, S) \setminus \{x_{w+1}, \ldots, x_{w+p}\} \\ &= S \setminus (S \setminus PW(R_N, S)) \\ &= PW(R_N, S). \end{aligned}$$

By conditional tops only (which can be applied because $CW(\overline{R}_N, S) = CCL(\overline{R}_N, S) = \emptyset$), it follows that

$$f(R_N, S) = f(\overline{R}_N, S) = PW(R_N, S) = f^D(R_N, S)$$

and the proof is complete. ■

Our characterization result can be modified easily if merely the strong Condorcet loser rather than the set of cumulative Condorcet losers is to be eliminated. In that case, the set $CCL(R_N, S)$ has to be replaced with $CL(R_N, S)$ in the axioms of cumulative Condorcet loser independence, conditional tops only and conditional reduction monotonicity. It is straightforward to verify that all arguments employed in the above proof remain valid for this alternative specification.

If conditional tops only and conditional reduction monotonicity are amended by removing the requirement that the sets of strong Condorcet winners and the cumulative set of strong Condorcet losers be empty, the resulting unconditional axioms in conjunction with anonymity and neutrality characterize the plurality rule.

A brief comparison with some earlier results on the plurality rule is in order. It appears difficult to adapt Ching's (1996) characterization of the plurality rule (which is a strengthening of a characterization by Richelson, 1978) to our conditional setting. Ching's (1996, pp. 299–301) first step consists of showing that if all top candidates in a profile are distinct, then all of them must be chosen by the voting rule. To establish this claim, he proceeds by induction on the number of voters. While the first step involving a single voter does not create any difficulty in our setting, the induction step cannot be reproduced in our conditional setting. Even if we were to restrict attention to profiles in which there are no strong Condorcet winners and no (cumulative) strong Condorcet losers, the profile with one less voter that is invoked may very well contain a strong Condorcet winner or a strong Condorcet loser. In such a situation, we cannot proceed on the basis of our axioms that only apply when such candidates are absent. Thus, our characterization cannot but be different from that of Ching (1996).

A second contribution that may be of relevance is Goodin and List's (2006) characterization of the plurality rule. In their setting, each voter can submit a unique candidate or abstain. This is operationalized by permitting the voter to choose either the zero vector (representing an abstention) or a unit vector with the interpretation that a single vote is assigned to the most desirable candidate and zero votes to all others. By defining a voting rule in this manner, a tops-only axiom is incorporated into their system of axioms. In addition to anonymity and neutrality, they impose a choice-theoretic version of May's (1952) positive responsiveness, a well-established monotonicity axiom. It is tempting to ask whether a conditional variant of positive responsiveness may be employed in place of our axiom of conditional reduction monotonicity. This is not the case owing to the nature of Daunou's rule—particularly, its crucial feature of respecting strong Condorcet winners and rejecting strong Condorcet losers. The very definition of strong Condorcet winners and losers necessitates the availability of each voter's entire (antisymmetric) ordering and, as a consequence, the domain employed by Goodin and List is not suitable for a rule such as that of Daunou. This is the case because the plurality rule by itself is perfectly well-defined once the most desirable candidate is identified for each voter, whereas this information is not sufficient to determine strong Condorcet winners and losers. In addition, the notion of an abstention is difficult to incorporate in our framework unless we were to allow voters to submit an empty relation instead of an ordering. This would be contrary to the notion of a voting rule as a social choice correspondence—and to the proper application of the Condorcet criteria.

We stress that our interest in Daunou's voting rule does not mean that we endorse it as the best rule in any sense of the word, even though we surely believe that it has some merit. In particular, Daunou succeeded in formulating a rule that uses an established voting mechanism (the plurality rule) after some of its major shortcomings have been eliminated by giving priority to other desiderata (those advocated by Condorcet). In this sense, Daunou's important contribution opened up a novel way of looking at potentially conflicting desiderata by accommodating them lexicographically, and we elaborate more on this point in the following section.

5 Lexicographic combinations of desiderata

The use of lexicographic combinations of potentially conflicting desiderata of social choice can be found in several contributions. A conspicuous example consists of Asimov's *three laws of robotics,* first spelled out explicitly in his short story *Runaround;* see Asimov (1942/1950). They read as follows.

First Law. A robot may not injure a human being or, through inaction, allow a human being to come to harm.

