Banking and Inside Money: Revisiting the Efficiency of Deposit Contracts

BSE Working Paper 1265

June 2021 (Revised: February 2022)

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Banking and Inside Money: Revisiting the Efficiency of Deposit Contracts*

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First version: June 29, 2021
This version: February 13, 2022

Abstract

In this paper we show that nominal demand deposits are not, in general, Pareto optimal contracts. We construct a variation of the Diamond-Dybvig model where bank intermediation is done through inside money. In this setting, we show that the interplay between non-contingent deposit contracts and price flexibility is not a sufficient mechanism to provide efficient risk-sharing. Furthermore, state-contingent deposit contracts do not expand the consumption possibility set to include the efficient allocation and could even be inferior to other market arrangements. Finally, we discuss the extent to which central banks can improve the banking allocation through their monetary policy and regulation.

JEL classification: G21, E42.
Keywords: deposit contracts; risk-sharing; money creation; state contingencies.

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*We thank Javier Fernández-Blanco, Luis Rojas and Jesús Vázquez for valuable comments on previous versions of this paper. Rodríguez Mendizábal would like to acknowledge the financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S) and through grant PID2019-108144GB-100 as well as that of the Generalitat de Catalunya through Grant 2017 SGR 1571. All errors are our responsibility.

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1 Introduction

A prevalent conclusion in the modern banking literature spurred by the seminal paper of Diamond and Dybvig [1983] (DD, henceforth) is the capacity banks have to write contracts that achieve an efficient allocation of real resources. According to this literature, maturity transformation is one of the fundamental reasons for banking institutions to exist. By engaging in the transformation service of converting illiquid assets into liquid liabilities, banks effectively pool idiosyncratic liquidity risks and deliver state-contingent payoffs to depositors which reproduce the first-best allocation.[1]

In this paper we show that, with unobservable liquidity shocks, the efficiency attributed to deposit contracts is, in general, misleading. The reason for this is that, once the model is written in nominal terms, it is not possible to have nominal deposits producing contingent real payoffs. Furthermore, even if we allow banks to write deposit contracts with contingent nominal rates, the first-best lays outside the consumption possibility set and is not achievable in our decentralized economy. In fact, other market arrangements such as bilateral trades could be superior in terms of welfare as compared with intermediation done by banks.

To address these issues, we build a nominal economy with just one deviation with respect to the real structure in the DD model. In particular, we assume production takes place by combining labor supplied by workers with technology owned by entrepreneurs. This deviation does not affect the characterization of the efficient allocation but provides a potential motive for money to be valued endogenously. In other words, workers and entrepreneurs demand inside money at some point because it is to be used as means of payment to trade the resources they supply and demand, namely, labor and consumption goods.

Unlike the existing literature on banking, at the heart of our theory is the idea that banks do not make direct decisions on real investments or their liquidation. Instead, banks issue nominal demandable deposits to smooth the consumption of agents and hedge their future liquidity needs. This intermediation function is what provides a rationale for the existence of banks in our model. It is only through the pricing of this intermediation process that banks can indirectly influence decisions taken by the real sector of the economy. However, for nominal deposits to be valued in equilibrium, they must provide with the same real return as other

[1] Although we focus on the role of financial intermediaries as insurance providers against liquidity shocks, alternative explanations on the specialness of commercial banks include the arguments that banks act as delegated monitors (Diamond [1984]), have some degree of market power in lending (Fama [1985]), have a superior ability to issue safe money-like liabilities (Gorton and Pennacchi [1990]), provide liquidity in the form of loan commitments (Holmstrom and Tirole [1998]), benefit from synergies between deposit-taking and lending operations (Kashyap et al. [2002]), have a competitive advantage at holding fixed-income assets with little risk (Hanson et al. [2015]), or take advantage of a deposit franchise to get market power (Drechsler et al. [2021]), among others.

existing assets which in our model corresponds to the production technology of entrepreneurs. It is this condition what prevents nominal deposits to support the efficient allocation of real resources. Introducing the possibility of state-contingent deposit rates does not help since, although it could drive demands in the right direction, the supply of goods are also affected by those rates but not in the direction of the efficient allocation. Unlike DD, in which banks take real decisions and internalize these effects, in a competitive equilibrium with nominal contracting this is not possible and nominal prices do not provide with the real contingency needed.

Methodologically, our main contribution is to provide a relatively simple model which includes realistic features of banks in modern economies. We then show how incorporating nominal contracting, with minimal modifications to the DD model so as to motivate a nominal environment with inside money, can overturn the efficiency result found in the existing banking literature. As a byproduct, in our setting the private banking sector services the demand for liquidity by issuing inside money without the need for a central bank. We then ask what elements the model would need to incorporate a central bank or the interbank market. With this exercise, we show the restrictions a monetary authority faces when trying to influence real allocations through its monetary policy and regulation.

The reminder of the paper proceeds as follows. The next section reviews the literature and compares it with our setup. In section 3 we develop the real environment and show how our model is equivalent to the model in DD. Section 4 introduces the nominal version of the model by incorporating banks and inside money. Section 5 solves the nominal model and presents the main results. Section 6 modifies the model in several directions to explore whether it is possible to reach efficiency. In particular, we include (i) state-contingent deposit contracts, and (ii) the possibility to liquidate loans before maturity. Neither of these modifications can improve on the allocation provided by banks. Furthermore, this section also elaborates on the conditions needed for a monetary authority to affect real outcomes. Finally, section 7 concludes.

2 Discussion and related literature

Following the seminal work of DD, the literature on banking has generally assumed that banks design real contracts specified in terms of consumption goods. According to this theory, banks pool resources to invest in projects with different maturities by issuing demandable deposits. This simple financial intermediation chain allows banks to provide liquidity insurance to risk averse consumers under the commitment that deposits can be withdrawn when required to meet liquidity needs. In this manner, banks decide on the investment mix between short-term and long-term projects to satisfy the aggregate consumption demands of their depositors, just as the social planner would do. The main result of DD is that the implementation of the efficient allocation comes at the cost of financial fragility. Although by pooling deposits and investments banks improve the market allocation as they facilitate risk-sharing across consumers, the banking system is prone to self-fulfilling runs when a significant mass of depositors coordinate to withdraw at the same time because of the fear of being rationed in the future. To meet the commitment of servicing this excess of deposit withdrawals, the long-term investment must be "called" before maturity and, therefore, inefficiently liquidated.

An exception in this literature on real deposits is Fahri et al. [2009]. These authors show that competitive equilibria is inefficient in a DD model with unobservable trades so that agents can borrow and lend among themselves on a private market. This information friction tightens incentive compatibility constraints and reduces risk-sharing. In such setup, a liquidity floor requiring financial intermediaries to hold a minimal share of their portfolio in short-term assets implements the constrained efficient allocation. This liquidity floor is reminiscent of the reserve requirements imposed on banks by financial regulators.

Notwithstanding its numerous contributions in the analysis of financial intermediaries and their regulation, a misleading element in this literature on real deposit contracts is the consideration of banking as a real intermediation activity. The implicit idea is that financial liquidity, namely, the easiness in which a financial asset may be exchanged for goods and services, can be approximated with technological liquidity, a concept which has to do with the degree of irreversibility of real investments. In practice, however, commercial banks provide financial liquidity to private customers through the issuance of deposits that are written in nominal terms. The liquidity of these bank liabilities is related with the perception depositors have about the acceptance of those deposits as payment means in future trades and not necessarily with the degree of irreversibility of the investments these deposits fund.

The observation that in reality deposit contracts are denominated in terms of money may have significant implications that are overlooked by real models because of the treatment of money as a veil. In this sense, a growing body of literature on nominal-demand deposit contracts has been developed recently to shed light on the consequences of introducing money into models à la DD that formalize the role of banks as liquidity insurance providers. A common result found in this strand of the literature is that nominal deposit contracts eliminate the suboptimal allocation that real deposits support. For these authors (see Allen and Gale [1998], Skeie [2008] or Allen et al. [2014], among others), it is price variations what hedge banks against expectational bank runs. The fundamentals behind this price mechanism rely on the fact that excessive withdrawals rise the demand for goods and, consequently, the underlying increase on nominal prices makes the real value of deposits to fall, which discourages depositors to withdraw before time. The stability of nominal deposit contracts shows that the maturity mismatch incurred by banking institutions is not enough to explain liquidity runs, so further financial frictions are required.[2] Furthermore, in Rivero and Rodríguez Mendizábal [2019] we shed light on the equivalence between deposit insurance and the lender of last resort function of central banks, which supports the fundamental view of bank runs and stresses the importance of lender of last resort policies to solve expectational bank panics.[3]

A second element of discussion in this strand of the banking literature, and the object studied in this paper, deals with the concept of *efficiency*. The main argument used so far to justify the optimality of nominal deposit contracts is that the price mechanism in the market for goods, apart from eliminating the suboptimal equilibria associated with a bank run, also facilitates efficient risk-sharing across depositors. At the heart of this efficiency result, however, there is the need for a central bank to adjust equilibrium prices. Allen and Gale [1998] state that fluctuations in the price level allow nominal debt to become effectively state contingent if the central bank regulates the price level so that, in each state, the real value of deposits equals the first-best allocation. Skeie [2008] argues that nominal deposit contracts are Pareto superior over real contracts since they not only offset pure liquidity-driven runs but also allow banks to ensure the optimal amount of real liquidity in the economy. To achieve efficiency, though, deposits are backed by fiat money issued on a first instance by the central bank, and banks need to call part of financial positions with private debtors to satisfy the payment orders from depositors. In an environment with aggregate return risk and liquidity shocks, Allen et al. [2014] also show that non-contingent nominal deposit contracts lead to first-best efficiency when there is a passive monetary policy that accommodates the demand for liquidity from banks to service withdrawals on an intraday basis. In such a case, the price level is proportional to the money supply provided by the central bank, so this price level can be adjusted to provide risk-sharing. Andolfatto et al. [2020] use a new monetarist framework wherein nominal deposit contracts facilitate both efficient liquidity insurance and prevent bank runs when combined with a central bank lender of last resort facility. Another example is Schilling et al. [2020] who take a step further to prove a trilemma where efficiency can be achieved at the cost of the stability in prices or financial markets (runs) when the central bank acts as an intermediary providing an alternative to demand-deposits offered by private banks. Despite the supply for money is decided *ex-ante*, the central bank still controls

[2] Shin [2009] provides reflections on the fundamentals required to explain bank instability.

