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LIQUIDITY RISK, MARKET POWER AND THE INFORMATIONAL EFFECTS OF POLICY

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Abstract. Using a structural approach, we combine bidding data from open market operations as well as macroeconomic information to recover the latent distribution of liquidity risk across financial institutions in Chile and how it is affected by policy. We find that unanticipated shocks to foreign reserve accumulation and interest rates have significant effects on aggregate beliefs about a liquidity shock in the near future. We demonstrate that accounting for market power is important for measuring the strength of this informational channel of macroeconomic policy.

Keywords: Multi-unit auction, Liquidity risk, Signalling Effects of Macroeconomic Policy

JEL codes: C57, D44, D80, E58, F30, G20

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1. Introduction

Managing the state of liquidity in the financial sector has been at the forefront of the policy responses to the 2008-2009 financial crisis. During and after the crisis, major Central Banks engaged in quantitative easing (QE) policies of an unprecedented scale. Between mid 2009 and mid 2013, the U.S. Federal Reserve purchased more than $1000 billion of medium to long term US Treasury securities. At the same time, in conjunction with the QE induced abundance of liquidity, emerging market countries accumulated foreign currency reserves. Indicatively, Mexico increased its international reserves to GDP ratio by 2.4% in 2008-2009, reaching 11.9% in 2011. Brazil's ratio increased from 11.4% to 15.1% during 2008-2011, while Chile increased its ratio by 4.2% in 2008, reaching 15.7% in 2011.[1]

There is of course considerable heterogeneity of liquidity risk across financial institutions, where this risk is defined as the probability that they suffer a large enough shock such that they are unable to meet their liabilities in a timely manner. This risk depends on individual strategic decisions as well as market and macroeconomic conditions which are exogenous to each institution.

While policy makers can possess information on the aggregate component of liquidity risk, and react to developments in the economy that are related to it, the idiosyncratic risk component is highly latent as financial institutions have private information about their own exposure in the near future.[2] Moreover,

[1] Such a practice was also prevalent before the crisis as many of these countries transitioned from fixed to floating exchange rates; e.g. Mexico increased its ratio from 3.5% to 7% during 1996-2007, after adopting a floating exchange rate regime in 1994. Thus, reserve accumulation may also serve other objectives apart from financial stability, such as managing the exchange rate.

[2] This actually explains the emergence of several measures of liquidity risk after the financial crisis. Examples of measures which are based on balance-sheet data are the Liquidity Coverage Ratio and the Net Stable Funding Ratio introduced by Basel III (BCBS (2013)). Other measures include the LIBOR-OIS spread or other composite financial market liquidity indicators (BoE, 2007). Balance sheet based measures may heavily depend on stress scenarios and can be sensitive to expert categorizations of assets and liabilities while they may not be available at high frequency. On the other hand high frequency market based measures may not disentangle liquidity risk from other risks e.g. Cassola, Hortaçsu, and Kastl (2013) show that CDS data do not seem to explain much of the willingness to pay for liquidity extracted from auctions of the European Central Bank.

financial institutions condition on public information to form beliefs about the distribution of risk in their market, which opens up the possibility of asymmetric information between policy makers and market participants. From an econometric perspective, the consequence is that learning about this distribution inevitably requires combining economic theory and data at the micro and macro level.

This paper develops and applies a structured approach in order to construct an estimate of the distribution of liquidity risk as expected by financial market participants and, additionally, to investigate whether Central Bank policies such as reserve accumulation and monetary policy convey information about its aggregate component. The latter is important as financial market perceptions about aggregate risk can potentially influence the decision of banks to expand lending and invest in riskier projects.[3] If policy makers can influence these perceptions, then this provides an *informational* channel through which policy can affect the economy.

In our study we analyze auction data from open market operations for domestic bonds issued by the Chilean Central Bank during 2002-2012. Since financial institutions have little incentives to reveal their individual risk exposure, we use their bids in these auctions in order to uncover it by exploiting the structure imposed by a theoretical model that closely reflects the auctions' features. A bank participating in such an auction will adjust both the quantity of bonds it wishes to buy and the interest rate it wishes to receive, according to the state of liquidity it expects to have until the bonds mature.

The case of Chile is a natural choice for studying liquidity risk and providing empirical evidence on the effects of policy, as it is one of the aforementioned countries that has historically used reserve accumulation, for precautionary and other reasons (Cabezas and Gregorio, 2019). In addition, since the 1990's Chile has adopted inflation targeting, and a standard practice to achieve the

[3]While providing an answer to this question is beyond the scope of the paper, Pflueger, Siriwardane, and Sunderam (2020) show that in a different context, perceived risk as measured by the price of volatile stocks can affect the macroeconomy by changing the cost of capital. In our context, banks' perception of liquidity risk may well be affected by the quality of the projects they finance, and whether risky firms will eventually repay or not. Of course, our measure encompasses other interpretations as well, such as a run by creditors.

desired interest rate is issuing domestic bonds of different maturities through open market operations. Open market operations can be tightly connected to foreign reserve accumulation in practice, as they drain liquidity from the market to achieve the interest rate target; on the other hand, buying foreign currency is part of the reason why there is excess liquidity in the financial system.

The paper makes several contributions. We build on the empirical literature on the intersection of finance and industrial organization (Kastl, 2017) by showing that identifying liquidity risk requires taking into account both market microstructure and the degree of privately and publicly provided insurance against this risk. While the non-competitive structure of financial markets is something that has been shown to matter in different settings e.g. in the primary market for liquidity provision by Central Banks (Cassola, Hortaçsu, and Kastl, 2013), we study its implications for learning about liquidity risk from bidding behavior of banks in liquidity *draining* auctions, where banks (and other financial institutions) buy bonds from the Central Bank.

Furthermore, we show that a bank's true valuation for the bond (or equivalently, its cost of giving up liquidity) depends on the probability of obtaining liquidity insurance either from the interbank market or the lender of last resort, which in turn depends on macroeconomic conditions and policy. In this sense, the paper makes a methodological contribution by combining individual level auction data across time, with macroeconomic time series to identify the latent distribution of liquidity risk. This framework allows us to investigate the effects of policy on financial market perceptions of aggregate risk. Beyond its descriptive usefulness, we also show how this distribution can be used to calibrate funding risk in structural macroeconomic models with banking sectors.

The paper's empirical results contribute to the strand of the empirical macroeconomic literature that studies asymmetric information between policy makers and the private sector. More particularly, we find evidence of a novel *informational* channel of foreign reserve policy; an increase in foreign reserves signals to private financial institutions the expectation of an aggregate liquidity shock in the next period. In addition, we do not find evidence that news about future interventions have significant effects. This could be explained by the costly nature of these interventions, as policy actions are more credible than promises. Additionally, we find that the financial sector also revises its beliefs about aggregate risk upwards after an increase in the nominal rate. Similar to reserve accumulation, this suggests that the private sector understands that a higher interest rate is meant to alleviate some of the pressure on international capital flight, signalling a higher likelihood of a liquidity shock in the near future.