Second Law. A robot must obey the orders given it by human beings except where such orders would conflict with the First Law.

Third Law. A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

Asimov's three laws presuppose the priority of the first desideratum that guarantees the protection of human life and safety over the second desideratum that demands the obedient conduct of a robot. Thus, these laws can be summarized by the phrase *human safety and well-being first, the robot's following of human orders second.* It is clear that, in this instance, a symmetric treatment of the two desiderata would raise serious issues, to put it mildly. If their order of priority were reversed, disastrous consequences (at least from the viewpoint of the humans) would ensue immediately. This simple example serves to illustrate that the order in which we accommodate the potentially conflicting desiderata can be of paramount importance.

Turning from the realm of science fiction to that of the social sciences, there are at least two prominent classes of issues in collective decision-making that stand out in the literature because of their evident need to determine a priority ranking of conflicting desiderata.

The first of these addresses what are usually referred to as *equity-efficiency tradeoffs* and their resolution. Problems of this nature have been studied extensively by Tinbergen (1946), Foley (1967), Kolm (1972), Varian (1974), Suzumura (1981, 1983a,b) and Tadenuma (2002, 2008), among others. The second major representative is the *Paretian liberal paradox,* which was first defined and addressed by Sen (1970a, 1970b/2017, 1976) and developed further by authors such as Nozick (1974), Sugden (1985), Deb, Pattanaik and Razzolini (1997), Gaertner, Pattanaik and Suzumura (1992) and Suzumura (1978/1979, 1996). Let us address these two issues in some more detail.

We begin with the potentially conflicting desiderata of equity and efficiency. In the setting discussed here, the notion of equity is captured by the *no-envy* principle. A social alternative x is referred to as *envy-free* if no member of society would prefer to be in someone else's shoes, given that this alternative materializes. This definition may seem somewhat abstract but its intention is immediately apparent in specific contexts, particularly if the alternatives under consideration consist of allocations of goods and services to the members of society. In this case, envy-freeness means that no one would prefer someone else's allocation to his or her own, and the above formulation is simply a general statement of this criterion. The notion of efficiency is usually expressed in terms of *Pareto efficiency.* A social alternative x is (Pareto) efficient if there is no other feasible alternative y such that all members of society prefer being themselves in y to being themselves in x.

That the two desiderata of envy-freeness and efficiency may conflict can be illustrated by means of a simple example. Suppose that there are two social alternatives x and y and two members of society labeled individual 1 and individual 2. Individual 1's first choice is to be in individual 2's shoes in alternative x, and being in her own shoes in alternative x is her second-best alternative. Being herself in alternative y is third-best and, finally, being in individual 2's shoes in alternative y is worst. For individual 2, the ranking is such that being himself in alternative x is best, followed by being himself in alternative y. The third-place choice is being in the other individual's shoes in alternative y and, at the bottom of his ranking, individual 2 is in individual 1's shoes in alternative x. A quick inspection of these rankings reveals that there is no social alternative that is equitable (in the sense of envy-freeness) and efficient (in the sense of Pareto) at the same time. Clearly, x cannot be envy-free because individual 1 would prefer to be in individual 2's shoes in this case, and y is not Pareto efficient because both individuals would prefer being themselves in alternative x.

The above example crystallizes the nature of the equity-efficiency tradeoff: there may be no social alternative that accommodates both desiderata at the same time. As a possible escape route, let us now examine the lexicographic use of the two desiderata.

(1) The efficiency-first rule applies Pareto efficiency first and excludes alternative y from consideration because it fails to be efficient. This leaves us with x as the only feasible alternative when applying the secondary desideratum of envy-freeness. Because x fails to be envy-free, we again end up with an impossibility: even if the two desiderata are applied lexicographically with priority given to Pareto efficiency, we are unable to make a choice.

(2) As an alternative, consider now the equity-first rule. Alternative x must be excluded according to the equity desideratum because it leads to envy. This leaves us with the reduced set of alternatives that contains y only for the application of the secondary desideratum—that of Pareto efficiency. Because there are no alternatives other than y in this reduced set, this alternative is automatically Pareto efficient and, thus, the second option of assigning priority indeed leaves us with a possibility in this example.

The gist of this example is that the capability of a lexicographic combination to resolve the impossibility hinges squarely on the order of priority assigned to the two conflicting desiderata.