[3] For a formal discussion on the contrast between panics and fundamental based runs, see Jacklin and Bhattacharya [1988].

nominal prices since it decides the *ex-post* liquidation of the real investment, which ensures the implementation of the optimal allocation.

In sharp contrast with the previous literature on banking with nominal contracts and money, we argue that price adjustments in the market for goods in response to withdrawal variations, by themselves, are not a sufficient condition for the efficiency of non-contingent nominal deposit contracts. A common misconception in the existing theoretical analysis is the characterization of nominal deposits as contracts written in terms of outside money. In other words, it is outside money the very object banks intermediate between borrowers and lenders. This modeling decision imposes a structure that is at odds with current monetary systems we find in actual economies. On the one hand, in these models, central banks are required in order to implement the competitive equilibrium so as to provide the economy with the outside money needed to be exchanged for goods. In reality, though, outside money does not have a direct effect on private real decisions and can only indirectly affect them through possible impacts on the pricing and production of inside money. On the other hand, in these models, by using outside money to buy goods, central banks have automatically the capacity to affect real equilibrium outcomes by engineering changes in the nominal price level. However, in reality, central banks do not engage in production and investment decisions, at least, in a magnitude large enough to affect agregate allocations.

In modern economies, nominal deposits are bank liabilities created *ex-nihilo* on the spot when loans are provided and not outside money introduced in the banking system.[4] The production of deposits in the provision of lending is what allows the borrower to solve its financial problem (the lack of liquid assets to make a purchase) and is based on the convention that these liabilities are generally acceptable means of payment. The credibility that these private assets can be used to settle accounts between third parties is what makes deposits liquid. Once created, these deposits then circulate in the economy as their owners pay for goods, services and other assets themselves. Inside money created in such a manner only extinguishes when it is used to pay back the loans that generated them in the first place, when they are exchanged for another bank liability such as bank debt, when they are exchanged for a bank asset such as outside money (cash) or a security previously owned by the bank, or when they are written off because of the liquidation of the originating bank.

Of course, outside money, in the form of cash and central bank reserves, circulates alongside inside money. But this outside money is not a necessary precondition for deposit issuance. The creation of nominal deposits exposes banks inherently to liquidity risk, which materializes

[4] Tobin [1963] provides the foundations on the hypothesis of commercial banks as money creator institutions. For a recent policy discussion on the basis of money creation in the current financial system and its link to loan origination see McLeay et al. [2014] and Jordan [2018].

when the liabilities created are transferred to a different banking institution or withdrawn in the form of cash. To settle interbank transactions or to convert deposits into cash, any bank needs to hold central bank reserves. Deposit creation may also induce a demand for reserves in monetary systems with reserve requirements. But these requirements are usually lagged with respect to the deposits they relate to. Therefore, while outside money might be required either *ex-post* to clear payment transactions or to satisfy reserve requirements, or *ex-ante* for precautionary reasons to prevent liquidity problems, it is the provision of deposits what causes the demand for outside money and not the other way around.[5]

In our model, as in reality, inside money is created in the form of deposits when loans are granted to borrowers. In particular, when a loan is given to an entrepreneur, the bank produces deposits so that the entrepreneur pays a worker, obtains the labor, and starts producing. Because these deposits can be used at any time in the future to purchase consumption goods, they serve as a hedge against liquidity preference shocks. However, as price takers, banks have no power to enhance the contingency of deposit contracts to improve liquidity risk-sharing among agents who need to consume at different random times. As compared with Fahri et al. [2009], our model also implies deposit contracts are inefficient but without the presence of further frictions. In other words, unlike Fahri et al. [2009], it is the set of possible trades achievable using money what prevents efficiency to be reached and not additional restrictions on existing trades.

For reasons that will become clear below, there is no need for a central bank to operate in our economy. Private banks are autonomous to provide agents with enough means of payments for them to trade with each other and, through the pricing of deposits and loans, effectively influence investment and liquidation decisions of their customers. In section 6 we discuss how to modify the model to incorporate a central bank that mimics these institutions in modern economies and the extent to which, within that role, central banks have the ability to move the economy to the first best. The general conclusion will be that efficiency is out of reach also for central banks.

3 The real model

The real model is based on DD with just one deviation, namely, output is produced by combining labor supplied by workers with technology owned by entrepreneurs. As we show below, this departure from the original DD model does not change the efficient allocation but introduces a potential motive for agents to hold inside money.

[5] See Carpenter and Demiralp [2012]. These authors show that deposits Granger cause reserves and not the other way around.

There are three periods indexed by $t = 0, 1, 2$. The economy is populated by a continuum of risk-averse agents with measure 2. Half of the agents, with measure 1, are workers while the other half, also with measure 1, are entrepreneurs. Workers are endowed with a unit of time which they inelastically supply at date 0. Entrepreneurs, on the other hand, have access to a perfectly divisible and risk-free production technology using labor as of period 0. This technology transforms each unit of labor employed at date 0 into $2R > 2$ units of goods as of period 2. If liquidated prematurely at $t = 1$, the long technology produces a scrap value of 2. Because the total amount of agents in the economy is 2, in per-capita terms, the long technology produces 1 at $t = 1$ and $R > 1$ at $t = 2$ just as in the original DD model. All agents have also access to storing at $t = 1$. Notice the fact that it takes two periods for the investment to provide a positive net return is what defines *technological illiquidity* in the DD model.

All agents face uncertainty at date 0 about their future consumption preferences. Let $\tau = \{1, 2\}$ denote the individual state that shows the timing of consumption for any agent. In the aggregate, a fraction 2λ of agents will be impatient ($\tau = 1$) and consume at date 1, while the remaining fraction $2(1 - \lambda)$ will be patient ($\tau = 2$) and value consumption exclusively at date 2. As in DD, we assume that types are privately observable but λ is common knowledge. All uncertainty is resolved at period $t = 1$ before consumption takes place. Independently of the timing of consumption, agents value consumption according to the utility function $u(c)$ being twice continuously differentiable, strictly concave, satisfying the Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$ together with

$$-\frac{cu''(c)}{u'(c)} > 1$$

everywhere.

3.1 The efficient allocation

Because there is no aggregate risk, a planner, treating all agents symmetrically, would use all available labor, equal to 1, in the production technology. The planner decides the fraction to be liquidated at $t = 1$, x, together with the per capita consumption at $t = 1$, c_1, and $t = 2$, c_2, to maximize total utility across agents

$$2\lambda u(c_1) + 2(1 - \lambda)u(c_2)$$

subject to the resource constraints

$$2\lambda c_1 \leq 2x, \tag{1}$$

$$2(1-\lambda)c_2 = 2(1-x)R + 2x - 2\lambda c_1, \tag{2}$$

and

$$0 \leq x \leq 1, \tag{3}$$

together with the incentive compatibility constraint

$$c_1 \leq c_2. \tag{4}$$

Expression (1) restricts the amount of goods provided to impatient agents to be below what is available at $t = 1$, namely, output from the liquidation of the technology. Similarly, in expression (2), whatever is consumed at $t = 2$ cannot exceed output of the production technology plus goods stored from $t = 1$. Furthermore, expression (3) restricts liquidation to be below the total amount invested in the production technology. Finally, expression (4) restricts the planner to make sure the allocation is incentive compatible so that patient agents do not pretend to be impatient ones.

Although the problem allows for storing, it will never be optimal for the planner to store goods from $t = 1$ to $t = 2$. With this in mind, the efficient allocation satisfies

$$c_1^* = \frac{x^*}{\lambda}, \tag{5}$$

$$c_2^* = \left(\frac{1-x^*}{1-\lambda}\right)R, \tag{6}$$

where x^* is obtained from

$$u'(c_1^*) = Ru'(c_2^*). \tag{7}$$

That is, the efficient allocation must equal the ratio of marginal utilities to the marginal return on the productive investment, R. Because the function $u(c)$ is assumed to have a coefficient of relative risk aversion larger than 1, the efficient solution implies $x^* > \lambda$, leading to $1 < c_1^* < c_2^* < R$. Thus, the incentive compatibility constraint is satisfied. This allocation corresponds to the one in the original model in DD. Efficiency then calls for providing insurance by liquidating the long technology at a rate larger than λ in order to reduce the distance between consumption of patient and impatient agents, so that the ratio between c_1^* and c_2^* fulfills

$$1 < \frac{c_2^*}{c_1^*} < R. \tag{8}$$

This expression will be used below to evaluate the extend to which the nominal model can reach efficiency.

3.2 Real banks

We could think how to decentralize production and consumption decisions. One possible arrangement could be, as in DD, to have real "financial" institutions intermediating between workers and entrepreneurs. Agents "deposit" their endowments of time and technologies while these banks organize production and consumption, meaning they decide how to allocate labor between all technologies, how much to liquidate at $t = 1$, and make promises about the consumption goods agents can withdraw at either $t = 1$ or $t = 2$ in return of their deposits.