The market we consider is oligopsonistic as the number of financial institutions participating in it is small and hence they can affect the market price on the margin. We show that this has implications for existing measures of liquidity risk that are based on observed market returns that do not account for market imperfections, as variation in the interest rates does not only reflect the cost of giving up liquidity, but also the strategic behaviour of banks with market power. We show that had we ignored the oligopsonistic structure of the primary market of bonds, we would have largely underestimated the aforementioned effects of policy interventions. The structure imposed by the auction model is therefore important for identification and for understanding the effects of open market operations which are an integral part of the implementation of conventional and unconventional monetary policy.

1.1. **Related Literature.** The market under consideration is one where a Central Bank sells bonds through an auction. The literature on multi-unit auctions was initiated by Wilson (1979), who considered a setting where buyers bid for shares of a divisible good. Bids are continuously differentiable demand schedules, specifying the highest quantity demanded at any price level, while winning bidders pay the market clearing price. Subsequent work modified this framework to better fit realistic features of multi unit auctions in markets for government securities, electricity distribution, electromagnetic spectrum and construction procurement, among others. For example Kastl (2012) characterizes the equilibria in multi unit auctions where bids are k-step functions, i.e bidders can submit up to k price-quantity combinations in order to buy shares of a divisible good. Since the implied bidding function is not differentiable, instead of using calculus of variations techniques like in Wilson (1979), Kastl (2012) uses local optimality conditions to characterise the equilibrium quantity bid at each step for two different price mechanisms. In Kastl (2011), an econometric model is also proposed in order to estimate the marginal valuations of bidders at each step; Cassola, Hortaçsu, and Kastl (2013) uses the same baseline model in order to study European banks' demand for short-term liquidity during the summer 2007 subprime market crisis.

Our paper is related to Kastl (2011, 2012); Cassola, Hortaçsu, and Kastl (2013), in the sense that the auction format we wish to replicate is a uniform multi-unit auction where banks submit interest rate-quantity tuples instead of differentiable functions. However, despite the discreteness of quantity bids, since in our dataset we do not observe banks placing more than one interest rate bid, we turn to the first order condition with respect to the price, where differentiability holds, in order to derive simpler and intuitive results about optimal behaviour.

Our model is one where banks are sellers of liquidity (or, equivalently, buyers of bonds) and thus the cost of giving up liquidity is one of the main drivers of their bidding behaviour. This cost is also connected to the state of insurance against an adverse liquidity shock, that is, the ability to obtain additional liquidity, which is itself a function of macroeconomic conditions. We show that using a resampling algorithm similar to the one proposed by Kastl (2011); Hortaçsu and McAdams (2010) is sufficient to identify the distribution of liquidity risk under partial insurance.

The paper also relates to the strand of the literature that studies the importance of market power in these auctions. Cassola, Hortaçsu, and Kastl (2013) show that strategic behavior matters for the identification of the cost of funding. We perform counterfactual exercises that illustrate that market power matters both for identifying liquidity risk as well as the effects of different macroeconomics shocks on its distribution in the economy.[4]

Our paper is also connected to the literature on foreign exchange interventions and their effectiveness. There is a growing literature that reevaluates the

[4]On the inefficiencies due to market power in analogous settings, see e.g. Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) and Vives (2011), as well as Ausubel (2004), Ausubel and Cramton (2004) and Kremer and Nyborg (2004) for mechanisms to address the issue.

ability of these interventions to influence the exchange rate, either empirically or theoretically, as well as on what is the optimal way of implementing them.[5] There are of course theories for reserve accumulation that go beyond a theory of exchange rate determination. Starting with Calvo (2006) and the credibility of the lender of last resort policy, the literature has proposed precautionary motives for foreign reserve accumulation (Obstfeld, Shambaugh, and Taylor (2010); Bocola and Lorenzoni (2017); Céspedes (2019)) and counteracting externalities from excessive foreign borrowing (e.g Arce, Bengui, and Bianchi (2019)) or externalities from rich household's consumption to that of the poor (Fanelli and Straub, 2021).[6]

Our contribution to the literature on reserve accumulation relates to its informational effects. To our knowledge, there is not much empirical evidence on the short run effects, if any, of reserve accumulation to beliefs about aggregate risk, which is an additional channel through which this policy can have an impact. Mussa (1981) focused on the signalling effects of foreign exchange interventions to market participants about future monetary policy, while Vitale (2003) argues that since foreign reserve accumulation is a costly action, it can reduce the uncertainty about future policy which can stabilise the economy. Dominguez and Frankel (1993) stress that public interventions, whether sterilized or not, can affect the exchange rate through market expectations.

Our paper makes also a contribution to the distinct but related literature of informational effects of monetary policy. Romer and Romer (2000) show that monetary-policy actions provide signals of the Federal Reserve's information about inflation, while Nakamura and Steinsson (2018) find that unexpected monetary tightening raises expected output growth, which provides further evidence on the existence of an informational channel of monetary policy actions. Melosi (2016) stresses the signalling effect of monetary policy by conveying the central bank's view about the macroeconomy to price setting firms. Similarly,

[5]See for example Kearns and Rigobon (2005) and Gabaix and Maggiori (2015), as well as Fanelli and Straub (2021); Cavallino (2019); Davis, Devereux, and Yu (2020); Amador, Bianchi, Bocola, and Perri (2019).

[6]See also Bianchi and Lorenzoni (2021) and Maggiori (2021) for a comprehensive review of the literature on the use of foreign exchange reserves and the effectiveness of foreign exchange interventions.

Jarociński and Karadi (2020) find that monetary policy announcements have a significant informational component. Our results suggest that monetary policy can also have signalling effects through influencing beliefs about aggregate risk, just as is the case of reserve accumulation.

The rest of the paper is organized as follows: Section 2 provides some institutional details regarding open market operations while Section 3 describes the auction dataset. In Section 4 we present the auction model, and in Section 5 we characterize optimal bidding behavior. Sections 6 and 7 discuss identification, the estimation methodology, the aggregate model and the resulting estimates of the distribution of liquidity risk. In Section 8 we present the identified policy effects and the counterfactual exercise. Section 9 discusses other potential applications of our estimates, and Section 10 concludes. The Appendix contains derivations, details about computation, the datasets and empirical results, as well as further details about the auction model and its connections to the more general optimization problem of the bank.