Let us now turn to the second example—the Paretian liberal paradox and its resolution. It was Sen (1970a, 1970b/2017, 1976) who introduced the novel concept of individual libertarian rights within social choice theory and revealed an intrinsic conflict between his formulation of rights and the Pareto principle. One of the idiosyncratic features of Sen's concept of rights is that his scheme confers to each individual the power to exclude a social state from the socially chosen set if that state is inferior to another state when both states belong to what Sen refers to as the individual's *protected sphere.* In this sense, Sen's rights may be referred to as the *preference-contingent power of rejection.* Sen showed that there exists no social choice rule with an unrestricted preference domain that satisfies the Pareto principle and the social respect of individual libertarian rights in his sense.

Many proposals were made in the literature to cope with this logical conflict of social desiderata. As a reflection of the dominant status of the Pareto principle at the time, most of these proposed resolution schemes confer priority to the Pareto principle over Sen's notion of individual rights. In a later contribution, Sen (1976) inverted the priority between the requisite social desiderata and identified a social choice rule that makes his formulation of rights and a contingent version of the Pareto principle compatible on an unrestricted preference domain. Sen's scheme was subsequently generalized by Suzumura (1978, 1983b).

One of the earliest critical responses to Sen's original contribution came from Nozick (1974, p. 166), who asserts that Sen's paradox should be resolved by assigning priority to individual rights:

> "Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice may be made by a social mechanism based upon a social ordering; if there are any choices left to make! Rights do not determine a social ordering but instead set the constraints within which a social choice is to be made, by excluding certain alternatives, fixing others, and so on. *How else can one cope with Sen's result?*"

Three features of Nozick's resolution scheme deserve special emphasis. In the first place, not only do Sen's and Nozick's schemes differ substantially but, moreover, the latter represents a fundamental criticism of the former. This criticism is focused on the conspicuous contrast between the preference-contingent power of rejection in Sen's description of rights, and the freedom of choice of protected personal features of the world in Nozick's proposal. The latter would seem to be more suitable to capture the essence of the classical libertarian tradition, *which is completely independent of individual preferences over social states.*

In the second place, Nozick's conception of rights is represented by means of an individual's *direct choice of his or her personal-feature alternatives.* This allows for a straightforward illustration but is too narrow as a general presentation of what these rights intend to capture. Later contributions by Sugden (1985) and Gaertner, Pattanaik and Suzumura (1992) generalized Nozick's basic insight by introducing a *game-form approach* to individual rights.

In the third place, although the game-form rights are meant to represent a general approach to individual libertarian rights, they are *not* intended to be a general resolution scheme for the Paretian liberal paradox. As shown by Deb, Pattanaik and Razzolini (1997), the paradox persist even if the game-form representation replaces Nozick's original formulation.

A natural question that presents itself at this point is whether it may be useful to reverse the priorities of the two desiderata in Daunou's method. Such a reversal of priorities would consist of applying the plurality rule first and, if there are multiple plurality winners, use a variant of the Condorcet winner criterion to choose the strong Condorcet winner *among these plurality winners* if one exists. In the absence of a strong Condorcet winner, a conditional version of the Condorcet loser criterion can then be invoked to reduce the set of plurality winners by eliminating candidates according to this criterion. It seems to us that the reverse procedure is less attractive because the strong Condorcet criteria appear to be on more solid ground as compared to the plurality rule, the negative attributes of which were what Daunou intended to prevent in the first place.

To the best of our knowledge, Daunou's voting rule as reinstated and characterized in this paper is the first example of the lexicographic combination of two conflicting desiderata in the specific context of designing voting rules. The secondary purpose of our contribution is to promote the further exploration of this lexicographic approach in the theory of social choice in general, and voting theory in particular. Applying his procedure systematically may have the capability of playing an instrumental role in many different contexts within economic and political theory. This belief seems to be well-warranted by our identification of the precise nature of his rule as well as our characterization thereof.

Appendix

Part A: Insufficiency of the cumulative Condorcet loser criterion. The formal definition of the cumulative Condorcet loser criterion is as follows.

Cumulative Condorcet loser criterion. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$, for all $S \in \mathcal{X}$ and for all $x \in S$, if $x \in CCL(R_N, S)$, then $x \notin f(R_N, S)$.

