Unemployment Risks and Intra-Household Insurance

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April 2020

Barcelona GSE Working Paper Series

Working Paper nº 1174

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April 22, 2020

Abstract

A spouse's income provides consumption insurance, but also increases the risks job-seekers take on in labor markets. We use a tractable directed search model with households to study how much public insurance should be provided in addition to the private insurance arrangements, and how it varies with the spouse's income. Private insurance is provided within the household through the spouse's labor supply and sought in the labor markets by applying to less risky jobs. Both insurance channels are used excessively in the laissez-faire equilibrium. In line with the empirical evidence, and in sharp contrast to the social planner's allocation, the equilibrium exhibits falling job-finding rates over the spouse's income distribution. If spouse's productivity is observable, the planner's allocation can be decentralized by implementing falling unemployment benefits.

Keywords: Unemployment Risks, Intra-Household Risk-sharing, Directed Search, Constrained Efficient Insurance
JEL Codes: J08, J22, J64, J65

*I thank Jeff Campbell, Yongsung Chang, Martin Kaae Jensen, Helmut Maurer, Jean-Baptiste Michau, Claudio Michelacci, Edgar Preugschat, Aysegul Sahin, Itay Saporta and Edward Schlee for their feedback. I am also grateful to numerous participants at conferences and seminars, and to Tommaso Santini for excellent research assistance. I acknowledge financial support from the Spanish Ministry of Science and Technology under Grant No. ECO 2013-46395, ECO2015-67602-P from MINECO/FEDER, UE, from the Generalitat de Catalunya through Grant 2017-SGR1765, and from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

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1 Introduction

How much public unemployment insurance should be provided in a world where consumers have various ways to buffer income shocks? And how should public provision vary with private insurance? We address these questions in an economy where a spouse's income plays a major role in consumption insurance in line with the empirical findings reported by Browning and Crossley (2001) and Blundell et al. (2016b). We consider two key features of the spouse's ability to provide insurance. First, it significantly determines the risks job-seekers take on in labor markets: We document that job-losers in the bottom quintile of the distribution of spouse's prior earnings are approximately 6 percentage points more likely to find a job than their counterparts whose spouse's earnings are above the median. We show that this pattern results from an ill-designed public insurance provision. Second, it varies greatly across households and is largely unobservable. This implies an additional burden on the design of the public insurance, and questions the standard single-replacement-rate public scheme. In this respect, our theory provides a rationale for the dependency allowance jobless workers with an unemployed spouse are entitled to in eight states of the US.[1]

In this paper, we first study the risk allocation in a laissez-faire economy populated by two-member, risk-averse households. We show that the equilibrium is consistent with the empirical evidence on private insurance. Then, we use the model to study the *constrained efficient* insurance provision. Specifically, we examine an incomplete markets, static economy where households are risk-sharing institutions formed by a job-seeker and his or her spouse, who is assumed to be employed. Households face unemployment risks and differ in their ability to provide insurance. The spouse is privately informed about his or her productivity and supplies labor after learning the search outcome. Job search is assumed to be directed to endogenize how much income risks households take on.

In this setting, consumption risks can be partially insured away through two different channels. First, insurance is privately arranged within the household by pooling income and adjusting the spouse's labor supply. As in Chetty and Saez (2010), we assume no moral hazard is generated as a result of such arrangements. The consumption insurance derived from the spouse's behavioral responses is defined as the difference between the income of the spouse of an unemployed worker and the income of the spouse of an employed one. Second, insurance can also be sought in the labor markets as job-seekers trade off higher wages and higher employment chances in the directed search framework. In equilibrium, workers whose

[1] The states are Connecticut, Illinois, Iowa, Maine, Michigan, New Jersey, Ohio and Pennsylvania. See the Department of Labor documentation: https://oui.doleta.gov/unemploy/pdf/uilawcompar/2018/monetary.pdf

spouse's productivity is above a certain threshold decide not to participate in the labor force as the gains from job search decline with the spouse's productivity. Furthermore, we show that both insurance channels are used more intensely the lower the spouse's productivity under limited substitutability between consumption and leisure and, loosely speaking, decreasing absolute risk aversion (DARA). The former ensures that both consumption and leisure are normal goods. With regard to the latter, directed job search can be thought of as investing in a risky asset. Then, job-seekers with less intra-household insurance are willing to pay a higher premium in forgone wages to insure themselves against unemployment risks under DARA, as in Acemoglu and Shimer (1999). Using SIPP data for the US, we show that such theoretical predictions are consistent with the empirical evidence. Furthermore, public provision of insurance crowds out private insurance provided through those two channels in line with the empirical literature,[2] but such crowding-out effects diminish with the spouse's income because of the concavity and limited substitutability assumptions.

When studying the constrained efficient provision of insurance, we focus on consumption risks and abstract from the redistributive motive of the social planner linked to the ex-ante heterogeneity across households. Specifically, we restrict the planner to make transfers only among ex-ante identical households. The planner faces a tension between the forgone leisure of the spouse, the vacancy creation costs and the distortions generated by the transfers scheme. Recall that households are assumed to be privately informed about the spouse's productivity and his or her labor supply as well as their job search. On the contrary, both the spouse's income and the worker's labor market status are observable to the planner. The optimal insurance provision is, hence, limited by these information frictions, which make the planner design an incentive compatible mechanism. We show that the insurance provision in the laissez-faire equilibrium falls short of the optimal level: Although the participation margin is efficiently set, the intra-household insurance and the labor market insurance margins are not because of the information frictions. To gain further insights, we remove such information frictions, and show that the equilibrium allocation does not solve the planner's problem with observable types either. This is because the participation constraint is slack in equilibrium, while the planner manages to eliminate consumption risks as much as ensuring participation permits through a system of transfers. Furthermore, and unlike the equilibrium outcome, the planner's allocation exhibits rising job-finding rates as transfers to one-earner households decline with the spouse's productivity because of concavity and limited substitutability. Importantly, the planner's allocation can be implemented through declining unemployment

[2] The crowding-out effects of public insurance provision are not negligible. See e.g. Cullen and Gruber (2000) and Engen and Gruber (2001). The latter estimate that the negative percentage effect of public insurance on asset holdings is twice as large for singles as for married individuals.

benefits over the distribution of households.

To assess quantitatively the constrained-efficient insurance provision and the welfare costs of private information, we calibrate our model to the US economy using the so-called standard macro preferences. The labor market and the intra-household insurance channels appeared both excessively used in equilibrium. The constrained efficient ratio of the transfers to the unemployed of a given type of household to the income obtained by their employed counterparts, a form of replacement rate, declines significantly with the spouse's income. Interestingly, the welfare costs of private information appear to be very small.

This paper contributes to several branches of the labor literature. First, following the lead of Burdett and Mortensen (1978) in the search literature, Guler et al. (2012) and Pilossoph and Wee (2018) address the joint search of couples in a McCall setting with an exogenous distribution of wages. Birinci (2019), using a random search model, and Wang (2019), in a setting with time-invariant exogenous wages, quantitatively examine the cyclicality of optimal unemployment benefits. The key contribution relative to their work is our focus on the constrained efficient allocation of endogenous consumption risks and how public provision of insurance varies over the distribution of households.

Second, Hansen and İmrohoroğlu (1992), Lentz (2009), Krusell et al. (2010), Lifschitz et al. (2018) and Braxton et al. (2019), among others, study numerically the optimal unemployment insurance replacement rate in search settings where workers are allowed to save and borrow.[3] With the exception of the latter, either wages are taken exogenously or search is assumed to be random. The optimal replacement rate is typically found to be fairly low because of the large distortions on job creation and the crowding out effects.[4] Haan and Prowse (2017) add the spouse's labor supply to a life cycle model with exogenous wages, and find that the optimal replacement rate is 20% when social assistance is close to 0 for Germany. Similarly, we find that the weighted average constrained efficient replacement rate is just below 22% in our numerical exercise. Ortigueira and Siassi (2013) and Choi and Valladares-Esteban (2016) have also numerically approached the role of the spouse's labor supply as an insurance mechanism in a Bewley framework with exogenous income risks.

Finally, the dependence of the constrained efficient transfers on the spouse's income is consistent with the aforementioned unemployment insurance schemes in some US states as

[3] Wealth holdings are typically fairly modest for newly unemployed workers (see e.g. Engen and Gruber (2001) and Chetty (2008) for the U.S. and Kolsrud et al. (2018) for Sweden). However, Braxton et al. (2019) document that the US constrained job losers default on their credit obligations and the unconstrained ones borrow.

[4] For example, Krusell et al. (2010) find that the optimal replacement rate is 30%. In Lifschitz et al. (2018), the welfare gains of unemployment insurance stem largely from redistribution across exogenously different workers, and the optimal replacement rate is around 10% if such exogenous heterogeneity is absent.

well as supported by the empirical evidence. For example, Browning and Crossley (2001) estimate a small effect of the replacement rate on household's expenditure, but significant for those households with an unemployed spouse. This theoretical result is also in line with a consistent finding in public economics: significant welfare gains can be obtained from making policy instruments contingent on worker's observable characteristics. Beyond Alesina et al. (2011) analysis of gender-based taxation, Kleven et al. (2009) find that optimal tax rates on an individual's labor income differ by the earnings of his or her spouse. Likewise, Weinzierl (2011) and Farhi and Werning (2013) show that the optimal tax rates vary over the life cycle, and Michelacci and Ruffo (2015) extend this result to unemployment benefits.

The paper proceeds as follows. Section 2 describes the economy. In Section 3, we study the market equilibrium. Section 4 analyzes the planner's solution. In Section 5, we undertake a numerical exploration. The last section concludes.

2 Economy

In this section, we describe a frictional model of the labor market that will be used to examine private insurance provision against unemployment risks as well as the constrained efficient insurance provision.

Consider a static economy populated by a measure one of two-member households and a large continuum of risk-neutral firms. Following e.g. Guler et al. (2012), we assume that households are the decision-making units. Households are formed by a jobless worker and his or her spouse, who is assumed to be employed. The former can search for a job and supplies indivisible labor in the market. The spouse is endowed with productivity $x \in X \equiv [\underline{x}, \overline{x}]$, and chooses labor supply $\ell \in [\underline{\ell}, \overline{\ell}]$.[5] Let $F(x)$ denote the measure of households with type below or equal to x.

Information Frictions. Unlike an individual's income, neither the spouse's productivity nor his or her labor supply is observable to the social planner.[6] Because it is convenient to work with observable variables, we mostly refer to the spouse as deciding income y instead of

[5]The labor supply asymmetry between the two household members resembles the one modeled by Kleven et al. (2009) to study optimal joint taxation. In their case, workers are endowed with some unobservable productivity and choose labor income, whereas spouses decide whether or not to participate at a fixed number of hours.

[6]Productivity x can be alternatively interpreted as ability or hourly wage. Our interpretation neglects general equilibrium effects on x, however. As pointed out by Salanie (2011), if labor supply were interpreted as hours worked, the government could force employers to report them.

labor supply $\ell = \frac{y}{x}$, but use both terms indistinctly.[7] Likewise, a worker's status is observable, whereas whether a household engages in job search is not.

Preferences. Households derive utility from consumption, c, and leisure of the spouse.[8] Income is pooled within a household. We impose the following assumptions on the utility function υ that describes households' preferences:

A1. υ is thrice continuously differentiable.

A2. Strict monotonicity in consumption and leisure: $\upsilon_c > 0$, $\upsilon_\ell < 0$.

A3. True concavity, i.e. the Hessian is negative definite: $\upsilon_{cc}, \upsilon_{\ell\ell} < 0$, and $\upsilon_{\ell\ell}\upsilon_{cc} - \upsilon_{c\ell}^2 > 0$.

A4. Limit conditions: $0 < \lim_{\ell \to \underline{\ell}} (\upsilon_c x + \upsilon_\ell)$ and $\lim_{\ell \to \overline{\ell}} (\upsilon_c x + \upsilon_\ell) < 0$, for all $x \in X$.

A5. Limited substitutability between consumption and leisure: $\upsilon_{c\ell} < \upsilon_{cc}\frac{\upsilon_\ell}{\upsilon_c}$, and $\upsilon_{c\ell} \leq \upsilon_{\ell\ell}\frac{\upsilon_c}{\upsilon_\ell}$.

The first four conditions are fairly standard. Limited Edgeworth-Pareto substitutability is a necessary and sufficient condition for both consumption and leisure to be normal goods, given concavity. It holds under additive separability between consumption and leisure, and more generally under weak complementarity, $\upsilon_{c\ell} \leq 0$. We will pay particular attention to the following family of preferences, which are standard in the macroeconomics literature:

$$\mathcal{F}_1 \equiv \left\{ \upsilon(c,\ell) = \frac{c^{1-\theta}}{1-\theta} - \gamma\frac{\ell^{1+\xi}}{1+\xi} \;\;\middle|\;\; \theta, \xi, \gamma > 0 \right\}$$

The analysis carried out here allows for non-separability and some extent of substitutability between consumption and leisure, in line with some empirical evidence. See e.g. Hall and Milgrom (2008) and Blundell et al. (2016b). In particular, we also consider the following family of preferences:

$$\mathcal{F}_2 \equiv \left\{ \upsilon(c,\ell) = \frac{\big(c \cdot exp(-\psi\ell)\big)^{1-\theta}}{1-\theta} \;\;\middle|\;\; \theta > 1, \psi > 0 \right\}$$

[7] For expositional reasons, it is convenient to assume that $\underline{x} > 0$. According to CPS data, not-in-the-labor-force wives amount to 30 percent of married women. To abstract from the underlying reasons of their non-participation decision -caring of children and elderly, etc.-, one-earner households can be thought of as those with the spouse's productivity x being arbitrarily small. We shall deal with a mass of households with zero spouse productivity in the quantitative exercise in Section 5.

[8] We are implicitly assuming that the worker derives no leisure regardless of the employment state as he or she works full time either in the market or at home.

We will refer to the former family as the standard macro preferences and to the latter family as the LMP preferences.[9] The latter preferences are of particular interest as the spouse's income adjustments provide full consumption insurance.

Timing. There are four stages. In stage one, unemployed workers direct their search, and choose a submarket or location where to submit a job application at cost k_w. Similarly, in stage two, firms decide on the submarket where to place their vacancies, and incur cost k_f. Market productivity of newly employed workers is denoted by y_w.[10] As usual in the search literature, each recruiting firm holds a single vacancy. To ensure that vacancy creation is a profitable activity, we assume that $y_w - z > k_f$. Meetings take place in stage three as described below. In this frictional economy, a number of workers and jobs remain unemployed and vacant, respectively, at the end of the period. In stage four, spouses decide their income y. Both production and consumption take place. Unemployed workers produce z units of output at home.

Matching. Meetings are bilateral. We denote by q the expected queue length or ratio of job-seekers to vacancies at a particular location. Although the ratio q depends on the characteristics of the jobs posted in that location, we eliminate this dependence notation unless necessary for the sake of readability. Workers find a job with probability $\nu(q)$, whereas firms fill their vacancies with probability $\eta(q)$. It must be the case that $\nu(q) = \frac{\eta(q)}{q}$ in any given location since the mass of newly employed workers equals the mass of newly filled vacancies. We assume that ν is a decreasing function to capture the intuition that it is harder to find a job in tighter labor markets. Similarly, η is assumed to be increasing. Likewise, the following limit conditions are necessary to ensure existence of the equilibrium and the planner's allocations: $\lim_{q\to 0} \nu(q) = \lim_{q\to\infty} \eta(q) = 1$ and $\lim_{q\to\infty} \nu(q) = \lim_{q\to 0} \eta(q) = 1$. Let $\phi(q) \equiv \frac{q\eta'(q)}{\eta(q)}$ denote the elasticity of the job-filling probability, which is assumed to be a decreasing function.

[9] Standard macro preferences are used e.g. by Heathcote et al. (2014) and Gayle and Shephard (2019). The LMP preferences extend those assumed in Low et al. (2010) to the intensive margin of labor supply. Variations of the LMP preferences are assumed e.g. by Blundell et al. (2016a) and Shephard (2019).

[10] Assortative mating is assumed away. We interpret this as a conservative assumption. Notice also the very modest earnings correlation within married couples.

3 Market Economy

In this section, we analyze an economy in which agents make decisions in a decentralized way. There is free entry of firms, and potentially infinitely many submarkets. Each submarket is defined by a wage offer, w, and its associated queue length, $q(w)$. Whereas firms decide the wage they commit to when creating a vacancy, a household's decision is threefold. First, it decides whether or not to participate in the labor force. Second, conditional on participating, the household chooses a submarket to submit a job application. Then, after learning the search outcome, it decides the labor supply of the spouse. We start with stage four, and proceed backwards.

Stage Four. Let w denote the income of the job-seeker at the end of the period, with $w = z$ if unemployed. We denote the indirect utility function of a household of type x by V_x, which is defined as

$$V_x(w) \quad \equiv \quad \max_y \ \upsilon\big(y + w, \frac{y}{x}\big) \tag{1}$$

The Weierstrass theorem together with Assumption A1 ensures that V_x is well-defined. The first order condition is also sufficient because of Assumptions A3 and A4. Therefore, by equating the marginal utilities of consumption and labor, the following equation uniquely determines the income of the spouse, $y_x(w)$.[11]

$$\upsilon_c\big(y + w, \frac{y}{x}\big)x \quad = \quad -\upsilon_\ell\big(y + w, \frac{y}{x}\big) \tag{2}$$

Stage Two. There is entry of firms in all submarkets as long as expected profits are positive, both in and out of equilibrium. That is, the following condition must hold for all $w \in [z, y_w]$:

$$\eta(q(w))(y_w - w) \leq k_f, \text{ and } q(w) \leq \infty, \text{ with complementary slackness.} \tag{3}$$

Intuitively, the larger the wage, the higher the ratio of job-seekers to vacancies. In the limit, no positive mass of firms commit to a wage equal to the market productivity of workers.

Stage One. The expected utility of a household of type x amounts to

$$V_x(z) + \max\{0, S(x) - k_w\},$$

[11] In the case of LMP preferences, Assumption A4 does not hold for spouse's productivity values below $w\psi$. In that case, a spouse's optimal labor supply is 0.

where the second argument in the max operator, $S(x) - k_w$, denotes the search value (or the expected gains to searching in the market economy) net of the participation costs. If the expected gains from a job application, $\nu(q(w))\big(V_x(w) - V_x(z)\big)$, do not outweigh the participation costs in any submarket w, i.e. if $S(x) < k_w$, then it is optimal for the household not to participate in the labor market. Otherwise, the household does seek job opportunities.

3.1 Equilibrium.

We now turn to the equilibrium definition.

Definition 1 *A directed search equilibrium consists of a search value $S^* : X \to \mathcal{R}_+$, the income of the spouse of an employed worker $\{y_x^{e*}\}_{x \in X} : [z, y_w] \to \mathcal{R}_+$ and the income of the spouse of an unemployed worker $\{y_x^{u*}\}_{x \in X} \in \mathcal{R}_+$, a set of labor force participants $X_p^* \subset X$, wages $\{w_x^*\}_{x \in X_p^*}$, and a queue length function $Q^* : [z, y_w] \to \mathcal{R}_+$ such that:*

i) Households' optimal decisions:

(a) labor force participation:

$$x \in X \setminus X_p^* \text{ if and only if } \nu(Q^*(w))\big(V_x(w) - V_x(z)\big) < k_w, \ \forall w \in [z, y_w]$$

(b) job search: $\forall x \in X_p^*$,

$$\begin{aligned} \nu(Q^*(w))\big(V_x(w) - V_x(z)\big) &\le S^*(x), \qquad \forall w \in [z, y_w], \text{ and} \\ \nu(Q^*(w_x^*))\big(V_x(w_x^*) - V_x(z)\big) &= S^*(x) \end{aligned}$$

(c) spouse's income: for all $w \in [z, y_w]$, $y_x^{e*}(w)$ and y_x^{u*} solve the household's problem (1), $\forall x \in X_p^*$ and $\forall x \in X$, respectively.

ii) Free entry of firms:

$\eta(Q^*(w))(y_w - w) \le k_f$, $\forall w \in [z, y_w]$, and $Q^*(w) \le \infty$, with complementary slackness. In particular, the first inequality is an equality for all w_x^*.

The first equilibrium condition is self-explanatory. The second condition states that workers form rational expectations about firms' decisions in this second stage. Specifically, they expect the ratio of job-seekers to firms in any submarket to be determined by the free entry condition. Thus, they trade off a higher wage and a higher job-finding probability.

