Compromising on Compromise Rules

Salvador Barberà
Danilo Coelho

June 2021

Barcelona GSE Working Paper Series

Working Paper nº 1263

COMPROMISING ON COMPROMISE RULES

Salvador Barberà (MOVE, UAB and Barcelona GSE) and

Danilo Coelho (IPEA)*

June 1, 2021

Abstract

We propose three mechanisms to reach compromise between two opposing parties. They are based on the use of Rules of k Names, whereby one of the parties proposes a shortlist and the other chooses from it. Methods of this class are used in practice to appoint Supreme Court justices and have been recently proposed for arbitration selection processes. Our mechanisms are flexible and allow the parties to participate in the endogenous determination of the role of proposer and the shortlist size. They involve few stages, weakly implement the Unanimity Compromise Set and are robust to the strategic inclusion of candidates.

Keywords: The Unanimity Compromise Set, Compromise Rule of k Names, Shortlisting Contest, Alternate Shortlists, Shortlisting, Voting by Alternating Offers and Vetoes and Fallback Bargaining. **JEL classification:** D02, D71, D72.

---

*salvador.barbera@uab.cat and danilo.coelho@ipea.gov.br. An earlier version of this paper has been circulated under the title "On the selection of compromise arbitrators". Salvador Barberà acknowledges financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centers of Excellence in R&D (SEV-2015—0563) and (CEX2019—000915-S), and grants ECO2014—53051-P, ECO2017— 83534-P and FEDER, and from the Generalitat de Catalunya, through grants 2014SGR—0515 and 2017SGR—0711. We thank Andrzej Skrzypacz, Douglas Bernheim, Florian Brandl, Hervé Moulin, Matthew Jackson, Fuhito Kojima, Raul Lopez, Matias Nunez, Remzi Sanver and Arkadii Slinko as well as participants at the Online Social Choice Seminar Series in 2020 organized by the Centre for Mathematical Social Science and Matthew Jackson's discussion group seminar in 2019 at Stanford University for useful comments.

1 Introduction

We propose three new methods to achieve compromise between two concerned parties. Compromises must be reached under many circumstances, but we mostly have in mind cases where someone has to be appointed to develop a task, and those involved in selecting the appointee represent conflicting points of view, though not always diametrically opposed ones.

Consider, for instance, the appointment of a new member of the Supreme Court. Judges and other members of the legal profession may form their preferences over candidates on the basis of characteristics that can be different than those used by politicians to evaluate them. And yet, a compromise solution between the possibly divergent opinions of these two groups must be achieved. One way to do it is by adopting the following rules that are used to select State Supreme Court justices in thirteen U.S. states and are known as the Missouri Plan (or the Merit Plan). A merit commission formed by lawyers and non-lawyers submits a qualified list of candidates to the governor of the state, who then appoints one candidate from the list. In fact, several countries use similar rules to appoint the members of their Supreme Courts. But who submits the shortlist varies across countries. Instead of a merit commission, it can be the Senate, the Court itself or the chief of the executive power (as in Mexico) (see Barberà and Coelho 2017).

A second example of situations where a similar procedure may be applied is the selection of an arbitrator. Arbitration is an alternative dispute resolution method that takes place outside of the courts, but results in a final and binding decision similar to a court's judgment. One of the main advantages of arbitration is the ability of the parties to participate in the choice of the arbitrator who will resolve the dispute (see Shavell 1995). Mainly for that reason, practically all cross-border commercial disputes are resolved by arbitration. Some contractual arbitrate clauses specify an arbitral institution provider that will administrate and supervise all future disputes between the parties . These institutions may specify a structured selection procedure to help the parties to exercise their right of choice. A similar procedure to the one used by the Merit Plan has been proposed recently by de Clippel, Eliaz, and Knight (2014) who call it Shortlisting, to appoint arbitrators: one of the parties start by selecting $((\mathbf{c}+1)/2)$ out of the c available candidates, and then the other party selects the arbitrator out of that shortlist.

Both rules of appointment refer to important decision problems and belong to a family of procedures that we have called Rules of k Names in previous work (Barberà and Coelho 2010, 2017 and 2018). Given a $k \in \{1, ..., \mathbf{c}\}$ exogenously fixed, one of the parties (the proposer) selects k candidates out of the c available ones, and then the other party (the chooser) picks one winner out of that shortlist proposed by the opponent .

Rules of that kind are reminiscent of divide and choose methods . They have been used for centuries, and are still very much resorted to in many countries for a variety of appointments, in particular within the judicial power.

In this article we accept that rules of k names, as we just described them, and shortlisting in particular, are good methods to achieve compromise, but that they can be improved upon. This is because when the parties' positions are not radically opposed, there may be situations in which the outcome of any fixed rule could be made more fair, in a sense to be made precise, by changing to a different rule among the class.

Fairness is a basic concern when designing methods to achieve compromise, and one that helps to get their use be agreed upon by the two sides in conflict. If a method is perceived to be unfair, it will be hardly accepted, and its rules unlikely to be respected. Hence the importance of proposing rules that are as fair as possible, and, in addition, of letting the parties to be involved in their choice.

Let us describe the rules that we propose, and explain, after that, in what sense their use would be advantageous when compared with that of others

The first method is the **Compromise Rule of k Names (CRK)**, which works as follows: Party 1 chooses $k \in \{(\mathbf{c}/2), ..., \mathbf{c}\}$. Once this choice is made public, Party 2 decides whether to act as the proposer or the chooser. Then the two parties play according to the resulting Rule of k Names.

The second method we propose is that of **Alternate Shortlists (ASL)**, which works as follows: Party 1 first proposes a non-empty subset of $\mathbf{C}$ with cardinality greater than or equal to $(\mathbf{c}/2)$. Then Party 2 decides whether to immediately select the winning candidate from the subset proposed by Party 1, or else counteract with a subset of cardinality one plus that of the set it rejects to choose from. In that case, 1 selects the winning candidate out of those presented by 2.

Our third proposal, that we call the **Shortlisting Contest (SLC)** works as follows: both parties simultaneously propose a non-empty subset of $\mathbf{C}$ with cardinality greater than or equal to $(\mathbf{c}/2)$. The subset with the highest cardinality prevails and whoever proposed the discarded subset shall select the winning candidate from the prevailing subset. If the cardinalities are the same and odd, the parties know that Party 1's proposed subset prevails, otherwise Party 2's proposed subset prevails.

Let us discuss why we propose three methods rather than one alone. Although the three mechanisms are not strategically equivalent, they share basically the same normative properties. They all introduce the possibility for parties to have an influence on the exact form of the Rule of k Names that they will eventually use to determine the chosen candidate. But the methods differ in the specifics of the extensive form game that parties play, and in the number of steps involved.

Figure 1 describes how the stages of our mechanisms are related, providing an intuition how we conceived them. The Compromise Rule of k Names transparently reflects the sequence by which one party selects the size of of k, then the other chooses the role that each one will play, and finally the resulting rule is used. The Alternating Shortlist method allows the first mover to not only indicate the size of k, but also the precise set from which her opponent may already pick an outcome, if accepting the role of chooser. But this second agent can also decide to take the role of proposer, by responding with a new set. The spirit is the same as before, but the exchanges regarding size and role are more direct, and that reduces the number of steps in the extensive game they play. The Shortlisting Contest method collapses the process even more, because it starts by a simultaneous game that determines both what set will be offered and by whom, and thus both the size of k and the roles of agents, after which the choice is determined in a second and final step.

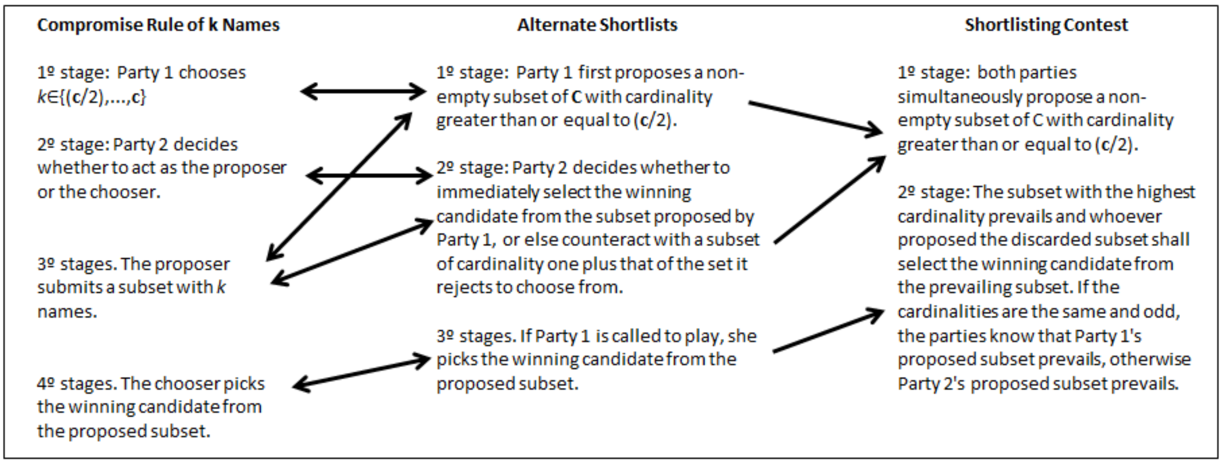

Figure 1

What makes these methods more attractive than the direct use of a fixed, exogenously imposed rule of k names?