Clearly, the problem of these institutions, if workers and entrepreneurs are treated equally, coincides with that of the planner stated in the previous subsection just as in the original DD model. Notice banks do not need to know types to implement this allocation. Because $c_1^* < c_2^*$ the equilibrium is incentive compatible at the individual level. However, similarly to DD, the model includes a bank run as a second equilibrium. In this inefficient equilibrium, patient agents withdraw their deposits at $t = 1$ and all banks totally liquidate the technology at $t = 1$.

3.3 Labor market

Another way to decentralize the economy is to include a labor market. Assume entrepreneurs and workers meet at the beginning of period 0 and bargain over how to split output in the future, possibly including contingencies associated with what types they turn out to be.

If we assume all agents have the same bargaining power, output would be divided equally between workers and entrepreneurs. Thus, at $t = 0$ each worker works in the technology of an entrepreneur. At $t = 1$ there are three possible outcomes associated with the realization of the individual liquidity demand uncertainty. First, it may happen that both the worker and the entrepreneur are impatient and prefer to consume at $t = 1$. This would happen with probability λ^2. In such a case, it is obvious they would liquidate the technology obtaining a total output of 2 and split it so that each agent consumes $c_1 = 1$. Second, it may happen that both agents are patient and prefer to consume at $t = 2$. This would happen with probability $(1 - \lambda)^2$. In such a case, they would maintain the production process until $t = 2$ obtaining a total output of $2R$ and split it so that each agent consumes $c_2 = R$. Finally, it may happen that one of the agents, no matter who, is impatient and prefers to consume in period $t = 1$ while the other is patient and prefers to consume at $t = 2$. This would happen with probability $2\lambda(1 - \lambda)$. In such a case, they will find the fraction of the technology to be liquidated, x, so that each consume the same amount independently of when they do it. It is easy to see that the equilibrium value for x is

$$x = \frac{R}{R+1}, \tag{9}$$

so that each consume

$$c_1 = c_2 = \frac{2R}{R+1}. \tag{10}$$

Thus, under a labor market arrangement, any agent achieves the following consumption schedule: $c_1 = 1$ with probability λ^2, $c_2 = R$ with probability $(1 - \lambda)^2$, and $c_1 = c_2 = 2R/(R+1)$ with probability $2\lambda(1-\lambda)$. As of period $t = 0$, the expected utility to be obtained under this labor market arrangement, denoted by $E(u^{LM})$, would then be

$$E(u^{LM}) = \lambda^2 u(1) + (1 - \lambda)^2 u(R) + 2\lambda(1 - \lambda)u\left(\frac{2R}{R+1}\right). \tag{11}$$

Notice, as this allocation does not agree with expression (8), it is inefficient.

4 The nominal economy

Next, we characterize the nominal version of a decentralized economy with banks and idiosyncratic liquidity risks. The production technology and consumption preferences remain as described in the previous section. Assume a competitive banking sector with a measure 1 of banks. Unlike other nominal models in banking, we assume banks cannot deal with production decisions. These decisions are the responsibility of entrepreneurs. What banks can do is to produce nominal claims and to intermediate between borrowers and lenders. In particular, banks offer the following contract. Any agent can borrow units of account at $t = 0$ to be returned to the bank at $t = 2$ together with an interest. In all periods, these deposits could be transferred to any other agent, at no cost, to make a payment. Furthermore, these banks liabilities earn an interest rate if left in the bank between periods.

At the heart of our theoretical setting is the idea that banks provide agents with *financial liquidity* to be used in the future. Banks create at date 0 an illiquid asset (loans) that takes two periods to mature and cannot be disposed of before, together with a liquid liability (deposits) to be used by workers any time. We point out that this liquidity mismatch in banks' balance sheets is completely independent from the *technological liquidity* entrepreneurs face in production. The financial liquidity is provided by banks in the form of nominal deposit contracts redeemable on demand. These deposits represent units of account denominated in terms of inside money that are transferable between agents for the settlement of economic transactions. But this idea of financial liquidity is a separated concept from the timing and returns of production in which the DD model is based upon.

The timing of events of the nominal model is presented in Figure 1. At date 0, banks provide the financial liquidity which serves as an insurance mechanism to hedge the uncertainty on future consumption preferences. Each entrepreneur borrows D_1 units of account (call it euros) from a bank and uses this loan to pay a worker at $t = 0$ for the labor used in production. The liability side of the initial loan of D_1 to entrepreneurs is the creation of the corresponding deposits in banks' balance sheets. Notice these claims in the form of deposits are created *ex-nihilo* and transferred to workers in exchange for labor. Workers will hold on to these deposits because they are to be used either at $t = 1$ or $t = 2$ to pay for consumption goods from entrepreneurs. This way, deposits help in hedging against liquidity preference shocks. On the other hand, entrepreneurs accept these deposits when selling goods because these deposits are the means to pay for the loans they asked for in the first place.[6]

Figure 1: Timeline

Date 0	**Date 1**	**Date 2**
∘ Banks extend loans to entrepreneurs.	∘ Workers receive interest i_1^d from sight deposits.	∘ Agents receive interest i_2^d from sight deposits.
∘ Entrepreneurs pay salary to workers saved in deposits.	∘ Agent $j = \{W, E\}$ observes τ.	∘ Entrepreneurs sell goods $y_2(\tau)$ and repay the loan at rate i^b.
∘ Entrepreneurs use labor in production.	∘ Entrepreneurs liquidate $x(\tau)$ and sell goods $y_1(\tau)$.	∘ Patient agents consume.
	∘ Impatient agents consume.	
	∘ Entrepreneurs and patient workers save.	

At the beginning of period $t = 1$, workers receive the return on deposits, i_1^d. Then, the liquidity shock realizes and each agent $j = \{W, E\}$ privately observes her type $\tau = \{1, 2\}$. Entrepreneurs have the control over the amount of goods supplied in the competitive good market. In the aggregate, the demand for goods at date 1 comes from λ workers that are impatient and withdraw deposits to purchase consumption goods at the nominal price P_1. On the other hand, the supply of goods, denoted by $y_1(\tau)$, derives from the fraction of the investment liquidated, $x(\tau)$, less the fraction that is consumed directly by λ entrepreneurs

[6]Of course, agents would use the intermediation provided by banks if their expected utility improves as compared with other market arrangements, in particular, the bilateral labor market described in the previous subsection.

that are impatient. That is,

$$\lambda y_1(1) + (1-\lambda)y_1(2) = \lambda c_1^W(1). \tag{12}$$

Agents transfer wealth to the next period either in real terms (i.e., storing goods) or in nominal terms through sight deposits. While patient workers (savers) carry money to date 2 for consumption reasons, entrepreneurs (either patient or impatient) are the borrowers in this economy and, as such, have to transfer financial resources not only for consumption but also to repay the contracted debt with the bank.

At the beginning of period $t = 2$ a fraction $2 - \lambda$ of depositors, namely, entrepreneurs and patient workers, receive the return on deposits, i_2^d. On this date, the supply of goods, $y_2(\tau)$, is accounted by the fraction $[1 - x(\tau)]$ of the production technology invested at date 0 and not liquidated in period 1, yielding the return R, together with any storage of goods that entrepreneurs may carry out from the previous period, less the consumption from $(1-\lambda)$ entrepreneurs that are patient. The demand for goods at date 2 is equivalent to the proportion $(1-\lambda)$ of patient workers that withdraw deposits to purchase consumption goods at price P_2. That is,

$$\lambda y_2(1) + (1-\lambda)y_2(2) = (1-\lambda)c_2^W(2). \tag{13}$$

Finally, entrepreneurs use the nominal savings carried out from date 1 and the earnings obtained from selling goods at date 2 to pay back the loans they asked at period $t = 0$ at rate i^b.

It is noteworthy to point that in the model workers act as savers, since they have positive net positions with the banks, while entrepreneurs act as borrowers, since they hold net liabilities with the banks. However, in this economy there is no sense in which workers' deposits are used to lend to entrepreneurs as is common in the literature of financial intermediation. Savings appear *ex-post* when an agent decides not to spend the units of account received as a payment after these claims have been created previously through a loan. In this sense, the need for deposits to fund lending has to do with the ability of banks to convince depositors to keep those deposits throughout the maturity of the loans maintained in the asset side of their balance sheets and not with objects originally in the hands of savers that are transferred to borrowers.

We next provide a formal characterization of the problems for the entrepreneurs, workers and banks.