To define our example of a voting rule f that illustrates the insufficiency of this axiom in our characterization, we require the notion of a weak Condorcet loser. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. A candidate $x \in S$ is a *weak Condorcet loser for R_N in S* if

$$|\{i \in N \mid yR_ix\}| \geq \frac{|N|}{2} \text{ for all } y \in S \setminus \{x\}.$$

Note that it is possible for there to be multiple weak Condorcet losers. The set of all weak Condorcet losers for R_N in S is denoted by $WCL(R_N, S)$.

Consider any population N, any profile R_N and any feasible set S. We construct the set of candidates chosen by f according to the procedure described below.

(i) In analogy with the first case in the definition of Daunou's rule, if there is a strong Condorcet winner for R_N in S, choose this candidate and only this candidate.

(ii) If there is no strong Condorcet winner for R_N in S and there is no cumulative strong Condorcet loser for R_N in S, all plurality winners for R_N in the set S are chosen.

(iii) If there is no strong Condorcet winner for R_N in S and there is a cumulative strong Condorcet loser for R_N in S and the set of plurality winners for R_N in $S \setminus CCL(R_N, S)$ contains at least one candidate who is not a weak Condorcet loser for R_N in $S \setminus CCL(R_N, S)$, the set of plurality winners for R_N in $S \setminus CCL(R_N, S)$ who are not weak Condorcet losers is chosen.

(iv) If there is no strong Condorcet winner for R_N in S and there is a cumulative strong Condorcet loser for R_N in S and the set of plurality winners for R_N in $S \setminus CCL(R_N, S)$ only contains weak Condorcet losers for R_N in $S \setminus CCL(R_N, S)$, the set of plurality winners for R_N in $S \setminus CCL(R_N, S)$ is chosen.

The above four cases (i) to (iv) can be defined more formally as follows. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$, and define $f(R_N, S)$ as follows.

(i) If $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = CCL(R_N, S) = \emptyset$, then $f(R_N, S) = PW(R_N, S)$;

(iii) if $CW(R_N, S) = \emptyset$ and $CCL(R_N, S) \neq \emptyset$ and

$$PW(R_N, S \setminus CCL(R_N, S)) \setminus WCL(R_N, S \setminus CCL(R_N, S)) \neq \emptyset,$$

then $f(R_N, S) = PW(R_N, S \setminus CCL(R_N, S)) \setminus WCL(R_N, S \setminus CCL(R_N, S))$;

(iv) if $CW(R_N, S) = \emptyset$ and $CCL(R_N, S) \neq \emptyset$ and

$$PW(R_N, S \setminus CCL(R_N, S)) \setminus WCL(R_N, S \setminus CCL(R_N, S)) = \emptyset,$$

then $f(R_N, S) = PW(R_N, S \setminus CCL(R_N, S))$.

The voting rule f satisfies the cumulative Condorcet loser criterion because a cumulative strong Condorcet loser is never chosen. Also, the rule satisfies the axioms of anonymity and neutrality because the labels assigned to the voters and the candidates are irrelevant, and it satisfies the Condorcet winner criterion because a (unique) strong Condorcet winner is always chosen; see case (i). Conditional tops only and conditional reduction monotonicity are satisfied because the axioms are silent in the presence of a cumulative strong Condorcet loser, and in case (ii)—the only case that is relevant for the two axioms—there cannot be a violation because the plurality rule applies in all requisite instances. That cumulative Condorcet loser independence is violated can be seen by examining the following example. Let $N = \{1, 2, 3, 4\}$, $S = X = \{x, y, z, w, v\}$ and consider the profile R_N given by

$$xR_1wR_1zR_1yR_1v,$$
$$yR_2zR_2wR_2xR_2v,$$
$$zR_3xR_3yR_3wR_3v,$$
$$wR_4xR_4zR_4yR_4v.$$

There is no strong Condorcet winner for R_N in S, candidate v is the unique strong Condorcet loser for R_N in S, and candidates y and w are weak Condorcet losers for R_N

in $S \setminus CCL(R_N, S) = \{x, y, z, w\}$. The set of plurality winners in $S \setminus CCL(R_N, S)$ is $PW(R_N, S \setminus CCL(R_N, S)) = PW(\{x, y, z, w\}) = \{x, y, z, w\}$. According to the above definition, it follows that

$$f(R_N, S) = f(R_N, \{x, y, z, w, v\}) = \{x, z\}$$

because case (iii) applies, and

$$f(R_N, S \setminus CCL(R_N, S)) = f(\{x, y, z, w\}) = \{x, y, z, w\}$$

because case (ii) applies. Clearly, $f(R_N, S) \neq f(R_N, S \setminus CCL(R_N, S))$ so that cumulative Condorcet loser independence is violated. Note that this example can also be employed if only strong Condorcet losers are to be eliminated but not the entire set of cumulative strong Condorcet losers; this is the case because there is no new strong Condorcet loser once v has been removed.