We now characterize a household's indirect utility function V_x and the optimal income of the spouse since they are two key objects in the analysis hereafter. Their properties, stated in the following proposition, are inherited from the assumptions on preferences. In particular, the limited substitutability assumption is central for a number of results. First, for any given productivity x, consumption increases and the spouse's income decreases with the worker's wage.[12] Put differently, spouse's leisure is a normal good. The reduction in spouse's income is smaller than the increase in wages for the standard macro preferences, whereas it is one-to-one for the LMP preferences. In this latter case, there is full consumption insurance through the spouse's labor supply as equation (2) becomes $w + y_x^e(w) = \frac{x}{\psi}$. Second, for a given wage $w \geq z$, the spouse's income (and hence the household's consumption) increases with his or her productivity. Third, a household's indirect utility function $V_x(w)$ is increasing and concave in w, and increasing in x. Importantly, concavity of $V_x(w)$ requires not only concavity of the utility function v, but also limited substitutability between consumption and leisure. Further, the marginal utility gains from higher wages fall with x.

Importantly, a lower spouse's productivity can be interpreted as a higher risk aversion (i.e. $V_{x'}$ is a concave transformation of V_x, for $x' < x$) if the absolute risk aversion of the household's indirect utility function is decreasing in x. In particular, this is the case for both the standard macro and the LMP preferences.

Proposition 3.1 ***Household's Indirect Utility Function and Spouse's Income.***

1. *For any productivity $x \in X$, the optimal solution $y_x^e(w)$ is twice continuously differentiable and strictly decreasing in wage w, while consumption $w + y_x^e(w)$ is non-decreasing. In particular, $y_x^e(w) < y_x^u$ for all $w > z$. Furthermore, function $V_x(w)$ is twice continuously differentiable, strictly increasing and concave in w.*

2. *For any wage $w \geq z$, the optimal solutions $y_x^e(w)$ and y_x^u (as well as consumption $w + y_x^e(w)$) is twice continuously differentiable and increasing in productivity x. Furthermore, function $V_x(w)$ is strictly increasing and its derivative with respect to wage w, $V_x'(w)$, strictly decreasing in the spouse's productivity x.*

3. *Consider $x, x' \in X$ such that $x' < x$. There exists a function $\mathcal{V}$ such that $V_{x'}(w) = \mathcal{V}\big(V_x(w)\big)$, and $\mathcal{V}' > 1$. Furthermore, $\mathcal{V}$ is concave (convex) if and only if the absolute risk aversion of V_x is decreasing (increasing) in x.*

[12] The negative cross-wage elasticity is in line with the empirical evidence. Hyslop (2001) estimates that a \$1 increase in a husband's hourly wages reduces the wife's annual earnings by \$300 and her labor supply by 35 annual hours. Devereux (2004) estimates the cross-wage elasticity of wife's hours worked at -0.4, while Blau and Kahn (2007) at -0.2. Likewise, Blundell et al. (2016b) find the Marshallian cross-wage elasticity to be -0.75 for women and -0.22 for men.

4. Consider an additively separable utility function.[13] *If $\frac{dA_c}{dc}, \frac{dA_\ell}{d\ell} \leq 0$ and $\frac{dR_\ell}{d\ell} \geq 0$ with some strict inequality, then the absolute risk aversion of V_x is decreasing in x. In particular, preferences that belong to $\mathcal{F}_1$ satisfy those conditions. Instead, if $\frac{dA_c}{dc}, \frac{dA_\ell}{d\ell} \geq 0$ and $\frac{dR_\ell}{d\ell} \leq 0$, then the absolute risk aversion of V_x is increasing in x. Furthermore, the absolute risk aversion of V_x is also decreasing in x if $\upsilon \in \mathcal{F}_2$.*

Next, we turn to the equilibrium characterization. A household's expected value at the beginning of the period is

$$V_x(z) + \max\Big\{0\ ,\ S(x) - k_w\Big\}, \tag{4}$$

where the max operator refers to the participation decision, and the search value is defined as

$$\begin{aligned} S(x) \equiv \quad & \max_{q\geq 0, w\in[z,y_w]} \quad \nu(q)\big(V_x(w) - V_x(z)\big) \\ & \quad\text{s. to} \qquad \text{condition (3)} \end{aligned} \tag{5}$$

where the free entry condition determines the relationship between wages and job-finding probabilities. No submarket with a promised expected value below $S(x)$ attracts applications from job-seekers of type x.

The following proposition states that there exists a unique equilibrium, and characterizes it. First, the equilibrium participation decision boils down to a reservation rule because the search value $S(x)$ decreases with productivity x. That is, $X_p^* = [\underline{x}, x^*]$. The returns to job search diminish with a spouse's productivity because of the income-pooling mechanism and monotone and concave preferences together with limited substitutability. Second, conditional on participating, equilibrium condition (6) results from combining the first order conditions of the household's problem (5). It equates the costs of creating a vacancy to the expected profits, which amount to the probability of filling a vacancy times the share $1-\phi(q)$ of the joint value of the firm-worker pair. The latter is the sum of the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$, and the firm's profits, $y_w - w$. Equilibrium equation (7) is the zero-profit condition. For notational simplicity (with some abuse of notation), we denote hereafter the equilibrium queue length and spouse's income at wage w_x^* as $q_x^* \equiv Q^*(w_x^*)$ and $y_x^{e*} \equiv y_x^{e*}(w_x^*)$, respectively.

Proposition 3.2 ***Equilibrium Characterization.***

There exists a unique equilibrium. Furthermore,

[13] To save on notation, $A_j \equiv |\frac{\upsilon_{jj}}{\upsilon_j}|$ and $R_j \equiv jA_j$, for $j \in \{c, \ell\}$, refer to the absolute and relative risk aversion, respectively.

1. *$S^*(x)$ decreases with x. Thus, there exists a reservation productivity $x^* \in X$ such that a household of type x participates in the labor force if and only if $x \leq x^*$. Therefore, $X_p^* = [\underline{x}, x^*]$.*

2. *for any $x \in X$, the equilibrium income y_x^{u*} is determined by equation (2) for $w = z$. Furthermore, for any $x \in X_p^*$, the equilibrium tuple (q_x^*, w_x^*, y_x^{e*}) is characterized by equation (2) and*

$$k_f = \eta(q)(1-\phi(q))\left(\frac{V_x(w)-V_x(z)}{V_x'(w)} + y_w - w\right) \tag{6}$$

$$k_f = \eta(q)\big(y_w - w\big) \tag{7}$$

Importantly, condition (6) implies that the equilibrium is generically separating: the equilibrium allocation exhibits wage dispersion insofar as the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$, depends on x even though workers are equally productive and firms are homogeneous. This highlights a source of wage dispersion that has been overlooked when using only individual data in a Mincerian regression.

Why does wage dispersion arise in equilibrium? Recall that job-seekers trade-off wages and employment risks in a directed search setting. Thus, wage dispersion and differences in risk attitudes are the two sides of the same coin. To obtain some insights, consider preferences that are quasi-linear in consumption and in leisure. In both cases, all households direct their search to the same submarket in equilibrium (to maximize expected income net of home production) because the household's surplus amounts to $w - z$. In the first case, households are risk neutral in consumption, whereas full consumption insurance through the spouse's labor supply arises in the second case. It is then tempting to conclude that wage dispersion results from consumption risk aversion and differences in private insurance arrangements within the household. Nonetheless, recall that full insurance against consumption risks also takes place with LMP preferences, whereas the household's surplus decreases with x.[14] Next section studies why some households take on more risks than others.

3.2 Private Insurance

In this section, we examine how private insurance varies across households who do participate in the labor force. A households' expected value, in excess of $V_x(z) - k_w$, amounts to

[14] Notice that the utility function $v(c,\ell) = \log(c) - \psi\ell$, which is quasi-linear in leisure, is the limit case of the LMP preferences (properly adjusted) as elasticity θ goes to 1. Also recall that spouse's income is positive provided that $z\psi < x$ with LMP preferences. Therefore, all households with spouse's productivity below $z\psi$ are identical, and apply to the same jobs.

$V_x(w) - V_x(z)$ with probability $\nu(q)$ and to 0 with the remaining probability. Our focus here is on consumption insurance arrangements beyond income-pooling in an incomplete markets economy. We first look at the insurance risk-averse agents seek in labor markets, which is linked to the equilibrium wage dispersion characterized above. Then, we examine the private insurance provided within the household through adjustments in the labor supply of the spouse for given labor market conditions. The main result is that, under some mild assumptions on preferences, the higher the spouse's productivity is, the less additional insurance is arranged through either channel, which is consistent with the empirical evidence.

3.2.1 Labor Market Insurance

How does job search differ across households? Attitudes towards unemployment risks may vary with both preferences and household's total income. It is widely accepted that the risk premium we are willing to pay to get rid of a risk decreases with our wealth, which is the case when preferences exhibit decreasing absolute risk aversion. See e.g. Pratt (1964) and Gollier (2004). In a similar fashion, in our setting, households of higher types apply to higher-wage jobs that are harder to obtain if the absolute risk aversion of the household's indirect utility function decreases with the spouse's productivity.

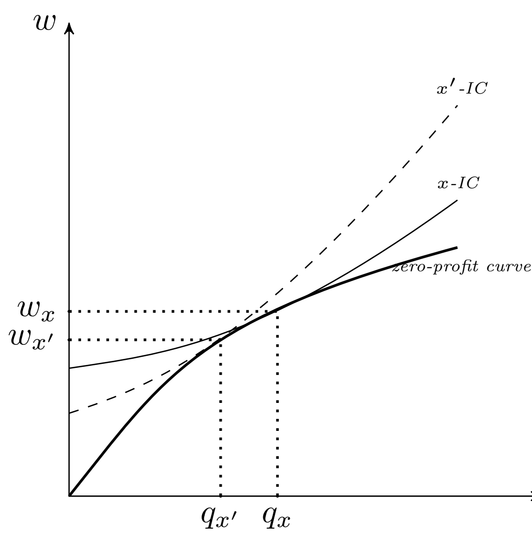


Figure 1: Equilibrium Sorting

To obtain some intuition on the relationship between decreasing absolute risk aversion and the equilibrium sorting outcome, consider the household's problem (5). Figure 1 depicts the indifference curves for two different types of households as well as a firms' isoprofit curve. The latter does not depend on a spouse's productivity as all job-seekers are assumed to be equally productive. The slope of a household's indifference curve amounts to $\frac{-\nu'(q)}{\nu(q)}$ times the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$. Proposition 3.3 states that the household's surplus declines with x if and only if the absolute risk aversion of the household's indirect utility function also declines with x. Flatter indifference curves lead to higher wages and higher unemployment risks through longer queue lengths. This is the case, in particular, for the standard macroeconomic and the LMP preferences. Interestingly, this proposition highlights an endogenous force for a positive correlation of earnings between the household's members.

This is in line with the findings in Acemoglu and Shimer (1999), where workers have some wealth endowment and direct their search,[15] and Guler et al. (2012), where married workers sequentially draw offers from a wage distribution and set a reservation wage as a function of the spouse's wage. In both economies, the transition rate from unemployment to employment decreases with the respective private insurance provision under DARA preferences.

Proposition 3.3 ***Sorting in Labor Markets.***

The household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(w)}$, is strictly decreasing (increasing) in x, and both w_x and q_x strictly increasing (decreasing), if and only if so is $\frac{-V_x''(w)}{V_x'(w)}$. In particular, if $v \in \mathcal{F}_1 \cup \mathcal{F}_2$, then both w_x^* and q_x^* increase with x.

3.2.2 Additional Intra-household Insurance

We now turn to the private provision of insurance within the household, and how it evolves over the spouse's productivity distribution. As a first attempt to study such dynamics, consider the variation of the difference $V_x(w) - V_x(z)$ over the distribution of participating households while keeping the market wage w fixed. Lemma 3.4 states that this difference declines with the spouse's productivity.[16] Because of the Envelope theorem, this pattern primarily results from the mere rise in productivity x and the insurance provided through income-pooling.

To focus on the additional insurance that stems from the spouse's behavioral responses, which can be empirically tested, we define the additional intra-household insurance as the spouse's income difference between a single-earner and a two-earner households, $\frac{y_x^{u*}-y_x^{e*}}{y_x^{e*}}$. This measures how much extra spouse's income is optimal upon the realization of the unemployment

[15] It is worth noticing that the absolute risk aversion of the indirect utility function with preferences that are CARA-type, $v(c,\ell) = \frac{-e^{-\gamma_c c}}{\gamma_c} - \gamma\frac{e^{\gamma_\ell \ell}}{\gamma_\ell}$, with $\gamma, \gamma_c, \gamma_\ell > 0$, is decreasing in x; hence, wages and queue lengths also increase with productivity x.

[16] Notice that this result does not imply that the household's surplus is decreasing in the spouse's productivity as the marginal value $V_x'(w)$ also declines with x as claimed in Proposition 3.1.

risks. Importantly, this measure captures a variation in the income risk that the insurance itself generates. For the sake of the argument, consider the LMP preferences, in which case $\frac{y_x^{u*}-y_x^{e*}(w_x^*)}{y_x^{e*}(w_x^*)} = \frac{w_x^*-z}{\frac{x}{\psi}-w_x^*}$. This measure declines with x for any given w_x^*. However, as stated in Propositions 3.1 and 3.3, households of higher types are less risk averse and apply to higher-wage, lower-meeting-rate jobs, thereby inducing a lower spouse's income $y_x^e(w_x)$ and a higher additional intra-household insurance. We address this issue below. Nonetheless, it is worth underscoring that the spouse's income does not generate moral hazard insofar as the household is the decision-making unit.

We examine the variation of the additional insurance with a spouse's productivity. Consider the following monotonic transformation of it,

$$\ln(y_x^{u*}) - \ln(y_x^{e*}) = -\int_z^{w_x^*} \frac{\frac{\partial y_x^{e*}(w)}{\partial w}}{y_x^{e*}(w)} dw$$

We differentiate it to obtain

$$\frac{\partial\big(\ln(y_x^{u*}) - \ln(y_x^{e*})\big)}{\partial x} = \underbrace{-\frac{\frac{\partial y_x^{e*}(w_x^*)}{\partial w}}{y_x^{e*}(w_x^*)}\frac{\partial w_x}{\partial x}}_{d_1(x)} \underbrace{-\int_z^{w_x^*} \frac{\frac{\partial^2 y_x^{e*}(w)}{\partial w \partial x} y_x^{e*}(w) - \frac{\partial y_x^{e*}(w)}{\partial x}\frac{\partial y_x^{e*}(w)}{\partial w}}{y_x^{e*}(w)^2} dw}_{d_2(x)} \tag{8}$$

Changes in productivity x affect both the upper limit w_x^* and the integrand of the expression for the log income difference. The former channel captures the labor market responses to a higher spouse's productivity as reasoned above. Thus, if a household's surplus falls with x, the term $d_1(x)$ in expression (8) is positive as the spouse's leisure is a normal good. When abstracting from such labor market responses, the term $d_2(x)$ in that expression is the primary object of our analysis. Consider the numerator of the integrand of $d_2(x)$. While the sign of the product of the two partial derivatives is negative as stated in Proposition 3.1, additional assumptions on higher order derivatives of the utility function are required to determine the sign of the cross derivative and of the sum itself. Nonetheless, Lemma 3.4 claims that the integrand is always positive if preferences are additively separable, or of the LMP sort. As a result, the additional intra-household insurance falls with the spouse's productivity.

Lemma 3.4 ***Differences in Intra-household Insurance.***

Given a wage w_x^*,

1. *the difference $V_x(w_x^*) - V_x(z)$ declines with spouse's productivity x,*

2. *the additional intra-household insurance, $d_2(x)$, declines with x if preferences either are additively separable or belong to set $\mathcal{F}_2$.*

The bottom line is that workers with lower spouse's productivity use both insurance channels more intensely. One interpretation of this result is that a public provision based on individual past income instead of the household's total income may be inefficiently biased towards those workers who value additional insurance less and benefit from it for a longer non-employment period.

3.2.3 Testable Implications

We now discuss whether the theoretical predictions regarding these two channels of private insurance provision are supported by the empirical evidence.

We use data from the Survey of Income and Program Participation (SIPP) for the U.S. Our dataset covers the period from 1996:6 to 2013:6, and comprises individuals aged 25-55 who have been employed at least twice within a 3-4 year period. In addition to demographics, we have precise information about their labor market status, earnings, occupation, non-employment duration if jobless, wealth, and their spouse's earnings if married. Labor market status is reduced to employment (E) and non-employment ($\not{E}$). See Appendix 7.3 for further details.

We first use Cox proportional hazard models to estimate the relationship between the hazard rate of a worker and his or her spouse's earnings prior to the non-employment spell. The results are shown in Table 3. In line with Guler et al. (2012), this relationship is negative, also when controlling for the household's net liquid wealth. We find that a worker whose spouse's earnings are in the bottom quintile of the distribution is over 6 percentage points more likely to find a job than workers whose spouse's earnings are above the median. The gap is even larger for those households with a spouse with no earnings, a group that amounts to 16.5% of our sample of married individuals.

According to these findings and Proposition 3.3, theory predicts that a spouse's earnings are positively related to the value of the job applied to. An indirect way to test this is by examining the relationship between occupation-switching rates and a spouse's earnings. To the extent that the returns to occupation-specific human capital are sizable (see e.g. Kambourov and Manovskii (2009)), it is reasonable to think that households with lower occupation-switching rates typically obtain higher job values. Table 4 reports the Probit estimates. Jobless workers whose spouse's earnings are above the median of the distribution are significantly more likely not to switch occupations upon reemployment despite their longer non-employment spells. This is a strong result since longer spells are associated with higher switching rates. See also e.g. Carrillo-Tudela and Visschers (2013). Therefore, this evidence suggests that higher spouse's prior earnings are associated with a higher job value.

Finally, we explore how much additional insurance is effectively arranged within the household during a non-employment spell by regressing the change in the spouse's log earnings between the end and the beginning of a $\not{E}$ spell of a worker on the distribution of the spouse's log earnings prior to the $\not{E}$ spell.[17] The first column of Table 5 displays the estimates for the whole sample of married workers. Spouses at the bottom of the distribution increase significantly more their market earnings than those at the top. It could be argued that spouses whose earnings are above the median have little margin to adjust their labor supply, while those in the lower tail of the distribution may be working part time. The second column of the table restricts the sample to those workers with full time spouses, and the aforementioned patterns still hold.[18] These findings are quite in line with Cullen and Gruber (2000), although, unlike them, we do find significant effects on the extensive margin.[19]

3.3 Does Public Provision Crowd Out Private Insurance?

We now examine how the public insurance provision affects the private insurance arrangements, and how such effects vary over the distribution of households. To capture the first-order effects and abstract from the tax-related general equilibrium effects, we study the implications derived from changes in home production, z.

The following proposition states that a more generous public provision of insurance (i.e. a higher z) increases wages and reduces job-finding rates, a result that is a well-known in search theory. Acemoglu and Shimer (1999) refer to this as the *market-generated moral hazard* as the public insurer cannot oblige workers not to apply to higher wages because job applications are private information. In the same fashion, the gains from job search fall with z, thereby

[17] In the specifications displayed in Table 5, we also control for the duration of the non-employment spell of the worker, which could capture the insurance-inducing extra risks. Insofar as this is not fully the case, theory suggests that the actual additional insurance differences across households are larger than what differences in raw log earnings capture. Furthermore, our estimates of the additional intra-household insurance arrangements may also be conservative as there is evidence that the behavioral responses of the spouses start before the non-employment spell, when information about job loss risks arrives, and may take place later upon learnings about the long-run impact on the worker's earnings. See e.g. Stephens (2002) and Hendren (2017).

[18] To test whether such patterns largely capture a mean-reversion effect, we regress the earnings changes on the distribution of the spouse's earnings averaged over the nine months prior to the job loss. As the third column shows, the estimates are hardly different from those in the first column.

[19] Although not shown here as reporting log earnings, the increase in earnings is also sizable on the extensive margin, i.e. for those spouses with no previous earnings. Likewise, the same pattern is observed if the sample is further constrained to observations of only one sex or when further restricting the sample to households with no children to abstract from the substitution effect in home production within the household.

reducing the mass of labor market participants. Labor supply and income of the spouses of the unemployed also drop because a spouse's leisure is a normal good. This together with the increase in wages implies that the labor supply of the spouse of an employed worker also decreases with z. These direct and indirect negative effects of unemployment benefits on a spouse's labor supply are consistent with the empirical evidence reported by Cullen and Gruber (2000). They estimate that each \$100 in potential benefits lowers the working hours of wives of employed and unemployed workers by 5.2 and 22.7 per month, respectively. It follows from both higher unemployment and lower spouse's labor supply that aggregate output falls with unemployment benefits. Further, by abstracting from the y_x^{e*} fall induced by workers taking on more risks as benefits rise, we conclude that the additional intra-household insurance, $\frac{y_x^{u*}-y_x^{e*}}{y_x^{e*}}$, also declines with the generosity of the public provision.