2. Institutional Details

Like other Central Banks, the Central Bank of Chile (BCCh) can inject money in the economy via repo operations. However this is not the main tool for liquidity management. Because of large capital inflows in the past and large foreign exchange reserves in the 1990s (before the adoption of a fully flexible foreign exchange rate regime in 1999), the BCCh controls the quantity of money in the Chilean market mostly by draining liquidity from the financial sector. Therefore, the BCCh conducts open market operations by issuing promissory notes and bonds at various maturities through auctions, performed weekly (or sometimes bi-weekly).[7] The BCCh determines the maximum number of participants that can be different in every auction: potential market participants include twenty-three banks, four pension fund administrators, the unemployment fund administrator, three insurance companies and four stock brokers. Among the bonds sold by the Central Bank, PDBC (the short term notes) are the most heavily used to manage and regulate the quantity of money

[7]Bonds sold include short-term notes due in 30 to 360 days, nominal bonds with maturities of 2, 5 and 10 years and inflation-indexed bonds with maturities of 5 and 10 years.

in circulation in the financial system within a given month or from one month to the next. The auction schedule for these notes is announced on a monthly basis. The program planning takes into account the liquidity demand expectations, maturing debt from previous periods, strategies for complying with reserve requirements and seasonal factors affecting liquidity in that period.[8]

The PDBC auctions are mainly uniform multi-unit auctions where banks bid for a discrete amount of bonds they wish to buy along with the minimum interest rate they would accept. The Central bank then decides the cutoff interest rate awarded to all winners of bonds by ranking interest rate bids from smallest to highest and assigning bonds up to capacity. During the period relevant to our paper, the BCCh retained the option to award a different amount than scheduled (which in the case of bonds was $\pm 20\%$ of the amount auctioned) and to unilaterally declare the auction as deserted if the rates asked by the banks were too high.

3. Bidding Data

The paper focuses on the 30 day discount promissory notes (PDBC30) for several reasons. First, in the absence of liquidity risk, the shorter maturity of these bonds makes them a better substitute for cash or reserves at the central bank. Second, we can safely assume that in this market there is no inflation premium asked, as would be the case with longer maturity bonds. Third, short term bonds are more likely to be kept to maturity and not resold in secondary markets. Moreover, Central Bank short term debt is considered as a safe asset as it is much less likely to carry a default premium.[9] Banks' liquidity risk should therefore be the main driver of the positive risk premium asked in these auctions. Finally, a more practical reason is that 30-day PDBC

[8]For more details, please refer to the liability management report published by the Central Bank e.g. BCCh (2012). Appendix A9 also contains a brief exposition of the main liquidity facilities provided by the Bank as part of its monetary policy.

[9]As noted in Reis (2015), the Central Bank is just one of many government agencies, and thus solvency of the Central Bank cannot be separated from the solvency of the government. In this sense, Central Bank issued securities can be comparable to government issued debt that also carries low default risk. Demand for the latter is high as it is valued for its safety and liquidity, at least in the case of the US Treasury Debt (Krishnamurthy and Vissing-Jorgensen, 2012), while similar reasoning could apply for government debt issued by other countries (Du, Im, and Schreger, 2018).

auctions are the most frequent open market operations of the BCCh, with the largest number of participants, most of which are banks.

Our dataset contains all bidding information for the open market operations of the BCCh from September 2002 to August 2012. The information includes the total volume of bonds allotted by the Central Bank in each auction, the marginal (or cut-off) interest rate, the bidders' identities and the rates and quantities of bonds asked by each bidder.[10]

# of Auctions: 934	1st Quartile	2nd Quartile	3rd Quartile
# Bids per auction	5	6	8
Interest Rate Bids	2.1199	3.5155	5.0328
Quantity Bids (Billion)	6.2	16	40
Total Vol. Demanded (Billion)	95	140	183
C.B. Supply (Billion)	90	200	300
Total Vol. Assigned (Billion)	60	90	200
Equilibrium Rate	1.8270	3.7836	5.12
# Bids per Month	38.75	48.5	59.25

Due to the high frequency of these auctions and the relatively thin market for bonds, the number of participants in each auction is small. We will deal with this by pooling data of different auctions as is typically done in the literature (see e.g. Hortaçsu and Kastl (2012)). More information on sample selection can be found in Appendix A4.

4. The Model

We consider a primary market for liquidity, where financial institutions, mainly banks, give up liquidity by buying bonds from the Central Bank through an auction. The behaviour of banks in the auction, which is the focus of this paper, can be thought of as a sub-problem of a more general expected utility maximization problem, where they optimally decide how much equity to hold over time and how to allocate it among net assets, deposits and reserves, subject to constraints such as reserve and capital requirements. For the interested

[10]This dataset is not publicly available and usually only the information on the total volume allotted and marginal rates are available on the BCCh web site.

reader, in Appendix B1, we present a formulation of this general maximization problem and show that making optimal bidding choices at the Central Bank bond auction while conditioning on other choices is entirely consistent with it. This allows us to focus the analysis of the auction model, which we now turn to.

Below we outline the auction model environment and the underlying assumptions about primitives, while we later turn to analyzing optimal bidding behaviour and equilibrium outcomes.

***A1: Market Participants, Endowments and Ownership*.**
The market is populated by N risk neutral buyers and a Central Bank (CB), the seller. N is common knowledge, and $N \leqslant \bar{N}$, where $\bar{N}$ is the maximum number of potential market participants, exogenously fixed by institutional constraints. The buyers are of two types: banks and non banks, with N_B and N_O members respectively, also common knowledge, where $N_B + N_O = N$.

Each buyer $i \in [1, N]$ is endowed with m_i monetary units of net external funds, with m_i private information. In the case of banks, these funds may be borrowed from creditors and are net of any other investments, excluding the bonds in the auction under consideration. Thus, m_i is the difference between the bank's available cash before the auction takes place and its reserve requirement.[11]

The seller (CB) issues $Q \in [0, \bar{Q}]$ bonds, each with nominal value equal to one, which are sold through an auction.

***A2: Information*.**
Public Information: We assume that all market participants observe the same information about the economy, Ω, that is possibly a proper subset of the complete information set about its aggregate state. Ω includes any policy actions taken by the CB.
Buyers' Signals: Before the auction takes place, each buyer privately learns $s_i \in [0, 1]$, to be interpreted as their individual probability of becoming illiquid in the next period conditional on Ω. In the case of banks, which are the focus of this paper, we define the probability of becoming illiquid as the probability

[11] In Appendix B2, we clarify how m_i is determined in the general optimization problem. Nevertheless, when the bank chooses its optimal bid in the auction, it takes m_i as given.

that excess reserves become negative, where excess reserves are defined as the difference between reserves, including cash from the sale of assets, either liquid or partially liquid, and the reserve requirement. It is common knowledge that signals $\{s_i\}_{i \in [1,N]}$ are independent draws, conditional on public information[12]:

$$s_i \underset{iid}{\sim} F_G(. \mid \Omega)$$

where $F_G(. \mid \Omega)$ is the conditional distribution of risk among buyers of type $G = \{\text{bank}, \text{non bank}\}$, which is atomless on [0,1] with a positive and continuous density $f_G(. \mid \Omega)$, almost everywhere.