Part B: Independence of the axioms used in Theorem 1. The following examples establish the independence of the axioms used in our characterization. In each of them, the axiom that is violated is indicated.

Anonymity. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. Define the *modified plurality score* $mps(R_N, S; x)$ of $x \in S$ for R_N in S by

$$mps(R_N, S; x) = \begin{cases} |\{i \in N \setminus \{1\} \mid t(R_i, S) = x\}| + 2 & \text{if } 1 \in N; \\ |\{i \in N \mid t(R_i, S) = x\}| & \text{if } 1 \notin N. \end{cases}$$

These scores reflect a special status accorded to voter 1: the top candidate of this voter receives twice the weight of all other voters' top candidates. A candidate $x \in S$ is a *modified plurality winner for R_N in S* if

$$mps(R_N, S; x) \geq mps(R_N, S; y) \text{ for all } y \in S.$$

The set of modified set of plurality winners for R_N is S is denoted by $MPW(R_N, S)$. Now define $f(R_N, S)$ as follows.

(i) If $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$, then $f(R_N, S) = MPW(R_N, S \setminus CCL(R_N, S))$.

Because voter 1 has a special status, anonymity is violated. All other axioms are satisfied.

Neutrality. Let $N \in \mathcal{N}$, $R_N \in \mathcal{R}_N$ and $S \in \mathcal{X}$. Suppose that x^* is a fixed candidate in X and that n^* is the plurality-winner score for R_N in $S \setminus CCL(R_N, S)$. Now define $f(R_N, S)$ as follows.

(i) If $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$ and

$$x^* \in S \setminus CCL(R_N, S) \quad \text{and} \quad |PW(R_N, S \setminus CCL(R_N, S))| \geq 2$$
$$\text{and} \quad |\{i \in N \mid t(R_N, S) = x^*\}| = n^* - 1, \qquad (3)$$

then $f(R_N, S) = PW(R_N, S) \cup \{x^*\}$;

(iii) if $CW(R_N, S) = \emptyset$ and (3) does not apply, then $f(R_N, S) = f^D(R_N, S)$.

Neutrality is violated because the candidate x^* has a special status—under the circumstances of case (ii), x^* is chosen even with a top-position score that falls short of the plurality-winner score by one vote. All other axioms are satisfied.

Condorcet winner criterion. The plurality rule f^p violates the Condorcet winner criterion and satisfies all other axioms.

Cumulative Condorcet loser independence. Define the voting rule f as follows. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$,

(i) if $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$, then $f(R_N, S) = PW(R_N, S)$.

This voting rule violates cumulative Condorcet loser independence because the cumulative Condorcet losers are not removed before applying the plurality rule in part (ii) and, therefore, they are chosen for some profiles. Clearly, all other axioms are satisfied.

Conditional tops only. For $i \in \mathbb{N}$ and $R_i \in \mathcal{R}_i$, let $top(R_i)$ be voter i's top candidate for R_i in X and, if X contains at least two elements, let $sec(R_i)$ be i's second-best candidate in X. For all $N \in \mathcal{N}$ and for all $R_N \in \mathcal{R}_N$, define an ordering $\succsim_{R_N}$ on X as follows. For all $x, y \in X$, $x \succsim_{R_N} y$ if and only if

$$|\{i \in N \mid top(R_i) = x\}| > |\{i \in N \mid top(R_i) = y\}|$$

or

$$|\{i \in N \mid top(R_i) = x\}| = |\{i \in N \mid top(R_i) = y\}| \text{ and}$$
$$|\{i \in N \mid sec(R_i) = x\}| \geq |\{i \in N \mid sec(R_i) = y\}|.$$

Now define, for all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$,

(i) if $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$, then $f(R_N, S)$ is the set of best elements in $S \setminus CCL(R_N, S)$ according to the ordering $\succsim_{R_N}$.