Next, we look at how changes induced by a more generous public provision vary over the distribution of households. First, the induced decline in the search value is more pronounced the lower the spouse's productivity partly because such workers have higher job-finding rates if the absolute risk aversion of the household's indirect utility function declines with x. Second, the induced wage increases always fall with the spouse's productivity; hence, the crowding-out effects of the public provision on private insurance appear to be larger for those households with lower spouse's productivity. These patterns rely on the monotonicity and concavity as well as limited substitutability assumptions. Third, for the standard macro and LMP preferences, the induced fall in the additional intra-household insurance is also steeper the lower the spouse's productivity.

Proposition 3.5 ***Crowding-out Effects.***

1. *Wages increase whereas job-finding probabilities, the search value and the reservation productivity x^* decrease with z. Further, the spouse's labor supply (and income) declines with z regardless of the worker's employment status, and, hence, so does the additional intra-household insurance (abstracting from the equilibrium effects through wages).*

2. *As a spouse's productivity increases, the wage rise induced by an increase in z falls, whereas the induced decline in the gains from search $S(x)$ also falls if the absolute risk aversion of the household's indirect utility function falls with x. Furthermore, if preferences either are additively separable or belong to set $\mathcal{F}_2$, then the fall in additional intra-household insurance, $\frac{y_x^{u*}-y_x^{e*}}{y_x^{e*}}$, abstracting from the effects through wages, also diminishes with spouse's productivity.*

The bottom line is that public insurance provision crowds out private insurance sought in labor markets and provided within the households, and such crowding-out effects appear to be stronger at the bottom of the household distribution.

4 Centralized Economy

In this section, we examine the constrained efficient insurance scheme set by a social planner that is assumed to abstract from redistributive motives. We show that constrained efficiency cannot be attained in the market economy. Information frictions prevent the private insurance channels from being efficiently set in equilibrium. When making the spouse's productivity observable, the planner's transfers to the unemployed workers decline with the spouse's productivity, while job-finding rates rise.

4.1 Constrained Efficiency

As usually assumed in the search literature, the social planner maximizes a utilitarian welfare function. It faces the same coordination and information frictions as agents encounter in the market economy, and observes neither a worker's job search nor his or her spouse's productivity and labor supply. Instead, both a worker's employment status and his or her spouse's income are observable. The planner sets a mass of vacancies, and dictates a worker's participation decisions and job search strategies and the spouse's income as well as assigns consumption bundles to households.

To be more specific, let subset X_p comprise all households types that engage in job search, and $X_{np} \equiv X \setminus X_p$. The planner designs a symmetric incentive compatible revelation mechanism that consists of a menu of contracts $\mathcal{C} \equiv \{(q_x, c_x^e, c_x^u, y_x^e, y_x^u)\}_{x \in X_p} \cup \{(c_x^u, y_x^u)\}_{x \in X_{np}}$ indexed by a household's announcement of its type. That is, for any reported type x, the mechanism specifies 1) whether or not to search for a job, 2) conditional on participating, a location where to submit an application and the associated job-finding probability, 3) consumption bundles contingent on both the participation status and the search outcome, and 4) the contingent income of the spouse.

The mechanism must be symmetric, feasible as well as incentive compatible, and must ensure a non-negative job-search value. We turn to these properties in order. The mechanism is symmetric in the sense that all households reporting the same type are treated identically. Because of our focus on constrained efficient insurance abstracting from redistributive motives among ex-ante different households, the planner is allowed to pool and redistribute resources only among ex-ante identical households. Formally, we say that a mechanism $\mathcal{C}$ is feasible if total consumption promises do not exceed total output net of vacancy creation costs for each household's type x, i.e. if the following type-specific resource constraint holds

$$\begin{aligned}
\frac{k_f}{q_x} &= \nu(q_x)\big(y_w + y^e_x - c^e_x\big) + (1-\nu(q_x))\big(z + y^u_x - c^u_x\big), \ \forall x \in X_p, \\
c^u_x &= z + y^u_x, \qquad \forall x \in X_{np}
\end{aligned} \tag{RC$_x$}$$

For readability reasons, we denote the expected utility of a type x household that reports type $x' \in X_p$ as

$$\mathcal{U}_x(x') \equiv \upsilon\big(c^u_{x'}, \frac{y^u_{x'}}{x}\big) + \max\big\{0, \nu(q_{x'})\big(\upsilon\big(c^e_{x'}, \frac{y^e_{x'}}{x}\big) - \upsilon\big(c^u_{x'}, \frac{y^u_{x'}}{x}\big)\big) - k_w\big\},$$

and $\mathcal{U}_x \equiv \mathcal{U}_x(x)$ for all $x \in X_p$. The max operator on the right hand side reflects the possibility of not participating in the labor market. Likewise, the utility of a type x household reporting type $x' \in X_{np}$ is $\upsilon(c^u_{x'}, \frac{y^u_{x'}}{x})$. The mechanism must be compatible with households' incentives. That is, for households to truthfully reveal their type, the following incentive compatibility constraints must hold.

$$\begin{aligned}
\mathcal{U}_x &\geq \mathcal{U}_x(x'), \upsilon(c^u_{x''}, \frac{y^u_{x''}}{x}), \qquad \forall x \in X_p, x' \in X_p, x'' \in X_{np} \\
\upsilon(c^u_x, \frac{y^u_x}{x}) &\geq \mathcal{U}_x(x'), \upsilon(c^u_{x''}, \frac{y^u_{x''}}{x}), \qquad \forall x \in X_{np}, x' \in X_p, x'' \in X_{np}
\end{aligned} \tag{ICC$_x$}$$

Furthermore, the net value of job search must exceed the search cost to ensure that participating in the market is desirable for those households who are asked to engage in job search. That is, the following set of participation conditions must also hold.[20]

$$\mathcal{U}_x \geq \upsilon(c^u_x, \tfrac{y^u_x}{x}), \quad \forall x \in X_p \tag{PC$_x$}$$

For simplicity, we will assume throughout this section that the cdf F has a continuous

[20]Notice that the participation constraint implies that, for all $x \in X_p$,

$$\mathcal{U}_x = \upsilon\big(c^u_x, \frac{y^u_x}{x}\big) + \nu(q_x)\big(\upsilon\big(c^e_x, \frac{y^e_x}{x}\big) - \upsilon\big(c^u_x, \frac{y^u_x}{x}\big)\big) - k_w$$

Thus, we shall remove the max operator from the $\mathcal{U}_x$ expression hereafter.

support, and is differentiable.[21] The planner's problem is

$$\begin{array}{ll} \max\limits_{X_p, \mathcal{C}} & \displaystyle\int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} v(c^u_x, \frac{y^u_x}{x}) dF(x) \\ \text{s. to} & (\text{RC}_x) \text{ and } (\text{ICC}_x) \text{ for a.e. } x \in X, \text{ and } (\text{PC}_x) \text{ for a.e. } x \in X_p \end{array}$$

Let $(\hat{X}_p, \hat{\mathcal{C}})$ denote the constrained efficient allocation. Notice that incentive compatibility ensures that households with more productive spouses are promised higher expected values.[22]

The following proposition characterizes the planner's allocation. First, the value of the non-participating households amounts to $V_x(z)$ as no transfer can take place within subset $\hat{X}_{np}$. Indeed, this value is naturally a lower bound for all households. Thus, no household has incentives to pretend to be a non-participant of a different type. Second, the planner sets a reservation productivity to define the participating group, with higher-type households not participating. The threshold is pinned down by the net returns to participating being just equal to the participation costs as in the equilibrium allocation. Indeed, as we shall claim below in Proposition 4.3, the planner's threshold coincides with the equilibrium one. Third, the equilibrium allocation belongs to the domain of the planner's problem as it is feasible and incentive compatible. Nonetheless, the laissez-faire equilibrium does not attain constrained efficiency, and all households are better off in the planner's allocation.[23]This is because of the superior tools the planner has to transfer resources among ex-ante identical households who differ after income risks being realized. Put differently, the private provision of insurance against consumption risks falls short of the constrained efficient level. Importantly, there is no other source of inefficiency since the equilibrium allocation would be constrained efficient if the planner were not allowed to make transfers to one-earner households.

Proposition 4.1 ***Planner's Allocation.***

[21] In the numerical analysis in Section 5, the support of F will be assumed to be discrete.

[22] To see this, consider e.g. $x \in \hat{X}_p$ and $x' \in X$ such that $x' < x$. Then,

$$\begin{array}{llll} \mathcal{U}_x & \geq & \mathcal{U}_x(x') = \nu(\hat{q}_{x'}) v\big(\hat{c}^e_{x'}, \frac{\hat{y}^e_{x'}}{x}\big) + (1 - \nu(\hat{q}_{x'})) v\big(\hat{c}^u_{x'}, \frac{\hat{y}^u_{x'}}{x}\big) - k_w > \mathcal{U}_{x'}, & \text{if } x' \in \hat{X}_p \\ \mathcal{U}_x & \geq & v(\hat{c}^u_{x'}, \frac{\hat{y}^u_{x'}}{x}) > v(\hat{c}^u_{x'}, \frac{\hat{y}^u_{x'}}{x'}), & \text{if } x' \in \hat{X}_{np} \end{array}$$

where the first inequality is condition (ICC_x), and the second inequality results from utility function v being strictly increasing in leisure.

[23] Davoodalhosseini (2019) studies constrained efficiency in a directed search framework with adverse selection and quasi-linear preferences. In the Online Appendix we show that if preferences are quasi-linear in consumption, then the constrained efficiency result in Moen (1997) carries over to an economy with households. With quasi-linear preferences in leisure, vacancy creation is also determined to maximize total output, yet the equilibrium allocation is not constrained efficient.

1. *There exists a solution $(\hat{X}_p, \hat{\mathcal{C}})$ of the planner's problem. The utility of those households with spouse productivity $x \in \hat{X}_{np}$ is $V_x(z)$, and $\mathcal{U}_x \geq V_x(z)$ for all households such that $x \in \hat{X}_p$. Furthermore, $\mathcal{U}_x$ converges to $V_x(z)$ in the boundary of $\hat{X}_p$.*

2. *There is no positive mass of households that are worse off in the planner's allocation than in equilibrium: $\mathcal{U}_x \geq V_x(z) + \max\{0, S^*(x)\}$ for almost every $x \in X$.*

3. *Consider differentiable mechanisms. There exists a threshold $\hat{x} \geq x^*$ such that $\hat{X}_p = [\underline{x}, \hat{x}]$ and $\hat{X}_{np} = (\hat{x}, \overline{x}]$, and $\mathcal{U}_{\hat{x}} = V_{\hat{x}}(z)$.*

4. ***Comparison with the Equilibrium Allocation:** The equilibrium allocation is not a solution of the planner's problem. If the planner is further constrained not to redistribute resources and, in particular, $c^u_x = y^u_x + z$ for all $x \in X$, then the laissez-faire equilibrium is constrained efficient.*

What are the insurance margins to blame for the inefficiency result? As claimed above, the participation margin is efficiently set in the market economy. Thus, the insufficient insurance provision does not generate welfare losses along the extensive margin of a worker's labor supply in equilibrium.

Regarding the additional intra-household insurance channel, let $D^e_{x'}(x) \equiv v_c(c^e_x, \frac{y^e_x}{x'}) + \frac{v_\ell(c^e_x, \frac{y^e_x}{x'})}{x'}$ denote the marginal utility that a two-member household of type x' derives from increasing the spouse's income while pretending to be of type x. Then, the planner's counterpart of equilibrium condition (2) is

$$D^e_x(x) \;=\; \int_{X_p} \hat{\lambda}^3_{x',x} D^e_{x'}(x) dF(x') + \int_{X_{np}} \hat{\lambda}^5_{x',x} D^e_{x'}(x) dF(x') \tag{9}$$

where $\hat{\lambda}^3_{x',x}$ and $\hat{\lambda}^5_{x',x}$ are a composite of the Lagrange multipliers of the participation conditions (PC_x) and the incentive-compatibility conditions (ICC_x).[24] Since $D^e_x(x) = 0$ in equilibrium, incentive compatibility introduces a wedge between the equilibrium spouse's labor supply decisions and their planner's counterparts.

Similarly, the insurance provided through the vacancy creation margin is affected by information frictions, but also by the planner's ability to redistribute resources. The planner's counterpart of equilibrium equation (6) is

$$R_x(x) = \int_{X_p} \hat{\lambda}^3_{x',x} R_{x'}(x) dF(x') + \int_{X_{np}} \hat{\lambda}^5_{x',x} R_{x'}(x) dF(x') \tag{10}$$

[24] See the Online Appendix for a detailed derivation of such expressions.

where $R_{x'}(x) \equiv v(c^e_x, \frac{y^e_x}{x'}) - v(c^u_x, \frac{y^u_x}{x'}) + v_c(c^e_x, \frac{y^e_x}{x'})\big(y_w + y^e_x - c^e_x - z - y^u_x + c^u_x + \frac{k_f}{q^2 \nu'(q_x)}\big)$.

Note that, if a household's type were observable to the planner (i.e. $\hat{\lambda}^3_{x',x} = \hat{\lambda}^5_{x',x} = 0$ for all $x, x' \in \hat{X}_p$), the equilibrium allocation would satisfy equations (9) and (10). Can we conclude that the laissez-faire equilibrium would solve the planner's problem if types were observable?

4.2 Planner's Allocation with Observable Types.

We turn to study the centralized economy under the assumption that a spouse's productivity is perfectly observable to the social planner.[25] In this scenario, the planner's problem can be rewritten as a sequence of type-specific maximization problems because of the elimination of the incentive compatibility conditions. That is, the planner's problem reduces to the set of maximization problems $V_x(z)$ for the non-participating households, and the set of problems (P^{OT}_x) for participating ones, where

$$(P^{OT}_x) \quad \max_{\mathcal{C}} \quad \mathcal{U}_x$$
$$\text{s. to} \quad (\text{RC}_x) \text{ and } (\text{PC}_x)$$

We will refer to the planner's solution in this alternative scenario as the planner's OT allocation to distinguish it from the constrained efficient one. Let $\hat{X}^{OT}_p$ and $\hat{X}^{OT}_{np}$ denote the set of types of participating and non-participating households, respectively. We also denote the planner's tuple for any given $x \in \hat{X}^{OT}_p$ as $(\hat{q}^{OT}_x, \hat{c}^{eOT}_x, \hat{c}^{uOT}_x, \hat{y}^{eOT}_x, \hat{y}^{uOT}_x)$.

The following proposition characterizes this planner's allocation. First, as in previous sections, non-participating households obtain $V_x(z)$, and this value determines the productivity threshold. Second, regarding a spouse's labor supply decision, planner's OT condition (11) coincides with equilibrium condition (2). Third, the participation constraint is binding for all $x \in \hat{X}^{OT}_p$ as the planner manages to eliminate consumption risks as much as ensuring participation permits because households dislike lotteries. Fourth, planner's vacancy creation condition (12) is equivalent to $R_x(x) = 0$. This equation equates the vacancy creation costs with the expected net social gains of an extra vacancy. The first factor of the latter, $\eta(q_x)\big(1 - \phi(q_x)\big)$, is the job-filling probability net of the negative effects on the other available jobs as their chances to be filled fall with the extra vacancy. The term in brackets amounts

[25] This would be the case if labor supply were observable, e.g. as in the case of hours worked. Also notice that the baseline economy with a two-point productivity distribution with 0 as the first mass point is indeed a setting with public information because households with productive spouses see no advantage in obtaining 0 income.

to the net value of a two-earner household, properly adjusted by the marginal utility, plus the match output minus the net income a two-earner household receives. Notice that, in the absence of a transfer system, this equation coincides with equilibrium condition (6), and the first term within brackets is the equilibrium household's surplus.

Although both the spouse's labor supply margin and the vacancy creation margin are efficiently set in the market economy, the equilibrium allocation does not solve the planner's OT problem because the participation constraint is slack in equilibrium. Put differently, the spouse's labor supply in one-earner households is inefficiently large as well as too-low-wage jobs are created in the market economy due to the absence of a transfer scheme.

Importantly, transfers to the unemployed (or one-earner households) decline with the spouse's productivity as so do the welfare gains of such transfers. In sharp contrast to the equilibrium allocation, job-finding rates rise over the distribution of households. So does spouse's income. These patterns are in line with the crowding out effects of public insurance provision over the household distribution stated in Proposition 3.5, and rely on the concavity of the utility function, the limited substitutability assumption and the income pooling mechanism.

Proposition 4.2 ***Planner's OT Allocation.***

Assume productivity x is observable to the planner.

1. *The utility of the non-participating households is $V_x(z)$. There is no positive mass of households that are worse off in the planner's OT allocation than in equilibrium: $\mathcal{U}_x \geq V_x(z) + \max\{0, S^*(x) - k_w\}$ for almost every $x \in X$. If differentiable mechanisms, there exists a threshold $\hat{x}^{OT}$ such that $\hat{X}_p^{OT} = [\underline{x}, \hat{x}^{OT}]$, and $\mathcal{U}_{\hat{x}^{OT}} = V_{\hat{x}^{OT}}(z)$.*

The planner's solution for each $x \in \hat{X}_p^{OT}$ is determined by the resource constraint (RC_x) and the following conditions:

2. *Additional intra-household insurance is determined by*

$$v_c\big(c_x^j, \frac{y_x^j}{x}\big)x + v_\ell\big(c_x^j, \frac{y_x^j}{x}\big) = 0, \text{ for } j \in \{e, u\} \tag{11}$$

3. *The participation constraint (PC_x) is binding, and $\hat{c}_x^{eOT} \geq \hat{c}_x^{uOT}$ and $\hat{y}_x^{eOT} < \hat{y}_x^{uOT}$.*

4. *Vacancy creation is determined by*

$$k_f \;=\; \eta(q_x)\big(1-\phi(q_x)\big)\left(\frac{v(c_x^e, \frac{y_x^e}{x}) - v(c_x^u, \frac{y_x^u}{x})}{v_c(c_x^e, \frac{y_x^e}{x})} + y_w + y_x^e - c_x^e - z - y_x^u + c_x^u\right) \tag{12}$$

Furthermore,

5. ***Pattern over the Household Distribution:*** $\forall x, x' \in \hat{X}_p^{OT} \mid x' < x$,

   (a) *Declining transfers to one-eaner households:*

$$\hat{c}_x^{uOT} - \hat{y}_x^{uOT} \leq \hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT}$$

   (b) *Increasing job-finding rates and spouse's income:*

$$\nu(\hat{q}_{x'}^{OT}) < \nu(\hat{q}_x^{OT}),\ \hat{y}_{x'}^{eOT} < \hat{y}_x^{eOT},\ y_{x'}^{uOT} < \hat{y}_x^{uOT},$$

$$\hat{c}_x^{eOT} - \hat{y}_x^{eOT} \leq \hat{c}_{x'}^{eOT} - \hat{y}_{x'}^{eOT},\ \textit{and}$$

$$v(\hat{c}_x^{eOT}, \frac{\hat{y}_x^{eOT}}{x}) - v(\hat{c}_x^{uOT}, \frac{\hat{y}_x^{uOT}}{x}) < v(\hat{c}_{x'}^{eOT}, \frac{\hat{y}_{x'}^{eOT}}{x'}) - v(\hat{c}_{x'}^{uOT}, \frac{\hat{y}_{x'}^{uOT}}{x'})$$

6. ***Comparison with the Equilibrium Allocation:*** *The equilibrium allocation does not coincide with the planner's OT allocation. Further,*

$$\hat{x}^{OT} = \hat{x} = x^*,\ \textit{and}\ \ q_x^* < \hat{q}_x^{OT},\ z + y_x^{u*} < \hat{c}_x^{uOT}\ \textit{and}\ \hat{y}_x^{uOT} < y_x^{u*},\ \forall x \in \hat{X}_p^{OT}.$$

Implementation of the Planner's OT Allocation. A natural question is whether the planner's OT allocation can be attained in the market economy, and, if so, what fiscal instruments are required. We claim that a sufficiently rich public policy can implement the planner's OT allocation. We provide here a sketch of the analysis, while further details are postponed to Appendix 7.2.1.

We consider a type-specific three-object public scheme: after-tax benefits, b_x, an income tax for newly employed workers, $T_x(w)$, and a proportional income tax rate for the employed spouses, τ_x. Needless to say, the government is also subject to a type-specific balanced-budget constraint. The tax-distorted equilibrium allocation must solve the household's problem (4), where the household's indirect utility function is now

$$V_x(w) \equiv \max_y\ v\big(w - T_x(w) + y(1-\tau_x), \frac{y}{x}\big),$$

and $w - T_x(w) = z + b_x$ if the worker remains unemployed at the end of the period.