Let us answer this question with a simple example. Consider two parties, 1 and 2, facing a set of five candidates and the following preferences: $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c3 \succ_2 c2 \succ_2 c1 \succ_2 c4 \succ_2 c5$. Each party knows its opponent's preferences. While they disagree on who is the best candidate, they share second best choice, $c2$, so this is a natural and obvious compromise candidate. However, under the rule of three names (which is de Clippel, Eliaz, and Knight (2014)'s shortlisting method, since $k = (\mathbf{c} + 1)/2 = 3$) the outcome would be $c1$ if Party 1 was the first mover, since by proposing $\{c1, c4, c5\}$ she would induce induces Party 2 to choose $c1$. Likewise, $c3$ would be the outcome if 2 was the proposer. All our methods would produce $c2$ as the unique equilibrium outcome[1].

This unfair treatment in favor of the proposer in such a case may have even further negative consequences in practice. Under this same preference profile in the example, Clippel, Eliaz, and Knight (2014), in a laboratory experiment, noticed that sometimes the outcome under the shortlisting method when 1 was the chooser was one of the two Pareto dominated candidates, $c4$ and $c5$, instead of $c1$. The likely explanation for that fact is that some of the second movers retaliated against the first by choosing a bad candidate, feeling that she should have facilitated the selection of the compromise alternative $c2$.

[1]Notice that if the parties had completely opposite preferences ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c5 \succ_2 c4 \succ_2 c3 \succ_2 c2 \succ_2 c1$), c3 would be a compromise choice and only k=3 would be able to induce it.

These authors attributed this relatively poor performance of their method in cases of that sort to the fairness concerns of some participants in the experiment, whose decisions could be explained through a theory of intentions-based reciprocity.

Our three methods always selection of $c2$ in this example is not accidental, but rather the result of a general property that they share: they always choose alternatives in the Unanimity Compromise Set, an attractive normative notion that we now define.

The Unanimity Compromise Set results from applying a method, called Fallback Bargaining, proposed and studied by Hurwicz and Sertel (1997) and Brams and Kilgour (2001). The definitions that follow are expressed for any number of parties but their use in our case will only be applied to two parties. Start by considering the set of alternatives that are best for some party. If all prefer the same alternative to all others, there is a depth 1 agreement, the procedure stops and that alternative is the Unanimity Compromise Set. If not all the parties agree on a most-preferred alternative, then their next-most preferred alternatives are also considered. I there exist some alternatives that are within the top two of every party, these would provide a depth 2 agreement, and the intersection of such alternatives become the Unanimity Compromise Set. Otherwise, the procedure continues, and as long as there is no common agreement of lower depth, the parties descend to lower and lower levels in their rankings, one at a time, until the intersection of their top-ranked alternatives becomes non-empty for the first time, at depth d^*. That set of common agreements, which always exists for some d^*, is the Unanimity Compromise Set.

Notice that under the preference profile of the previous example, the Unanimity Compromise Set is $\{c2\}$ and $d^* = 2$. The Unanimity Compromise Set has attracted a lot of attention on its normative grounds. It has been proven to contain at most two elements (for example, under this profile $c1 \succ_1 c2 \succ_1 c3$ and $c2 \succ_2 c1 \succ_2 c3$, this set is doubleton and $d^* = 2$) and coincides with the set of all Pareto efficient candidates that maximize the welfare of the worst-off party when each party's payoff from a candidate x is the cardinality of that party's lower contour set at x. As we shall see, our proposed methods will naturally lead to the choice of elements in the Unanimity Compromise Set[2], that we adopt it as a criterion of fairness.

[2] See Kibris and Sertel (2007), Sprumont (1993), Congar and Merlin (2012), Kibris and Sertel (2007) and de Clippel and Eliaz (2012) for axiomatic characterizations of the Unanimity Compromise Set.

Notice that, although we shall argue that our methods diminish the bias in favor or against the parties, some uneven treatment between them remains, depending on who plays the role of first or second mover in the Compromise Rule of k Names or the Alternate Shortlists mechanisms, or who gets her way when both parties offer same-sized sets in the Shortlisting Contest case. Yet, the remaining asymmetry is limited, because in our case the Unanimity Compromise Set will consist of at most two alternatives, and the advantage of a party playing one role or the other is just that one of them gets to determine which one of the two elements in the Unanimity Compromise Set will prevail. For the sake of symmetry, these roles could be determined by a previous uniform lottery.

Here we avoid the further analytical complications that would result from introducing outcome uncertainty, and concentrate on the modelling and the properties of our mechanisms that hold independently of the roles that parties are assigned by chance. Thus, we study the extensive form games that become fully specified once the order of play or the tie breaking rule is determined. Following de Clippel, Eliaz, and Knight (2014) and Anbarci (1993 and 2006), we consider that the complete information assumption is appropriate in the case of choosing arbitrators[3]. Our analysis focuses on the pure strategy subgame perfect equilibria. We now announce some important facts that result from our analysis.

As already stated, the equilibria of the games induced by each one of our rules may be multiple, but their outcomes are unique for all possible preferences of the parties. Their unique subgame perfect equilibrium outcome always belongs to the Unanimity Compromise Set, and thus shares some of its important features: it is always Pareto efficient, is never ranked below the median alternatives of any of the two parties whenever c is odd, and maximizes the welfare of the worst-off party, when each party's payoff from an alternative x is the cardinality of that party's lower contour set at x.

Moreover, their equilibrium outcomes of our first two mechanisms do not change if the set **C** expands in such a way that all parties consider all added candidates to be worse than those in **C**, a property that we term Invariance with respect to undesirable candidacies. This property seems to be obvious and natural. But surprisingly, as we show

[3]de Clippel, Kfir, and Knight (2014) wrote on this assumption: "...arbitration occur between parties that have a long-term relationship (e.g., unions and management). In addition, the arbitration agencies provide both parties with the same information about the potential arbitrators." p.2

later in this paper, only another known method satisfies this property.

The fact that our mechanisms implement the Unanimity Compromise Set in subgame perfect equilibria is especially relevant, because only one mechanism had previously been shown to achieve this goal. We refer to an ingenious method proposed by Anbarci (1993), called Voting Alternating Offers and Vetoes. VAOV works as follows: the parties take turns making offers until one candidate is accepted. Any offer that has been rejected is eliminated from further consideration. If no offer has been accepted before and only one candidate remains, that one becomes the chosen candidate.

But even that remarkable method is subject to criticisms. One of them comes from the computational social choice community. It is related to the computational complexity of finding an equilibrium strategy of VAOV. Since there can be $\mathbf{c} - 1$ rounds, it trivial exploration of the whole game tree in order to find a subgame perfect equilibrium strategy would take at least $O(2^{\mathbf{c}})$ operations (Erlich, Hazon, and Kraus 2018). Along the same line, de Clippel, Eliaz, and Knight (2014), advocating in favor of the use of shorter methods, argue that VAOV involves multiple stages and hence the rationality assumptions behind the use of subgame perfection might be too strong and unrealistic as a description of actual behavior. We believe that our methods are simple enough to avoid such criticisms.

Now let us give an intuitive explanation of how our mechanisms induce the parties to a compromise decision.