4.1 Entrepreneurs

Entrepreneurs are the only agents who take decisions on the real side of the economy. Let $\tau = \{1, 2\}$ denote the type of the agent as indexing the period in which consumption is preferred the most, and let ρ be an index variable taking value 1 if $\tau = 1$ or value 0 if $\tau = 2$. Given prices, the problem of an entrepreneur of type τ consists on choosing the allocation set $\{y_t(\tau), x(\tau), c_t^E(\tau), D_2(\tau)\}$, where $y_t(\tau)$ is the amount of consumption goods that are sold by the entrepreneur at date $t = \{1, 2\}$, $x(\tau)$ is the fraction of the productive technology liquidated at date 1, $c_t^E(\tau)$ is the demand for goods made by entrepreneur of type τ at date $t = \{1, 2\}$, and $D_2(\tau)$ represents the nominal savings transferred as deposits from period 1 to date 2; and storage from $t = 1$ to $t = 2$, to maximize[7]

$$u\left[c_t^E(\tau)\right]$$

subject to the resource constraint on the availability of consumption, if impatient, and of production that is sold,

$$0 \leq y_1(\tau) + \rho c_1^E(\tau) \leq 2x(\tau), \tag{14}$$

the financial amount that the entrepreneur saves at the end of period 1,

$$0 \leq D_2(\tau) \leq P_1 y_1(\tau), \tag{15}$$

in the form of deposits in the commercial bank, and the financial constraint that at $t = 2$ the entrepreneur pays back the loan

$$(1 + i^b)D_1 = (1 + i_2^d)D_2(\tau) + P_2 y_2(\tau), \tag{16}$$

with revenues arising from return on existing deposits, and the selling of output available in the period,

$$y_2(\tau) = [1 - x(\tau)]\, 2R + 2x(\tau) - y_1(\tau) - c_t^E(\tau). \tag{17}$$

Notice consumption enters in the storage carried from $t = 1$ for impatient entrepreneurs and as use of resources in period $t = 2$ for patient entrepreneurs. Additionally, the problem is

[7] Consistent with consumption preferences, for an entrepreneur of type $\tau = \{1, 2\}$ we have that at date 1 $c_1^E(1) > 0$ and $c_1^E(2) = 0$, while at date 2 $c_2^E(1) = 0$ and $c_2^E(2) > 0$.

subject to the non-negativity constraint

$$0 \leq y_1(\tau), \tag{18}$$

and the boundary conditions

$$0 \leq x(\tau) \leq 1. \tag{19}$$

4.2 Workers

Workers receive D_1 units of account at date 0 as a demandable deposit in a bank that allows them to withdraw funds in the future. At the beginning of $t = 1$ workers enter with deposits $\left(1 + i_1^d\right) D_1$, where i_1^d is the interest rate on deposits from period 0 to period 1. Once liquidity risk is resolved and types are observed, impatient ($\tau = 1$) workers purchase $c_1^W(1)$ consumption goods at the nominal price P_1 satisfying the deposit in advance constraint

$$c_1^W(1) = (1 + i_1^d)\frac{D_1}{P_1}, \tag{20}$$

which shows how the purchasing power of impatient workers at date 1 equals the real value of their nominal claims. If the worker turns out to be patient ($\tau = 2$), then the funds are kept in the bank until $t = 2$, when they are withdrawn and used to buy $c_2^W(2)$ consumption goods at nominal price P_2 according with the deposit constraint

$$c_2^W(2) = (1 + i_1^d)(1 + i_2^d)\frac{D_1}{P_2}, \tag{21}$$

where the right-hand side represents the real value of nominal deposits as of period 2.

In our economy, workers do not take any decision on the real side of the economy; they just make use of the financial liquidity provided by the bank (i.e., withdraw deposits) on due time according with their time preferences for consumption.[8] As stated by constraints (20) and (21), they spend all the nominal wealth to consume when they have to, so only patient workers ($\tau = 2$) transfer nominal savings from date 1 to date 2 to consume in the last period. Impatient workers ($\tau = 1$), though, do not pass deposits on to $t = 2$ since they do not have to satisfy any loan obligation nor consume at $t = 2$.

[8] Patient workers could also withdraw at $t = 1$, buy goods and store then until $t = 2$. However, as it will be clear below, this is not an optimal choice for them in an equilibrium with valued deposits.

4.3 Banks

Banks start date 0 with zero net worth. As suppliers of means of payment, banks provide at date 0 financial liquidity equivalent to D_1 in nominal loans backed by deposits created on the spot. By the beginning of period $t = 1$, workers are the only agents who hold deposits with the bank. These deposits are remunerated at the rate i_1^d. During period 1, banks satisfy payment orders from impatient depositors associated with the purchases of consumption goods. Impatient workers spend $P_1 c_1^W(1)$ of deposits while entrepreneurs of type $\tau = \{1, 2\}$ decide to maintain $D_2(\tau)$ in deposits from the revenues from selling goods. This means that the end-of-period net worth of the bank will be

$$NW_1 = D_1 - (1-\lambda)(1+i_1^d)D_1 - \lambda[(1+i_1^d)D_1 - P_1 c_1^W(1)] - \lambda D_2(1) - (1-\lambda)D_2(2). \quad (22)$$

In equilibrium, because of symmetry, the net flow associated with these payments is zero. In terms of deposits, deposits from impatient workers are renamed in favor of entrepreneurs as follows

$$\lambda(1+i_1^d)D_1 = \lambda D_2(1) + (1-\lambda)D_2(2). \quad (23)$$

At date 2, banks enter the period with maturing loans D_1 paying a loan rate i^b, together with deposit holdings in the hands of a mass $2 - \lambda$ of agents to whom they have to pay an homogeneous deposit rate of i_2^d. Again, banks satisfy payment orders from patient depositors during period $t = 2$. Patient workers spend $P_2 c_2^W(2)$ of their deposits. Impatient entrepreneurs receive a total revenue of $\lambda P_2 y_2(1)$ while patient entrepreneurs have a total revenue of $(1 - \lambda)P_2 y_2(2)$. Then, the net worth of banks at the end of $t = 2$ equals to

$$\begin{aligned} NW_2 =& (1+i^b)D_1 - (1-\lambda)[(1+i_1^d)(1+i_2^d)D_1 - P_2 c_2^W(2)] \\ & - \lambda[(1+i_2^d)D_2(1) + P_2 y_2(1)] - (1-\lambda)[(1+i_2^d)D_2(2) + P_2 y_2(2)]. \end{aligned} \quad (24)$$

Once more, in equilibrium, because of symmetry, the net flow associated with these payments is zero.

5 Results

We discuss now the equilibrium allocations of the nominal economy. First, we evaluate whether, in the presence of privately observable liquidity shocks, nominal demand deposits allow for reaching the efficient allocation of real resources. Then, we ask whether the nominal contracting designed by the bank is the best arrangement disposable for agents.

5.1 Are nominal deposit contracts efficient?

We concentrate on equilibria with valued deposits defined as follows.

Definition (Competitive equilibrium with valued deposits). *A competitive equilibrium with valued deposits is a collection of allocations* $\{y_1(\tau), y_2(\tau), x(\tau), c_t^j(\tau), D_2(\tau)\}$, *for all* $\tau = \{1,2\}$, $t = \{1,2\}$, *and* $j = \{W, E\}$, *and prices* $\{P_1, P_2, i_1^d, i_2^d, i^b\}$ *such that, (i) given prices, allocations solve the individual problems of agents and banks, (ii) market of goods and deposits clear in periods 1 and 2, and (iii)* $D_2(\tau) > 0$ *for at least one* $\tau \in \{1,2\}$.

We assume deposits are always convertible into cash. Thus, for an equilibrium with valued deposits to exist we need $i_t^d \geq 0$ so that deposits dominate cash in rate of return. A consequence of these deposit rates is that it is never optimal for any agent to store from period 1 to period 2. It will always be preferable to sell the goods and deposit the proceeds. The competitive equilibrium is characterized as follows.

Proposition 1. *There exists a unique equilibrium with valued deposits in which interest rates are given by*

$$i_1^d = 0, \tag{25}$$

and

$$(1 + i^b)\frac{P_1}{P_2} = (1 + i_2^d)\frac{P_1}{P_2} = R, \tag{26}$$

consumption allocations are

$$c_1^E(1) + c_1^W(1) = 2, \tag{27}$$

$$c_2^E(2) + c_2^W(2) = 2R, \tag{28}$$

and production decisions satisfy

$$\lambda x(1) + (1 - \lambda)x(2) = \lambda. \tag{29}$$

This allocation is inefficient.

Proof. See the Appendix A.

The intuition of Proposition 1 is the following. First, deposits should provide with the same real return as the productive technology, $(1+i_2^d)P_1/P_2 = R$, as stated in the left equality of (26). To see this, assume first that $(1+i_2^d)P_1/P_2 > R$. If this was to be the equilibrium value for the real deposit rate, both types of entrepreneurs would totally liquidate the productive technology at $t = 1$ since it would be more profitable for them to sell the proceeds at $t = 1$ and deposit the revenues in the bank rather than to wait until $t = 2$ to reap the output of the technology. This would imply that the supply of goods in period $t = 2$ would be zero which cannot be an equilibrium. On the contrary, assuming that $(1 + i_2^d)P_1/P_2 < R$, entrepreneurs will find it profitable to cut the supply of goods in period $t = 1$ to zero. Impatient entrepreneurs would only liquidate the amount needed for consumption. This, again, cannot be an equilibrium either. Thus, the only possible equilibrium involves $(1 + i_2^d)P_1/P_2 = R$.

Second, because of perfect competition in the banking sector, the only set of prices consistent with equilibrium are given by expressions (25) and (26). These are the rates that ensure the net worth of the banks to be zero both at $t = 1$ and $t = 2$.

Third, with these prices, it is not optimal for workers or entrepreneurs to store, so that the market clearing conditions for goods read

$$2\left[\lambda x(1) + (1-\lambda)x(2)\right] = \lambda\left[c_1^W(1) + c_1^E(1)\right], \tag{30}$$

and

$$2\left[\lambda(1-x(1)) + (1-\lambda)(1-x(2))\right]R = (1-\lambda)\left[c_2^W(2) + c_2^E(2)\right]. \tag{31}$$

Furthermore, using the equilibrium prices and equating the budget constraints for impatient and patient entrepreneurs, (16), as well as the budget constraints of workers (20) and (21), produces

$$\frac{c_2^E(2)}{c_1^E(1)} = (1+i_2^d)\frac{P_1}{P_2} = R = \frac{c_2^W(2)}{c_1^W(1)}. \tag{32}$$

By comparing (32) with (8), it is immediate to see that this equilibrium allocation does not coincide with the first best. Therefore, the banking solution is not efficient.