The following proposition states that the planner's OT allocation can be decentralized through a system of unemployment benefits falling over the spouse's productivity distribution. As we already learned from the comparison with planner's condition (9), it is not efficient to distort private insurance provision within the household; hence, the spouse's income cannot be taxed. Put differently, transfers must depend on household's overall income, yet joint taxation reduces welfare.

Proposition 4.3 *Constrained efficiency can be attained in the market economy through the implementation of a public unemployment insurance with benefits falling with spouse's income, and financed with an income tax on newly employed workers. No tax can be levied on spouse's income.*

The planner's OT allocation can also be obtained in the market economy without the government's intervention. For example, consider a wider contracting space such that firms are allowed not only to commit to a wage, w, to successful applicants, but also to reward unsuccessful applications with a payment, s. Conditional on searching, a household's problem in this setting would be

$$\begin{aligned} \max_{q,w,s} \quad & \nu(q)V_x(w) + \big(1-\nu(q)\big)V_x(z+s) \\ \text{s. to} \quad & k_f = \eta(q)(y_w - w) - sq\big(1-\nu(q)\big) \\ & k_w \leq \nu(q)\big(V_x(w) - V_x(z+s)\big) \end{aligned}$$

The first constraint is the free-entry condition. The last term of the right hand side amounts to the payment s times the expected mass of unsuccessful applicants that corresponds to a given firm in that submarket. It is easy to show that this constraint is indeed the resource constraint (RC_x); hence, this household's program turns out to be problem (P_x^{OT}). Such firms are indeed offering labor and insurance contracts at once. Golosov et al. (2013) argue that such contracts are not enforceable because of the information frictions, and offer an alternative decentralization that consists of having an insurance market alongside the labor market.

5 Quantitative Exploration

The goal of this section is primarily to assess quantitatively the constrained-efficient insurance provision, and, in particular, the slope of the transfer schedule the planner sets. Furthermore, we aim to explore the welfare implications of private information. The main result is twofold: first, a form of replacement rate steadily falls from just below 30% to 0 over the household distribution, and, second, the costs of the private insurance arrangements being unobservable are fairly small.

Table 1: Parameter Values

Parameter	Value	Interpretation	Target
θ	1.730	inverse of Frisch elasticity of consumption wrt price	Blundell et al. (2016b)
ξ	1.451	Frisch elasticity of labor supply wrt own wages	Blundell et al. (2016b)
m_0	0.194	mass of $x = 0$	mass of spouses with no earnings (SIPP)
γ	$5.9 \cdot 10^{-4}$	scale labor disutility parameter	ratio of avg hours worked by household members (SIPP)
s_1	10.371	scale factor of the matching function	monthly job-finding rate (SIPP)
s_2	1.908	elasticity of the matching function	elasticity of job-filling rate
(m_x, d_x)	(8.092, 1.022)	mean and standard deviation of F	mean and st. dev. of spouse earnings dist. (SIPP)
y_w	7063.454	worker's productivity	ratio of avg worker's and avg spouse's earnings (SIPP)
k_f	1445.142	vacancy creation cost	14% average quarterly wage per hire
k_w	$2.6 \cdot 10^{-5}$	participation cost	percentage of non-emp. spells over 1 year (SIPP)
z	1642.020	home production	consumption ratio c_u/c_e
b	1242.841	unemployment benefits	replacement rate for average wage

5.1 Baseline Calibration

We set a model period to be 1 month. The following unemployment insurance policy is assumed: benefits b are collected by all unemployed workers, and the insurance scheme is funded though a proportional tax rate, τ, on worker wages and spouse earnings to balance the government's budget. We also assume the following functional forms. First, we consider standard macro preferences, $v \in \mathcal{F}_1$. Second, the matching function is CES, $\nu(q) = \frac{1}{(s_1+q^{s_2})^{1/s_2}}$. Third, we assume that the spouse's productivity is log-normally distributed, with mean m_x and standard deviation d_x. Furthermore, there is a mass m_0 of households with no spouse's earnings. We use 100 grid points with equal distance over $(0, \overline{x}]$, where $\overline{x}$ is set to rule out the top 2.5% of the actual spouse's earnings distribution.[26]

Table 1 summarizes the exercise. Regarding the preference parameters, and along the lines of Krueger and Wu (2018), we take the Frisch elasticities θ and ξ directly from Blundell et al. (2016b),[27] and the scale factor γ is calibrated to match the ratio of average hours worked by the newly employed in the first month to their employed spouses' counterpart, which in our sample is 1.065. Furthermore, in our subsample of married jobless individuals, 19.38% of the spouses have no earnings after the worker transits back to employment; hence, we set $m_0 = 0.1938$.

The remaining parameters are jointly calibrated together with the scale factor γ. We first refer to the targets determined using our SIPP dataset, and then to those values taken from the literature. To calibrate the parameters of distribution F, we target the empirical distribution of the spouse's log earnings at a worker's re-employment, conditional on earnings

[26] Notice that a grid of 100 points implies aproximately 10^4 incentive compatibility constraints in the planner's problem. Increasing the number of grid points adds no much to the results while increasing dramatically the computation time.

[27] As we do not distinguish by gender, we take averages of such estimates.

being positive and normalized to the median spouse's earnings. Specifically, we target its mean and st. deviation, -0.109 and 0.960, respectively. Workers' market productivity y_w is set to match the ratio of worker's average log earnings after reemployment within the first month to spouse's average log earnings, 0.966. The proportion of jobless individuals after 1 year of non-employment (more than twice the average non-employment spell in our sample), 10.47%, informs the participation cost, k_w. Regarding the two parameters of the matching technology, we target the job-finding rate at the first month, 0.206,[28] and the elasticity of the job-filling rate with respect to the unemployment-to-vacancy ratio at 0.5 as widely used in the literature.

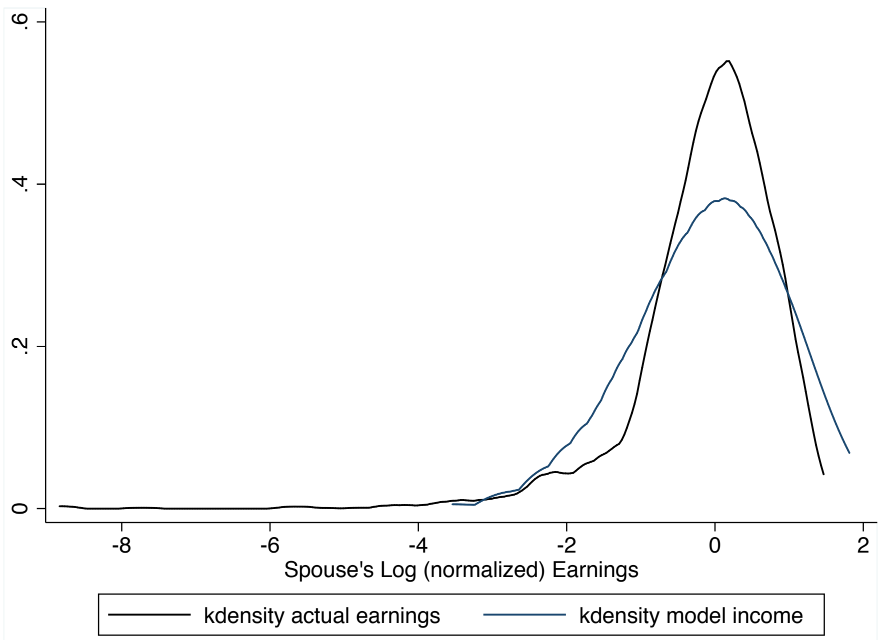


Figure 2: Actual vs. Model Spouse Earnings Data

Finally, we follow Hall and Milgrom (2008) to pin down the vacancy-creation cost, k_f, home production, z, and the unemployment benefits, b. The average replacement rate informs the latter, and is set at 25%.[29] We set the costs of vacancy-posting to match 14% of the average quarterly wages per hire, and home production to match the ratio of the average

[28]This rate is rather low (e.g. below the 0.248 reported by Krusell et al. (2011)) mostly because of the elimination of non-employment spells shorter than 3 weeks. Because of this, we compute the job-finding rate for the first 5 weeks of non-employment.

[29]We take a rather low estimate of the replacement rate because of our use of non-employment instead of unemployment spells. For example, Anderson and Meyer (1997) estimate a pre-tax rate to be around 40%, although they also document quite low take-up rates. Hornstein et al. (2005) argue that 20% would be an upper bound since the unemployed workers' salaries are below average wages.

consumption in one-earner households to the average consumption in two-earner households at 0.85 as estimated by Browning and Crossley (2001).

The model matches the set of targets fairly well. In particular, the two first moments of the distribution of the spouse's log normalized earnings are very precisely met. Nonetheless, as the Kernel density estimates plotted in Figure 2 show, the model-generated earnings appear to be more dispersed than the actual ones.

5.2 Comparison with the Planner's Allocations

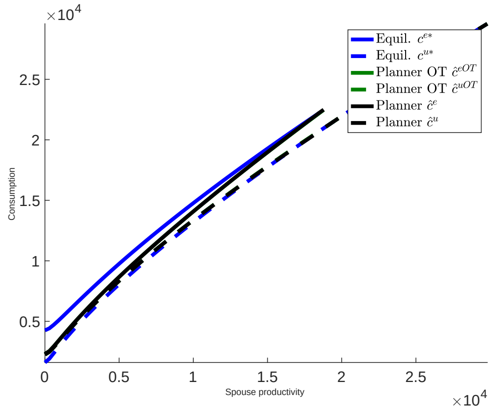
We now turn to the comparison of the laissez-faire and centralized economies. To this end, we take the calibrated parameter values except the policy ones, i.e. unemployment benefits and taxes.[30] Figure 3 displays the laissez-faire equilibrium as well as the two planner's allocations.

The consumption gap between one- and two-earner households is larger in equilibrium than in the planner's allocation. This is the case despite the fact that the two private insurance margins are excessively used in equilibrium: first, the spouse's labor supply in one- (two-) earner households is inefficiently large (small), and, second, job creation is also excessive over the whole household distribution. Indeed, these two insurance margins are less used the higher the spouse's productivity in sharp contrast with the planner's allocation. This is closely related to the falling transfers system the planner sets.

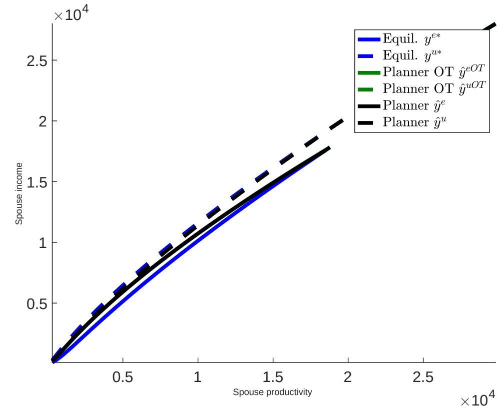
We are interested in the variation of transfers $\hat{c}_x^u - z - \hat{y}_x^u$ over the household distribution. While it is 0 in equilibrium, it is positive and steadily declines in the planner's allocation. Figure 4e plots the ratio $\frac{\hat{c}_x^u - z - \hat{y}_x^u}{\hat{c}_x^e - \hat{y}_x^e}$, which defines the type-specific replacement rate in the planner's allocation as the ratio of the net transfers received by an unemployed worker to the net income earned by his or her employed counterpart. It steadily declines with the spouse's productivity from just above 28% to 0. Conditional on participating in the labor force, the planner's average replacement rate is 21.68%, while it goes down to 20.06% when excluding households with non-employed spouses. Thus, a back-of-the-envelope calculation of the dependency allowance at work in some states of the US points to a flat 20% replacement rate and a rate 8 percentage points higher for workers with an unemployed spouse. The latter is fairly close to the 4 to 7 percentage point increase in the replacement rate for a dependent in the state of New Jersey.

The difference between the planner's allocation and its OT allocation captures how large the costs of private information are. They appear to be fairly small. For example, the consumption difference $\frac{\hat{c}_x^{OT} - \hat{c}_x}{\hat{c}_x}$ and the income difference $\frac{\hat{y}_x^{OT} - \hat{y}_x}{\hat{y}_x}$ are below 0.02 and 0.2%,

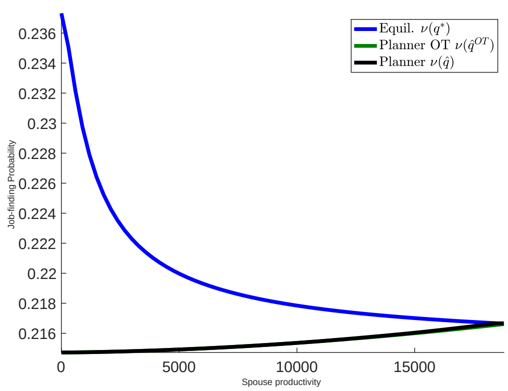
[30] To compute the planner's solutions, we use AMPL, which is a modeling language to solve large-scale non-linear optimization problems.



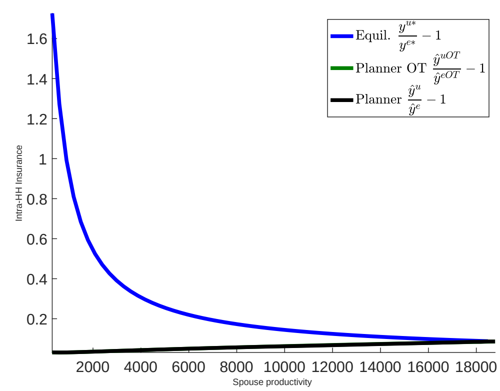
(a) Consumption



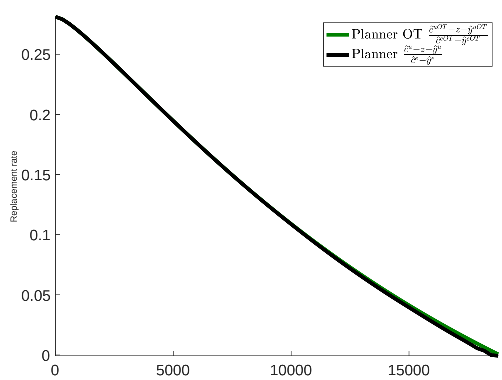
(b) Spouse Income



(c) Job-finding Probability



(d) Intra-Household Insurance



(e) Replacement Rate

Figure 3: Planner's and Laissez-Faire Equilibrium Allocations

respectively, and increase with the spouse's productivity. The replacement rate is always higher with observable types, and also monotonically rises with productivity x as can be appreciated in Figure 4e, yet the difference is again quantitatively small.

Finally, the same quantitative exercise is performed with LMP preferences. See Appendix 7.4 for details. Recall that full consumption insurance is arranged within the household through the spouse labor supply both in equilibrium and the planner's OT allocation. This not only leads to lower replacement rates (just above 12%), but also to an almost flat replacement rate schedule over the household distribution.

6 Conclusions and Final Discussions

This paper studies the constrained efficient combination of public and private provision of insurance against unemployment and, hence, consumption risks. In this setting, insurance is both sought in labor markets by applying to low wage jobs and provided at home through the spouse's labor supply. In the absence of public provision, these two insurance margins are operated inefficiently. The planner's allocation with observable types exhibits both rising job-finding rates and falling transfers over the household distribution.

There are many factors we abstract from to gain tractability. First, it may be argued that private insurance is superior by construction because spouses are assumed to be employed. However, we do model severe frictions on the extensive margin by allowing for a mass point at (or arbitrarily close to) the zero productivity in addition to costs in the form of forgone leisure. Furthermore, frictions on the additional income provided by the spouse can be easily accommodated in this setting. For example, a spouse's income could respond to unemployment news only with some probability. Second, assortative mating may have an effect on the constrained efficient risk allocation. In this respect, we believe our results are likely to be conservative since more productive workers would face better employment prospects, and, hence, would require even relatively less insurance from both their spouses and the government. Third, as shown by Chetty and Szeidl (2007), consumption commitments are relevant for the optimal design of consumption insurance, and they may differ significantly over the distribution of households in various ways. We also abstract from the fact that US jobless worker's search incentives may also be driven by their lack of health insurance unless they can enroll in their spouse's job-based plan. However, such benefits are positively correlated with the spouse's income, and such health coverage is ultimately an income insurance.

Finally, our setting hosts a unitary model of the household, in which households are the decision-making units that maximize a utility function subject to a budget constraint. This modeling has been questioned on empirical and theoretical grounds. Although not shown here, we have explored how robust some results are when considering a cooperative model of the household instead. In such alternative models, income is pooled and each member of the household has their own preferences and decides on their individual consumption and labor supply. Such cooperative models ensure Pareto efficient intra-household outcomes. Our work with this alternative model suggests that the qualitative results shown in this paper are robust.

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7 Appendix

7.1 Appendix. Proofs of Section 3

Proof of Proposition 3.1.

As an abuse of notation, we denote $\upsilon = \upsilon(c, \ell)$ and similarly for the partial derivatives throughout this proof. Likewise, we write y_x^u as $y_x^e(z)$.

1. Consider the first order condition (2). Let $f_x(w, y) \equiv \upsilon_c\big(y+w, \frac{y}{x}\big)x+\upsilon_\ell\big(y+w, \frac{y}{x}\big)$, where $w \in [z, y_w]$. Notice that $\frac{\partial f_x(w,y)}{\partial y} = \upsilon_{cc}x + 2\upsilon_{c\ell} + \frac{\upsilon_{\ell\ell}}{x}$. This derivative is strictly negative at any zero of function f_x because of Assumption A5. Therefore, the Implicit Function theorem ensures that there exists a unique function $y_x^e(w)$ such that $f_x(w, y_x^e(w)) = 0$. Indeed, y_x^e is twice continuously differentiable since so is f_x because of Assumption A1.

   To show that $y_x^e(w)$ is a strictly decreasing function, we differentiate equation (2) with respect to w, and obtain

$$\begin{aligned}\big(\upsilon_{cc}x + \upsilon_{\ell c}\big)\left(\frac{dy_x}{dw}+1\right) + \upsilon_{c\ell}\frac{dy_x}{dw} + \upsilon_{\ell\ell}\frac{dy_x}{dw}\frac{1}{x} = 0 \\ \Leftrightarrow \frac{dy_x}{dw} = -x\frac{\upsilon_{cc}x + \upsilon_{c\ell}}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} < 0\end{aligned} \tag{13}$$

   The inequality results from replacing x in the numerator by using the first order condition (2) that defines $y_x^e(w)$ together with Assumption A5. Similarly,

$$\frac{d(w+y_x)}{dw} = \frac{\upsilon_{c\ell}x + \upsilon_{\ell\ell}}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} \geq 0.$$

   We now make use of these results to derive some properties of function V_x. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = \upsilon\big(y_x^e(w) + w, \frac{y_x^e(w)}{x}\big)$, and, hence, so is it. To show that function V_x is strictly increasing and concave, we compute its first and second derivatives.

$$\begin{aligned}\frac{dV_x(w)}{dw} &= \upsilon_c > 0 \\ \frac{d^2V_x(w)}{dw^2} &= \upsilon_{cc}\left(\frac{dy_x}{dw}+1\right) + \upsilon_{c\ell}\frac{dy_x}{dw}\frac{1}{x} = \frac{\upsilon_{cc}\upsilon_{\ell\ell} - \upsilon_{c\ell}^2}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} < 0\end{aligned}$$

   The first derivate is determined using the Envelope theorem. To compute the second derivative, we have used expression (13). Assumptions A3 and A5 ensure that the second derivative is negative.