Consider again the preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c3 \succ_2 c2 \succ_2 c1 \succ_2 c4 \succ_2 c5$) and, now, the Compromise Rule of k Names mechanism. The last two stages of this mechanism forms different subgames that are characterized by a value of k and who submits the k candidates (the proposer). When Party 2 playing as the proposer and $k = 3$, the equilibrium outcome would be $c3$ and it would be $c2$ if$k = 4$ or $c1$ if $k = 5$. And if it was Party 1, the proposer, the equilibrium outcome would be $c1$ if $k = 3$, $c2$ if $k = 4$ or $c3$ if $k = 5$. Knowing it, Party 2 would opt to be the proposer if $k = 3$. Consequently, Party 1's best strategy is to choose $k = 4$ in order to ensure the election of $c2$. Under the Alternate Shortlist method, in equilibrium, Party 1 proposes $\{c1, c2, c4, c5\}$ and Party 2 decides to pick $c2$ since she knows that cannot induce a better outcome by proposing a subset with five alternatives. Under the Shortlisting Contest mechanism, in equilibrium Party 1 proposes $\{c1, c2, c4, c5\}$ and Party 2 proposes $\{c2, c3, c4, c5\}$, in the last stage Party 1 picks $c2$ out of $\{c2, c3, c4, c5\}$.

The Compromise Rule of k Names gives to the second-mover (Party 2) the advantage of selecting the role, as chooser or as proposer, to play at the stage where the Rule of k Names will be used. Thus it forces the first-mover (Party 1) to choose a k that minimizes that advantage which is equal to $k^* \equiv \mathbf{c} - d^* + 1$. The number k^* we call the mirrored depth of the preference profile, as it is related to its depth, as defined by Fallback Bargaining, and it is important in our further analysis of how much compromise is possible at different profiles (See Definition 2).

The Shortlisting Contest and Alternate Shortlists methods give to each party the possibility of assuming the role of proposer by playing a subset larger than the one advanced by the opponent. The shortlist size contest of the Shortlisting Contest method and the counteract option of the Alternate Shortlists method force at least one of the parties to play a subset with cardinality large enough to discourage the other from playing an even larger one. The optimal subsets that accomplish it are precisely those that induce an outcome belonging to the Unanimity Compromise Set. These subsets proposed by the parties share the same cardinality k^*.

The equilibrium outcome of Compromise Rule of k Names is Party 2's best candidate in the Unanimity Compromise Set. In the case of Alternate Shortlists method, the equilibrium outcome is Party 1's best candidate in the Unanimity Compromise Set. While the Shortlisting Contest, if k^* is odd is Party 1's best candidate in the Unanimity Compromise Set, otherwise is Party 2's best candidate in the Unanimity Compromise Set.

Therefore, as Unanimity Compromise Set can be a doubleton, the equilibrium outcome of our mechanisms depends on who moves first or the parity of k^*.

We shall compare our proposed methods with other interesting ones that are either used in practice or proposed in the literature. Veto-rank and Alternate Strikes schemes, used in practice to appoint arbitrators in many arbitral institutions, were first studied deeply by Bloom and Cavanagh (1986) and Moulin (1981), respectively. Regarding them, Anbarci (1993 and 2006) argues that VAOV leads to a faster and fairer decisions and is less subject to strategic manipulation of the pool of candidates than Alternate Strike. In turn, de Clippel, Eliaz, and Knight (2014) point out that Veto-Rank can induce a Pareto inefficient outcome. As far we know, the other four methods, proposed by Núñez and Laslier (2015), Laslier, Núñez, and Sanver (2021), Anbarci (1993 and 2006) and de

Clippel, Eliaz, and Knight (2014), respectively, have not yet been used in practice.

The paper is organized as follows. In the next Section 2, we present the formal model and results. We will prove the preceding statements, and for that we should first characterize the equilibria of the games induced by our new methods, and prove the uniqueness of their outcomes. In Section 3, we compare our methods with others already proposed. In Section 4, we check whether the different methods we have considered are invariant with respect to undesirable candidacies.

2 The model and results

Consider any finite set of candidates, $\mathbf{C} = \{1, ..., \mathbf{c}\}$. There are two parties, 1 and 2. Let $\mathbf{P}$ be the set of all strict orders on $\mathbf{C}$.[4] Preferences profiles are elements of $\mathbf{P} \times \mathbf{P}$, denoted as $(\succ_1, \succ_2)$. These two components are interpreted to be the preferences of parties 1 and 2, respectively.

2.1 The Unanimity Compromise Set and the Rules of k Names

Definition 1 (Brams and Kilgour 2001) *Given any preference profile $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, the depth of a preference profile, denoted by $d^*(\succ_1, \succ_2)$, is the smallest value of q in $\{1, ..., \mathbf{c}\}$ for which the intersection between the parties' q-top candidates is non-empty. The Unanimity Compromise Set, denoted by $\mathbf{UC}(\succ_1, \succ_2)$, is the set of all alternatives that belong to this intersection.*

Remark 1 *Brams and Kilgour (2001) prove that the $\mathbf{UC}(\succ_1, \succ_2)$ has at most two elements and coincides with the set of all Pareto efficient candidates that maximize the welfare of the worst-off party when each party's payoff from a candidate x is the cardinality of that party's lower contour set at x.*

Definition 2 *Given any preference profile $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, the mirrored depth of the preference profile is denoted by $k^*(\succ_1, \succ_2)$ and we defined as follows:*
$k^*(\succ_1, \succ_2) \equiv \mathbf{c} - d^*(\succ_1, \succ_2) + 1.$

[4] Transitive: For all $x, y, z \in \mathbf{C}$: $(x \succ y$ and $y \succ z)$ implies that $x \succ z$. Asymmetric: For all $x, y \in \mathbf{C}$: $x \succ y$ implies that $\neg(y \succ x)$. Irreflexive: For all $x \in \mathbf{C}$,$\neg(x \succ x)$. Complete: For all $x, y \in \mathbf{C}$: $x \neq y$ implies that $(y \succ x$ or $x \succ y)$.

Remark 2 *Brams and Kilgour (2001) prove that* $d^*(\succ_1, \succ_2) \leq \frac{\mathbf{c}+1}{2}$ *if* $\mathbf{c}$ *is odd,* $d^*(\succ_1, \succ_2) \leq \frac{\mathbf{c}+2}{2}$*, otherwise. This implies that* $k^*(\succ_1, \succ_2) \geq \frac{\mathbf{c}+1}{2}$ *if* $\mathbf{c}$ *is odd and* $k^*(\succ_1, \succ_2) \geq \frac{\mathbf{c}}{2}$*, otherwise.*

Definition 3 (Barberà and Coelho 2010) *The Rule of* k *Names works as follows: one of the parties (the proposer) selects* k *candidates out of the* $\mathbf{c}$ *available ones, and then the other party (the chooser) picks one winner out of the shortlist proposed by the opponent.*

Proposition 1 (Barberà and Coelho 2010) *Consider any preference profile* $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$*. Suppose that Party* $i \in \{1, 2\}$ *is the proposer and* $j \in \{1, 2\} \backslash \{i\}$ *is the chooser. The unique subgame perfect equilibrium outcome of the game of complete information induced by the Rule of* k *Names is the proposer's preferred candidate among the chooser's* $(\mathbf{c} - k + 1)$*-top candidates. There may be several subgame perfect strategy profiles leading to the unique common outcome. A strategy profile is a subgame perfect equilibrium of this game if and only if its strategies satisfy the following two conditions:*
C1. The chooser always picks its preferred candidate out of any subset submitted by the proposer.
C2. The proposer always submits a subset that contains its preferred candidate among the chooser's $(\mathbf{c} - k + 1)$*-top candidates, and any other* $k - 1$ *candidates that are ranked below that one according to the chooser's preferences.*

Corollary 1 *The unique subgame perfect equilibrium outcome of the Rule of* $k^*(\succ_1, \succ_2)$ *Names is the proposer's preferred candidate in the Unanimity Compromise Set.*

Proposition 2 *Given any preference profile* $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$ *and any* $k^{'} \in \{1, ..., \mathbf{c}\}$*, under any subgame perfect equilibrium strategy profile, if* $k^{'}$ *is not greater than mirroed depth,* $k^*(\succ_1, \succ_2)$*, then both parties are weakly better off playing as the proposer under the Rule of* $k^*(\succ_1, \succ_2)$ *Names than playing as the chooser under the Rule of* $k^{'}$ *Names. Otherwise, both parties are better off playing as the chooser under the Rule of* $k^*(\succ_1, \succ_2)$ *Names than playing as the proposer under the Rule of* $k^{'}$ *names.*

It is worth noting that Proposition 2 implies that, given any k and preference profile over candidates, both agents share the same preferences between the roles of proposer and chooser.

Our proofs of the characterizations of the subgame perfect equilibria of our mechanisms will be heavily based on propositions 1 and 2. The proofs of Proposition 2 and of theorems 1, 2 and 3 below are in Appendix.