This is a lexicographic rule that applies the second-place scores as a tie-breaker if the top scores are the same, in violation of conditional tops-only. Because the top scores have priority, conditional reduction monotonicity is satisfied. That the remaining axioms are satisfied is immediate.

Conditional reduction monotonicity. Define a voting rule f as follows. For all $N \in \mathcal{N}$, for all $R_N \in \mathcal{R}_N$ and for all $S \in \mathcal{X}$,

(i) if $CW(R_N, S) \neq \emptyset$, then $f(R_N, S) = CW(R_N, S)$;

(ii) if $CW(R_N, S) = \emptyset$, then

$$f(R_N, S) = \{x \in S \setminus CCL(R_N, S) \mid \{i \in N \mid t(R_i, S) = x\} \neq \emptyset\}.$$

According to case (ii) of this voting rule, all candidates who appear at least once in a top position are chosen. This violates conditional reduction monotonicity because there are profiles such that the elimination of a preference relation with a specific chosen candidate at the top from the profile does not remove this candidate from the chosen set. That the remaining axioms are satisfied is straightforward to verify.

Part C: Illustration of the proof method of Theorem 1. To make the argument employed in the proof of the only-if part of our theorem easier to follow, we provide an informal explanation of the proof structure, along with an illustrative example.

Consider a profile R_N and a feasible set S. First, observe that if there is a strong Condorcet winner, then this candidate must be chosen uniquely; this is an immediate consequence of the Condorcet winner criterion. If there is no strong Condorcet winner, we can, without loss of generality, assume that there are no cumulative strong Condorcet losers; this follows from cumulative Condorcet loser independence. What remains to be shown is that, in the absence of strong Condorcet winners and of cumulative strong Condorcet losers, the set of candidates chosen by the voting rule must be given by the set of plurality winners.

If the set of plurality winners coincides with the set of candidates S, it follows that each candidate in S has the same plurality-winner score n^* of top positions for the profile under consideration in the set S. Now an auxiliary profile $\overline{R}_N$ can be defined that preserves the top candidates of all voters in N and, moreover, is such that each candidate in S (and, thus, in the set of plurality winners) appears in each position n^* times so that $\overline{R}_N$ is a replica of a completely symmetric profile. By anonymity and neutrality, all candidates in S (and, thus, all plurality winners) must be chosen for the profile $\overline{R}_N$. Because the top candidates are the same for R_N and for $\overline{R}_N$, conditional tops only implies that these plurality winners must be chosen for R_N as well.

The last (and most subtle) case is obtained if the set of plurality winners is a strict subset of the set of feasible candidates S. Again, let n^* be the plurality-winner score for R_N in S. In contrast to the previous case, now there are candidates in S with a number of top positions assigned to them that is less than n^*. To deal with this situation, we first augment the profile R_N by adding as many voters as required to arrive at a profile that is an n^*-fold replica of a completely symmetric profile; that this augmentation is well-defined is established in the formal proof. Denote this augmented profile by $\overline{R}_{N\cup N'}$, where N' is the set of added voters. By anonymity and neutrality, it follows that all candidates in S must be chosen for the profile $\overline{R}_{N\cup N'}$. We then successively reduce the profile $\overline{R}_{N\cup N'}$ to a profile $\overline{R}_N$ that contains only the original voters in N. Each step in this reduction corresponds to one of the candidates that are in S but not in the set of plurality winners. Applying the axiom of conditional reduction monotonicity in each step, we conclude that, once the profile $\overline{R}_N$ is reached, the only candidates that remain chosen are the plurality winners for R_N in S. By construction, the profile $\overline{R}_N$ is such that all top candidates are identical to those in R_N and, using conditional tops only, we arrive at the desired conclusion that the chosen set for R_N is the set of plurality winners in S. We also need to show that, at any stage in the reduction process, the set of strong Condorcet winners and the cumulative set of strong Condorcet losers remain empty; this observation is, of course, essential in order to invoke conditional reduction monotonicity. The following example illustrates the proof method just outlined.