2. Next, we turn to the properties of $y_x(w)$ and $V_x(w)$ as a function of x, for a given wage $w \geq z$. Regarding the spouse's income, the proof of its twice differentiability is analogous to the one carried out in the previous point; hence, omitted. By differentiating equation (2) with respect to productivity x and grouping terms, we obtain

$$\left(\upsilon_{cc}x + \upsilon_{\ell\ell}\frac{1}{x} + 2\upsilon_{\ell c}\right)\frac{\partial y_x(w)}{\partial x} = \frac{y_x(w)}{x}\left(\upsilon_{c\ell} + \frac{\upsilon_{\ell\ell}}{x}\right) - \upsilon_c$$

$$\Longrightarrow \frac{\partial y_x(w)}{\partial x} = \frac{-\upsilon_c x + \upsilon_{c\ell}y_x(w) + \upsilon_{\ell\ell}\frac{y_x(w)}{x}}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} > 0 \tag{14}$$

This derivative is positive because of Assumptions A2 and A5. Obviously, $\frac{w+\partial y_x(w)}{\partial x} > 0$. Indeed, for the LMP preferences, condition (2) becomes

$$w + y_x(w) = \frac{x}{\psi} \Longrightarrow \frac{\partial y_x(w)}{\partial x} = \frac{1}{\psi}.$$

Now, consider $V_x(w) \equiv \max_y \upsilon(y + w, \frac{y}{x})$. Then, because of the Envelope theorem,

$$\frac{\partial V_x}{\partial x} = -\upsilon_\ell \frac{y_x(w)}{x^2} > 0$$

Thus, V_x is increasing in x for a given wage w. Recall that $\frac{dV_x(w)}{dw} = \upsilon_c$. Then,

$$\begin{aligned}
\frac{\partial \frac{dV_x(w)}{dw}}{\partial x} &= \upsilon_{cc}\frac{\partial y_x(w)}{\partial x} + \upsilon_{c\ell}\left(\frac{\partial y_x(w)}{\partial x}\frac{1}{x} - \frac{y_x(w)}{x^2}\right) = \\
&= \frac{\left(\upsilon_{cc} + \frac{\upsilon_{c\ell}}{x}\right)\left(-\upsilon_c x + \upsilon_{c\ell}y_x(w) + \upsilon_{\ell\ell}\frac{y_x(w)}{x}\right) - \frac{y_x(w)}{x^2}\upsilon_{c\ell}\left(\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}\right)}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} \\
&= \frac{(\upsilon_{cc}\upsilon_{\ell\ell} - \upsilon_{c\ell}^2)\frac{y_x(w)}{x} - \upsilon_c(\upsilon_{cc}x + \upsilon_{c\ell})}{\upsilon_{cc}x^2 + 2\upsilon_{c\ell}x + \upsilon_{\ell\ell}} < 0
\end{aligned}$$

where the last expression is obtained after some simplifications. The inequality follows from Assumptions A2, A3 and A5 after replacing x in the last term of the numerator by using the first order condition (2).

3. Consider now types x and x' such that $x' < x$. Given that $V_x(w)$ is strictly increasing and differentiable in w, there exists an inverse function V_x^{-1}, which is also differentiable. Define function $\mathcal{V}(s) \equiv \max_y \upsilon\left(V_x^{-1}(s) + y, \frac{y}{x'}\right)$, which is twice continuously differentiable.

Notice that $V_{x'}(w) = \mathcal{V} \circ V_x(w)$. By differentiating with respect to w, we obtain

$$\frac{dV_{x'}(w)}{dw} = \mathcal{V}'(V_x(w))\frac{dV_x(w)}{dw}, \text{ and } \frac{d^2V_{x'}(w)}{dw^2} = \mathcal{V}''(V_x(w))\frac{dV_x(w)}{dw}^2 + \mathcal{V}'(V_x(w))\frac{d^2V_x(w)}{dw^2}$$

Thus, $\mathcal{V}' > 1$ because $\frac{dV_x(w)}{dw}$ is decreasing in x given w. Now, using the first derivative, we can rewrite the second expression as

$$\mathcal{V}''(V_x(w))\frac{dV_x(w)}{dw}^2 = \frac{d^2V_{x'}(w)}{dw^2} - \frac{d^2V_x(w)}{dw^2}\frac{\frac{dV_{x'}(w)}{dw}}{\frac{dV_x(w)}{dw}} = \frac{dV_{x'}(w)}{dw}\left(\frac{-\frac{d^2V_x(w)}{dw^2}}{\frac{dV_x(w)}{dw}} - \frac{-\frac{d^2V_{x'}(w)}{dw^2}}{\frac{dV_{x'}(w)}{dw}}\right)$$

Therefore, $\mathcal{V}$ is a concave (convex) function and, hence, $V_{x'}(w)$ is a concave (convex) transformation of $V_x(w)$ if and only if $\frac{-\frac{d^2V_{x'}(w)}{dw^2}}{\frac{dV_{x'}(w)}{dw}}$ is greater (lower) than $\frac{-\frac{d^2V_x(w)}{dw^2}}{\frac{dV_x(w)}{dw}}$.

4. Consider, first, additively separable preferences. Then, we can rewrite

$$\frac{-\frac{d^2V_x(w)}{dw^2}}{\frac{dV_x(w)}{dw}} = \frac{A_cA_\ell}{A_cx + A_\ell}$$

where $A_j \equiv |\frac{\upsilon_{jj}}{\upsilon_j}|$ and $R_j \equiv jA_j$, for $j \in \{c, \ell\}$. Given w, its derivative with respect to x times $(A_cx + A_\ell)^2$ is

$$\begin{aligned}
&\left(\frac{\partial A_c}{\partial x}A_\ell + A_c\frac{\partial A_\ell}{\partial x}\right)(A_cx + A_\ell) - A_cA_\ell\left(\frac{\partial A_c}{\partial x}x + A_c + \frac{\partial A_\ell}{\partial x}\right) = \frac{\partial A_c}{\partial x}A_\ell^2 + \frac{\partial A_\ell}{\partial x}A_c^2x - A_c^2A_\ell \\
&= \frac{dA_c}{dc}\frac{\partial c_x}{\partial x}A_\ell^2 + \frac{dA_\ell}{d\ell}\frac{\partial \ell_x}{\partial x}A_c^2x - A_c^2A_\ell = \frac{dA_c}{dc}\frac{\partial c_x}{\partial x}A_\ell^2 + \frac{dA_\ell}{d\ell}\left(\frac{\partial y_x}{\partial x} - \ell_x\right)A_c^2 - A_c^2A_\ell \\
&= \frac{dA_c}{dc}\frac{\partial c_x}{\partial x}A_\ell^2 + \frac{dA_\ell}{d\ell}\frac{\partial y_x}{\partial x}A_c^2 - A_c^2\frac{dR_\ell}{d\ell} < 0 \text{ if } \frac{dA_c}{dc}, \frac{dA_\ell}{d\ell} \leq 0 \text{ and } \frac{dR_\ell}{d\ell} \geq 0,
\end{aligned}$$

with at least one strict inequality.

The derivative is strictly negative if that sufficient condition holds because, given w, $\frac{\partial c_x}{\partial x}, \frac{\partial y_x}{\partial x} > 0$ as shown above. To show that preferences that belong to $\mathcal{F}_1$ satisfy these conditions, we write

$$\frac{-\frac{d^2V_x(w)}{dw^2}}{\frac{dV_x(w)}{dw}} = \frac{\theta\xi}{\theta y + \xi c} \text{ and } (A_c, A_\ell, R_\ell) = \left(\frac{-\upsilon_{cc}}{\upsilon_c}, \frac{\upsilon_{\ell\ell}}{\upsilon_\ell}, \frac{\ell\upsilon_{\ell\ell}}{\upsilon_\ell}\right) = \left(\frac{\theta}{c}, \frac{\xi}{\ell}, \xi\right).$$

On the contrary, if $\frac{dA_c}{dc}, \frac{dA_\ell}{d\ell} \geq 0$ and $\frac{dR_\ell}{d\ell} \leq 0$, then that derivative is positive.

Finally, consider preferences $\upsilon \in \mathcal{F}_2$. The absolute risk aversion of the household's indirect utility function is

$$\frac{-\frac{d^2V_x(w)}{dw^2}}{\frac{dV_x(w)}{dw}} = -\frac{\psi(1-\theta)}{x},$$

which is decreasing in x. $\|$

Proof of Proposition 3.2.

1. Consider a worker's search value (5). We are to show that it decreases with productivity x. As the domain does not depend on x, it suffices to prove that the value difference $V_x(w) - V_x(z)$ also falls with x for any given wage w. Consider $x, x' \in X$ such that $x' < x$, and suppose instead that

$$V_x(w) - V_x(z) \geq V_{x'}(w) - V_{x'}(z) = \mathcal{V}(V_x(w)) - \mathcal{V}(V_x(z))$$

where the equality results from $V_{x'}$ being a monotonic transformation of V_x. The Mean Value theorem along with the monotonicity of function V_x implies that, for some $\omega \in [V_x(z), V_x(w)]$,

$$V_x(w) - V_x(z) \geq \mathcal{V}'(\omega)\big(V_x(w) - V_x(z)\big) \Longrightarrow 1 \geq \mathcal{V}'(\omega)$$

This contradicts Proposition 3.1, which states $\mathcal{V}'(V) > 1$. Therefore,

$$X_p^* = \{x \in X | S^*(x) \geq k_w\} = \{x \in X | x \leq x^*\}$$

where $x^* = \begin{cases} \underline{x}, & \text{if } S^*(\underline{x}) < k_w \\ S^{-1*}(k_w), & \text{if } S^*(\underline{x}) \geq k_w \text{ and } S^*(\overline{x}) < k_w \\ \overline{x}, & \text{if } S^*(\overline{x}) \geq k_w \end{cases}.$

2. We now rewrite problem (5) by replacing q as an increasing function of w using the complementary slackness condition (3). This is an unconstrained maximization problem in w. Since the resulting objective function is continuous in w and the domain $[z, y_w]$ is compact, the Weierstrass theorem ensures the existence of a solution. Notice that the objective function is non-negative and values 0 at the two extremes of the domain. Moreover, the first derivative with respect to w is

$$-\nu(q)\eta(q)\frac{1-\phi(q)}{\phi(q)}\frac{V_x(w) - V_x(z)}{k_f} + \nu(q)\frac{dV_x(w)}{dw},$$

which is strictly positive at $w = z$. Thus, the wage solution must be an interior point. The first order condition becomes

$$\frac{V_x(w) - V_x(z)}{V_x'(w)} = \frac{k_f}{\eta(q)} \frac{\phi(q)}{1 - \phi(q)},$$

where $V_x'(w) \equiv \frac{dV_x(w)}{dw}$ (and similarly for the second derivative). The right hand side of this expression is decreasing in q and, hence, also in w. The derivative of the left hand side is

$$1 - \frac{(V_x(w) - V_x(z))V_x''(w)}{V_x'(w)^2} > 0$$

This expression is positive because function V_x is concave as stated in Proposition 3.1. Therefore, the solution of the first order condition must be unique.

Incomes y_x^{u*}, for any $x \in X$, and $y_x^{e*}(w)$, for any $x \in X_p$ and w, are determined by equation(2), which has a unique solution because of the concavity of the utility function.

Therefore, $\{(q_x^*, w_x^*, y^{e*}(w_x^*), y_x^{u*})\}_{x \in X_p^*} \cup \{(y_x^{u*})\}_{x \in X_{np}^*}$ take part of equilibrium allocation, and it is unique.$\|$

Proof of Proposition 3.3

We first show that a decreasing absolute risk aversion of $\frac{-V_x''(w)}{V_x'(w)}$ is a necessary and sufficient condition for the the household's surplus, $\frac{V_x(w)-V_x(z)}{V_x'(z)}$, also to decline with x. Notice that $\frac{V_x(w)-V_x(z)}{V_x'(w)} = \frac{V_x'(\hat{w})(w-z)}{V_x'(z)}$, for some $\hat{w} \in (z, w)$. Then,

$$\begin{aligned} \frac{\partial}{\partial x}\left(\frac{V_x(w) - V_x(z)}{V_x'(w)}\right) &= (w - z)\frac{V_x'(\hat{w})}{V_x'(w)}\left(\frac{\frac{\partial V_x'(\hat{w})}{\partial x}}{V_x'(\hat{w})} - \frac{\frac{\partial V_x'(w)}{\partial x}}{V_x'(w)}\right) < (>)0 \\ &\iff \frac{-V_x''(w)}{V_x'(w)} \text{ is decreasing (increasing) in } x \end{aligned}$$

Next, we show the dynamics of wages and queue lengths over the space of the spouses' productivity. Using the constraint to replace q in the objective function of the search problem (5), we can rewrite it as

$$\max_w H(w, x) \equiv \nu(q(w))\big(V_x(w) - V_x(z)\big)$$

The necessary condition with respect to w is

$$\frac{\partial H(w, x)}{\partial w} = 0 \iff \left(\nu'(q(w))\frac{dq}{dw}\frac{V_x(w) - V_x(z)}{V_x'(w)} + \nu(q(w))\right)V_x'(w) = 0,$$

where $V_x'(w) \equiv \frac{dV_x(w)}{dw}$. We now evaluate the change of the first derivative with respect to a marginal increase in x at the solution candidate, and obtain

$$\begin{aligned}
\frac{\partial^2 H(w,x)}{\partial w \partial x} &= \nu'(q(w))\frac{dq}{dw}\frac{\partial \frac{V_x(w)-V_x(z)}{V_x'(w)}}{\partial x} > (<) 0 \\
&\iff \frac{-V_x''(w)}{V_x'(w)} \text{ is decreasing (increasing) in } x
\end{aligned}$$

Thus, the absolute risk aversion of the household's indirect utility function being decreasing in x is a necessary and sufficient condition for wages to increase with x. The direction of the queue lengths dynamics is the same as for wages since the equilibrium zero-profit condition establishes a positive relationship between q and w.

Proof of Lemma 3.4.

First, the decline of the value difference $V_x(w) - V_x(z)$ with x, for a given wage w, was proved in Proposition 3.2.

Second, consider expression (8) and an additively separable utility function. The second element of the sum in the numerator in $d_2(x)$ is always positive as stated in Proposition 3.1. We now show that the cross derivative in the numerator is 0 for such preferences.

Let $G(y,w,x) \equiv v(w+y, \frac{y}{x})$. Its derivative $\frac{\partial G}{\partial y} = v_c + v_\ell \frac{1}{x}$ pins down the solution of the household's problem (1). As stated in Proposition 3.1, increases in w lead to reductions in the spouse's income y: $\frac{\partial^2 G}{\partial y \partial w} = v_{cc} + v_{\ell c}\frac{1}{x} < 0$. However, such declines do not depend on the spouse productivity x for such preferences as $v_{\ell c} = 0$. $\|$

Proof of Proposition 3.5.

1. To show that wages increase with z, we proceed along the same lines as in the proof of Proposition 3.3. Using the constraint to replace q in the objective function, we can rewrite the maximization problem (5) as

$$\max_w H(w,z), \text{ where } H(w,z) \equiv \nu(q(w))\big(V_x(w) - V_x(z)\big)$$

   The necessary condition with respect to w is

$$\frac{\partial H(w,z)}{\partial w} = 0 \iff \nu'(q(w))\frac{dq}{dw}\big(V_x(w) - V_x(z)\big) + \nu(q(w))V_x'(w) = 0,$$

   where $V_x'(w) \equiv \frac{\partial V_x(w)}{\partial w}$. We now evaluate the change of the first derivative with respect to a marginal increase in z, and obtain

$$\frac{\partial^2 H(w,z)}{\partial w \partial z} \quad = \quad -\nu'(q(w))\frac{dq}{dw}V_x'(z) > 0$$

This derivative is positive, in particular, at the candidate solutions. Thus, wages increase with z. So do queue lengths since the equilibrium zero-profit condition establishes a positive relationship between q and w. Further, the Envelope theorem implies $\frac{\partial S(x)}{\partial z} = -\nu(q_x)\upsilon_c\big(z + y_x^u, \frac{y_x^u}{x}\big) < 0$. Thus, the reservation value x^* lowers with z, increasing the mass of non-participating individuals.

2. We turn now to the dynamics of the variation with respect to z over the distribution of households. We can rewrite the above lines in terms of $H(w,z,x)$, and then differentiate with respect to x as well to obtain

$$\frac{\partial^3 H(w,z,x)}{\partial w \partial z \partial x} = -\nu'(q(w))\frac{dq}{dw}\frac{\partial V_x'(z)}{\partial x} < 0$$

The last term is negative according to Proposition 3.1; hence, so is this expression. That is, the higher the x, the lower the increases in wages induced by increases in z are. Furthermore,

$$\frac{\partial^2 S(x)}{\partial z \partial x} = -\nu'(q_x)\frac{\partial q_x}{\partial x}V_x'(z) - \nu(q_x)\frac{\partial V_x'(z)}{\partial x} > 0,$$

where the first term is positive if the absolute risk aversion of the indirect utility function falls with x as stated in Proposition 3.3, and Proposition 3.1 claims the second term is also positive.

Finally, we study changes in the dynamics of the intra-household additional insurance over the distribution of households, $\frac{\partial^2 \frac{y_x^{u*} - y_x^{e*}}{y_x^{e*}}}{\partial x \partial z}$. Consider, first, additively separable preferences. Term $d_2(x)$ in expression (8) becomes

$$d_2(x) = \int_z^{w_x} \frac{\frac{\partial y_x^{e*}(w)}{\partial x}\frac{\partial y_x^{e*}(w)}{\partial w}}{y_x^{e*}(w)^2} dw$$

Thus,

$$\frac{\partial d_2(x)}{\partial z} = -\frac{\frac{\partial y_x^{e*}(w)}{\partial x}\frac{\partial y_x^{e*}(w)}{\partial w}}{y_x^{e*}(w)^2} + \int_z^{w_x} \frac{\partial}{\partial z}\frac{\frac{\partial y_x^{e*}(w)}{\partial x}\frac{\partial y_x^{e*}(w)}{\partial w}}{y_x^{e*}(w)^2} dw,$$

which is positive because so is the first term as stated in Proposition 3.1, whereas the second term is 0 once the effects of z through wages are discarded.

In particular, consider $v \in \mathcal{F}_1$. Then,

$$\frac{\partial \frac{y_x^{u*}}{y_x^{e*}}}{\partial z} = \frac{-x^2 v_{cc}}{x^2 v_{cc} + v_{\ell\ell}} \frac{1}{y_x^{e*}} = \frac{1}{y_x^{e*}} \frac{-1}{1 + \frac{1}{x^2}\frac{v_{\ell\ell}}{v_{cc}}} = \frac{y_x^{u*}}{y_x^{e*}} \frac{-1}{y_x^{u*} + \frac{\eta(z+y_x^{u*})}{\sigma}}$$

Thus,

$$\frac{\partial^2 \frac{y_x^{u*}}{y_x^{e*}}}{\partial z \partial x} = \frac{\partial \frac{y_x^{u*}}{y_x^{e*}}}{\partial x} \frac{-1}{y_x^{u*} + \frac{\eta(z+y_x^{u*})}{\sigma}} + \frac{y_x^{u*}}{y_x^{e*}} \frac{\partial \frac{-1}{y_x^{u*} + \frac{\eta(z+y_x^{u*})}{\sigma}}}{\partial x} > 0$$

The first term is positive as stated in Proposition 3.4, while Proposition 3.1 claims that the second term is also positive.

Likewise, if $v \in \mathcal{F}_2$, then we know that $y_x^{e*} + w = \frac{x}{\psi} = y_x^{u*} + z$. Therefore,

$$\frac{y_x^{u*}}{y_x^{e*}} = \frac{\frac{x}{\psi} - z}{\frac{x}{\psi} - w} \Longrightarrow \frac{\partial \frac{y_x^{u*}}{y_x^{e*}}}{\partial z} = \frac{-1}{\frac{x}{\psi} - w} \Longrightarrow \frac{\partial^2 \frac{y_x^{u*}}{y_x^{e*}}}{\partial z \partial x} = \frac{1}{\psi\big(\frac{x}{\psi} - w\big)^2} > 0.\|$$

7.2 Appendix. Proofs of Section 4

Proof of Propostion 4.1

1. Consider the planner's problem. Notice that the equilibrium allocation belongs to its domain as households' equilibrium decisions ensure incentive compatibility, the feasibility constraint becomes the zero-profit condition and the participation constraint holds. Thus, because of the continuity of the functional forms and the closedness of the domain, the Weierstrass theorem ensures the existence of a solution to this problem.