2.2 Characterizing the equilibria of our proposed mechanisms

Theorems 1, 2 and 3 provide characterizations of some subgame perfect equilibrium strategies of the game induced by our three mechanisms.

Theorem 1 *Given any preference profile* $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$*, the game induced by the Compromise Rule of k Names method has a subgame perfect equilibrium such that*

(i) *in the first stage, Party 1 chooses* $k^*(\succ_1, \succ_2)$*, and*

(ii) *in the second stage, for any value of* k *chosen by Party 1, Party 2 opts to be the proposer unless* $k > k^*(\succ_1, \succ_2)$*, and*

(iii) *in the third stage, for any value of* k *chosen by Party 1, whoever is the proposer proposes a subset that contains its preferred candidate among the chooser's* $(\mathbf{c}-k+1)$ *top candidates, plus the chooser's* $k-1$ *worst candidates, and*

(iv) *in the fourth stage, whoever is the chooser picks its preferred candidate out of opposing party's proposed subset.*

As a consequence, a candidate is a subgame perfect equilibrium outcome if and only if it is Party 2's best candidate in the Unanimity Compromise Set.

Before presenting theorems 2 and 3, Remark 3 provides an intuition for Theorem 1.

Remark 3 *The first two stages of the Compromise Rule of k names determine the parameters of the Rule of k Names that will be used by the parties in the resulting subsequent subgame. Note that the strategy profile of that subgame has the a similar format of that characterized by Proposition 1. In the first two stages, Party 1 proposes a* k *equal to* $k^*(\succ_1, \succ_2)$*, and Party 2 opts to be the proposer unless* k *is greater than* $k^*(\succ_1, \succ_2)$*. By choosing a* $k > k^*(\succ_1, \succ_2)$*, Party 1 could assume the role of the proposer if Party 2's strategy remains unchanged. However, this would not not be a profitable deviation as Proposition 2 states that playing as the chooser under* $k^*(\succ_1, \succ_2)$ *gives a higher payoff* *than playing as the proposer under* $k^*(\succ_1, \succ_2) + 1$*. This same proposition states that under any* $k \leq k^*(\succ_1, \succ_2)$ *the role of the proposer is at least as good than the role of the chooser. Thus, Party 2 also has no incentive to deviate. Finally, it is worth noting that when the chosen* k *is equal to* $k^*(\succ_1, \succ_2)$*, by Corollary 1, the equilibrium outcome of the resulting subsequent subgame is the proposer's preferred candidate in the Unanimity Compromise Set .*

Theorem 2 *Given any preference profile* $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$*, the game induced by the Alternate Shortlists method has a subgame perfect equilibrium such that*

(i) *in the first stage, Party 1 submits a subset with cardinality equal to* $k^*(\succ_1, \succ_2)$ *that contains its preferred candidate among Party 2's* $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ *top candidates, plus Party 2's* $k^*(\succ_1, \succ_2) - 1$ *worst candidates, and*

(ii) *in the second stage, for any Party 1's proposed subset* $\mathbf{S}$*, Party 2 picks its preferred candidate out of* $\mathbf{S}$ *only if this set contains at least one candidate weakly preferred to its preferred candidate among Party 1's* $(\mathbf{c} - \#|\mathbf{S}|)$ *top candidates. Otherwise, it counter-offers with a subset with cardinality* $\#|\mathbf{S}| + 1$ *that contains its preferred candidate among Party 1's* $(\mathbf{c} - \#|\mathbf{S}|)$ *top candidates plus Party 1's* $\#|\mathbf{S}|$ *worst candidates, and*[5]

(iii) *in the third stage, whenever Party 1 assumes the role of the chooser it picks its preferred candidate out of the opposing party's proposed subset.*

As a consequence, a candidate is a subgame perfect equilibrium outcome if and only if it is Party 1's best candidate in the Unanimity Compromise Set.

Remark 4 *In contrast to Theorem 1, on the equilibrium path of Theorem 2's strategy profile, Party 2 picks the winning candidate out of the subset proposed by Party 1. The reason is the following one: in doing so and by Corollary 1, the winning candidate is Party 1's best candidate in The Unanimity Compromise Set. Denote by* x *this candidate. According to its strategy, Party 2 does not counter-offer with another subset only if* x *is weakly preferred to its preferred candidate among Party 1's* $(\mathbf{c} - k^*(\succ_1, \succ_2))$ *top candidates.*

[5] The reader can get an intuition for this counter offer strategy by comparing it with the proposer's equilibrium strategy of the game induced by the rule of k names characterized by Proposition 1.

This condition is satisfied for the following reason: by definition, $k^*(\succ_1, \succ_2) \equiv \mathbf{c} - d^*(\succ_1, \succ_2) + 1$, *so* $\mathbf{c} - k^*(\succ_1, \succ_2) = d^*(\succ_1, \succ_2) - 1$. *Thus, in case that condition was not satisfied then it would imply that the intersection between the parties'* $(d^*(\succ_1, \succ_2) - 1)$ *top candidates is not empty which contradicts the definition of* $d^*(\succ_1, \succ_2)$.

Theorem 3 *Given any preference profile* $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, *the game induced by the Shortlisting Contest method has a subgame perfect equilibrium such that*

(i) *in the first stage, each party proposes a subset with cardinality equal to* $k^*(\succ_1, \succ_2)$ *that contains its preferred candidate among the opposing party's* $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ *top candidates plus the* $k^*(\succ_1, \succ_2) - 1$ *worst candidates according to the opposing party's preference, and*

(ii) *in the second stage, whoever party is the chooser picks its preferred candidate out of the opposing party's proposed subset.*

As a consequence, a candidate is a subgame perfect equilibrium outcome when $k^*(\succ_1, \succ_2)$ *is odd (respectively,* $k^*(\succ_1, \succ_2)$ *is even) if and only if it is Party 1's (respectively Party2's) best candidate in the Unanimity Compromise Set.*

Remark 5 *The equilibrium outcomes of our mechanisms are unique. Note that in the games induced by our first two mechanisms only one player moves at each stage. Hence, subgame perfect equilibria and backward induction equilibria coincide, and any backward induction equilibrium outcome is unique as long as the parties' preferences are strict. Regarding the Shortlisting Contest mechanism, the argument for uniquess, to be in the appendix, starts from the realization that any subgame perfect equilibrium outcome must be Pareto efficient. Otherwise, the party who proposed the subset from which the outcome is picked would have a profitable deviation, by changing its composition and avoid the election of the Pareto inefficient candidate. Given that fact, we suppose by contradiction that besides the equilibrium outcome described in Theorem 3 there was another one, and we prove that no strategy profile could sustain it.*

We hope that the next example and remark will clarify theorems 1, 2 and 3.

Example 1 *Consider two parties, 1 and 2, facing a set of five candidates and the following preferences:* $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ *and* $c2 \succ_2 c3 \succ_2 c1 \succ_2 c4 \succ_2 c5$.
First notice that the depth of this preference profile is $d^*(\succ_1, \succ_2) = 2$, *its mirrored depth is* $k^*(\succ_1, \succ_2) = 4$ *and its Unanimity Compromise Set is* $\mathbf{UC} = \{c2\}$. *Thus, according to theorems 1, 2 and 3, the unique subgame perfect equilibrium outcome of the games induced by our proposed methods is c2.*
Applying Theorem 1, the following strategy profile is a subgame perfect equilibrium of the Compromise Rule of k Names method: Party 1 proposes $k = k^*(\succ_1, \succ_2)$. *As for Party 2, at stage 2, its strategy is to opt to be the proposer unless* $k > k^*(\succ_1, \succ_2)$. *Once the parameters of the Rule of k Names are fully determined at stages 1 and 2, the parties play the stages 3 and 4 according to the subgame equilibrium of the game induced by this rule. Thus, at stage 3, Party 2 proposes* $\{c2, c3, c4, c5\}$ *and stage 4, Party 1 chooses* $c2$.
We now turn to the Alternate Shortlists method. Applying Theorem 2, the following strategy profile is a subgame perfect equilibrium: Party 1 proposes $\{c1, c2, c4, c5\}$ *and, at stage 3, it picks its preferred candidate out of Party 2's proposed subset, if called to do so. As for Party 2, at stage 2, its strategy is to pick its preferred candidate out of Party 1's proposed subset only if its cardinality is larger than three or it contains c2. Otherwise, it counteracts by proposing* $\{c2, c3, c4, c5\}$.
Finally, let us consider the the Shortlisting Contest method. Applying Theorem 3, the following strategy profile is a subgame perfect equilibrium: Party 1 proposes $\{c1, c2, c4, c5\}$ *at stage 1, and at the second stage it picks its preferred candidate out of Party 2's proposed subset, if called to do so. Party 2 proposes* $\{c2, c3, c4, c5\}$ *at the first stage, and at the second stage it picks its preferred alternative out of Party 1's proposed subset, if called to do so.*
In both previous strategy profiles, the equilibrium outcome is Party 1's preferred candidate in the Party 2's proposed subset, $\{c2, c3, c4, c5\}$, *which is* $c2$.