Bond supply by the CB: The density of Q conditional on Ω, is also common knowledge.[13]

A3: Auction Format and Market Structure.

The CB sells bonds through a *uniform multi-unit* auction. After the CB announces the amount of bonds it wishes to sell in the auction, buyers submit a two dimensional bid specifying the share q_i of total volume Q of bonds that they wish to buy, which is restricted to take discrete values, along with b_i, their corresponding minimum acceptable interest rate.

Uniform Interest Rate Auction: Given the bids $(b_i, q_i)_{i \in [1,N]}$, the CB ranks the interest rate bids from the lowest to the highest: $b_1 \leqslant b_2 \leqslant ... \leqslant b_N$. Denote the corresponding shares asked by $q_1, q_2, ..., q_N$. The buyer with b_1 is assigned q_1, then the buyer with b_2 is assigned q_2, and so on and so forth, until Q bonds have been assigned. All winning buyers receive the same interest rate once the bonds mature -to be referred to as the equilibrium cutoff bid B^c-, which is defined as the highest winning interest rate bid.

A4: Action Set.

Each buyer i submits a pair of an interest rate bid (b_i) and a quantity bid (q_i) from the action set A_i.

$$A_i = \{(not\ bid) \cup \{(b_i, q_i) : b_i \geqslant 0, q_i \in [0,1]\}\}$$

[12]By conditional independence, we mean that $\Pr(s_i \leqslant u_i, s_k \leqslant u_k \mid \Omega) = \Pr(s_i \leqslant u_i \mid \Omega) \Pr(s_k \leqslant u_k \mid \Omega)$.

[13]In Appendix A1, we plot the planned versus the realized supply of bonds in the BCCh auctions. After the Lehmann Brothers episode, supply uncertainty measured as the difference between the planned and realized supply, increased.

Whereas in Kastl (2011), Kastl (2012), Hortaçsu and Kastl (2012), and Cassola, Hortaçsu, and Kastl (2013), banks are allowed to submit several steps of an implied demand function for bonds, we assume that buyers are allowed to submit only one interest rate-quantity bid in the auction. Although allowing for more steps is entirely reasonable within some contexts, we have not observed this kind of behaviour in our dataset.

Since each bank submits an interest rate-quantity pair, the residual demand for bonds is a step function, and thus rationing will happen with probability one in equilibrium. Thus, a buyer having placed the equilibrium cutoff interest rate receives a share of bonds equal to

$$q_i^c \equiv \frac{q_i}{q_i + \sum_{l:b_l=b_i} q_l} \left(1 - \sum_{j:b_j<b_i} q_j\right)$$

In the case of ties, the cutoff bidders proportionally share the residual supply, while when buyer i is the only cutoff bidder, q_i^c is equal to the residual supply of bonds it faces, $q_i^c = 1 - \sum_{j:b_j<b_i} q_j$.

The quantity received by buyer i in equilibrium is therefore summarized as follows:

$$q_i^*(b_i, q_i) = \begin{cases} 0 & \text{if } b_i > B^c \\ q_i^c & \text{if } b_i = B^c \\ q_i & \text{if } b_i < B^c \end{cases}$$

A5: Timing.

We divide the game in four stages: In stage 0, Ω is updated and each buyer learns m_i and s_i and then bids in the auction in stage 1. In stage 2 the liquidity shock materializes (or not) while in stage 3 the bonds mature. Therefore, an institution that had a liquidity shock might not be able to meet its obligations to its creditors.

A6: Maturity Structure and Costs for Banks.

While we do take into account the bidding behavior of non-bank institutions for the determination of equilibrium prices and quantities in the auction, we do not need to specify their payoff related primitives as we are not interested in identifying the liquidity risk for this group. We next characterize the maturity structure and costs for banks.

Maturity Structure: If the liquidity shock does not materialize for bank i, the institution is sure to be able to repay its creditors once the bonds mature, as its liabilities in m_i mature after the bonds bought in the auction do. If however the bank suffers the liquidity shock, then its liabilities mature before the bonds do. In this case, a bank that has successfully bid in the auction may not be able to fully repay its creditors and is in need of additional liquidity.

Costs: There are no transaction costs associated with participating in the auction. On the contrary, there is an opportunity cost: if a bank does not participate in the auction, it can deposit its cash endowment m_i at the Central Bank in return for an interest rate ι. In addition, there is a cost associated to the event that a bank is hit by the liquidity shock, which mainly depends on whether the buyer can obtain the necessary funding or not. In case the bank can find lenders, which happens with probability p, the cost of borrowing funds before the bonds mature is pinned down by an exogenous interest rate d. When the bank cannot obtain additional funding in order to repay its creditors, which happens with probability $1 - p$, it is forced to liquidate the bonds at face value and forgo any returns.[14] In this case, the expected cost of having bought an amount of bonds $q_i Q$ in the case in which a liquidity shock materializes is $s_i q_i Q$.

Note that the probability of obtaining funding p is exogenous to the bank but endogenous to the aggregate state of the economy. We view p as a reduced form that captures different events. In Appendix B1, we provide an analysis of p, which includes the possibility of borrowing from the interbank market and the lender of last resort.

[14]See for example Holmström and Tirole (1998) in the context of partial liquidation of firms.

5. Equilibrium Bidding Behavior: Banks

In this section we characterize the expected payoff for each bank (bidder i) when there is uncertainty about the ability of the bank to obtain liquidity when in need. Since bidders are risk neutral, the utility function representing their preferences over wealth is linear. Therefore, each bidder i chooses (b_i, q_i) to maximize its expected profits (Π_i) subject to the constraint that total investment should be less than or equal to its endowment m_i:

$$\begin{aligned} \max_{b_i, q_i} \quad & \mathbb{E}(\Pi_i \mid s_i) \\ s.t. \quad & q_i Q \leqslant m_i \end{aligned}$$

We next analyze the bank's payoff. When the liquidity shock does not materialize, a winning bidder receives a return equal to the value of the bonds it was allotted plus interest, B^c, discounted by the deposit rate (ι), which captures the opportunity cost of investment. Its per unit profit is thus B^c minus the nominal value of the bonds.