Corollary. *The particular distribution of consumption between entrepreneurs and workers is not pinned down and will depend on the value of the ratio* $p_1 = P_1/D_1 \in (0.5, \infty)$ *so* *that the consumption of workers reaches*

$$c_1^W(1) = \frac{1}{p_1}, \tag{33}$$

and

$$c_2^W(2) = \frac{R}{p_1}, \tag{34}$$

while the consumption of entrepreneurs equals

$$c_1^E(1) = 2 - \frac{1}{p_1}, \tag{35}$$

and

$$c_2^E(2) = \left(2 - \frac{1}{p_1}\right) R. \tag{36}$$

This equilibrium is neutral with respect to the nominal size of the loan D_1*.*

An inherent feature of deposit contracts that explains why the equilibrium in the nominal economy is not efficient relies on its incompleteness. While deposit contracts provide an interest rate remuneration that is independent of the state, the first-best allocation of resources is only reachable by a risk-sharing contract that is contingent on the state τ. However, the demand deposit contract offered by the bank is a simple arrangement which is remunerated at a particular interest rate i_t^d for period $t = \{1, 2\}$ but independently of the idiosyncratic liquidity preference for consumption of each depositor. As such, the nominal value of deposits at dates 1 and 2 does not depend on the individual state τ.

Notwithstanding the incompleteness of the nominal deposit contract, a fundamental difference between our inefficiency result and the previous literature of nominal models of banking is that price level variability in the market for goods is not sufficient to support efficient risk-sharing contracts. With free entry in the banking sector, in our competitive equilibrium commercial banks are price-takers with nominal prices determined in the market for goods. Hence, deposits written in nominal terms that promise fixed payments on inside money do not allow risk to be shared because price adjustments do not introduce *per se* state contingency to contracts. To introduce a desirable level of contingency that serves as risk-sharing mechanism, there is the need of an institution that exerts a direct influence on market prices. This is why further institutional assumptions that are not applicable to the reality of a modern monetary system, such as commercial and central banks engaging in real investments, are required in the previous literature to achieve the efficiency result.

5.2 Do nominal deposit contracts provide the best decentralized allocation?

Although nominal deposit contracts are not efficient, a different question is whether it is the best market allocation. For simplicy, assume a symmetric equilibrium in which workers and entrepreneurs obtain the same allocation and prices adjust accordingly. In such a case, $P_1 = D_1$ and allocations are

$$c_1^W(1) = c_1^E(1) = 1, \tag{37}$$

and

$$c_2^W(2) = c_2^E(2) = R. \tag{38}$$

Notice, since $c_1^j(1) < c_2^j(2)$ for both $j = \{E, W\}$, the allocation is incentive compatible and banks do not need to know types when servicing the withdrawal of deposits.

In this symmetric equilibrium with banks intermediating between workers and entrepreneurs, the expected utility of any agent, denoted by $E(u^B)$, would be

$$E(u^B) = \lambda u(1) + (1 - \lambda)u(R). \tag{39}$$

At $t = 0$ workers and entrepreneurs would accept the deposit contract and do not bilaterally trade among themselves as long as expected utility (39) is larger than expected utility (11)

$$E(u^B) \geq E(u^{LM}). \tag{40}$$

After some rearrangement, for bank intermediation to be preferred to bilateral labor trade, it must be the case that

$$u\left(\frac{2R}{R+1}\right) \leq \frac{1}{2}[u(1) + u(R)]. \tag{41}$$

Because we have assumed $R > 1$, we have that

$$1 < \frac{2R}{R+1} < \frac{R+1}{2} < R. \tag{42}$$

Then, there will be a threshold for the degree of relative risk aversion above which expression (41) does not hold. In other words, if the economy is populated with relatively more risk averse agents, intermediation by banks does not provide as much risk sharing as bilateral labor trade. In such a case, agents would not want to take a loan or deposit at the bank and would prefer to directly trade among themselves. This threshold in risk aversion is increasing in the productivity of the technology R. The larger R is, the more risk averse agents have to be to prefer the bilateral trade allocation.

In summary, the banking allocation with nominal deposits is not only inefficient but also could be inferior in welfare terms to other market arrangements involving direct bilateral trade in economies with high enough degree of relative risk aversion and/or low enough returns on the productive technology. In the next section we introduce several alternatives to explore the possibility to reach efficiency or, at least, to improve the welfare implications from bank intermediation.

6 Augmenting the nominal model

In this section, we include additional elements in the model to see whether this nominal economy with privately observable liquidity shocks can reach efficiency. In the first subsection we allow banks to write state-contingent contracts conditioned on observables. Then, we allow entrepreneurs to pay part of the loan in period $t = 1$. How much they pay in advance of maturity is their choice. Finally, we discuss how a central bank and monetary policy could be included in the model and to what extent it could help the economy to approximate the efficient solution.

6.1 Contingent deposit contracts

So far, we have shown how non-contingent nominal deposit contracts are not able to decentralize an efficient allocation. Thus, the obvious reply to this statement would be to analyze to what extent banks could write deposit contracts contingent on types of their customers. Although banks cannot observe types directly, they observe the timing of withdrawals and deposits and could potentially infer types accordingly. In particular, impatient workers withdraw all their deposits at $t = 1$ while patient workers keep them in the banks until $t = 2$. Furthermore, impatient entrepreneurs end up with a smaller deposit balance at the end of $t = 1$ as compared with patient entrepreneurs.

If banks could infer types this way, they could pay a different deposit rate from $t = 1$ until $t = 2$, denoted by $i_2^d(\tau)$, to each type $\tau = \{1, 2\}$. Notice we assume neither the lending rate, i^b, nor the deposit rate from $t = 0$ until $t = 1$, i_1^d, can include this contingency. The deposit rate i_1^d is paid before types are revealed. On the other hand, the lending rate i^b only affects one side of the economy, entrepreneurs, and not workers. Furthermore, it would be awkward to condition a rate paid to an asset of the bank on the use of their liabilities, which may include the use of liabilities of a different bank. Also, making deposit rates contingent on types is all that is needed to achieve efficiency in the DD model. For all these reasons we concentrate only on the deposit rate i_2^d.

The first element to notice would be to understand the nature of the contingency needed to approximate the decentralized equilibrium to an efficient allocation. For that, consider the budget constraints of workers (20) and (21) which imply

$$c_2^W(2) = [1 + i_1^d(2)]c_1^W(1), \tag{43}$$

together with those of entrepreneurs (16), for $\tau = 1$

$$(1 + i^b)D_1 = [1 + i_2^d(1)]D_2(1) + P_2[1 - x(1)]2R, \tag{44}$$

and $\tau = 2$,

$$(1 + i^b)D_1 = [1 + i_2^d(2)]D_2(2) + P_2[1 - x(2)]2R - P_2c_2^E(2). \tag{45}$$

In these expressions we have already included the contingency of deposit rates together with the idea that an efficient solution involves no storing.

Given the competitive equilibrium characterized above, the only way to approximate an efficient allocation would be by raising the nominal deposit rate for impatient agents above that used to remunerate deposits of patient agents, namely, $i_2^d(1) > i_2^d(2)$. This will allow impatient entrepreneurs to increase consumption at $t = 1$ and still pay back the loan at $t = 2$ while reducing the consumption at $t = 2$ of patient entrepreneurs who now have become poorer. Similarly, it will also decrease the purchasing power of patient workers relative to the impatient ones.

The first consideration is whether an interest rate scheme in which $i_2^d(1) > i_2^d(2)$ is incentive compatible. Banks not knowing the type of their customers can only rely on the timing and amount of withdrawals and deposits. This means, banks would pay a larger interest rate on remaining deposit balances to those depositors withdrawing funds at $t = 1$ or those depositing smaller amounts on that period. These are the impatient types. However, for any given difference between deposit rates, $i_2^d(1) - i_2^d(2)$, it would be in the interest of patient workers to withdraw some amount in cash, or to buy some small amount of goods to be stored, and claim the high rate on the remaining balances in their deposit accounts. Another possibility would be for patient entrepreneurs to split their deposits between different banks to have balances close to those of impatient entrepreneurs. The only way for banks to prevent this outcome would be to know the exact amount to be withdrawn by impatient workers and deposited by impatient entrepreneurs and to cross that information with other banks. That would mean knowing preferences of their customers, something we believe is beyond the information possibilities of current depository institutions.

In any case, within the theoretical boundaries of the model, it would be possible for banks to gather all necessary information on withdrawals and deposits to tell one type apart from the other so that the contingent deposit rates could be implementable. In fact, if banks knew the exact amount to be withdrawn by impatient workers or to be deposited by impatient and patient entrepreneurs, and condition rates on these exact amounts, the proposed interest rate scheme would be incentive compatible. No patient agent would pretend to be impatient.

The problem, however, is more fundamental than incentive compatibility or how much information banks have about their customers as the following proposition states.

Proposition 2. *A nominal economy with contingent deposits rates $i_2^D(1) > i_2^D(2)$ yields the following aggregate allocation:*

$$c_1^W(1) + c_1^E(1) \leq 2, \tag{46}$$

and

$$c_2^W(2) + c_2^E(2) \geq 2R. \tag{47}$$

This allocation is not compatible with efficiency.

Proof. See the Appendix B.

Proposition 2 suggests the efficient allocation is not inside the consumption possibilities set of the competitive equilibrium even allowing for contingent nominal deposit rates. It is also remarkable that, in terms of welfare, comparing expressions (46) and (47) with expressions (27) and (28), this allocation is not preferable to the resulting allocation obtained without contingent deposit rates since it involves no less consumption polarization between patient and impatient agents. So, allowing for this contingency does not improve the welfare of all agents as compared with the non-contingent allocation.