   Next, consider the following alternative problem:

$$\begin{aligned}
(P_a) \quad & \max_{X_p, \mathcal{C}} \int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} v\big(z + y_x^u, \frac{y_x^u}{x}\big) dF(x) \\
\text{s. to} \quad & \frac{k_f}{q_x} = \nu(q_x)\big(y_w + y_x^e - c_x^e\big) + (1 - \nu(q))(z + y_x^u - c_x^u), \text{for } x \in X_p \\
& \mathcal{U}_x \geq v\big(c_x^u, \frac{y_x^u}{x}\big), \text{for } x \in X_p \\
& \mathcal{U}_x \geq \mathcal{U}_x(x'), \ \text{for } x, x' \in X_p \\
& v\big(z + y_x^u, \frac{y_x^u}{x}\big) \geq \mathcal{U}_x(x'), \ \text{for } x \in X_{np}, \text{for } x' \in X_p
\end{aligned}$$

We are to show that it has the same solution as the planner's. This alternative problem only differs from the planner's in that the possibility of misreporting a type from set

X_{np} has been eliminated, i.e. the following constraints have been removed:

$$\mathcal{U}_x \geq v\big(z + y^u_{x'}, \frac{y^u_{x'}}{x}\big), \text{ for } x \in X_p, \text{for } x' \in X_{np} \tag{15}$$

$$v\big(z + y^u_x, \frac{y^u_x}{x}\big) \geq v\big(z + y^u_{x'}, \frac{y^u_{x'}}{x}\big), \text{ for } x, x' \in X_{np} \tag{16}$$

Therefore, the planner's problem yields a lower value than the solution to this alternative problem. It is convenient to write the incentive compatibility conditions of problem (P_a) as separate conditions depending on whether the household participates or not. Thus, let $\mathcal{L}$ denote the Lagrangian of problem (P_a), and

$$\begin{aligned}
\mathcal{L} \ &= \ \int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} v\big(z + y^u_x, \frac{y^u_x}{x}\big) dF(x) \\
&+ \ \int_{X_p} \lambda^1_x \Big(\nu(q_x)\big(y_w + y^e_x - c^e_x\big) + (1 - \nu(q))(z + y^u_x - c^u_x) - \frac{k_f}{q_x} \Big) dF(x) \\
&+ \ \int_{X_p} \lambda^2_x \Big(\nu(q_x)\big(v\big(c^e_x, \frac{y^e_x}{x}\big) - v\big(c^u_x, \frac{y^u_x}{x}\big)\big) - k_w \Big) dF(x) \\
&+ \ \int_{X_p}\int_{X_p} \lambda^3_{x,x'} \Big(\mathcal{U}_x - \Big(\nu(q_{x'}) v(c^e_{x'}, \frac{y^e_{x'}}{x}) + \big(1 - \nu(q_{x'})\big) v(c^u_{x'}, \frac{y^u_{x'}}{x}) - k_w \Big)\Big) dF(x') dF(x) \\
&+ \ \int_{X_p}\int_{X_p} \lambda^4_{x,x'} \Big(\mathcal{U}_x - v(c^u_{x'}, \frac{y^u_{x'}}{x}) \Big) dF(x') dF(x) \\
&+ \ \int_{X_{np}}\int_{X_p} \lambda^5_{x,x'} \Big(v\big(z + y^u_x, \frac{y^u_x}{x}\big) - \Big(\nu(q_{x'}) v(c^e_{x'}, \frac{y^e_{x'}}{x}) + \big(1 - \nu(q_{x'})\big) v(c^u_{x'}, \frac{y^u_{x'}}{x}) - k_w \Big)\Big) dF(x') dF(x \\
&+ \ \int_{X_{np}}\int_{X_p} \lambda^6_{x,x'} \Big(v\big(z + y^u_x, \frac{y^u_x}{x}\big) - v(c^u_{x'}, \frac{y^u_{x'}}{x}) \Big) dF(x') dF(x)
\end{aligned}$$

where the Lagrange multipliers $\lambda^2_x, \lambda^3_{x,x'}, \lambda^4_{x,x'}, \lambda^5_{x,x'}$ and $\lambda^6_{x,x'}$ are non-negative. The necessary condition with respect to y^u_x for $x \in X_{np}$ is

$$\Big(v_c\big(z + y^u_x, \frac{y^u_x}{x}\big) + \frac{v_\ell(z + y^u_x, \frac{y^u_x}{x})}{x} \Big) \Big(dF(x) + \int_{X_p} \lambda^5_{x,x'} dF(x') + \int_{X_p} \lambda^6_{x,x'} dF(x') \Big) = 0$$

Since the first factor must be zero, the solution of this alternative problem assigns value $V_x(z)$ to all households in set X_{np}.

Moreover, the solution of problem (P_a) satisfies that $\mathcal{U}_x \geq V_x(z)$ for $x \in X_p$. To see this, suppose that $\mathcal{U}_x < V_x(z)$ for $x \in \tilde{X} \subset X_p$, where $\tilde{X}$ is a subset of positive mass. Then, consider the alternative allocation that only differs from the solution in that households in subset $\tilde{X}$ are now reassigned to X_{np}. This alternative allocation trivially satisfies all the constraints, and delivers a strictly higher value than the solution, which is a contradiction.

Next, notice that conditions (15) and (16) also hold for the allocation that solves the alternative problem. This is because no household of type $x \in X_{np}$ would be better off by reporting type $x' \in X_{np}$ by definition of $V_x(z)$. Neither would any other household of type $x \in X_p$ because the solution ensures it gets a value greater than or equal to $V_x(z)$ if participating in the search activity. Therefore the solution of the alternative problem (P_a) also belongs to the domain of the planner's problem, and, hence, such an allocation must coincide with the constrained efficient one.

Finally, let $\hat{X}_p$ and $\hat{X}_{np}$ denote the set of participating and non-participating households in the planner's allocation, respectively. To see the continuity of $\mathcal{U}_x$ at the boundary of $\hat{X}_p$, suppose instead that it does not converge to $V_x(z)$ for some $x \in \partial \hat{X}_p$, and, without loss of generality, assume that $x \in \hat{X}_p$. That is, $\mathcal{U}_x > V_x(z) + \epsilon$ for some $\epsilon > 0$. Consider a sequence $\{x_n\}_n \subset \hat{X}_{np}$ such that $x_n \to x$. By the continuity of $V_t(z)$ in t, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{U}_x > V_{x_n}(z) + \epsilon/2 \geq \mathcal{U}_{x_n}(x) + \epsilon/2$ for all $n \geq n_0$, where the last inequality follows from incentive compatibility. This contradicts the continuity of $\mathcal{U}_t(x)$ as a function of t.

2. Consider the solution to problem (P_a), and let $\tilde{X} \subset X_p^*$ denote the subset of values such that $V_x(z) + S^*(x) - k_w > \mathcal{U}_x$. This is without loss of generality because $\mathcal{U}_x \geq V_x(z)$ holds in the planner's allocation for all $x \notin X_p^*$. Suppose that $\tilde{X}$ is of positive mass. Consider now the alternative allocation that only differs from the planner's within subset $\tilde{X}$, where it takes the equilibrium values. We are to show that this other allocation belongs to the domain of problem (P_a), thereby reaching a contradiction. It clearly satisfies the feasibility and participation constraints. As for incentive compatibility, we distinguish two cases. First, for any $x \in \tilde{X}$, $V_x(z) + S^*(x) - k_w > \mathcal{U}_x \geq \mathcal{U}_x(x')$, for all $x' \in \hat{X}_p$, where the last inequality is the (ICC) that holds for the planner's allocation. Further, such households are strictly better off than not participating. Second, for all $x' \notin \tilde{X}$ and $x \in \tilde{X}$,

$$\mathcal{U}_{x'} \geq V_{x'}(z)+S^*(x')-k_w \geq v(z+y_x^{u*}, \frac{y_x^{u*}}{x'})+\nu(q_x)\big(v(w_x^*+y_x^{e*}, \frac{y_x^{e*}}{x'})-v(z+y_x^{u*}, \frac{y_x^{u*}}{x'})\big)-k_w,$$

$$\text{and} \quad \mathcal{U}_{x'} \geq V_{x'}(z) > v(z + y_x^{u*}, \frac{y_x^{u*}}{x'}),$$

where $y_x^{u*} \equiv \underset{y}{\operatorname{argmax}}\ v(z + y, \frac{y}{x})$ and $y_x^{e*} \equiv \underset{y}{\operatorname{argmax}}\ v(w_x^* + y, \frac{y}{x})$. The last inequality of the first line holds because the equilibrium allocation is incentive compatible. Thus, incentive compatibility holds for the alternative allocation.

3. We next focus on the subset $\hat{X}_p$. Let $(\hat{q}_x, \hat{c}^e_x, \hat{c}^u_x, \hat{y}^e_x, \hat{y}^u_x)$ denote the planner's solution for any $x \in \hat{X}_p$. We first show that, for differentiable mechanisms, there exists a threshold $\hat{x}$ such that $\hat{X}_p = [\underline{x}, \hat{x}]$ and $\hat{X}_{np} = (\hat{x}, \overline{x}]$. Suppose instead that there exists $\tilde{x} \in X$ such that $[\tilde{x} - \delta, \tilde{x}) \subset \hat{X}_{np}$ and $[\tilde{x}, \tilde{x} + \epsilon] \subset \hat{X}_p$ for some $\delta, \epsilon > 0$. Assume that $\tilde{x} \in \hat{X}_p$ for simplicity. As shown above, $\mathcal{U}_{\tilde{x}} = V_{\tilde{x}}(z)$. It follows that $\lambda^3_{x',\tilde{x}} = \lambda^4_{x',\tilde{x}} = \lambda^5_{x',\tilde{x}} = \lambda^6_{x',\tilde{x}} = 0$ for all $x' \in X$ because $\mathcal{U}_{x'} \geq V_{x'}(z) > \upsilon\big(c^u_{\tilde{x}}, \frac{y^u_{\tilde{x}}}{x'}\big)$ if $x' \in X_p$ and $V_{x'}(z) > \upsilon\big(c^u_{\tilde{x}}, \frac{y^u_{\tilde{x}}}{x'}\big)$ $x' \in X_{np}$. That is, no household has incentives to pretend to be $\tilde{x}$. Then, the first order conditions of the Lagrangian $\mathcal{L}$ with respect to consumption and income in each labor market status at $\tilde{x}$ are

$$\begin{aligned}
\upsilon_c\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\left(1 + \lambda^2_{\tilde{x}} + \int_{\hat{X}_p} \big(\lambda^3_{\tilde{x},x'} + \lambda^4_{\tilde{x},x'}\big) dF(x')\right) &= \lambda^1_{\tilde{x}} \\
\upsilon_c\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\left(1 - \lambda^2_{\tilde{x}}\frac{\nu(\hat{q}_{\tilde{x}})}{1-\nu(\hat{q}_{\tilde{x}})} + \int_{\hat{X}_p} \big(\lambda^3_{\tilde{x},x'} + \lambda^4_{\tilde{x},x'}\big) dF(x')\right) &= \lambda^1_{\tilde{x}} \\
\upsilon_\ell\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\left(1 + \lambda^2_{\tilde{x}} + \int_{\hat{X}_p} \big(\lambda^3_{\tilde{x},x'} + \lambda^4_{\tilde{x},x'}\big) dF(x')\right) &= -\lambda^1_{\tilde{x}}\tilde{x} \\
\upsilon_\ell\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\left(1 - \lambda^2_{\tilde{x}}\frac{\nu(\hat{q}_{\tilde{x}})}{1-\nu(\hat{q}_{\tilde{x}})} + \int_{\hat{X}_p} \big(\lambda^3_{\tilde{x},x'} + \lambda^4_{\tilde{x},x'}\big) dF(x')\right) &= -\lambda^1_{\tilde{x}}\tilde{x}
\end{aligned}$$

By combining these necessary conditions, we obtain

$$\upsilon_c\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\tilde{x} + \upsilon_\ell\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big) = 0;\ \upsilon_c\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\tilde{x} + \upsilon_\ell\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big) = 0$$

Thus, we can write $\upsilon\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big) = V_{\tilde{x}}(z)$ and $\upsilon\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big) = V_{\tilde{x}}(\omega)$, where $\hat{c}^e_{\tilde{x}} = \omega + \hat{y}^e_{\tilde{x}}$. Because of the properties of the indirect utility function $V_{\tilde{x}}$ and the optimal spouse's income stated in Proposition 3.1, it follows that $\hat{c}^e_{\tilde{x}} \geq \hat{c}^u_{\tilde{x}}$ and $\hat{y}^e_{\tilde{x}} \leq \hat{y}^u_{\tilde{x}}$. It also follows from the above necessary conditions that $0 > \upsilon_\ell\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big) \geq \upsilon_\ell\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)$. Thus,

$$\upsilon_\ell\big(\hat{c}^e_{\tilde{x}}, \frac{\hat{y}^e_{\tilde{x}}}{\tilde{x}}\big)\hat{y}^e_{\tilde{x}} \geq \upsilon_\ell\big(\hat{c}^u_{\tilde{x}}, \frac{\hat{y}^u_{\tilde{x}}}{\tilde{x}}\big)\hat{y}^u_{\tilde{x}}$$

Moreover, for all $x \in [\tilde{x}, \tilde{x} + \epsilon]$, the right derivative

$$\frac{d\big(\mathcal{U}_x - V_x(z)\big)}{dx^+} = -\nu(q_x)\upsilon_\ell\big(c^e_x, \frac{y^e_x}{x}\big)\frac{y^e_x}{x^2} - (1-\nu(q_x))\upsilon_\ell\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x^2} + \upsilon_\ell\big(z + y^{u*}_x, \frac{y^{u*}_x}{x}\big)\frac{y^{u*}_x}{x^2}$$

$$= \frac{\nu(q_x)}{x}\Bigg(\upsilon_\ell\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x} - \upsilon_\ell\big(c^e_x, \frac{y^e_x}{x}\big)\frac{y^e_x}{x}\Bigg) - \upsilon_\ell\big(c^u_x, \frac{y^u_x}{x}\big)\frac{y^u_x}{x^2} + \upsilon_\ell\big(z + y^{u*}_x, \frac{y^{u*}_x}{x}\big)\frac{y^{u*}_x}{x^2}$$

where $y^{u*}_x \equiv \underset{y}{\operatorname{argmax}}\ \upsilon(z + y, \frac{y}{x})$. The first line results from the fact that incentive compatibility implies

$$\frac{d\mathcal{U}_x(x')}{dx'}_{|_{x'=x}} = 0 \Leftrightarrow \nu'(q_x)\dot{q}_x(\upsilon^e - \upsilon^u) + \nu(q_x)\big(\upsilon^e_c\dot{c}^e_x + \upsilon^e_\ell\frac{\dot{y}^e_x}{x}\big) + (1 - \nu(q_x))\big(\upsilon^u_c\dot{c}^u_x + \upsilon^u_\ell\frac{\dot{y}^u_x}{x}\big) = 0$$

where the dot symbol denotes the derivative with respect to x; hence, the derivative $\frac{d\mathcal{U}_x}{dx^+}$ boils down to the first two summands of the first line. The second line is a mere reorganization of the terms. Notice that this derivative evaluated at $x = \tilde{x}$ is strictly negative because so is the first term while the sum of the last two terms vanishes. This is a contradiction because $\mathcal{U}_x \geq V_x(z)$ in $\hat{X}_p$.

Finally, to see that $\hat{x} \geq x^*$, notice that the participation constraint for any x arbitrarily close to, but below x^* delivers $\mathcal{U}_x \geq V_x(z) + S^*(x) - k_w > V_x(z)$. This implies that $x \in \hat{X}_p$ and, hence, $x^* \in \hat{X}_p$.

4. Assume that the equilibrium threshold satisfies $\underline{x} < x^*$ as this is the only interesting case, and that the equilibrium allocation is the solution of problem (P$_a$). Recall that the participation constraint and the incentive compatibility constraints are slack for participating households in the equilibrium allocation. That is, all multipliers but λ^1_x must be 0 within X^*; hence, problem (P$_a$) can be reduced to

$$\begin{aligned} \max_{X_p,\mathcal{C}} \quad & \int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} \upsilon\big(z + y^u_x, \frac{y^u_x}{x}\big)dF(x) \\ \text{s. to} \quad & \frac{k_f}{q_x} = \nu(q_x)\big(y_w + y^e_x - c^e_x\big) + (1 - \nu(q))(z + y^u_x - c^u_x), \text{for } x \in X_p \end{aligned}$$

and its Lagrangian

$$\begin{aligned} \mathcal{L} \;=\; & \int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} \upsilon\big(z + y^u_x, \frac{y^u_x}{x}\big)dF(x) \\ + \; & \int_{X_p} \lambda^1_x\Bigg(\nu(q_x)\big(y_w + y^e_x - c^e_x\big) + (1 - \nu(q))(z + y^u_x - c^u_x) - \frac{k_f}{q_x}\Bigg)dF(x) \end{aligned}$$

The following equations are necessary conditions for all $x \in X_p$:

$$\upsilon_c\big(c_x^e, \frac{y_x^e}{x}\big) = \lambda_x^1$$
$$\upsilon_c\big(c_x^u, \frac{y_x^u}{x}\big) = \lambda_x^1$$
$$\upsilon_\ell\big(c_x^e, \frac{y_x^e}{x}\big) = -\lambda_x^1 x$$
$$\upsilon_\ell\big(c_x^u, \frac{y_x^u}{x}\big) = -\lambda_x^1 x$$

That is, $\upsilon_c\big(c_x^e, \frac{y_x^e}{x}\big) = \upsilon_c\big(c_x^u, \frac{y_x^u}{x}\big)$ and $\upsilon_\ell\big(c_x^e, \frac{y_x^e}{x}\big) = \upsilon_\ell\big(c_x^u, \frac{y_x^u}{x}\big)$. The following lemma claims that this leads to $(c_x^e, y_x^e) = (c_x^u, y_x^u)$, which implies that the participation conditions do not hold at the equilibrium allocation. Therefore, the laissez-faire equilibrium is not a solution of the planner's problem.

Lemma 7.1 *Let $\upsilon : \mathbb{R}_+^2 \longrightarrow \mathbb{R}$ be a continuously differentiable, strictly increasing and concave function, and $(x_1, x_2), (x_1, x_2') \in \mathbb{R}_+^2$. If $\upsilon_1\big(x_1, x_2\big) = \upsilon_1\big(x_1', x_2'\big)$ and $\upsilon_2\big(x_1, x_2\big) = \upsilon_2\big(x_1', x_2'\big)$, then $(x_1, x_2) = (x_1', x_2')$.*

Proof of Lemma 7.1.

Define function $\tilde{\upsilon}(z_1, z_2) = \upsilon\big(z_1, z_2\big) - z_1 \upsilon_1\big(x_1, x_2\big) - z_2 \upsilon_2\big(x_1, x_2\big)$. $\tilde{\upsilon}$ is strictly concave because so is function υ, and non-monotone. That is, the necessary first-order conditions are also sufficient to determine its maximum, and the absolute maximizer of $\tilde{\upsilon}$ is unique; hence, $(x_1, x_2) = (x_1', x_2')$.‖

Next, we assume that the planner's set of tools is further restricted. Using the previous results in this proof, we can write the planner's restricted problem as

$$
\begin{array}{llll}
(\mathrm{P}_r) & \max\limits_{X_p, (q_x, c_x^e, y_x^e, y_x^u)} & \displaystyle\int_{X_p} \mathcal{U}_x dF(x) + \int_{X_{np}} V_x(z) dF(x) & \\
& \text{s. to} & \dfrac{k_f}{q_x} = \nu(q_x)\big(y_w + y_x^e - c_x^e\big), \quad \text{for } x \in X_p & \\
& & c_x^u = z + y_x^u, \quad \text{for } x \in X_p & \\
& & \mathcal{U}_x \geq \upsilon\big(c_x^u, \frac{y_x^u}{x}\big), \quad \text{for } x \in X_p & \\
& & \mathcal{U}_x \geq \mathcal{U}_x(x'), \quad \text{for } x, x' \in X_p & (17) \\
& & V_x(z) \geq \mathcal{U}_x(x'), \quad \text{for } x \in X_{np}, x' \in X_p & (18)
\end{array}
$$

We remove the last two constraints and consider the following alternative (less restricted) problem

$$\begin{aligned}
\max_{X_p,(q_x,c_x^e,y_x^e,y_x^u)} \quad & \int_{X_p}\Big(\nu(q_x)v\big(c_x^e,\frac{y_x^e}{x}\big)+(1-\nu(q_x))v\big(z+y_x^u,\frac{y_x^u}{x}\big)-k_w\Big)dF(x)+\int_{X_{np}}V_x(z)dF(x)\\
\text{s. to} \quad & \frac{k_f}{q_x}=\nu(q_x)\big(y_w+y_x^e-c_x^e\big), \qquad \text{for } x\in X_p\\
& \nu(q_x)\Big(v\big(c_x^e,\frac{y_x^e}{x}\big)-v\big(z+y_x^u,\frac{y_x^u}{x}\big)\Big)\geq k_w, \qquad \text{for } x\in X_p
\end{aligned}$$

We shall show that the solution of this alternative problem must also solve the original problem (P_r). Indeed, this is a set of maximization problems that can be solved separately. The necessary conditions with respect to c_x^e, y_x^e and y_x^u, for any $x\in\hat{X}_p$, are

$$\begin{aligned}
(dF(x)+\lambda_x^2)v_c\big(c_x^e,\frac{y_x^e}{x}\big) &= \lambda_x^1\\
(dF(x)+\lambda_x^2)v_\ell\big(c_x^e,\frac{y_x^e}{x}\big) &= -\lambda_x^1 x\\
\big(dF(x)-\frac{\lambda_x^2\nu(q_x)}{1-\nu(q_x)}\big)\Big(v_c\big(z+y_x^u,\frac{y_x^u}{x}\big)x+v_\ell(z+y_x^u,\frac{y_x^u}{x})\Big) &= 0
\end{aligned}$$

where λ_x^1 and λ_x^2 are the multipliers of the first and second constraints, respectively. The first two conditions imply $v_c\big(c_x^e,\frac{y_x^e}{x}\big)x+v_\ell\big(c_x^e,\frac{y_x^e}{x}\big)=0$. The last condition also leads to $v_c\big(z+y_x^u,\frac{y_x^u}{x}\big)x+v_\ell\big(z+y_x^u,\frac{y_x^u}{x}\big)=0$. This is the case even if the first factor were 0 because then the maximum attained would be below $V_x(z)$ if this condition did not hold. Therefore, this last problem can be rewritten as

$$\begin{aligned}
\max_{X_p,(q_x,w)} \quad & \int_{X_p}\Big(\nu(q_x)\big(V_x(w)-V_x(z)\big)-k_w\Big)dF(x)+\int_X V_x(z)dF(x)\\
\text{s. to} \quad & \frac{k_f}{q_x}=\nu(q_x)\big(y_w-w\big), \qquad \text{for } x\in X_p\\
& \nu(q_x)\big(V_x(w)-V_x(z)\big)\geq k_w, \qquad \text{for } x\in X_p
\end{aligned}$$

Notice that, for each $x\in\hat{X}_p$, this problem coincides with the household's problem (4) in the laissez-faire economy. Therefore, the laissez-faire equilibrium allocation solves this problem and $\hat{X}_p=[\underline{x},x^*]$ and $\hat{X}_{np}=X\setminus\hat{X}_p$. Finally, notice that by applying to submarket (q_x,w_x), the household with type $x\in\hat{X}_p$ reveals that $\mathcal{U}_x\geq\mathcal{U}_x(x')$ for $x'\in\hat{X}_p$; hence, constraint (17) holds. Similarly, no household with productivity $x>\hat{x}$ would be better off by misreporting their type; hence, constraint (18) also holds. Therefore, the laissez-faire equilibrium must coincide with the solution of the planner's restricted problem (P_r). $\|$

Proof of Proposition 4.2

1. The planner's OT problem divides into two different sets of problems. Given that the resource constraint must hold for non-participating households, the consumption-income pair (c_x^u, y_x^u) assigned to them is determined by the indirect utility function $V_x(z)$. As for the participating households, the planner's problem reduces to the set of type-specific maximization problems $\{(P_x^{OT})\}_{x\in \hat{X}_p^{OT}}$. Notice that the equilibrium allocation belongs to the domain of such maximization problems. Thus, no positive mass of households can be worse off in the planner's OT allocation than in equilibrium because assigning to such households the equilibrium allocation would be a Pareto improvement.