In the case of a liquidity shock, the payoff is different depending on the state of insurance, which affects the cost of illiquidiy. When the bank can insure itself against the liquidity shock (an event that happens with probability p), it can obtain the necessary liquidity by borrowing from other banks through the interbank market or from the lender of last resort. In this way, the bank will be able to repay its creditors, obviating the need to default on its debt. A bidder that suffered a liquidity shock in stage 2 can therefore still receive the full amount of profits when bonds mature in stage 3, while it will have to repay the loan it took at an interest rate d. When, on the contrary, the bank cannot find lenders (an event that happens with probability $(1-p)$, it is forced to liquidate its bonds at face value and forgo any returns. In this case, the institution's loss is equal to the nominal value of the bonds.

A bidder can win some, all or none of the bonds they bid for, depending on the relative ranking of their interest rate bid in comparison to the rest. When the bidder's interest rate is smaller than the cutoff bid, the bidder is assigned the total quantity of bonds it asked for. When the bidder's interest rate is the cutoff bid, the bidder is assigned the residual supply of bonds, or a proportion of it, when there are more than one cutoff bidders. Finally, if the bidder's interest rate is higher than the cutoff bid, the bidder is not assigned any bonds, and thus has a payoff from participating in the auction equal to zero.

Expected payoffs are therefore given by:

$$\begin{aligned}
\mathbb{E}(\Pi_i \mid s_i) &= \mathbb{P}(B^c > b_i \mid s_i)\mathbb{E}(\Pi_i \mid s_i, B^c > b_i) + \mathbb{P}(B^c = b_i \mid s_i)\mathbb{E}(\Pi_i \mid s_i, B^c = b_i) \\
&= \mathbb{P}(B^c > b_i \mid s_i)\mathbb{E}(Q)\big[(1-s_i)q_i\left(\frac{1}{1+\iota}\left(\mathbb{E}(B^c \mid B^c > b_i, s_i)+1\right)-1\right) \\
&\quad +s_i q_i\left(p\left(\frac{1}{1+\iota}\mathbb{E}(B^c - d \mid B^c > b_i, s_i)-1\right)-(1-p)\right)\big] \\
&\quad +\mathbb{P}(B^c = b_i \mid s_i)\mathbb{E}(Q)\times \\
&\quad \big[(1-s_i)\mathbb{E}(q_i^c \mid b_i = B^c, s_i)\left(\frac{1+b_i}{1+\iota}-1\right) \\
&\quad +s_i\mathbb{E}(q_i^c \mid b_i = B^c, s_i)\left(p\left(\frac{b_i-\mathbb{E}_t d}{1+\iota}-1\right)-(1-p)\right)\big]
\end{aligned} \tag{1}$$

where we use that the total quantity of bonds supplied by the Central Bank is exogenous, i.e. $\mathbb{E}(Q \mid s_i, B^c > b_i) = \mathbb{E}(Q \mid s_i, B^c = b_i) = \mathbb{E}Q$.[15]

Bidder i is the cutoff bidder under two conditions: a) the total demand of bonds of all bidders with a lower interest rate is smaller than supply, and b) the sum of bidder i's demand for bonds (or the total demand of all bidders with an interest rate equal to b_i, if bidder i ties in interest rate with others) and the total demand of bidders with a lower interest rate surpasses the CB's supply. Similarly, bidder i will not be the cutoff bidder but will win the whole amount of bonds demanded under one condition: the residual supply from bidders with a lower or equal interest rate than bidder i is greater than its demand (or the total demand of everyone with a bid equal to b_i, in case of ties). Thus, the above probabilities depend on both b_i and q_i, as

$$\mathbb{P}(B^c > b_i \mid s_i) = \mathbb{P}\left(\sum_{j:b_j \leqslant b_i} q_j + q_i < 1 \mid s_i\right) \tag{2}$$

$$\mathbb{P}(B^c = b_i \mid s_i) = \mathbb{P}\left(\sum_{l:b_l < b_i} q_l < 1, \sum_{j:b_j \leqslant b_i} q_j + q_i \geqslant 1 \mid s_i\right) \tag{3}$$

[15]Note that since the Central Bank sets aggregate supply depending on its *expectations* of aggregate conditions conditional on Ω, Q does not directly depend on future interest rates, and thus Q and d are conditionally independent as well: $\mathbb{E}(dQ|\Omega) = \mathbb{E}(Q|\Omega)\mathbb{E}(d|\Omega)$.

Since the choice of quantity is restricted to lie in a discrete set, the derivation of the optimality condition with respect to quantity should use local perturbation arguments like in Kastl (2012). We focus on the optimality condition with respect to the interest rate, which is a continuously differentiable variable, and has an intuitive interpretation.[16] The first order condition yields:[17]

$$(1-s_i(1-p))\left(q_i\frac{\partial\mathcal{E}(B^c \mid B^c>b_i,s_i)}{\partial b_i}+b_i\frac{\partial\mathcal{E}(q_i^c \mid B^c=b_i,s_i)}{\partial b_i}+\mathcal{E}\left(q_i^c \mid B^c=b_i,s_i\right)\right)$$
$$-(\iota+s_i(1+p\mathbb{E}_t d))\left(q_i\frac{\partial\mathbb{P}(B^c>b_i \mid s_i)}{\partial b_i}+\frac{\partial\mathcal{E}(q_i^c \mid B^c=b_i,s_i)}{\partial b_i}\right)=0 \quad (4)$$

In order to provide some useful intuition for the first order condition, we analyze the two extreme cases: when $p=0$ and when $p=1$.

5.1. **No Insurance against the shock ($p=0$).** After a few manipulations, the condition which characterizes the bidding behaviour of banks in a Bayes Nash equilibrium when there is no insurance against the shock can be expressed as follows:

$$\frac{b_i-\frac{s_i+\iota}{1-s_i}c_i}{b_i} = -\frac{1}{b_i}\left(\frac{\partial\mathcal{E}_c(q_i^c)}{\partial b_i}\right)^{-1}\left(\mathcal{E}_c(q_i)+q_i\frac{\partial\mathcal{E}_{nc}(B^c)}{\partial b_i}\right) \quad (5)$$

where we have defined $c_i := q_i\frac{\partial\mathbb{P}(B^c>b_i)}{\partial b_i}\left(\frac{\partial\mathcal{E}_c(q_i^c)}{\partial b_i}\right)^{-1}+1$, which is the ratio of the marginal cost of participating in the auction, and the marginal cost in the case in which the bank is the cutoff bidder[18]. Then, the left hand side (LHS) of (5) is the markup asked by Bank i as a percentage of the interest rate asked, within a stochastic environment. Correspondingly, the RHS represents the market power of bank i.