Remark 6 *If the Unanimity Compromise Set is a singleton then there always exists a subgame perfect strategy profile where Party 1 proposes a subset with cardinality equal to* $k^*(\succ_1, \succ_2) - 1$, *unless* $k^*(\succ_1, \succ_2) - 1 < \frac{\mathbf{c}}{2}$. *For instance, in the example above, the readers can check that if Party 1 proposes* $\{c1, c4, c5\}$ *instead of* $\{c1, c2, c4, c5\}$ *that strategy profile is still an equilibrium of the Shortlisting Contest and Alternate Shortlists mechanisms.*

Corollary 2 *Under the games induced by Compromise Rule of k Names, Shortlisting Contest and Alternate Shortlists methods, the subgame perfect equilibrium outcome is always Pareto efficient and maximizes the welfare of the worst-off party when each party's payoff from a candidate x is the cardinality of that party's lower contour set at x.*

2.3 An implementation point of view

Definition 4 *A social choice correspondence, $f : \mathbf{P} \times \mathbf{P} \rightarrow \mathbf{C}$, is weakly implementable if there is a mechanism such that, for any $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, the set of pure strategy Nash equilibrium outcomes is a subset of $f(\succ_1, \succ_2)$.*

The next proposition derives from Hurwicz and Schmeidler (1978) and Maskin (1999) prove that there exists no deterministic mechanism, except for dictatorship, guaranteeing that every Nash equilibrium is Pareto efficient. Given that any element in the Unanimity Compromise Set is Pareto efficient, this result implies there is no one shot deterministic mechanism that can weakly implement this set.

Proposition 3 *There exists no deterministic mechanism with less stages than those of the Shortlisting Contest that weakly implements the Unanimity Compromise Set.*

Definition 5 *A social choice correspondence, $f : \mathbf{P} \times \mathbf{P} \rightarrow \mathbf{C}$, is weakly implementable via backward induction if there exist an extensive form mechanism such that, for any $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, the set of backward induction Nash equilibrium outcomes associated with its extensive form game is a subset of $f(\succ_1, \succ_2)$.*

Our next result is a direct consequence of our Theorem 2 and de Clippel, Kfir, and Knight's (2014) Proposition 3. This proposition states that $f(\succ_1, \succ_2) \equiv \cup_{i \in \{1,2\}} \max\{x \in \mathbf{C} | \#\{y \in \mathbf{C} | x \succ_{j \neq i} y\} \geq \frac{\mathbf{c}-1}{2}\}$ is the unique social choice correspondence that is Pareto efficient, its outcome is never dominated by the parties' median choice alternatives and is weakly implementable by backward induction via a two-stage deterministic mechanism. Given that this social choice correspondence is not the Unanimity Compromise Set, this result implies there is no two-stage deterministic mechanism that can weakly implement by backward induction the Unanimity Compromise Set.

Proposition 4 *There exists no deterministic mechanism with less stages than those of the Alternate Shortlists method that weakly implements via backward induction the Unanimity Compromise Set.*

As we mentioned in the introduction, one could think of more complex mechanisms than ours where nature decides who plays the role of player 1 or 2, and then each of the games defined by our methods is used thereafter in one of the branches. Having in mind this possibility leads us to discuss the following notion of role robust implementation.

As a corollary of theorems 1,2 and 3, the sets of potential subgame perfect equilibrium outcomes of our methods coincide with the Unanimity Compromise Set. The following definition is a direct adaptation of role robust implementation by backward induction proposed by de Clippel, Eliaz, and Knight (2014).

Definition 6 *A social choice correspondence, $f : \mathbf{P} \times \mathbf{P} \rightarrow \mathbf{C}$, is role-robust implementable via subgame perfection if there exist a two-player extensive form mechanism such that, for each $(\succ_1, \succ_2) \in \mathbf{P} \times \mathbf{P}$, $f(\succ_1, \succ_2)$ coincides with the union of the two pure strategy subgame perfect equilibrium outcomes associated with the two extensive-form games obtained when assigning either Party 1 or Party 2 to the role of the first-mover or the priority in case of ties.*[6]

Corollary 3 *The Compromise Rule of k names, Alternate Shortlists and Shortlisting Contest methods role-robust implement via subgame perfection the Unanimity Compromise Set.*

3 Literature review

We now describe and compare six other methods that have been previously used in practice or discussed in the literature. The following three methods, the Veto-rank, the Approval and the Strike methods, involve only one stage. They rely on a uniform lottery after the parties have already taken their actions, and the possible outcomes of that lottery depend

[6] As a direct corollary of Anbarci's (1993) characterization, VAOV also role-robust implements by subgame perfection the Unanimity Compromise set. See Nunez and Sanver (2021) to some results on the subgame perfect implementability of the Unanimity Compromise set with more than two players.

on these actions. The Nash equilibrium outcomes of these methods depend on the parties' attitudes toward risk.

Veto-rank: the number of candidates needs to be odd. The parties simultaneously veto $\frac{\mathbf{c}-1}{2}$ candidates and rank the remaining $\frac{\mathbf{c}+1}{2}$ candidates. The selected candidate is the one with the minimal sum of ranks among those who have not been vetoed. Eventual ties are broken by means of a uniform lottery.

Approval Mechanism: the parties simultaneously choose non-empty subsets of candidates. If their intersection is not empty, then a uniform lottery shall select the candidate from it. Otherwise, that lottery shall select the candidate from the union of those sets.

Strike Mechanism: the parties simultaneously veto $\frac{\mathbf{c}-1}{2}$ candidates. A uniform lottery shall select the candidate from the set of unvetoed candidates.

The three methods that follow are deterministic, like ours. Therefore, the subgame perfect equilibrium outcomes of the games they play are independent of their attitudes toward risk.

Alternate Strikes: the parties alternatively veto one name from the set of candidates until only one option remains, and this is the chosen candidate.

Voting by Alternating Offers and Vetoes (VAOV): the parties take turns making offers until one candidate is accepted. Any offer that has been rejected is eliminated from further consideration. If no offer has been accepted before and only one candidate remains, that one becomes the chosen candidate.

Shortlisting: the number of candidates needs to be odd. One party chooses a subset containing $\frac{\mathbf{c}+1}{2}$ candidates, and the other party subsequently picks an candidate out of that subset.

We now summarize the different virtues and shortcomings that have been pointed at by the authors who contributed to the literature.

The veto-rank method satisfies two appealing properties whenever the parties act sincerely: the selected candidate is never below the parties' median ranking choices (if $\mathbf{c}$ is odd) and is Pareto efficient. However, if the parties act strategically, there may exist multiple equilibrium outcomes and some of them may be Pareto inefficient (see de Clippel, Eliaz, and Knight 2014 and Bloom and Cavanagh 1986).

The Approval Mechanism pure strategy Nash equilibria ensure that both parties obtain at least their average and median utility level in equilibrium. In addition, no equilibrium of the game can be Pareto dominated by an alternative as long as the players are partially honest. However, there may exist multiple equilibrium outcomes and some of them may be Pareto inefficient lotteries (see Núnez and Laslier 2015).

In order to circumvent Hurwicz and Schmeidler (1978) and Maskin (1999) impossibility result, Laslier, Núñez, and Sanver (2021) propose the Strike Mechanism that is deterministic-in-equilibrium while lotteries are allowed off-equilibrium. Under very mild conditions on preferences over lotteries, they show that it has multiple pure strategy Nash equilibrium outcomes such that all of them are Pareto efficient candidates and that, if $\mathbf{c}$ is odd, are never below the parties' median ranking choices. As far we know, it is the first one stage mechanism that has these properties.

Anbarci (1993) proposes VAOV and proves that its subgame perfect equilibrium outcome maximizes the welfare of the worst-off party, when each party's payoff from an alternative x is the cardinality of that party's lower contour set at x. This finding implies that this method weakly implements the Unanimity Compromise Set as pointed out by Erlich, Hazon, and Kraus (2018). These same authors provide an elegant new characterization of some subgame perfect equilibria of the game induced by VAOV.