   To show that there exists a threshold $\hat{x}$ such that $\hat{X}_p^{OT} = [\underline{x}, \hat{x}]$ and $\hat{X}_{np}^{OT} = (\hat{x}, \overline{x}]$, suppose instead that there exists $\tilde{x} \in X$ such that $[\tilde{x}-\delta, \tilde{x}) \subset \hat{X}_{np}^{OT}$ and $[\tilde{x}, \tilde{x}+\epsilon] \subset \hat{X}_p^{OT}$ for some $\delta, \epsilon > 0$. Assume that $\tilde{x} \in \hat{X}_p^{OT}$ for simplicity. By continuity, $\mathcal{U}_{\tilde{x}} = V_{\tilde{x}}(z)$. As shown below, the necessary conditions with respect to consumption and spouse's income for each labor market status at $\tilde{x}$ lead to

$$\upsilon_c\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big)\tilde{x} + \upsilon_\ell\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big) = 0;\ \upsilon_c\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big)\tilde{x} + \upsilon_\ell\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big) = 0$$

   Thus, we can write $\upsilon\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big) = V_{\tilde{x}}(z)$ and $\upsilon\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big) = V_{\tilde{x}}(\omega)$, where $\hat{c}_{\tilde{x}}^{eOT} = \omega + \hat{y}_{\tilde{x}}^{eOT}$. Because of the properties of the indirect utility function $V_{\tilde{x}}$ and the optimal spouse's income stated in Proposition 3.1, it follows that $\hat{c}_{\tilde{x}}^{eOT} \geq \hat{c}_{\tilde{x}}^{uOT}$ and $\hat{y}_{\tilde{x}}^{eOT} \leq \hat{y}_{\tilde{x}}^{uOT}$. Thus, $0 > \upsilon_\ell\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big) \geq \upsilon_\ell\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big)$, and $\upsilon_\ell\big(\hat{c}_{\tilde{x}}^{eOT}, \frac{\hat{y}_{\tilde{x}}^{eOT}}{\tilde{x}}\big)\hat{y}_{\tilde{x}}^{eOT} \geq \upsilon_\ell\big(\hat{c}_{\tilde{x}}^{uOT}, \frac{\hat{y}_{\tilde{x}}^{uOT}}{\tilde{x}}\big)\hat{y}_{\tilde{x}}^{uOT}$.

   Moreover, for all $x \in [\tilde{x}, \tilde{x}+\epsilon]$, because of the Envelope theorem, the right derivative

$$\begin{aligned}\frac{d\big(\mathcal{U}_x - V_x(z)\big)}{dx^+} \quad = \quad & \frac{\nu(\hat{q}_x^{OT})}{x}\Bigg(\upsilon_\ell\big(\hat{c}_x^{uOT}, \frac{\hat{y}_x^{uOT}}{x}\big)\frac{\hat{y}_x^{uOT}}{x} - \upsilon_\ell\big(\hat{c}_x^{eOT}, \frac{\hat{y}_x^{eOT}}{x}\big)\frac{\hat{y}_x^{eOT}}{x}\Bigg) \\ & -\upsilon_\ell\big(\hat{c}_x^{uOT}, \frac{\hat{y}_x^{uOT}}{x}\big)\frac{\hat{y}_x^{uOT}}{x^2} + \upsilon_\ell\big(z + y_x^{u*}, \frac{y_x^{u*}}{x}\big)\frac{y_x^{u*}}{x^2}\end{aligned}$$

   where $y_x^{u*} \equiv \underset{y}{\operatorname{argmax}}\ \upsilon(z+y, \frac{y}{x})$. This derivative evaluated at $x = \tilde{x}$ is strictly negative, which is a contradiction because $\mathcal{U}_x \geq V_x(z)$ in $\hat{X}_p^{OT}$.

   Next, let $(\hat{q}_x^{OT}, \hat{c}_x^{eOT}, \hat{c}_x^{uOT}, \hat{y}_x^{eOT}, \hat{y}_x^{uOT})$ denote the planner's OT allocation for participating households, and $\mathcal{L}_x^{OT}$ the Lagrangian of problem (P_x^{OT}).

$$\begin{aligned}\mathcal{L}_x^{OT} &= \mathcal{U}_x + \lambda_x^1\Big(\nu(q)(y_w + y^e - c^e) + \big(1-\nu(q)\big)(z + y^u - c^u) - \frac{k_f}{q}\Big) \\ &\quad + \lambda_x^2\Big(\nu(q)\big(v\big(c^e, \frac{y^e}{x}\big) - v\big(c^u, \frac{y^u}{x}\big)\big) - k_w\Big),\end{aligned}$$

where λ_x^1 and λ_x^2 are the Lagrange multipliers associated to the respective constraints. The following necessary conditions must be satisfied by the planner's solution:

$$0 = \big(v\big(c^e, \frac{y^e}{x}\big) - v\big(c^u, \frac{y^u}{x}\big)\big)\big(1 + \lambda_x^2\big) + \lambda_x^1\Big(y_w + y^e - c^e - z - y^u - c^u + \frac{k_f}{q^2\nu'(q)}\Big) \tag{19}$$

$$\lambda_x^1 = v_c\big(c^e, \frac{y^e}{x}\big)(1 + \lambda_x^2) \tag{20}$$

$$\lambda_x^1 = v_c\big(c^u, \frac{y^u}{x}\big)\Big(1 - \lambda_x^2\frac{\nu(q)}{1-\nu(q)}\Big) \tag{21}$$

$$-\lambda_x^1 = v_\ell\big(c^e, \frac{y^e}{x}\big)\frac{1 + \lambda_x^2}{x} \tag{22}$$

$$-\lambda_x^1 = v_\ell\big(c^u, \frac{y^u}{x}\big)\frac{1 - \lambda_x^2\frac{\nu(q)}{1-\nu(q)}}{x} \tag{23}$$

2. The planner's equation (11) is obtained from putting together conditions (20)-(23).

3. To see that the participation constraint is binding, suppose that the multiplier $\lambda_x^2 = 0$ instead. Then, it follows from conditions (20)-(23) that

$$v_c\big(c^e, \frac{y^e}{x}\big) = v_c\big(c^u, \frac{y^u}{x}\big), \text{ and } v_\ell\big(c^e, \frac{y^e}{x}\big) = v_\ell\big(c^u, \frac{y^u}{x}\big)$$

   As stated in Lemma 7.1, it follows that $(c^e, y^e) = (c^u, y^u)$; hence, a contradiction since the (PC) fails to hold.

   Now, given the necessary condition (11), we can write $V_x(\omega^j) = v\big(c^j, \frac{y^j}{x}\big)$ for $j \in \{u, e\}$, where $\omega^j \equiv c^j - y^j$. The participation constraint implies that $V_x(\omega^u) < V_x(\omega^e)$. Furthermore, we know from Proposition 3.1 that V_x is increasing, $\frac{dy}{d\omega} < 0$ and $\frac{d(\omega+y)}{d\omega} \geq 0$. Thus, $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} < \hat{c}_x^{eOT} - \hat{y}_x^{eOT}$, $\hat{y}_x^{eOT} < \hat{y}_x^{uOT}$ and $\hat{c}_x^{eOT} \geq \hat{c}_x^{uOT}$. Indeed, if $v \in \mathcal{F}_2$, the necessary conditions (11) become $\hat{c}_x^{uOT} = \hat{c}_x^{eOT} = x/\psi$; hence, $\hat{y}_x^{eOT} < \hat{y}_x^{uOT}$.

4. Likewise, by combining equations (19) and (20), we obtain condition (12).

5. We turn to the pattern over the household distribution.

(a) Transfers, $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} - z$, decline with x. Let $x' < x$, and consider the alternative tuple $(\hat{q}_x^{OT}, \hat{c}_x^{eOT} - \hat{y}_x^{eOT} + \tilde{y}_{x'}^e, \hat{c}_x^{uOT} - \hat{y}_x^{uOT} + \tilde{y}_{x'}^u, \tilde{y}_{x'}^e, \tilde{y}_{x'}^u)$, where $\tilde{y}_{x'}^j$ is the solution of the maximization problem associated to the indirect utility function $V_{x'}(\hat{c}_x^{jOT} - \hat{y}_x^{jOT})$, for $j \in \{e, u\}$. This tuple satisfies the feasibility condition of problem $(\mathrm{P}_{x'}^{OT})$. It also satisfies the participation constraint because

$$\frac{k_w}{\nu(\hat{q}_x^{OT})} \le V_x(\hat{c}_x^{eOT} - \hat{y}_x^{eOT}) - V_x(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}) \le V_{x'}(\hat{c}_x^{eOT} - \hat{y}_x^{eOT}) - V_{x'}(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}),$$

where the first inequality results from the necessary conditions (11), and the second one follows from Lemma 3.4. Therefore, this tuple belongs to the domain of problem $(\mathrm{P}_{x'}^{OT})$; hence, $V_{x'}(\hat{c}_x^{uOT} - \hat{y}_x^{uOT}) \le V_{x'}(\hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT})$, and then $\hat{c}_x^{uOT} - \hat{y}_x^{uOT} \le \hat{c}_{x'}^{uOT} - \hat{y}_{x'}^{uOT}$ because of the monotonicity of function $V_{x'}$.

(b) Define $H(\omega, s, x) \equiv \nu(q(\omega, s))V_x(\omega) + (1 - \nu(q(\omega, s)))V_x(z + s)$, where q is pinned down by the resource constraint $k_f = \eta(q)(y_w - \omega) - sq(1 - \nu(q))$. Problem (P_x^{OT}) is equivalent to

$$\max_{\omega, s} H(\omega, s, x)$$

The necessary condition with respect to ω is

$$\frac{\partial H}{\partial \omega} = 0 \Longleftrightarrow \nu'(q)\frac{\partial q}{\partial \omega}\big(V_x(\omega) - V_x(z + s)\big) + \nu(q)V_x'(\omega) = 0$$

Let $(\hat{\omega}_x^{OT}, \hat{s}_x^{OT})$ denote a solution of the maximization problem. When evaluating the change of the first derivative with respect to a marginal increase in x at the solution, we obtain

$$\left.\frac{\partial^2 H}{\partial \omega \partial x}\right|_{(\omega,s)=(\hat{\omega}_x^{OT}, \hat{s}_x^{OT})} = \nu(\hat{q}_x^{OT})\frac{\partial V_x'(\hat{\omega}_x^{OT})}{\partial x} < 0$$

because $\nu(\hat{q}_x^{OT})\big(V_x(\hat{\omega}_x^{OT}) - V_x(z + \hat{s}_x^{OT})\big) = k_w$ in the planner's OT allocation and the sign of the cross derivative follows from Proposition 3.1. The analysis with respect to s is analogous. Therefore, the higher the spouse's productivity x, the lower ω and s are. Then, condition (11) together with Proposition 3.1 implies that spouse's income increases with x as $y_{x'}^{eOT}(\hat{\omega}_{x'}^{OT}) < y_{x'}^{eOT}(\hat{\omega}_x^{OT}) < y_x^{eOT}(\hat{\omega}_x^{OT})$, for any pair $x, x' \in \hat{X}_p^{OT}$ such that $x' < x$. Analogously for the consumption, $z + s$, and spouse's income of one-earner households.

To see that $\hat{q}_x^{OT} < \hat{q}_{x'}^{OT}$, it suffices to use the resource constraint $k_f = \nu(q)(y_w -$

$\omega) - sq(1-\nu(q))$ to pin down a positive relationship between both ω and s and q. Finally, since the participation constraint is binding in the planner's OT allocation, and $\nu(\hat{\omega}^{OT}_{x'}) < \nu(\hat{\omega}^{OT}_{x'})$, it must be the case that $V_x(\hat{\omega}^{OT}_x) - V_x(z+\hat{s}^{OT}_x) < V_{x'}(\hat{\omega}^{OT}_{x'}) - V_{x'}(z+\hat{s}^{OT}_{x'})$.

6. The equilibrium allocation is not a solution of the planner's OT problem as the (PC) is slack in equilibrium within X^*_p. We next prove that $\hat{x}^{OT} = x^*$ by contradiction. Suppose, first, that $\hat{x}^{OT} < x^*$. Then, consider the alternative allocation that only differs from the planner's within the interval $(\hat{x}^{OT}, x^*)$ where it is the equilibrium tuple $(q^*_x, w^*_x + y^{e*}_x, z + y^{u*}_x, y^{e*}_x, y^{u*}_x)$. This alternative allocation is feasible and delivers a higher social value. Suppose, instead, that $\hat{x}^{OT} > x^*$. Then, for all $x \in (x^*, \hat{x}^{OT})$, there exists a tuple (q_x, w_x, s_x) such that

$$k_w = \nu(q_x)\big(V_x(w_x) - V_x(z+s_x)\big), \quad \text{and } \frac{k_f}{q_x} = \nu(q_x)(y_w - w_w) - s_x\big(1-\nu(q_x)\big).$$

Then, let $\tilde{w}_x = w_x + s_x \frac{1-\nu(q_x)}{\nu(q_x)}$. Note that the tuple $(q_x, \tilde{w}_x, 0)$ also satisfies (the zero-profit) condition $k_f = \eta(q_x)(y_w - \tilde{w}_x)$. Likewise, by the monotonicity of the indirect utility function, we have

$$k_w = \nu(q_x)\big(V_x(w_x) - V_x(z+s_x)\big) < \nu(q_x)\big(V_x(\tilde{w}_x) - V_x(z)\big)$$

Thus, $S^*(x) > k_w$; hence, $(x^*, \hat{x}^{OT}) \subset X^*_p$, which is a contradiction.

We had proved that $x^* \leq \hat{x}$ in Proposition 4.1. We are now to prove by contradiction that they are the same. Suppose that $\hat{x}^{OT} < \hat{x}$. Consider an alternative allocation that only differs from the planner's OT one in that it takes the tuples $(\hat{q}_x, \hat{c}^e_x, \hat{c}^u_x, \hat{y}^e_x, \hat{y}^u_x)$ for all $x \in (\hat{x}^{OT}, \hat{x})$. This alternative allocation belongs to the domain of the planner's OT problem, and delivers a strictly higher value.

To see that $q^*_x < \hat{q}^{OT}_x$, assume the opposite holds. Then,

$$w^*_x \geq \hat{\omega}^{OT}_x + \hat{s}^{OT}_x \frac{1-\nu(\hat{q}^{OT}_x)}{\nu(\hat{q}^{OT}_x)} \Longrightarrow w^*_x > \hat{\omega}^{OT}_x,$$

which follows from the equilibrium free-entry condition (7) and the planner's resource constraint (RC_x). Notice that the planner's condition (12) can be rewritten as

$$k_f \quad = \quad \eta(q)\big(1-\phi(q)\big)\left(\frac{V_x(w) - V_x(z+s)}{V'(w)} - w + z + s + y_w - z\right), \tag{24}$$

and is satisfied by the equilibrium allocation (condition (6)). From the monotonicity and concavity of function V_x stated in Proposition 3.1, it is easy to show that the term in brackets on the right hand side increases with w and decreases with s. Since $w_x^* > \hat{\omega}_x^{OT}$ and $\hat{s}_x^{OT} > 0$, we obtain $q_x^* < \hat{q}_x^{OT}$, which is a contradiction.

Finally, since the equilibrium tuple belongs to the domain of problem (P_x^{OT}) together with equation (2), it follows that $V_x(z) \leq V_x(c_x^{uOT} - y_x^{uOT})$ and then $z \leq c_x^{uOT} - y_x^{uOT}$; hence, $z + y_x^{u*} \leq c_x^{uOT}$ and $y_x^{uOT} \leq y^{u*}$ from Proposition 3.1. The strict inequalities result from the fact that the equilibrium allocation is not a solution of the planner's OT problem. ‖

7.2.1 Appendix. Market Economy with taxes and transfers.

Consider the economy described in Sections 2 and 3 now with a government that must balance its budget by household's type and can use the following type-specific instruments: an after-tax unemployment benefit, b_x, an income tax on newly employed workers, $T_x(w)$, and a proportional tax rate on a spouse's income, τ_x. Thus, for any given worker's income w at the last stage of the period, a household's indirect utility function is

$$V_x(w) \equiv \max_y \ v\big(w - T_x(w) + y(1-\tau_x), \frac{y}{x}\big),$$

where $w - T_x(w) = z + b_x$ if the worker remains unemployed. We turn to the definition of the tax-distorted equilibrium.

Definition 2 *Given public policy $\{(b_x, T_x, \tau_x)_x\}$, a directed search equilibrium consists of a search value $S^* : X \to \mathcal{R}_+$, income of the spouse of an employed worker $\{y_x^{e*}\}_{x\in X} : [z, y_w] \to \mathcal{R}_+$ and income of the spouse of an unemployed worker $\{y_x^{u*}\}_{x\in X} \in \mathcal{R}_+$, a set of labor force participants $X_p^* \subset X$, wages $\{w_x^*\}_{x\in X_p^*}$, and a queue length function $Q^* : [z, y_w] \to \mathcal{R}_+$ such that:*

i) Households' optimal decisions:

(a) labor force participation:

$$x \in X \setminus X_p^* \text{ if and only if } \ \nu(Q^*(w))\big(V_x(w) - V_x(z)\big) < k_w, \ \forall w \in [z, y_w]$$

(b) job search: $\forall x \in X_p^*$,

$$\nu(Q^*(w))\big(V_x(w) - V_x(z)\big) \leq S^*(x), \qquad \forall w \in [z, y_w], \text{ and}$$
$$\nu(Q^*(w_x^*))\big(V_x(w_x^*) - V_x(z)\big) = S^*(x)$$

(c) a spouse's income: for all $w \in [z, y_w]$, $y_x^{e*}(w)$ *and* y_x^{u*} *satisfy condition (2)* $\forall x \in X_p^*$ *and* $\forall x \in X$*, respectively.*

ii) Free entry of firms:

$\eta(Q^*(w))(y_w - w) \leq k_f$, $\forall w \in [z, y_w]$, *and* $Q^*(w) \leq \infty$, *with complementary slackness. In particular, the first inequality is an equality for all* w_x^*.

iii) Government's budget is balanced for all $x \in X_p^*$:

$$b_x\big(1 - \nu(Q^*(w_x^*))\big) = \tau_x\big(y_x^{e*}\nu(Q^*(w_x^*)) + y_x^{u*}\big(1 - \nu(Q^*(w_x)))\big) + T_x(w_x^*)\nu(Q^*(w_x^*))$$

Apart from the government's balanced budget constraint, this is the same definition as the one stated in Section 3 for the laissez-faire economy.

Proof of Proposition 4.3.