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[16]We abstract from the possibility of ties as rarely observe them in the data and thus they are not relevant for the estimations done in the next sections. For an explicit analysis of the implications of ties on equilibria see Kastl (2012), Kastl (2011)

[17]We can always decompose expectations as follows:

$$\begin{aligned}\mathbb{E}(.|s_i) &= \mathbb{P}(B^c>b_i \mid s_i)\mathbb{E}(.|s_i,B^c>b_i)+\mathbb{P}(B^c=b_i \mid s_i)\mathbb{E}(.|s_i,B^c=b_i)+\\ &\quad\ \mathbb{P}(B^c<b_i \mid s_i)\mathbb{E}(.|s_i,B^c<b_i)\\ &:= \mathcal{E}(.\mid B^c>b_i)+\mathcal{E}(.\mid B^c=b_i)+\mathcal{E}(.\mid B^c>b_i) := \mathcal{E}_{nc}(.)+\mathcal{E}_c(.)+\mathcal{E}_{nb}(.)\end{aligned}$$

[18]More details on c_i can be found in Appendix A2.

Moreover, recall that when the bank is the cutoff bidder, its bid b_i is equal to the equilibrium bid, while the quantity it receives is the residual supply of bonds at b_i. Conversely, when the bank is not the cutoff bidder it receives the full quantity it bid for, and an interest rate that is higher than its own bid.

The term $\mathcal{E}_c(q_i) + q_i \frac{\partial \mathcal{E}_{nc}(B^c)}{\partial b_i}$ is the price effect on the total revenue of an infinitesimal change in the interest rate asked. The first and second components are the effects in the case in which the bank is the cutoff bidder and a non-cutoff bidder respectively. The term $b_i \frac{\partial \mathcal{E}_c(q_i^c)}{\partial b_i}$ is the quantity effect on the total revenue of an infinitesimal change of the interest rate asked in the cutoff case. Since the quantity effect in the non cutoff case is zero, the term is equal to the quantity effect overall. Consequently, the RHS of (5) can be thought of as the interest rate elasticity of residual supply, within our setting which is that of an oligopsonistic market.

In the absence of market power, or as $N \to \infty$, the right hand side (RHS) of (5) becomes zero, while $\lim_{N\to\infty} c_i = 1$. Therefore, (5) implies that

$$b_i = \frac{s_i + \iota}{1 - s_i}$$

Note that when $s_i = 0$, the bidder's interest rate becomes equal to ι, the interest rate that the bidder would have received by depositing q_i instead.

The above has important implications regarding the identification of risk. With perfect competition, high interest rates are directly translated as high risk. When banks have market power, this is not the case anymore, as the bid will also contain a mark-up, which also depends on risk. We will revisit this issue when we deal with policy evaluation.

An additional key observation is related to the risk premium bank i asks, which is defined as the difference between the interest rate asked under uncertainty and the interest rate asked when there is no liquidity risk, $b_i(s_i) - b_i(0)$. Since $b_i(0) \in [0, \infty)$, then as the probability of having the liquidity shock goes to 1, the bank asks for an infinite risk premium: $\lim_{s_i \to 1} b_i(s_i) = \infty$.

5.2. **Full Insurance against the shock ($p = 1$).** Using the same definitions as in the previous section, the first order condition with respect to b_i can be written as follows:

$$\frac{b_i - (s_i(1+\mathbb{E}(d)) + \iota)c_i}{b_i} = -\frac{1}{b_i}\left(\frac{\partial \mathcal{E}_c(q_i^c)}{\partial b_i}\right)^{-1}\left(\mathcal{E}_c(q_i) + q_i\frac{\partial \mathcal{E}_{nc}(B^c)}{\partial b_i}\right) \tag{6}$$

Notice that since the right hand side of equations (5) and (6) is identical, the premium asked behaves differently in the full insurance case: even when s_i tends to 1, full insurance implies that the premium asked remains finite.

Moreover, when $N \to \infty$, (6) becomes

$$b_i = s_i(1+\mathbb{E}(d)) + \iota$$

Again, there is a proportional relationship between risk and return. The main difference from the no insurance case is that now whether high returns are indicative of risk depends on expectations of policy response, which determines the cost of illiquidity. For example, if the Central Bank announces that it will reduce policy rates in anticipation of a liquidity shock, then bond returns are not going to rise as much. Equivalently, high returns are not directly translated to high risk if there is a negative shock to $\mathbb{E}(d)$. An analogous interpretation holds when we account for market power.

The next section discusses the identification of s_i using bidding and macroeconomic information and the corresponding econometric procedure.

6. Identifying Individual Liquidity Risk

Despite the non-trivial prediction problem that the banks have to solve, the necessary conditions for optimal price bidding provide enough information to point identify liquidity risk from bid observations $\{(b_i, q_i)\}_{i=1..N}$ conditional on the state of insurance. More specifically, (4) can be re-expressed in terms of s_i as follows:

$$\begin{aligned} s_i &= \frac{q_i(S_{1,i} - \iota_t S_{4,i}) + (b_i - \iota)S_{2,i} + S_{3,i}}{(1+p\mathbb{E}(d))\,(S_{2,i} + q_i S_{4,i}) + (1-p)\,(q_i S_{1,i} + b_i S_{2,i} + S_{3,i})} \\ &= \frac{s_i^{NI}}{1 + p\left(\frac{\mathbb{E}(d)-\iota}{1+\iota} - s_i^{NI}\frac{1+\mathbb{E}(d)}{1+\iota}\right)} \end{aligned} \tag{7}$$

where

$$s_i^{NI} = \frac{q_i(S_{1,i} - \iota S_{4,i}) + (b_i - \iota)S_{2,i} + S_{3,i}}{q_i(S_{1,i} + S_{4,i}) + (b_i + 1)S_{2,i} + S_{3,i}} \tag{8}$$

is the identifying condition in the case of no insurance ($p = 0$).[19] The objects $S_{1,i}, S_{2,i}, S_{3,i}, S_{4,i}$[20] have an intuitive interpretation:

$$S_{1,i} := \frac{\partial \left(\int_{B^c:B^c>b_i} B^c f(B^c|s_i) dB^c \right)}{\partial b_i}$$

measures the ability of the bank to affect the expected equilibrium price by infinitesimally changing the bid in the states in which it is not a cutoff bidder,

$$S_{2,i} := \frac{\partial \left(\int_{q_i^c:B^c=b_i} q_i^c f(q_i^c|s_i) dq_i^c \right)}{\partial b_i}$$

measures the ability of the bank to influence the expected quantity of bonds it receives in the states in which it is the cutoff bidder, while

$$S_{3,i} := \int_{q_i^c:B^c=b_i} q_i^c f(q_i^c|s_i) dq_i^c$$

measures the expected quantity received when the bank is the cutoff bidder, and

$$S_{4,i} := \frac{\partial \left(\int_{B^c:B^c>b_i} f(B^c|s_i) dB^c \right)}{\partial b_i}$$

measures the marginal effect of the bank's bidding behavior on the probability of not being a cutoff bidder.