Example 1 Let $N = \{1, 2, 3, 4, 5, 6\}$ and $S = \{x, y, z, w, v\}$, and suppose that the profile R_N is given by

$$\begin{gathered} xR_1zR_1yR_1wR_1v, \\ xR_2zR_2yR_2wR_2v, \\ yR_3vR_3xR_3wR_3z, \\ yR_4wR_4vR_4xR_4z, \\ zR_5vR_5xR_5yR_5w, \\ wR_6zR_6vR_6xR_6y. \end{gathered}$$

Clearly, $CW(R_N, S) = CCL(R_N, S) = \emptyset$. We have $PW(R_N, S) = \{x, y\}$, $S \setminus PW(R_N, S) = \{z, w, v\}$, and the plurality-winner score is $n^* = 2$. We now add a set of four voters $N' = \{7, 8, 9, 10\}$ with preferences such that z appears $1 = 2 - 1 = n^* - |\{i \in N \mid t(R_i, S) = z\}|$ times at the top of a preference ordering, w is the top candidate in $1 = n^* - 1$ preferences, and v is the best candidate for $2 = n^* - 0$ voters. The reason why the number of added voters is equal to four is that this choice allows us to obtain a replica of a completely symmetric profile. By construction, in the augmented profile, each of the five candidates in S appears $n^* = 2$ times at the top of a preference ordering. Note that the top alternatives of the six initial voters 1 to 6 are unchanged in the augmented profile. We denote this augmented profile by $\overline{R}_{N \cup N'} = (\overline{R}_N, \overline{R}_{N'})$, and it is defined as

$$\begin{gathered} x\overline{R}_1y\overline{R}_1z\overline{R}_1w\overline{R}_1v, \\ x\overline{R}_2y\overline{R}_2z\overline{R}_2w\overline{R}_2v, \\ y\overline{R}_3z\overline{R}_3w\overline{R}_3v\overline{R}_3x, \\ y\overline{R}_4z\overline{R}_4w\overline{R}_4v\overline{R}_4x, \\ z\overline{R}_5w\overline{R}_5v\overline{R}_5x\overline{R}_5y, \\ w\overline{R}_6v\overline{R}_6x\overline{R}_6y\overline{R}_6z, \\ z\overline{R}_7w\overline{R}_7v\overline{R}_7x\overline{R}_7y, \\ w\overline{R}_8v\overline{R}_8x\overline{R}_8y\overline{R}_8z, \\ v\overline{R}_9x\overline{R}_9y\overline{R}_9z\overline{R}_9w, \\ v\overline{R}_{10}x\overline{R}_{10}y\overline{R}_{10}z\overline{R}_{10}w. \end{gathered}$$

In the augmented profile $\overline{R}_{N \cup N'}$, every candidate appears $n^* = 2$ times in each position and, therefore, the profile is a two-fold replica of a completely symmetric profile. By anonymity and neutrality, it follows that all five candidates must be chosen so that

$$f(\overline{R}_{N \cup N'}, S) = S = \{x, y, z, w, v\}.$$

We now show that, at each stage of the successive reduction process that leads us back to a profile involving the set of voters N, the set of strong Condorcet winners and the cumulative set of strong Condorcet losers remains empty. This allows us to invoke conditional reduction monotonicity in each iteration. To do so, let M be any non-empty subset of $N' = \{7, 8, 9, 10\}$. Observe that, in the profile $\overline{R}_{N \cup (N' \setminus M)}$, candidate x appears above candidate y at least $n^* = 2$ times and candidate y appears above candidate x at most $n^* = 2$ times. Analogously, y appears at least twice above z and z appears at most twice above y; z appears at least twice above w and w appears at most twice above z; w appears at least twice above v and v appears at most twice above w; and, finally, v appears at least twice above x and x appears at most twice above v. Thus, there are no strong Condorcet winners and no (cumulative) strong Condorcet losers in any subprofile of $\overline{R}_{N \cup N'}$ that contains N. As a consequence, conditional reduction monotonicity can be applied in the following process of successively eliminating the added voters. We perform this reduction one top candidate at a time in order to iteratively reduce the set of voters back to the original set N.