To understand why allowing for contingent deposit rates is not compatible with efficiency, first focus on the ratio of budget constraints for the workers (43). For this ratio to be efficient it must be the case that

$$[1 + i_1^d(2)]\frac{P_1}{P_2} < R. \tag{48}$$

However, for an equilibrium to exist, entrepreneurs should supply goods both at $t = 1$ and $t = 2$. This means that, in real terms, the only way to have deposit rates satisfying both (48) and $i_2^d(1) > i_2^d(2)$ is that

$$[1 + i_2^d(2)]\frac{P_1}{P_2} < R \leq [1 + i_2^d(1)]\frac{P_1}{P_2}. \tag{49}$$

The reason is simple. If both rates are below R in real terms, no entrepreneur would supply goods at $t = 1$ while if both rates are above R in real terms, no entrepreneur would supply goods at $t = 2$. Thus, (49) must hold. However, with that set of rates in expression (49), only impatient entrepreneurs would supply goods in $t = 1$. Thus, the aggregate supply schedule is characterized by (46) and (47) and the allocation is not compatible with efficiency.

6.2 Early liquidation of loans

We next allow for the possibility that entrepreneurs decide which part of the loan can be liquidated with the bank at date $t = 1$. Let $L_1(\tau)$ be the part of the loan to be paid for in period $t = 1$ by entrepreneur τ. Notice this decision is taken at $t = 1$ and may be different for different types $\tau = \{1, 2\}$. The interest rate in that case is i_1^b and will be determined in equilibrium. For notational purposes we denote now the interest rate of the loan to be paid at $t = 2$ to be i_2^b. As with i_2^b, and for the same reasons, we do not allow i_1^b to be contingent on types. We want to check whether allowing early liquidation of the loan would induce entrepreneurs to sell different amounts of goods at $t = 1$ and make the supply of goods in each period to reach their efficient level.

It is important to stress that the amount of early loan liquidation is a choice of the borrower. In the traditional banking literature, both real and nominal, cited in section 2, it is assumed that banks are the ones calling loans prematurely. However, the vast majority of bank loans cannot be recalled nor banks have any say about the liquidation decision of the investment projects pursued by borrowers. This is the case of basically all mortgages and, according to the Board of Governors of the Federal Reserve System, of 87.5 percent of all C&I loans.[9] In contrast, loans specify a regular calendar of payments and typically include provisions for borrowers to pay them in advance of that schedule, possibly charging a fee.

The possibility of anticipated liquidation of the loan affects both the maximization problems of entrepeneurs and banks. Regarding the problem of entrepeneurs, constraints (15) and (16) are replaced, respectively, by

$$0 \leq D_2(\tau) + (1 + i_1^b)L(\tau) \leq P_1 y_1(\tau), \tag{50}$$

[9]See Board of Governors of the Federal Reserve System [2003]. This figure is the average fraction of noncallable loans, weighted by volume, between the second quarter of 1997 and the first quarter of 2003. In 2003 the Board stopped including the amount of C&I loans that are callable because, representing a small fraction of total loans, their behavior did not significantly differ from loans which are not callable.

and

$$(1+i_2^b)[D_1 - L(\tau)] = (1+i_2^d)D_2(\tau) + P_2 y_2(\tau), \tag{51}$$

since now part of the revenues from selling goods in period $t = 1$ can be used to pay the loan in that period and deducted from the amount to be paid at $t = 2$.

Early liquidation of the loan also affects the net worth of banks. Specifically, the net worth of banks at the end of period $t = 1$ becomes

$$\begin{aligned} NW_1 = & D_1 - [\lambda L(1) + (1-\lambda)L(2)] - \lambda[(1+i_1^d) - P_1 c_1^W(1)] - (1-\lambda)(1+i_1^d)D_1 \\ & - \lambda D_2(1) - (1-\lambda)D_2(2). \end{aligned} \tag{52}$$

Total assets are deducted by the amount of the early liquidation but entrepreneurs reduce also their deposits by that amount together with the interest of the loan in $t = 1$ as expressed in (50). Furthermore, the net worth of the bank at $t = 2$ becomes now

$$\begin{aligned} NW_2 = & (1+i_2^b)[D_1 - \lambda L(1) - (1-\lambda)L(2)] - (1-\lambda)[(1+i_1^d)(1+i_2^d)D_1 - P_2 c_2^W(2)] \\ & - \lambda[(1+i_2^d)D_2(1) + P_2 y_2(1)] + (1-\lambda)[(1+i_2^d)D_2(2) + P_2 y_2(2)]. \end{aligned} \tag{53}$$

In this case, only the unpaid part of the loan produces revenues while the evolution of deposits is the same as in the baseline model.

As the next proposition shows, including early liquidation of the loan in the model does not change the equilibrium in real terms.

Proposition 3. *There exists a unique equilibrium with valued deposits and early liquidation of loans in which interest rates are given by*

$$i_1^d = i_1^b = 0, \tag{54}$$

and

$$(1+i_2^b)\frac{P_1}{P_2} = (1+i_2^d)\frac{P_1}{P_2} = R, \tag{55}$$

entrepreneurs are indifferent about when to liquidate the loan. With these rates, equilibrium allocations are

$$c_1^E(1) + c_1^W(1) = 2, \tag{56}$$

$$c_2^E(2) + c_2^W(2) = 2R, \tag{57}$$

and production decisions satisfy

$$\lambda x(1) + (1 - \lambda)x(2) = \lambda. \tag{58}$$

This allocation is inefficient.

Proof. See the Appendix C.

As before, the supply of goods must be positive in both periods, which means that the deposit rate from $t = 1$ to $t = 2$ must still satisfy the left equality in (55). On the other hand, lending rates must be such that they also do not induce entrepreneurs to accumulate sales in only one period. This means that

$$R\frac{P_2}{P_1} = \frac{1 + i_2^b}{1 + i_1^b}, \tag{59}$$

as stated in the right equality in (55). Finally, for the net worth of the bank to be zero in $t = 1$ we need $i_1^d = i_1^b = 0$.

Thus, allowing early liquidation of the loan does not change the real equilibrium of the model and the fact that it is not efficient.

6.3 Central banks and monetary policy

In this situation one may ask whether a central bank could reproduce the needed contingency in deposit rates through its monetary policy. Although our model does not include a central bank and an active monetary policy, we can still use it to provide an answer to this question. In our economy, there is no role for a central bank because neither agents nor banks demand the very object the central bank produces, namely, outside money. Entrepreneurs and workers prefer to hold deposits which dominate cash in rate of return. Similarly, because the net transfer of funds between banks is zero, these institutions do not need reserves to settle accounts among them. Without this demand for outside money, monetary policy has no leverage on the economy.

There are several channels by which to make agents demand outside money. One way would be to have agents demand cash alongside deposits. This could be done by imposing preferences for different payment means. But central banks do not design monetary policy through the management of cash so this does not seem a promising avenue of research.

A second way to incorporate monetary policy is to include a demand for reserves by private banks. This could be easily achieved by assuming that the distribution of patient and impatient agents is exogenous to banks and differ across financial intermediaries. Then the net flow of payments across banks would be uneven and reserves would be demanded to settle accounts between them. These reserves could be borrowed from the central bank at an interest rate which becomes the policy rate of the economy. Once reserves are injected in the system, an interbank market could also be used to exchange reserves between banks at an interest rate. Although designing such model is beyond the scope of this paper, it does not seem the central bank will be able to produce nominal deposits to have real state-contingent payoffs. This is because, at the end of the day, competition will equalize both, deposit and lending rates across banks. Then, the issue of contingent deposit rates inducing an intertemporal supply of goods inconsistent with efficiency still will be there.

A third avenue would be for the central bank to impose reserve requirements on banks. Thus, banks would need to maintain at $t = 1$, as current accounts at the central bank, a fraction of the created deposits D_1. These reserves would be loaned at $t = 1$ at the refinancing rate to be paid by banks at $t = 2$. Because reserve requirements impose a cost on banks, it will affect both deposit and lending rates. It could be possible that a combination of contingent deposits, early liquidation of loans (possibly with a minimum requirement) and reserve requirements could make the economy move towards the efficient allocation. In any case, this question is beyond the scope of the present paper and is left for future research.

7 Conclusion

In this paper we cast doubts about the ability of nominal deposit contracts to achieve an efficient allocation in an economy à la DD. The reason is the impossibility to have nominal deposit contracts producing the needed contingency in real terms. This also includes the situation in which banks offer nominal rates contingent on individual states or allow borowers to liquidate the loan before maturity.

The observation that nominal deposit contracts do not reproduce the efficient risk-sharing solution of the planner cannot be interpreted as an argument against the role of banking institutions in today's financial system. Indeed, our results support the socially valuable function of the maturity and liquidity transformation banks engage in. Demandable deposits nominally issued in inside money are designed to bridge the financial gap between buyers and sellers and to allow certain degree of risk-sharing across agents. However, we have shown that the ability of nominal deposit contracts to hedge against liquidity shocks could be inferior, in welfare terms, to other arrangements such as bilateral trades. The inferiority of deposits is more likely the more risk averse agents are and/or the lower the return on the illiquid real investment is.

Our nominal framework provides a new perspective that complements the existing theories about the specialness of depository institutions. Under our view, an institutional feature that is important in distinguishing commercial banks from other forms of financial intermediation such as mutual funds, insurance or fintech companies is the autonomous capacity of depository institutions for the on-balance sheet creation of money-like claims that are redeemable on demand. Central banks and interbank markets can be easily added to the model once we incorporate an uneven distribution of payment flows across banks. At the end of the day, the main role of these institutions is the management of outside money used for the settlement of accounts between depository institutions. The implications of extending the model in that direction are left for future research.