In a later contribution, Anbarci (2006) characterizes the unique subgame perfect equilibrium outcome of each of the two extensive form games induced by Alternate Strikes. His characterization implies that it is Pareto efficient and that, if $\mathbf{c}$ is odd, is never below the parties' median ranking choices. He also shows that these two methods do not induce a generic first-mover advantage[7].

de Clippel, Eliaz, and Knight (2014) propose the use of Shortlisting, which is actually a specific Rule of k Names for $k = \frac{\mathbf{c}+1}{2}$. They argue in favor of the use of methods with a small number of stages and proved that Shortlisting was a minimizer in a precise sense. Under this method, once the first-mover is known, the subgame perfect equilibrium outcome is unique and coincides with the first-mover preferred alternative among its opponent's $\frac{\mathbf{c}+1}{2}$ top alternatives. Thus, it is Pareto efficient and is never below the parties' median ranking choices. They also show that this method induces a generic first-mover advantage.

[7] In a more general analysis, Moulin (1981) shows that Alternate Strikes belongs to a family of rules such that the equilibrium outcome under "prudent" strategies coincides with the outcome under sophisticated strategies.

Yet, Shortlisting, like Alternate Strikes, does not always induce the selection of a unanimity compromise candidate, as shown by the following example.

Example 2 *Consider two parties, 1 and 2, facing a set of seven candidates and the following preferences:* $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6 \succ_1 c7$ *and* $c7 \succ_2 c5 \succ_2 c3 \succ_2 c1 \succ_2 c2 \succ_2 c4 \succ_2 c6$.

Under Shortlisting, $\frac{\mathbf{c}+1}{2} = 4$*, if Party 1 selected its four best candidates, which are* $c1, c2, c3, c4$*, then Party 2 would choose* $c3$*. Knowing it, Party 1 selects* $c1, c2, c4$ *and* $c6$*, and then Party 2 chooses* $c1$ *as the winner. Under the Alternate Strikes scheme, no matter who moves first,* $c1$ *would also be the winner.*[8] *Under our three methods and VAOV,* $c3$*, the compromise candidate would be the winner.*

But one can argue in favor of Shortlisting and Alternate Strikes by pointing out that, when the preferences of parties are completely opposite, these methods always select the compromise candidate. Notice that on this subject de Clippel, Eliaz, and Knight (2014) wrote: "Fourth, it is reasonable to assume that the parties do not necessarily have completely opposed rankings of all arbitrators. This is because arbitrators differ in their fees, their expertise, their past rulings and their delays in reaching a decision" (p. 2).

4 Remarks on invariance property

Now let us check whether the different methods we have considered are robust to strategic manipulation by altering the set of candidates. This is relevant in a general analysis, since almost all existing methods are vulnerable to the strategic inclusion of unacceptable candidates in the pool of candidates as a way to favor a party. However, potential violations of this property are less bothersome in those cases where the size of the pool of candidates is fixed from the start. as remarked in the introduction, that is the case in many processes used to select arbitrators.

Definition 7 *We say that a mechanism satisfies invariance with respect to undesirable candidacies if and only if their equilibrium outcomes do not change if the set* $\boldsymbol{C}$ *expands in such a way that all parties consider all added candidates to be worse than those in* $\boldsymbol{C}$*.*

[8] Anbarci (2006) proves that the equilibrium outcome of the Alternate Strikes is Party 1's best candidate among those who survive a Simultaneous Naive elimination Algorithm, in which, at each stage, each party simultaneously eliminates its worst alternative among those that have not yet been eliminated.

The next result is also a direct corollary of theorems 1, 2 and 3.

Proposition 5 *The Compromise Rule of k Names and Alternate Shortlists methods satisfy invariance with respect to undesirable candidacies and Shortlisting Contest method does not.*

Our Shortlisting Contest method fails to satisfy this property because the priority of parties in case of a tie depends on the cardinality of the proposed sets of candidates, and the size of the equilibrium proposals, hence the priority, switches when a dominated candidate is added to the set of alternatives. a simple way to modify this mechanism so that it meets the invariance property would be to establish that in case of ties, the same party's proposed subset always prevails. There is at tradeoff here, because this change would reduce the symmetry of the mechanism.

The two methods proposed by Anbarci (2006)'s VAOV and de Clippel, Eliaz, and Knight (2014)'s Shortlisting method are not immune either to strategic manipulation of the set of candidates.

The following examples illustrate the comments our preceding comments.

Example 3 *Let us consider first VAOV, a set of three alternatives* $\{c1, c2, c3\}$ *and the following preference* $c1 \succ_1 c2 \succ_1 c3$ *and* $c2 \succ_2 c1 \succ_2 c3$. *It is easy to see that if Party 1 is the first-mover, in the first round, it offers c3 to Party 2 and is rejected. In the next round Party 2 offers c2 and is rejected. So, only c1 remains and it is the subgame perfect equilibrium outcome. For the same reason, candidate c2 is the outcome if Party 2 is the first-mover. Now let us add candidate c4 such that:* $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4$ *and* $c2 \succ_2 c1 \succ_2 c3 \succ_2 c4$. *Now, if Party 1 is the first-mover, candidate c2 is the subgame perfect equilibrium outcome. And if Party 2 is the first-mover, candidate c1 is the equilibrium outcome. Therefore, this method is not invariant with respect to undesirable candidates.*

Example 4 *Now let us consider Shortlisting and the following preference profile:* $c1 \succ_1 c2 \succ_1 c3$ *and* $c3 \succ_2 c2 \succ_2 c1$. *If Party 1 is the first-mover, in equilibrium, it chooses* $\{c2, c1\}$, *Party 2 selects its preferred candidate from it and c2 is outcome. Now, let us add two undesirable candidates c4 and c5: Party 1:* $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ *and Party 2:* $c3 \succ_2 c2 \succ_2 c1 \succ_2 c4 \succ_2 c5$. *Now, Party 1 chooses* $\{c1, c4, c5\}$, *Party 2 selects c1 to be the outcome.*

Other methods satisfy the property, though.

Proposition 6 *Alternate Strikes is invariant with respect to undesirable candidacies.*

This is a consequence of Anbarci's (2006) characterization of its subgame perfect equilibrium outcome: it is the first-mover preferred candidate among those that survive the Simultaneous Naive Elimination Algorithm (see footnote 8), as the addition of candidates worse than those in $\mathbf{C}$ does not affect the outcome of this algorithm.

Remark 7 *Anbarci (2006) considers a different invariance property. He says that a method satisfies invariance with respect to bad candidates if and only if, for any preference profile, its set of potential subgame perfect equilibrium outcomes of the game induced by that method does not change if the set* $\mathbf{C}$ *expands in such a way that all added alternatives are Pareto dominated by all elements of the set of potential equilibrium outcomes when only candidates in* $\mathbf{C}$ *were available. These two apparently related invariance conditions are in fact independent of each other. Anbarci (2006) proves that VAOV satisfies invariance with respect to bad alternatives and Alternate Strikes does not. Notice that Example 4, shows that Shortlisting method fails to satisfy both properties. Therefore, none of them satisfies simultaneously both properties, while all our methods do.*

5 Final remarks

As far we know, all the institutions that used the Rule of k Names adopt a fixed value of k for every decision. Recently de Clippel, Eliaz, and Knight (2014) and Barberà and Coelho (2017) proposed that the parameter k should be a function of the number of candidates. The first authors prescribe $k(\mathbf{c}) = \frac{\mathbf{c}+1}{2}$ to guarantee that the chosen alternative is never below the parties' median ranking choices. Barberà and Coelho (2017) adopt an ex ante point of view and recommend $k(\mathbf{c}) = \mathbf{c} + 2 - \sqrt[2]{(2\mathbf{c} + 2}$ to equalize the parties' expected utilities.

Here we claim that the use of Rules of k Names can become even more effective to achieve compromise if the parties involved were allowed to actively participate in the selection of the particular rule to be used under each potential solution of conflict. In particular, this would add flexibility to the choice of rules, contribute to the fairness of the choices under all potential degrees of disagreement among parties, and facilitate the acceptance of the rules by both parties.

We have exhibited the common properties of our three proposed mechanisms and shown that they are not simultaneously satisfied by any of the previously used or proposed procedures. The game induced by each of our mechanisms has a unique subgame perfect equilibrium outcome that is Pareto efficient and maximizes the welfare of the worst-off party. Our Compromise Rule of k Names and Alternate Shortlists methods are robust to strategic manipulation of the set of candidates, in contrast with the fact that almost all existing methods are vulnerable to the strategic inclusion of unacceptable candidates in the pool of candidates as a way of favoring a party. Our Shortlisting Contest method fails this test, but induces a more symmetric treatment of the parties, which can play in favor of its acceptance by them. The tradeoff between these two features should be examined in the light of particular applications of each rule.