As long as $S_{1,i}, S_{2,i}, S_{3,i}, S_{4,i}$ are non-parametrically identified from the data, then liquidity risk is point identified.

The above quantities depend on the distributions of the equilibrium interest rate and quantity received by bank i. In order to estimate these distributions,

[19] Correspondingly, when $p = 1$,

$$s_i^{FI} = \frac{s_i^{NI}}{1 - s_i^{NI}} \frac{1+\iota}{1+\mathbb{E}(d)} = \frac{q_i(S_{1,i} - \iota S_{4,i}) + (b_i - \iota)S_{2,i} + S_{3,i}}{(1+\mathbb{E}(d))\,(S_{2,i} + q_i S_{4,i})} \tag{9}$$

[20] All elements on the right hand side are indeed functions of s_i, but we can estimate them non-parametrically.

and hence derive the estimates for s_i^{NI} and consequently s_i, we employ a resampling algorithm similar to Kastl (2011); Hortaçsu and McAdams (2010).

This procedure draws different realizations of residual supply faced by bank i from the pool of observed bids, and therefore all the possible realizations of the auction outcomes, ($B^c, q_i^\star$, "Cutoff/No Cutoff").

Due to the relatively small number of market participants in each auction, we pool auctions that occurred during the first 15 days of the each month and after the 20th day of the previous month. Not pooling the data in different auctions would produce noisier estimates, while by pooling we invoke the implicit assumption that the main unobservable driver of bidding, liquidity risk, is constant across the auctions we pool together. Given that public information (Ω) evolves at a monthly frequency e.g. interest rate decisions by the CB, we believe that this is an innocuous assumption[21].

Moreover, banks that participate more than once within a month are considered as different banks. Since we assume that liquidity risk (s_i) is a draw from a common distribution within the group of banks, including observations from bank i in the same dataset amounts to including an additional realization from this distribution. Effectively, including an additional observation gives a better approximation for the expected equilibrium price and quantities.

We have already entertained the possibility that the distribution of liquidity risk for banks is potentially different from that of the non-banks. We take this into account by separately resampling bids from these two groups. Moreover, since the number of participants varies across auctions, we resample with a weighting scheme that gives higher weight on observations from auctions in which the number of participants is closer to the number of participants in the auction that bank i places its bids. Another potential dimension of heterogeneity is the variability of the number of each of the two types of financial institutions we observe in the data. Given that we already control for variation in the total number, we do not need to control for variation in the within

[21]Most of the monetary policy meetings in our sample period took place from the second to the third week of the month, thus banks in the effective month face a given policy rate.

group size if the share of these types is relatively constant over the sample. In Appendix B3 we provide evidence that this is indeed the case.

We thus employ the following resampling algorithm: Given a pool of T auctions, and a bank i that belongs to a group with size $N_B \leqslant N$ where N is the number of participants in the auction where bank i participates, for $m = 1..M$:

(1) Fix bid vector by bank i: (q_i, b_i).
(2) Draw $N_B - 1$ bid vectors $\{(q_j, b_j)\}_{j=1,\dots,N_B-1}$ from the pool of bids in the group of banks, and N_O bid vectors from the non-bank group, both with weight $w_m = \frac{\mathcal{K}(\frac{N_\tau - N}{h})}{\sum_{\tau=1..T}\mathcal{K}(\frac{N_\tau - N}{h})}$ where $\mathcal{K}$ is the Normal kernel with bandwidth h.[22]
(3) Construct the residual supply curve (share) at each interest rate bid: $q_i^r(b) = 1 - \sum_{j\neq i} q_j(b)$. (Aggregate horizontally)
(4) Compare (q_i, b_i) with q_i^r(b) to compute the equilibrium price, B^c, and then compute the quantity received by bank i at B^c, $q_i^\star$. Bank i is the cutoff bidder if $b_i = B^c$.
(5) Go to 2.

The above algorithm generates a sample $\{B^c_m, q^\star_{i,m},\}_{m=1..M}$, whose empirical distribution can be used to compute $S_{1,i}$, $S_{2,i}$, $S_{3,i}$ and $S_{4,i}$ and therefore s_i.

Partial derivatives in $S_{1,i}$, $S_{2,i}$, $S_{3,i}$ and $S_{4,i}$ are computed by perturbing b_i by an arbitrarily small step ϵ and computing the corresponding numerical derivative of each integral e.g :

$$\hat{S}_{4,i} = \frac{1}{\epsilon}\left(\frac{1}{M}\sum_{m=1..M} 1[B^c_m(q_i, b_i+\epsilon) > b_i + \epsilon)] - \frac{1}{M}\sum_{m=1..M} 1[B^c_m(q_i, b_i) > b_i]\right)$$

where $\{B^c_m(q_i, b_i+\epsilon)\}_{m=1..M}$ is the sample generated by repeating the resampling procedure above with the perturbed demand for bank i.[23] In Figure 1,

[22]We set $h = 2\sigma_{N_\tau} N^{-0.2}$ where σ_{N_τ} is the standard deviation of the number of auction participants.
[23]In the application, we set the $M = 500000$ and $\epsilon = \kappa\sqrt{eps}$ where eps is machine precision error. We have used different step sizes, which are multiples $\kappa \in (10, 100, 1000)$ of $eps = 2^{-52}$, and get similar results. In the aggregate model, we employ the average of the results obtained from the different step sizes.

we plot the realization of a few residual supply curves faced by a specific bank according to the algorithm outlined above.

Interacting the bank's demand with these alternative residual supplies effectively generates a sample of possible equilibrium prices and quantities. A detailed derivation of the equilibrium interest rate and quantity for all possible cases can be found in Appendix A3, along with more graphical examples, including the cases in which there is positive residual supply of bonds for bank i at any level of the interest rate.