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A Proof of Proposition 1

First, start with the problem faced by entrepreneurs. In these problems, it is important to notice that as long as $i_2^d \geq 0$ it is never optimal for agents to store between $t = 1$ and $t = 2$. Because in equilibria with valued deposits $i_2^d \geq 0$, we impose no storing in the solution so that expression (14) is satisfied with equality, which for impatient entrepreneurs implies

$$0 < y_1(1) + c_1^E(1) = 2x(1), \tag{60}$$

while for patient entrepreneurs we have that

$$0 \leq y_1(2) = 2x(2). \tag{61}$$

Furthermore, agents will not waste resources so expression (15) is also satisfied with equality

$$D_2(\tau) = P_1 y_1(\tau), \tag{62}$$

for both $\tau = \{1, 2\}$. Using these expressions to substitute for $D_2(\tau)$ and $x(\tau)$, the problem of impatient entrepreneurs become to maximize

$$u\left[c_1^E(1)\right]$$

subject to

$$(1 + i^b)D_1 = (1 + i_2^d)P_1 y_1(1) + 2P_2 R - P_2 R y_1(1) - P_2 R c_1^E(1), \tag{63}$$

together with the boundary constraints

$$0 \leq y_1(1), \tag{64}$$

and

$$y_1(1) + c_1^E(1) \leq 2. \tag{65}$$

Similarly, the problem of a patient entrepreneur becomes to maximize

$$u\left[c_2^E(2)\right]$$

subject to

$$(1+i^b)D_1 = (1+i_2^d)P_1 y_1(2) + 2P_2R - P_2Ry_2(2) - P_2c_1^E(2), \tag{66}$$

together with the boundary constraints

$$0 \le y_1(2) \le 2. \tag{67}$$

Let $\beta(1)$, $\xi(1)$ and $\chi(1)$ be the multipliers associated with constraints (63), (64) and (65), respectively. Also, let $\beta(2)$ be the multiplier associated with (66) while let $\xi(2)$ and $\chi(2)$ be the multipliers associated with left and right inequalities in (67). The FOCs of the two problems are:

$$u'\left[c_1^E(1)\right] = \chi(1) + \beta(1)P_2R, \tag{68}$$

$$u'\left[c_2^E(2)\right] = \beta(2)P_2, \tag{69}$$

$$\xi(1) - \chi(1) = \beta(1)[P_2R - (1+i_2^d)P_1], \tag{70}$$

and

$$\xi(2) - \chi(2) = \beta(2)[P_2R - (1+i_2^d)P_1], \tag{71}$$

together with

$$\xi(1)y_1(1) = 0, \tag{72}$$

$$\chi(1)\left[y_1(1) + c_1^E(1) - 2\right] = 0, \tag{73}$$

$$\xi(2)y_1(2) = 0, \tag{74}$$

and

$$\chi(2)\left[y_1(2) - 2\right] = 0. \tag{75}$$

Now we can evaluate different values for prices to see whether they support an equilibrium.

First consider the case

$$(1+i_2^d)\frac{P_1}{P_2} < R.$$

Then, from (70) and (71) we have that both $\xi(1) > 0$ and $\xi(2) > 0$ with $\chi(1) = \chi(2) = 0$ which means $y_1(1) = y_1(2) = 0$. In other words, no entrepreneur sells at $t = 1$. Impatient entrepreneurs liquidate just enough to consume at $t = 1$ and still pay back the loan at $t = 2$ while patient ones do no liquidate at all. Thus, the supply of goods at $t = 1$ is zero which cannot be an equilibrium.

Next, there is the case

$$1 < R < (1+i_2^d)\frac{P_1}{P_2}.$$

However, from (70) and (71), $\chi_1(\tau) > 0$ for $\tau = \{1, 2\}$ so that $x(1) = x(2) = 1$ and all goods are sold at $t = 1$ and none at $t = 2$. Again, this cannot be an equilibrium.

Finally, we are left with the case

$$(1+i_2^d)\frac{P_1}{P_2} = R.$$

From (70) and (71) we have that $\xi(\tau) = \chi(\tau) = 0$ for $\tau = \{1, 2\}$. Substituting this rate in the budget constraint of both entrepreneurs, (63) and (66), we obtain

$$(1+i^b)D_1 = 2P_2R - P_2Rc_1^E(1),$$

and

$$(1+i^b)D_1 = 2P_2R - P_2c_2^E(2),$$

so equating both expressions produce

$$c_2^E(2) = Rc_1^E(1).$$

Substituting the deposit rate in the budget constraints of workers (20) and (21) we also get

$$c_2^W(2) = Rc_1^W(1).$$

Substituting these consumption levels in the the market clearing condition in the good market at $t = 1$

$$2\lambda x(1) + 2(1-\lambda)x(2) = \lambda \left[c_1^E(1) + c_1^W(1)\right],$$

and that of period $t = 2$

$$\begin{aligned} 2\left[\lambda\left(1 - x(1)\right) + (1-\lambda)\left(1 - x(2)\right)\right] R &= (1-\lambda)\left[c_2^E(2) + c_2^W(2)\right] \\ &= (1-\lambda) R \left[c_1^E(1) + c_{12}^W(1)\right], \end{aligned}$$

implies

$$\lambda x(1) + (1-\lambda)x(2) = \lambda.$$

Then, substitute back this expression in the market clearing conditions above to obtain

$$c_1^E(1) + c_1^W(1) = 2,$$

and

$$c_2^E(2) + c_2^W(2) = 2R.$$

Finally, using the fact that in equilibrium

$$\lambda(1 + i_1^d)D_1 = \lambda D_2(1) + (1-\lambda)D_2(2) = \lambda P_1 y_1(1) + (1-\lambda)P_1 y_1(2),$$

the net worth of any bank at $t = 1$, (22), becomes

$$NW_1 = D_1 - (1 + i_1^d)D_1.$$

For the net worth to be equal to 0, it must be the case that $i_1^d = 0$. Furthermore, using the fact that also in equilibrium

$$(1-\lambda)P_2 c_2^W = \lambda P_2 y_2(1) + (1-\lambda)P_2 y_2(2), \tag{76}$$

the net worth of banks at the end of $t = 2$, (24), becomes

$$NW_2 = (1 + i^b)D_1 - (1 + i_1^d)(1 + i_2^d)D_1.$$

For this net worth to be equal to 0, it must be the case that $i^b = i_2^d$.

B Proof of Proposition 2

Recalling expression (8)

$$1 < \frac{c_2^*}{c_1^*} < R,$$

and the ratio of budget constraints for the workers (43), the deposit rate paid to patient agents must be

$$[1 + i_1^d(2)]\frac{P_1}{P_2} < R \tag{77}$$

so as to have the efficient relation between consumptions for the workers. With these rates it would not be worth for patient entrepreneurs to sell goods at $t = 1$ so that $y_1(2) = x(2) = 0$. These entrepreneurs would prefer to wait until $t = 2$ and sell all output from production then. Because in equilibrium impatient workers must consume at $t = 1$, we need to induce impatient entrepreneurs to liquidate at least some the technology at $t = 1$ and sell some of the proceeeds then, which means that

$$[1 + i_1^d(1)]\frac{P_1}{P_2} \geq R \tag{78}$$

so that both $x(1) > 0$ and $y_1(1) > 0$.

With this set of rates and choices, looking at the market clearing condition at $t = 1$,

$$\lambda \left[c_1^W(1) + c_1^E(1)\right] = 2\left[\lambda x(1) + (1-\lambda)x(2)\right] = 2\lambda x(1),$$

which implies

$$c_1^W(1) + c_1^E(1) \leq 2, \tag{79}$$

while the market clearing condition at $t = 2$ reads

$$\begin{aligned}(1-\lambda)\left[c_2^W(2) + c_2^E(2)\right] &= 2\left[\lambda(1 - x(1)) + (1-\lambda)(1 - x(2))\right] R \\ &= 2\left[\lambda(1 - x(1)) + (1-\lambda)\right] R,\end{aligned}$$

or

$$c_2^W(2) + c_2^E(2) \geq 2R. \tag{80}$$

Expressions (79) and (80) mean it is not possible to achieve the efficient ratio of consumptions (8) for both workers and entrepreneurs simultaneously.

C Proof of Proposition 3

When entrepreneurs are allowed to liquidate part of the loan at $t = 1$ it is still true that as long as $i_2^d \geq 0$, there will be no storing between $t = 1$ and $t = 2$. Then, still expression (14) is satisfied with equality, so that

$$0 < y_1(1) + c_1^E(1) = 2x(1), \tag{81}$$

and

$$0 \leq y_1(2) = 2x(2). \tag{82}$$

Furthermore, still agents will not waste resources so expression (50) is also satisfied with equality

$$D_2(1) + (1 + i_1^b)L(1) = P_1 y_1(1), \tag{83}$$

and

$$D_2(2) + (1 + i_1^b)L(2) = P_1 y_1(2). \tag{84}$$

Using these expressions to substitute for $y_1(\tau)$ and $x(\tau)$, the problem of impatient entrepreneurs become to maximize

$$u\left[c_1^E(1)\right]$$

subject to

$$(1+i_2^b)[D_1 - L(1)] = (1+i_2^d)D_2(1) + 2P_2R - P_2R\frac{D_2(1)}{P_1} - P_2R(1+i_1^b)\frac{L(1)}{P_1} - P_2Rc_1^E(1), \tag{85}$$

together with the boundary constraints

$$0 \leq D_2(1), \tag{86}$$

$$0 \leq L(1), \tag{87}$$

$$\frac{D_2(1)}{P_1} + (1+i_1^b)\frac{L(1)}{P_1} + c_1^E(1) \leq 2. \tag{88}$$

Similarly, the problem of a patient entrepreneur becomes to maximize

$$u\left[c_2^E(2)\right]$$

subject to

$$(1+i_2^b)[D_1 - L(2)] = (1+i_2^d)D_2(2) + 2P_2R - P_2R\frac{D_2(2)}{P_1} - P_2R(1+i_1^b)\frac{L(2)}{P_1} - P_2c_2^E(2), \tag{89}$$