Which one we would recommend to be used in practice? In the case of arbitration selection, we lean to recommend the use of Shortlisting Contest. First, because having few stages is a desirable feature of arbitration procedures, in order to avoid the possibility of delaying tactics by the respondent party, in an attempt to block the start of the arbitration. Second, this method is more symmetric, so there is no dispute over who plays as Party 1 or 2. One caveat could be that this mechanism is not invariant with respect to undesirable candidacies. However, this property is not relevant when the selection of arbitrators is supervised by arbitral institutions, because, in general, they establish not only the method but also the cardinality of the set of candidates before the disputes arise.

On the other hand, the Compromise Rule of k Names seems to us the easiest for the players to understand and play. However, it is an empirical matter that we have not explored and certainly deserves a future research involving laboratory experiments.

As for future research on arbitration selection, it would be also relevant to extend our mechanisms to cover cases of more complex situations involving several parties, the choice of panels, rather than single arbitrators or the possibility of delaying tactics from the respondent party to block the start of the arbitration, and to find ways to avoid their uses (see Hosang 2014). Another promising direction would be to model how different methods of selection can affect the behavior of the arbitrators, given that the parties may form preferences over them based on previous decisions of these arbitrators in other cases (see Bloom and Cavanagh 1986).

Although we explicitly concentrate on the appointment of arbitrators, our methods are well suited for many other cases where people must be appointed and more than one party is concerned by the outcome (in addition to other situations where compromise must be sought). Since each case will be somewhat different, we think that the range of application of our methods, even if we restrict attention to appointment problems, is wide. What we have tried is to be more explicit about how our initial analysis points at different aspects that an analyst should pay attention to when focusing on a particular problem. Specifically, we emphasize that the flexibility that our methods provide is important when parties in conflict have nonetheless some level of agreement. And we also discuss when it is that violation of our invariance condition is problematic, and when it is not, depending on the applications.

6 References

Anbarci, N. "Noncooperative Foundations of the Area Monotonic Solution." *Quarterly Journal of Economics*, 108 (1993): 245-258.

Anbarci, N. "Finite Alternating-move Arbitration Schemes and the Equal Area Solution." *Theory and Decision*, 61 (2006), 21-50.

Barberà, S., and Coelho, D. "How to Choose a Non-controversial List with k Names." *Social Choice and Welfare*, 31 (2010),79–96.

Barberà, S. and Coelho, D. "On the Rule of k Names." *Games and Economic Behavior*, 70 (2010), 44-61.

Barberà, S. and Coelho, D. "Balancing the Power to Appoint Officers." *Games and Economic Behavior*, 101 (2017), 189-203.

Barberà, S. and Coelho, D. "On the Advantages and Disadvantages of Being the First Mover under Rules of k Names." *International Journal of Economic Theory*, 14 (2018), 51-60.

Bloom, D. E. and Cavanagh, C. L. "An Analysis of the Selection of Arbitrators." *American Economic Review*, 76 (1986), 408-422.

Brams, S. and Kilgour, DM. "Fallback Bargaining." *Group Decision Negotiation*, 10 (2001), 287–316.

de Clippel, G. and Eliaz, F. "Reason Based Choice: a Bargaining Rationale for the Attraction and Compromise Effects." *Theoretical Economics*, 7 (2012),125-162

de Clippel, G., Eliaz, F. and Knight, B. "On the Selection of Arbitrators." *American Economic Review*, 104 (2014), 3434–3458.

Erlich, S., Hazon, N. and Kraus, S."Negotiation Strategies for Agents with Ordinal Preferences." ArXiv e-prints arXiv:1805.00913, 2018.

Hosang, A. *Obstructionist Behavior in International Commercial Arbitration : Legal Analysis and Measures Available to the Arbitral Tribunal.* Utrecht: Eleven International Publishing, 2014.

Hurwicz, L. and Sertel, M. R. "Designing Mechanisms, in Particular for Electoral Systems: the Majoritarian Compromise." Department of Economics, Bogaziçi University, Istanbul (preprint), 1997.

Hurwicz, L. and Schmeidler, D. "Construction of Outcome Functions Guaranteeing Existence and Pareto Optimality of Nash equilibria." *Econometrica*, 46 (1978), 1447–1474.

Kibris, O. and Sertel, M. R., "Bargaining over a Finite Set of Alternatives." *Social Choice and Welfare*, 28 (2007), 421-437.

Laslier, F., Númez, M. and Sanver, R."A solution to the Two-person Implementation Problem." Journal of Economic Theory, 194 (2021), 105-261.

Maskin, E. "Nash equilibrium and Welfare Optimality." *Review of Economic Studies*, 66 (1999), 23–38.

Moulin, H. "Prudence Versus Sophistication in Voting Strategy." *Journal of Economic Theory*, 24 (1981), 398-412.

Númez, M. and Laslier, F. "Bargaining Through Approval." *Journal of Mathematical Economics*, 60 (2015), 63-73.

Númez, M. and Sanver, R. "The Subgame Perfect Implementability of Voting Rules." *Social Choice and Welfare*, 56 (2021), 421–441.

Shavell, S. "Alternative Dispute Resolution: An Economic Analysis." The Journal of Legal Studies, 24 (1995), 1-28

Sprumont, Y. "Intermediate Preferences and Rawlsian Arbitration Rules." *Social Choice and Welfare*, 10 (1993), 1–15.

APPENDIX

Proof of Proposition 2. Suppose that $k^*(\succ_1, \succ_2) \geq k'$. Denote by x the subgame equilibrium outcome when Party i is the proposer under Rule of $k^*(\succ_1, \succ_2)$ Names. Denote by y the subgame equilibrium outcome when Party i is the chooser under Rule of k' Names. Suppose by contradiction that $y \succ_i x$. By definition of $k^*(\succ_1, \succ_2)$, x is among Party i's $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ top candidates. Since $k^*(\succ_1, \succ_2) \geq k'$, it implies that x is also among Party i's $(\mathbf{c} - k' + 1)$ top candidates. Thus, $y \succ_j x$, it follows because by Proposition 1, y is Party j's preferred candidate among Party i's $(\mathbf{c} - k' + 1)$ top candidates. Therefore, we have that $y \succ_i x$ and $y \succ_j x$. Notice that it implies that x cannot be a subgame perfect equilibrium outcome under rule of Rule of $k^*(\succ_1, \succ_2)$ Names. This is a contradiction.

Now, suppose that $k' > k^*(\succ_1, \succ_2)$. Denote by x the subgame equilibrium outcome when Party i is the chooser under Rule of $k^*(\succ_1, \succ_2)$ Names. Denote by y the subgame equilibrium outcome when Party i is the proposer under Rule of k' Names. Suppose by contradiction that $y \succeq_i x$. By Proposition 1, y is among Party j's $(\mathbf{c} - k' + 1)$ top candidates. Since $k' > k^*(\succ_1, \succ_2)$, it implies that y is also among Party j's $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ top candidates. Notice also by Proposition 1 that x is among Party i's $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ top candidates. Since $y \succeq_i x$, it implies that y is also among Party i's $(\mathbf{c} - k^*(\succ_1, \succ_2) + 1)$ top candidates. Notice that these facts imply that x and y belong to the Unanimity Compromise Set and, by Corollary 1, we have that Party j weakly prefers x to y. Given that $y \succeq_i x$ and $x \succeq_j y$, by Remark 1, it implies that $\#\{a \in \mathbf{C} | y \succ_j a\} = k^*(\succ_1, \succ_2) - 1$. Notice that $k^*(\succ_1, \succ_2) \geq \mathbf{c} - k^*(\succ_1, \succ_2) + 1$ (due Remark 2) and $k' > k^*(\succ_1, \succ_2)$ imply that $k^*(\succ_1, \succ_2) > \mathbf{c} - k' + 1$. Thus, $\#\{a \in \mathbf{C} | y \succ_j a\} = k^*(\succ_1, \succ_2) - 1$ implies that y does not belong to Party j's $(\mathbf{c} - k' + 1)$ top candidates. This is a contradiction. ■

Proof of Theorem 1. Denote by x the subgame perfect equilibrium outcome of the strategy profile described in Theorem 1. First notice that given the first-mover's choice $k^*(\succ_1, \succ_2)$ and Corollary 1, x is the proposer preferred candidate in the Unanimity Compromise Set and, at the fourth stage, the chooser picks it out of the subset proposed by the proposer.