Let $M_1 = \{9, 10\}$. Thus, the profile $\overline{R}_{N \cup (N' \setminus M_1)}$ is given by

$$
\begin{array}{c}
x\overline{R}_1 y\overline{R}_1 z\overline{R}_1 w\overline{R}_1 v, \\
x\overline{R}_2 y\overline{R}_2 z\overline{R}_2 w\overline{R}_2 v, \\
y\overline{R}_3 z\overline{R}_3 w\overline{R}_3 v\overline{R}_3 x, \\
y\overline{R}_4 z\overline{R}_4 w\overline{R}_4 v\overline{R}_4 x, \\
z\overline{R}_5 w\overline{R}_5 v\overline{R}_5 x\overline{R}_5 y, \\
w\overline{R}_6 v\overline{R}_6 x\overline{R}_6 y\overline{R}_6 z, \\
z\overline{R}_7 w\overline{R}_7 v\overline{R}_7 x\overline{R}_7 y, \\
w\overline{R}_8 v\overline{R}_8 x\overline{R}_8 y\overline{R}_8 z.
\end{array}
$$

Because $t(\overline{R}_i, S) = v$ for all $i \in M_1$, conditional reduction monotonicity implies

$$f(\overline{R}_{N \cup (N' \setminus M_1)}, S) = f(\overline{R}_{N \cup N'}, S) \setminus \{v\} = \{x, y, z, w, v\} \setminus \{v\} = \{x, y, z, w\}.$$

Now let $M_2 = \{8\}$. The profile $\overline{R}_{N \cup (N' \setminus (M_1 \cup M_2))}$ is given by

$$
\begin{array}{c}
x\overline{R}_1 y\overline{R}_1 z\overline{R}_1 w\overline{R}_1 v, \\
x\overline{R}_2 y\overline{R}_2 z\overline{R}_2 w\overline{R}_2 v, \\
y\overline{R}_3 z\overline{R}_3 w\overline{R}_3 v\overline{R}_3 x, \\
y\overline{R}_4 z\overline{R}_4 w\overline{R}_4 v\overline{R}_4 x, \\
z\overline{R}_5 w\overline{R}_5 v\overline{R}_5 x\overline{R}_5 y, \\
w\overline{R}_6 v\overline{R}_6 x\overline{R}_6 y\overline{R}_6 z, \\
z\overline{R}_7 w\overline{R}_7 v\overline{R}_7 x\overline{R}_7 y.
\end{array}
$$

Because $t(\overline{R}_i, S) = w$ for all $i \in M_2$, conditional reduction monotonicity implies

$$f(\overline{R}_{N \cup (N' \setminus (M_1 \cup M_2))}, S) = f(\overline{R}_{N \cup (N' \setminus M_1)}, S) \setminus \{w\} = \{x, y, z, w\} \setminus \{w\} = \{x, y, z\}.$$

Finally, let $M_3 = \{7\}$. The profile $\overline{R}_{N \cup (N' \setminus (M_1 \cup M_2 \cup M_3))} = \overline{R}_N$ is given by

$$
\begin{array}{l}
x \overline{R}_1 y \overline{R}_1 z \overline{R}_1 w \overline{R}_1 v, \\
x \overline{R}_2 y \overline{R}_2 z \overline{R}_2 w \overline{R}_2 v, \\
y \overline{R}_3 z \overline{R}_3 w \overline{R}_3 v \overline{R}_3 x, \\
y \overline{R}_4 z \overline{R}_4 w \overline{R}_4 v \overline{R}_4 x, \\
z \overline{R}_5 w \overline{R}_5 v \overline{R}_5 x \overline{R}_5 y, \\
w \overline{R}_6 v \overline{R}_6 x \overline{R}_6 y \overline{R}_6 z.
\end{array}
$$

Because $t(\overline{R}_i, S) = z$ for all $i \in M_3$, conditional reduction monotonicity implies

$$
\begin{aligned}
f(\overline{R}_N, S) &= f(\overline{R}_{N \cup (N' \setminus (M_1 \cup M_2 \cup M_3))}, S) \\
&= f(\overline{R}_{N \cup (N' \setminus (M_1 \cup M_2))}, S) \setminus \{z\} \\
&= \{x, y, z\} \setminus \{z\} = \{x, y\} \\
&= PW(R_N, S).
\end{aligned}
$$

By conditional tops only (which can be applied because $CW(\overline{R}_N, S) = CCL(\overline{R}_N, S) = \emptyset$), it follows that

$$f(R_N, S) = f(\overline{R}_N, S) = \{x, y\} = PW(R_N, S) = f^D(R_N, S),$$

as desired. ■

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