Let x^* denote the tax-distorted equilibrium threshold. For any $x \leq x^*$, the set of conditions that characterize the tax-distorted equilibrium are

$$\begin{aligned}
v_c\big(c_x^e, \frac{y_x^e}{x}\big)(1-\tau_x)x + v_\ell\big(c_x^e, \frac{y_x^e}{x}\big) &= 0 && (25)\\
v_c\big(c_x^u, \frac{y_x^u}{x}\big)(1-\tau_x)x + v_\ell\big(c_x^u, \frac{y_x^u}{x}\big) &= 0 && (26)\\
\big(1-\phi(q_1)\big)\left(\frac{v\big(c_x^e, \frac{y_x^e}{x}\big) - v\big(c_x^u, \frac{y_x^u}{x}\big)}{v_c\big(c_x^e, \frac{y_x^e}{x}\big)(1 - T_x'(w_x))} + y_w - w_x\right) &= \frac{k_f}{\eta(q_x)} && (27)\\
\eta(q_x)(y_w - w_x) &= k_f && (28)\\
\big(T_x(w_x) + \tau_x y_x^e\big)\nu(q_x) &= \big(b_x - \tau_x y_x^u\big)\big(1 - \nu(q_x)\big) && (29)
\end{aligned}$$

The first two equations are the first-order conditions of a household's indirect utility function, and only differ from equation (2) in the tax wedge. Equations (27)-(28) are the counterparts of equilibrium equations (6)-(7). The last condition ensures that the government's budget is balanced for each household type separately.

We now show that the planner's OT tuple, $(\hat{q}_x^{OT}, \hat{c}_x^{eOT}, \hat{c}_x^{uOT}, \hat{y}_x^{eOT}, \hat{y}_x^{uOT})$, satisfies the tax-distorted equilibrium conditions, and also determine the necessary fiscal instruments.

First, the associated equilibrium wages, w_x^{OT}, are uniquely pinned down by equation (28). Second, it follows directly from the comparison of equations (25)-(26) with condition (11) that a spouse's income must be tax free, $\tau_x = 0$. Further, it must be the case that

$$\hat{c}_x^{eOT} = w_x^{OT} - T_x(w_x^{OT}) + \hat{y}_x^{eOT}, \text{ and } \hat{c}_x^{uOT} = z + b_x + \hat{y}_x^{uOT}$$

By using these expressions to replace taxes $T_x(w_x^{OT})$ and benefits b_x into the government's budget constraint (29), we obtain

$$\begin{aligned}
\nu(\hat{q}_x^{OT})\big(w_x^{OT} + \hat{y}_x^{eOT} - \hat{c}_x^{eOT}\big) + \big(1 - \nu(\hat{q}_x^{OT})\big)(z + \hat{y}_x^{uOT} - \hat{c}_x^{uOT}) = 0 \underset{(28)}{\Longleftrightarrow} \\
\nu(\hat{q}_x^{OT})(y_w + \hat{y}_x^{eOT} - \hat{c}_x^{eOT}) + \big(1 - \nu(\hat{q}_x^{OT})\big)\big(z + \hat{y}_x^{uOT} - \hat{c}_x^{uOT}\big) = \frac{k_f}{\hat{q}_x^{OT}}
\end{aligned}$$

which is the planner's feasibility condition (RC_x). Therefore, the planner's OT allocation also satisfies equation (29). Finally, the equilibrium equation (27) coincides with the planner's condition (12) if and only if

$$\big(1 - \phi(\hat{q}_x^{OT})\big)\big(T_x(w_x^{OT}) + b_x\big) \quad = \quad \phi(\hat{q}_x^{OT})T_x'(w_x^{OT})(y_w - w_x^{OT})$$

Consider the following tax schedule $T_x(w) = T_{x,0} + T_{x,1}w$. Then, there are three fiscal instruments $(T_{x,0}, T_{x,1}, b_x)$ to be determined as the unique solution of the following system of equations

$$\begin{aligned}
\big(1 - \phi(\hat{q}_x^{OT})\big)\big(T_{x,0} + b_x\big) &= T_{x,1}(\phi(\hat{q}_x^{OT})y_w - w_x^{OT}) \\
\hat{c}_x^{eOT} &= w_x^{OT}(1 - T_{x,1}) - T_{x,0} + \hat{y}_x^{eOT} \\
\hat{c}_x^{uOT} &= z + b_x + \hat{y}_x^{uOT}
\end{aligned}$$

After manipulating the above equations, we determine the tax rate $T_{x,1}$:

$$\begin{aligned}
\hat{c}_x^{eOT} - \hat{c}_x^{uOT} &= w_x^{OT}(1 - T_{x,1}) + \hat{y}_x^{eOT} - T_{x,0} - z - \hat{y}_x^{uOT} - b_x \\
&= w_x^{OT}(1 - T_{x,1}) - z - T_{x,1}\frac{\phi(\hat{q}_x^{OT})y_w - w_x^{OT}}{1 - \phi(\hat{q}_x^{OT})} + \hat{y}_x^{eOT} - \hat{y}_x^{uOT} \\
&= w_x^{OT} - z - T_{x,1}\frac{\phi(\hat{q}_x^{OT})}{1 - \phi(\hat{q}_x^{OT})}(y_w - w_x^{OT}) + \hat{y}_x^{eOT} - \hat{y}_x^{uOT} \\
\Longleftrightarrow \qquad & T_{x,1} = \frac{w_x^{OT} - z - \hat{c}_x^{eOT} + \hat{c}_x^{uOT} + \hat{y}_x^{eOT} - \hat{y}_x^{uOT}}{y_w - w_x^{OT}} \frac{1 - \phi(\hat{q}_x^{OT})}{\phi(\hat{q}_x^{OT})}
\end{aligned}$$

Table 2: Summary Statistics

	Full Sample		Subsample with only first spell	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Age	38.355	8.796	38.185	8.864
Female	0.520	0.500	0.526	0.499
Married	0.565	0.496	0.570	0.495
No. of children under 18	1.049	1.253	1.038	1.245
Spouse employed[1]	0.765	0.424	0.769	0.421
if men	0.641	0.480	0.646	0.478
if women	0.884	0.320	0.883	0.321
Spouse NILF[1]	0.164	0.370	0.160	0.367
if men	0.277	0.448	0.271	0.445
if women	0.055	0.228	0.057	0.232
Spouse's earnings[1]	3120.892	3939.473	3169.170	3968.896
if men	1872.476	2678.013	1961.715	2896.010
if women	4300.378	4539.483	4288.965	4469.704
HH net liquid wealth[2]	48 956.23 (median 57.40)	498 022.80	53 317.52 (median 171.87)	568 983.30
non-employment duration	20.831	23.695	21.940	25.227
No. of observations	39,640		29,389	

Note.- (1): Spouse-related variables are conditional on being married. (2): Net liquid wealth is defined as total wealth minus home, vehicles and business equity and also net of unsecured debt. The number of observations for this variable is 26,645 in the full sample and 19,933 in the restricted one.

Then, $b_x = \hat{c}_x^{uOT} - z - \hat{y}_x^{uOT}$ and $T_{x,0} = w_x^{OT}(1 - T_{x,1}) - \hat{c}_x^{eOT} + \hat{y}_x^{eOT}$. $\|$

7.3 Data Appendix.

We use data from the 1996, 2001, 2004 and 2008 SIPP panels, covering from 1996:6 to 2013:6. Surveyed individuals are interviewed every four months and report a number of demographic and economic variables for the previous four months, in particular their labor market status and earnings. We consider two labor market status: employment, E, and non-employment, $\not{E}$.

We restrict our dataset to individuals aged 25-55, who are quite attached to the labor market, and for which we have precise information of the duration of their non-employment spell. An observation is a $E\not{E}E$ spell. We eliminate observations with a $\not{E}$ spell shorter than 3 weeks as usual. The length of the $\not{E}$ spell varies largely, but is longer than 1 year for less than 10% of the sample. We have 39,640 spells. Table 2 shows the summary statistics for the whole sample. Since only 46.17% of the individuals in our sample have a single spell, dealing with all spells would overweight short spells. Therefore, we further restrict our dataset to the very first spell of all individuals in our sample, which accounts for 74.14% of the observations.[31] We use total earned income. Gross earnings in SIPP are reported monthly and defined as wages and salary and self-employment income, including earnings from all jobs in case of multi-job holders, before any deductions for taxes, health insurance, etc.[32] All monetary values are CPI-adjusted and reported in 2010 dollars. Unfortunately, usual hours are reported only once per wave. Finally, following Chetty (2008), net liquid wealth is defined as household's liquid wealth net of unsecured debt, where liquid wealth amounts to total wealth minus home, business and vehicles equity.

We first use Cox proportional hazard models to estimate the relationship between the hazard rate of a worker and his or her spouse's earnings prior to the non-employment spell. The results are shown in Table 3. Notice that the estimates do not vary much when information on net liquid wealth is removed in the second specification.

Next, we examine the relationship between the spouse's earnings and (1- and 3-digit) occupation-switching rates. Table 4 reports the Probit estimates.

Finally, we estimate the additional insurance arrangements within the household during a non-employment spell by regressing the change in the spouse's log earnings between the end and the beginning of a $\not{E}$ spell of a worker on the distribution of the spouse's log earnings prior to the $\not{E}$ spell. Table 5 shows the results.

[31] Cullen and Gruber (2000) deals with this issue by equally weighting all the observations of a given individual so that her total weight is one.

[32] Around 10% of the observations for married individuals have no data on the spouse's earnings, and above 15% spouse's earnings are 0. Most of the results are quite robust to the chosen spouse earnings variable, e.g. total wages or the difference between the family and the worker's earnings as a proxy of the spouse's earnings.

Table 3: Cox Hazard Model Estimates

Married	0.567***	(.081)				
Spouse's log earnings	−0.016	(.012)				
Married with no data on spouse's earnings			−0.021	(.030)	−0.033	(.029)
Spouse has no earnings			0.031	(.025)	0.043*	(.025)
Spouse's earnings Q1			0.009	(.026)	0.020	(.025)
Spouse's earnings Q2			−0.023	(.026)	−0.011	(.026)
Spouse's earnings Q3			−0.061**	(.025)	−0.050**	(.025)
Spouse's earnings Q4			−0.011	(.025)	−0.010	(.025)
Spouse's earnings Q5			−0.068**	(.028)	−0.074***	(.028)
HH net liquid wealth Q1					−0.478***	(.030)
HH net liquid wealth Q2					−0.538***	(.029)
HH net liquid wealth Q3					−0.508***	(.032)
HH net liquid wealth Q4					−0.467***	(.030)
HH net liquid wealth Q5					−0.458***	(.031)
Female	−0.248***	(.025)	−0.146***	(.016)	−0.137***	(.016)
Previous log earnings	0.095***	(.011)	0.092***	(.008)	0.090***	(.007)
No. of observations	11,824		28,398		28,398	

Note.- Dataset comprises the first $E\cancel{E}E$ spell for all individuals in the sample. The first coefficient in first specification reports the elasticity of the hazard rate with respect to the spouse's average log earnings of the previous three months prior to the non-employment spell. The first set of coefficients in second and third specifications can be interpreted as the percentage change in hazard rate associated with the quintile of the spouse's average earnings of the previous three months prior to the $\cancel{E}$ spell. All models also include log unemployment rate, a time line and monthly dummies, a quadratic polynomial of age, and dummies for the number of children under age 18, seam effects, white and black, high-school, college degree and post-college degree, homeownership, state, occupation and industry. The net liquid wealth dummies are indicator variables for whether the household wealth falls into the corresponding quintile of the distribution, and the reference group comprises those households for which there is no wealth information. Standard errors are in parenthesis.

Table 4: Occupation-switching Probit Model Estimates

	Major Occupations		3-Digit Occupations	
Married with no data on spouse's earnings	−0.034	(.042)	0.008	(.042)
Spouse has no earnings	−0.066*	(.037)	−0.054	(.036)
Spouse's earnings Q1	−0.100***	(.036)	−0.064*	(.036)
Spouse's earnings Q2	−0.074**	(.036)	−0.050	(.036)
Spouse's earnings Q3	−0.125***	(.038)	−0.117***	(.037)
Spouse's earnings Q4	−0.158***	(.038)	−0.099***	(.036)
Spouse's earnings Q5	−0.175***	(.042)	−0.207***	(.039)
HH net liquid wealth Q1	−0.064	(.046)	0.008	(.045)
HH net liquid wealth Q2	−0.147***	(.044)	−0.085**	(.044)
HH net liquid wealth Q3	−0.145***	(.049)	−0.076	(.048)
HH net liquid wealth Q4	−0.170***	(.046)	−0.117***	(.045)
HH net liquid wealth Q5	−0.219***	(.048)	−0.122***	(.046)
Female	−0.066***	(.024)	−0.038	(.023)
Previous log earnings	−0.062***	(.012)	−0.055***	(.011)
(log) $\not{E}$ duration	0.361***	(.011)	0.474***	(.011)
No. of observations	25,934			

Note.- The first set of coefficients in the specifications can be interpreted as the occupation-switching probability change associated with the quintile of the spouse's average earnings of the previous three months prior to non-employment. See Table 3 for the details on controls. Standard errors are in parenthesis.

Table 5: Estimates of Changes in Spouse's Earnings of Married Workers

	All workers		Workers with FT spouses		All workers - SPE	
Spouse's earnings Q1			0.633***	(.032)		
Spouse's earnings Q2	−0.443***	(.029)	0.332***	(.025)	−0.371***	(.031)
Spouse's earnings Q3	−0.535***	(.029)	0.249***	(.023)	−0.440***	(.031)
Spouse's earnings Q4	−0.607***	(.030)	0.178***	(.020)	−0.511***	(.031)
Spouse's earnings Q5	−0.795***	(.034)			−0.655***	(.035)
HH net liquid wealth Q1	0.064	(.042)	0.016	(.026)	0.014	(.026)
HH net liquid wealth Q2	−0.016	(.043)	−0.045	(.028)	−0.053*	(.028)
HH net liquid wealth Q3	0.049	(.045)				
HH net liquid wealth Q4	0.086**	(.041)	0.038	(.026)	0.025	(.026)
HH net liquid wealth Q5	0.130***	(.042)	0.072***	(.027)	0.060**	(.027)
Female	0.134***	(.018)	0.089***	(.019)	0.115***	(.019)
Previous log earnings	0.002	(.007)	−0.001	(.007)	−0.001	(.008)
(log) $\not{E}$ duration	0.0185**	(.008)	0.008	(.008)	0.017**	(.008)
No. of observations	10,803		8,509		10,800	

Note.- The earnings change is defined as the difference between the spouse's log earnings before and after a $\not{E}$ spell in 2010 dollars. The first column corresponds to the subsample of married workers. The second column refers to the subsample of workers whose spouses usually worked at least 35 hours per week in the month prior to the $\not{E}$ spell. The third column refers to all married workers with spouse's earnings ranked over the nine-month period prior to the job loss. The earnings change is regressed against a number of household's characteristics. See Table 3 for the details on controls. Standard errors are in parenthesis.

Table 6: Alternative calibration (LMP preferences). Parameter Values

Parameter	Value	Interpretation	Target
m_0	0.194	mass of $x = 0$	mass of spouses with no earnings (SIPP)
θ	3.402	inverse of Frisch elasticity of consumption wrt price	Blundell et al. (2016b)
ψ	0.810	scale labor disutility parameter	ratio of avg hours worked by household members (SIPP)
s_1	1.034	scale factor of the matching function	monthly job-finding rate (SIPP)
s_2	0.261	elasticity of the matching function	elasticity of job-filling probability
(m_x, d_x)	(7.407,0.490)	mean and standard deviation of F	mean and st. dev. of spouse earnings dist. (SIPP)
y_w	944.736	worker's productivity	ratio of avg worker's and avg spouse's earnings (SIPP)
k_f	15.603	vacancy creation cost	14% average quarterly wage per hire
k_w	$2 \cdot 10^{-10}$	participation cost	percentage of non-emp. spells over 1 year (SIPP)
z	239.966	home production	consumption ratio c_u/c_e
b	172.224	unemployment benefits	replacement rate for average wage

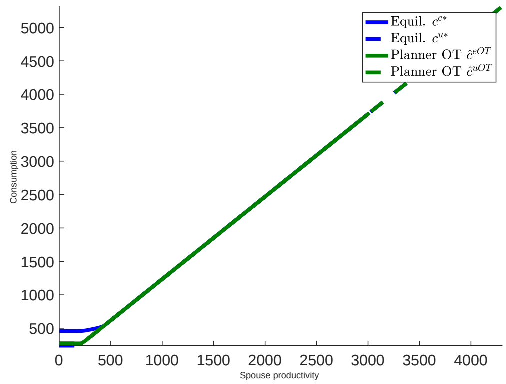
7.4 Alternative Calibration.

In the alternative calibration, we use LMP preferences. The calibration strategy only differs from the baseline one in the utility function, for which we use the same targets, namely the Frisch elasticity of labor with respect to wages for all couples formed by an employed spouse and a job-seeker, and the scale factor ψ is calibrated to match the normalized average annual hours worked.[33] Table 6 summarizes the exercise. The fit to the targeted data is good except for the average job-finding rate, for which the model does a very poor job.

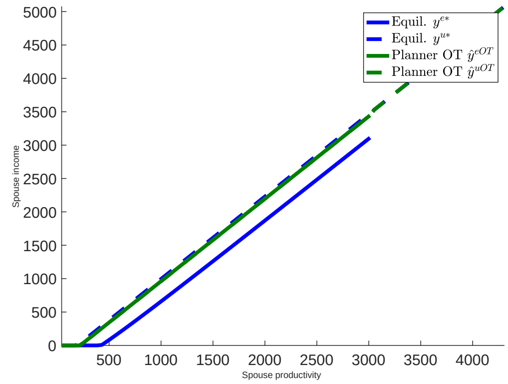
Figure 4 plots the distributions for some key variables for the equilibrium and the planner's OT allocation.[34] In equilibrium, all households with spouse's productivity below $z\psi$ behave in the same way as spouses find it optimal not to work. Above that threshold, consumption risks are fully insured away through the spouse's labor supply. This pattern is the same in the planner's OT allocation, yet the spouse's income difference are fairly small. Importantly, the full consumption insurance arranged within the household leads the planner's OT vacancy creation and replacement rates to be almost flat over the household distribution. Replacement rates are just above 12%.

[33]Under LMP preferences, the Frisch own elasticity of consumption with respect to price is 1, away from the 0.417 that Blundell et al. (2016b) estimate from the data.

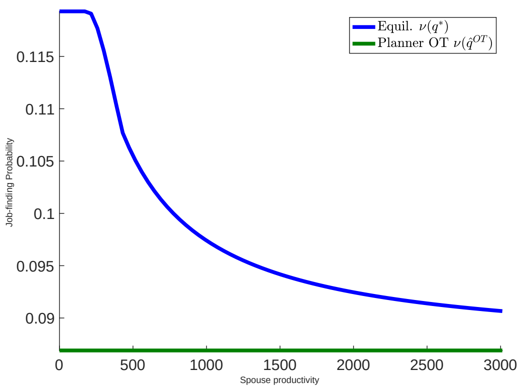
[34]The planner's allocation is not displayed here due to computational issues related to the corner solutions to the planner's problem.



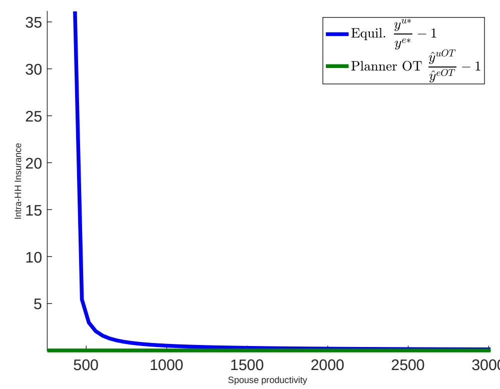
(a) Consumption



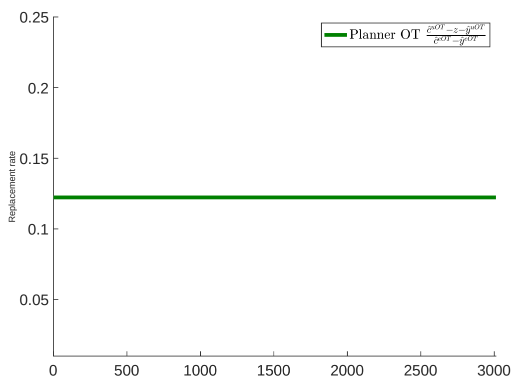
(b) Spouse Income



(c) Job-finding Probability



(d) Intra-Household Insurance



(e) Replacement Rate

Figure 4: Planner's OT and Laissez-Faire Equilibrium Allocations with LMP Preferences