First let us prove that the strategy profile stated in Theorem 1 is a subgame perfect equilibrium. Notice that the strategies adopted in the second, third and fourth stages are direct consequences of propositions 1 and 2.

Now, let us prove that Party 1 does not have a profitable deviation. Given Party 2's strategy, it is enough to consider only $k^{'} > k^{*}$. If $k^{'} > k^{*}(\succ_1, \succ_2)$, Party 1 will become the proposer. It follows by Proposition 2 that it would be not a profitable deviation. Therefore, our initial strategy profile is a subgame perfect equilibrium.

Now let us argue that the equilibrium outcome x is unique. Under this mechanism, only one player moves at each stage. Hence, subgame perfect equilibria and backward induction equilibria coincide, and any backward induction equilibrium outcome is unique as long as the parties' preferences are strict. ■

Proof of Theorem 2. Given the characterization of the strategy profile described in Theorem 2, let us show that Party 2 opts to not counter-offer and the winning candidate is Party 1's preferred candidate in the Unanimity Compromise Set. Denote by x this candidate. First note that, according to Party 2's strategy, it picks its preferred candidate out of the proposed subset only if this set contains at least one candidate weakly preferred to its preferred candidate among the first-mover's $(\mathbf{c} - \#|\mathbf{S}|)$ top candidates. Suppose that Party 2 does not counter-offer. It follows by Corollary 1 that the winning candidate will be x. We need to prove that x is weakly preferred to its preferred candidate among Party 1's $(\mathbf{c} - \ k^{*}(\succ_1, \succ_2))$ top candidates. This condition is satisfied for the following reason: by definition, $k^{*}(\succ_1, \succ_2) \equiv \mathbf{c} - d^{*}(\succ_1, \succ_2) + 1$, so $\mathbf{c} - k^{*}(\succ_1, \succ_2) = d^{*}(\succ_1, \succ_2) - 1$. Thus, in case that that condition was not satisfied then it would imply that the intersection between the parties' $d^{*}(\succ_1, \succ_2) - 1$-top candidates is not empty which contradicts the definition of $d^{*}(\succ_1, \succ_2)$.

Now let us prove that the strategy profile stated in Theorem 2 is a subgame perfect equilibrium. We use propositions 1 and 2. Suppose by contradiction that there is a profitable deviation. This means that if Party 2 counteroffers with a subset of cardinality equal to $k^{*}(\succ_1, \succ_2) + 1$, the winning candidate is preferred to x according to that Party 2's preferences. Proposition 2 states that when $k = k^{*}(\succ_1, \succ_2) + 1$, it is better to be the chooser. So, this winning candidate is also preferred to x according to the first-mover preferences. Hence, x is a Pareto inefficient candidate, a contradiction because any element in the Unanimity Compromise Set is Pareto efficient.

Now, let us prove that there is no profitable deviation to the first-mover. Denote by $\mathbf{Z}$ the subset proposed by the first-mover. We need to prove there is no subset $\mathbf{S} \subset \mathbf{C} \backslash \mathbf{Z}$, such that $\#|\mathbf{S}| \geq (\frac{\mathbf{c}}{2})$ and $\mathbf{S} \neq \mathbf{Z}$, that would make Party 1 better off by choosing $\mathbf{S}$ instead of

$\mathbf{Z}$, when Party 2's strategy remains unchanged. Proposition 1 implies that it is enough to consider only deviations with subsets such that $\mathbf{S} \subset \mathbf{C}$ with $\#|\mathbf{S}| \geq (\frac{\mathbf{c}}{2})$, containing Party 1's preferred candidate among Party 2's $(\mathbf{c} - \#|\mathbf{S}| + 1)$ top candidates plus the second-mover*'s* $\#|\mathbf{S}| - 1$ worst candidates.

Given the rules of our method and Party 2's strategy, if Party 1 deviates by choosing a subset with cardinality $k^{'} < k^{*}(\succ_1, \succ_2)$, it will become the chooser, because Party 2 will propose a subset with cardinality $k^{'} + 1$. Finally, if $k^{'} > k^{*}(\succ_1, \succ_2)$, Party 2 will choose the winning candidate from this subset. It follows by Proposition 2 that none of these two possible types of deviations would be profitable. Therefore, our initial strategy profile is a subgame perfect equilibrium.

Now let us argue that the equilibrium outcome x is unique. Under this mechanism, only one player moves at each stage. Hence, subgame perfect equilibria and backward induction equilibria coincide, and any backward induction equilibrium outcome is unique as long as the parties' preferences are strict. ■

Proof of Theorem 3. Without loss of generality suppose that $k^{*}(\succ_1, \succ_2)$ is odd. First let us prove that the strategy profile stated in Theorem 3 is a subgame perfect equilibrium. Denote by x its outcome and by $\mathbf{Z}^{i}$ the subset proposed by Party $i \in \{1, 2\}$ under this strategy profile. First notice that given $k^{*}(\succ_1, \succ_2) = \mathbf{c} - d^{*}(\succ_1, \succ_2) + 1$ and Corollary 1, x is Party 1's best candidate in the Unanimity Compromise Set.

We need only to prove for each $i' \in \{1, 2\}$ there exists no subset $\mathbf{S} \subset \mathbf{C} \backslash \mathbf{Z}^{i'}$, such that $\#|\mathbf{S}| \geq (\frac{\mathbf{c}}{2})$ and $S \neq \mathbf{Z}^{i'}$, that would make Party i' better off by choosing S instead of $\mathbf{Z}^{i'}$, while the other player's strategy remains unchanged. Proposition 1 implies that it is enough to consider only deviations with subsets S such that: $\mathbf{S} \subset \mathbf{C}$,with $\#|\mathbf{S}| \geq (\frac{\mathbf{c}}{2})$, that contains Party $i' \in \{1, 2\}$ preferred candidate among the opposing party's $(\mathbf{c} - \#|\mathbf{S}| + 1)$ top candidates plus the opposing party's $\#|\mathbf{S}| - 1$ worst candidates.

Given the rules of the mechanism and the other player strategy, if Party i deviates by choosing a subset with cardinality smaller than $k^{*}(\succ_1, \succ_2)$, it will pick the winning candidate out of the subset proposed by its opponent. And if it deviates by choosing a subset with cardinality higher than $k^{*}(\succ_1, \succ_2)$, its opponent will pick the winning candidate out of its subset. It follows from Proposition 2 that neither of these two possible types of deviations would be profitable.

Having proved that our proposed strategy profile is a subgame perfect equilibrium, let us show that x is the unique subgame perfect equilibrium outcome of the game. Note that any subgame perfect equilibrium outcome of this method needs to be Pareto efficient. Otherwise, the party who proposed the subset from which the outcome is picked would have a profitable deviation, by changing its composition and avoid the election of the Pareto inefficient candidate. Given that fact, we suppose by contradiction that besides the equilibrium outcome described in Theorem 3 there was another one. Now, we will prove that no strategy profile could sustain it.

Let us denote by **SX** the strategy profile described in Theorem 3 that sustains x as an equilibrium outcome. Suppose by contradiction that x is not unique. Let $y \neq x$ be another subgame perfect equilibrium outcome. Denote by **SY** the strategy profile that sustains y as a subgame perfect equilibrium outcome and by k' the cardinality of the subset from which one of the parties picks y on its equilibrium path. Suppose that $k' < k^*(\succ_1, \succ_2)$. It implies that the cardinality of the other subset proposed by the opponent is equal or smaller than k'. Given that x and y are Pareto efficient, denote by i the party that prefers x to y and by j the party that prefers y to x. Notice that Party i would have a profitable deviation by proposing a subset with cardinality $k^*(\succ_1, \succ_2)$, because x would be elected from this subset. So, we reach a contradiction. If $k' > k^*(\succ_1, \succ_2)$, there is also a contradiction because **SX** would not be a subgame perfect equilibrium given that Party j would have a profitable deviation by proposing k' instead of $k^*(\succ_1, \succ_2)$. Finally, given **SX**, if $k' = k^*(\succ_1, \succ_2)$, it implies that y was chosen from the subset proposed by Party 2, the one whose proposed subset does not prevail in case of ties. In addition, it implies that Party 2 prefers y to x and Party 1 prefers x to y. This is a contradiction because Party 1 would have incentive to deviate by proposing a subset with $k^*(\succ_1, \succ_2)$ to induce the election of x. ■