together with the boundary constraints

$$0 \leq D_2(2), \tag{90}$$

$$0 \leq L(2), \tag{91}$$

$$\frac{D_2(2)}{P_1} + (1+i_1^b)\frac{L(2)}{P_1} \leq 2. \tag{92}$$

Let $\beta(1)$, $\delta(1)$, $\xi(1)$ and $\chi(1)$ be the multipliers associated with constraints (85), (86), (87) and (88), respectively. Also, let $\beta(2)$, $\delta(2)$, $\xi(2)$ and $\chi(2)$ be the multipliers associated with constraints (89), (90), (91) and (92), respectively. The FOCs of the two problems are:

$$u'\left[c_1^E(1)\right] = \chi(1) + \beta(1)P_2R, \tag{93}$$

$$u'\left[c_2^E(2)\right] = \beta(2)P_2, \tag{94}$$

$$\delta(1) - \frac{\chi(1)}{P_1} = \beta(1)\left[R\frac{P_2}{P_1} - (1+i_2^d)\right], \tag{95}$$

$$\delta(2) - \frac{\chi(2)}{P_1} = \beta(2)\left[R\frac{P_2}{P_1} - (1+i_2^d)\right], \tag{96}$$

$$\xi(1) - (1+i_1^b)\frac{\chi(1)}{P_1} = \beta(1)(1+i_1^b)\left[R\frac{P_2}{P_1} - \frac{1+i_2^b}{1+i_1^b}\right], \tag{97}$$

$$\xi(2) - (1+i_1^b)\frac{\chi(2)}{P_1} = \beta(2)(1+i_1^b)\left[R\frac{P_2}{P_1} - \frac{1+i_2^b}{1+i_1^b}\right], \tag{98}$$

together with

$$\delta(1)D_2(1) = 0, \tag{99}$$

$$\xi(1)L(1) = 0, \tag{100}$$

$$\chi(1)\left[\frac{D_2(1)}{P_1} + (1+i_1^b)\frac{L(1)}{P_1} + c_1^E(1) - 2\right] = 0, \tag{101}$$

$$\delta(2)D_2(2) = 0, \tag{102}$$

$$\xi(2)L(2) = 0, \tag{103}$$

and

$$\chi(2)\left[\frac{D_2(1)}{P_1} + (1+i_1^b)\frac{L(1)}{P_1} - 2\right] = 0. \tag{104}$$

Again, we can evaluate different values for prices to see whether they support an equilibrium. First consider the case

$$R < (1+i_2^d)\frac{P_1}{P_2}.$$

However, from (95) and (96), $\chi_1(\tau) > 0$ for $\tau = \{1, 2\}$ so that $x(1) = x(2) = 1$ and all goods are sold at $t = 1$ and none at $t = 2$. This cannot be an equilibrium since patient workers will not consume.

Next, there is the case

$$(1+i_2^d)\frac{P_1}{P_2} < R.$$

Then, from (95) and (96) we have that both $\delta(1) > 0$ and $\delta(2) > 0$ which means $D_2(1) = D_2(2) = 0$. In other words, no entrepreneur sells at $t = 1$ to make deposits. However, they could sell to advance payments on the loan. That would depend on the loan rates i_1^b and i_2^b. There are several possibilities. If

$$R\frac{P_1}{P_2} < \frac{1+i_2^b}{1+i_1^b}$$

it is in the advantage of entrepreneurs to advance the loan payment as much as possible. This means they will liquidate all the technology at $t = 1$ so $x(1) = x(2) = 1$, to get revenues in $t = 1$ with which to pay the loan at the relatively smaller rate i_1^b instead of waiting to $t = 2$ and pay the relatively high rate i_2^b. However, again, this cannot be an equilibrium with positive consumption in $t = 2$ by patient workers. Second, there is the possibility that

$$R\frac{P_1}{P_2} > \frac{1+i_2^b}{1+i_1^b}.$$

In this case it would be benefitial for entrepreneurs to postpone payment of the loan until $t = 2$. We would have $\xi(\tau) > 0$, so that $y(\tau) = 0$ for $\tau = \{1, 2\}$. Again, this could not be an equilibrium with positive consumption in $t = 1$ by impatient workers. Finally, it must be the case that

$$R\frac{P_1}{P_2} = \frac{1+i_2^b}{1+i_1^b},$$

so entrepreneurs are indifferent regarding when to liquidate the loan and $\xi(\tau) = \chi(\tau) = 0$ for $\tau = \{1, 2\}$. Because $D(\tau) = 0$, we have that

$$(1+i_1^b)L(\tau) = P_1 y_1(\tau).$$

However, in this case, the net worth of the banks in period 1 after using this expression to substitute for $L(\tau)$ and imposing market clearing in the goods market at $t = 1$ is

$$NW_1 = D_1 - \lambda\frac{1+i_1^d}{1+i_1^b}D_1 - (1-\lambda)(1+i_1^d)D_1.$$

Equating this expresion to 0 implies deposit and lending rates at $t = 1$ satisfy

$$1+i_1^d = \frac{1+i_1^b}{\lambda+(1-\lambda)(1+i_1^b)}.$$

Furthermore, the net worth of banks at $t = 2$, again, after substituting the relation of rates at $t = 1$ and the market clearing condition equals

$$NW_2 = (1+i_2^b)D_1 - (1-\lambda)(1+i_1^d)D_1 - (1-\lambda)(1+i_2^d)(1+i_1^d)D_1.$$

Making this expression equal to 0 implies $i_2^b = i_2^d$. But this is a contradiction with the assumed rates

$$R\frac{P_1}{P_2} = \frac{1+i_2^b}{1+i_1^b}$$

and

$$(1+i_2^d)\frac{P_1}{P_2} < R.$$

Finally, we are left with the case

$$(1+i_2^d)\frac{P_1}{P_2} = R.$$

Again, from (95) and (96) we have that $\delta(\tau) = \chi(\tau) = 0$ for $\tau = \{1, 2\}$. Regarding lending rates we have now only two possibilities. The first is

$$R\frac{P_1}{P_2} > \frac{1+i_2^b}{1+i_1^b}.$$

However, in this case it would be benefitial for entrepreneurs to postpone payment of the loan until $t = 2$. In this case, $\xi(\tau) > 0$, so that $y(\tau) = 0$ for $\tau = \{1, 2\}$. Again, this could not be an equilibrium with positive consumption in $t = 1$ by impatient workers. The second case is

$$R\frac{P_1}{P_2} = \frac{1+i_2^b}{1+i_1^b}.$$

Substituting this rate in the budget constraint of both entrepreneurs, (85) and (89), we obtain

$$(1+i_2^b)D_1 = 2P_2R - P_2Rc_1^E(1),$$

and

$$(1+i_2^b)D_1 = 2P_2R - P_2c_2^E(2),$$

so equating both expressions produce

$$c_2^E(2) = Rc_1^E(1).$$

Substituting the deposit rate in the budget constraints of workers (20) and (21) we also get

$$c_2^W(2) = Rc_1^W(1).$$

Substituting these consumption levels in the the market clearing condition in the good market at $t = 1$

$$2\lambda x(1) + 2(1-\lambda)x(2) = \lambda\left[c_1^E(1) + c_1^W(1)\right],$$

and that of period $t = 2$

$$\begin{aligned} 2\left[\lambda\left(1 - x(1)\right) + (1-\lambda)\left(1 - x(2)\right)\right] R &= (1-\lambda)\left[c_2^E(2) + c_2^W(2)\right] \\ &= (1-\lambda) R \left[c_1^E(1) + c_{12}^W(1)\right], \end{aligned}$$

so that

$$\lambda x(1) + (1-\lambda)x(2) = \lambda.$$

Substituting back this expression in the market clearing conditions above, obtain

$$c_1^E(1) + c_1^W(1) = 2,$$

and

$$c_2^E(2) + c_2^W(2) = 2R.$$

Finally, using the fact that in equilibrium

$$\lambda(1 + i_1^d)D_1 = \lambda D_2(1) + (1-\lambda)D_2(2) = \lambda P_1 y_1(1) + (1-\lambda)P_1 y_1(2),$$

the net worth of any bank at $t = 1$, (52), becomes

$$NW_1 = D_1 + i_1^b[\lambda L(1) + (1-\lambda)L(2)] - (1 + i_1^d)D_1.$$

For the net worth to be equal to 0, it must be the case that

$$i_1^d = i_1^b \left(\frac{\lambda L(1) + (1-\lambda)L(2)}{D_1}\right).$$

Furthermore, using the fact that also in equilibrium

$$(1-\lambda)P_2 c_2^W(2) = \lambda P_2 y_2(1) + (1-\lambda)P_2 y_2(2), \tag{105}$$

the net worth of banks at the end of $t = 2$, (53), becomes

$$NW_2 = (1 + i_2^b)D_1 - (1 + i_1^d)(1 + i_2^d)D_1.$$

For this net worth to be equal to 0, it must be the case that

$$1 + i_2^b = (1 + i_1^d)(1 + i_2^d).$$

However, because rates also satisfy

$$(1 + i_2^d)\frac{P_1}{P_2} = R,$$

$$R\frac{P_1}{P_2} = \frac{1 + i_2^b}{1 + i_1^b},$$

and

$$i_1^d = i_1^b \left(\frac{\lambda L(1) + (1 - \lambda)L(2)}{D_1} \right),$$

the only possibility for which alll these expressions are met is when $i_1^d = i_1^b = 0$.