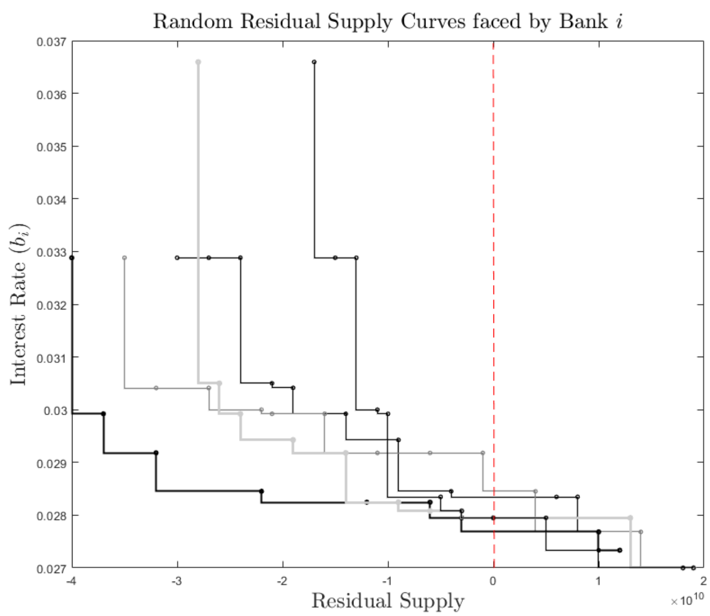

FIGURE 1

In the rest of the paper, we employ the predictions of the model in order to estimate the time varying distribution of liquidity risk in the case of Chile. Since (4) depends on the probability of insurance p, which in turn depends on macroeconomic factors, in the next section we consider an aggregate model which will allow us to take them into consideration. More importantly, the model we develop can also be used to shed some light on the effects of policy shocks on the aggregate component of this distribution.

7. Estimation using Micro and Macro Data

Since we are interested in the evolution of liquidity risk, from now on, we consider a sequence of auctions, and thus index idiosyncratic risk also by time (t). Note also that we have not imposed any restriction on the relationship between the distribution function of liquidity risk F_t and its past, $F_{t-l,l>0}$.[24] Our maintained assumption is that $s_{i,t} \underset{iid}{\sim} F_t(. \mid \Omega)$, and thus aggregate shocks can affect all cross sectional moments of $s_{i,t}$. In what follows, we adapt the previous analysis in order to build a model that can provide empirically plausible measures of this risk and its variation over time.

From the earlier discussion of the identification of individual liquidity risk, it is clear that for identifying $s_{i,t}$ we need to obtain estimates of $s_{i,t}^{NI}$, and p_t. The first can be readily estimated using the algorithm described in the previous section. As for the probability of insurance p_t, while exogenous to the bank, it is likely to be a variable that reacts to macroeconomic conditions. Thus, in order to estimate it, we consider a joint model of aggregate risk, policy and other macroeconomic variables.

We define aggregate risk to be the first moment $\mathbb{S}_t := \int s_{i,t} dF_t(s_{i,t}|\Omega)$, which is the average probability of a liquidity shock in the next period conditional on any public information, including the Central Bank's actions. To facilitate aggregation and matching with observable moments, we first order approximate (7) along the p_t dimension, which yields that[25]

$$s_{i,t} \approx s_{i,t}^{NI}\left(1 - p_t\left(\frac{d_t - \iota_t}{1+\iota_t} - s_{i,t}^{NI}\frac{1+d_t}{1+\iota_t}\right)\right)$$

Aggregating s_i across banks yields that aggregate risk can be linked to the distribution of the no-insurance measure as follows:

$$\mathbb{S}_t = \mathbb{S}_t^{NI}\left(1 - p_t\left(\frac{1+d_t}{1+\iota_t} - 1\right)\right) + p_t\left(\frac{1+d_t}{1+\iota_t}\left(\mathbb{V}_t^{NI} + \left(\mathbb{S}_t^{NI}\right)^2\right)\right) \quad (10)$$

[24] As the object of interest is the distribution for the banking sector, for brevity we drop the group index on the distribution.

[25] The first order approximation is only done for the purpose of estimating the aggregate state space model, and is not employed when we later on recover individual $s_{i,t}$ using the estimated p_t.

where $\mathbb{S}_t^{NI} = \int s_i^{NI} dF_t(s_i^{NI})$ and $\mathbb{V}_t^{NI} = \int (s_i^{NI} - \mathbb{S}_t^{NI})^2 dF_t(s_i^{NI})$. The first two moments of the distribution of $s_{i,t}^{NI}$ are therefore sufficient statistics for adjusting the raw estimate S_t^{NI} to the possibility of insurance.

We view (10) as an observation equation, meaning that once all quantities on the right hand side are known, we can readily estimate the level of aggregate risk $\mathbb{S}_t$. While the algorithm proposed in section 6 provides us with estimates of $\mathbb{S}_t^{NI}$ and $\mathbb{V}_t^{NI}$, p_t remains unknown. In order to estimate it, we make certain additional modeling assumptions regarding its link to the aggregate state of the economy, summarized by a state vector X_t. We will thus allow this probability to depend on the elements of X_t that can affect the credibility and ability of the state to intervene and provide emergency liquidity as well as the ability of the rest of the financial system to privately provide such liquidity to the individual bank.

7.1. **Modeling partial insurance.** Following the large literature on logistic modeling[26], we model this probability as a logistic function, where the log-linear index depends on aggregate states as follows:

$$p_t = \frac{e^{\boldsymbol{\alpha}'\Theta_{\mathfrak{J}} ln(X_t)}}{1 + e^{\boldsymbol{\alpha}'\Theta_{\mathfrak{J}} ln(X_t)}} \in (0,1) \tag{11}$$

$\Theta_{\mathfrak{J}}$ is a matrix that selects the relevant elements $j \in \mathfrak{J}$ of X_t and $\boldsymbol{\alpha}$ the corresponding coefficients that load on $ln(X_t)$. This specification implies that the elasticity of p_t with respect to factor X^j is equal to

$$e_{p,j} = \alpha_j(1 - p_t)$$

Therefore, the elasticity is not constant, but is instead low (high) in absolute value at high (low) levels of this probability. This is a reasonable implication as it reflects both diminishing returns to policy at very high levels of this probability, as well as the fact that it takes a large shock in factor X_t to make a difference if the economy is already doing well in providing liquidity to its financial institutions, both privately and publicly. Furthermore, as we will be analyzing fluctuations at a monthly frequency, the relevant determining factors are less likely to be those that influence the possibility of insurance in

[26]See e.g. Ch.11 in Stock and Watson (2011).

the longer term, such as the institutional design of the Central Bank liquidity facilities and of the interbank market. What this also implies though is that we cannot identify the contribution of these slow moving factors.

Given our assumptions, identifying the probability boils down to identifying $\boldsymbol{\alpha}$. In order to do achieve this, we introduce additional information through a medium scale macroeconomic model. The model will generate enough restrictions that will enable us to estimate $\boldsymbol{\alpha}$, p_t, and thus $s_{i,t}$ in (7) at a monthly freequency.

7.2. **The Aggregate Model.** The set of variables that will summarize the joint dynamics of financial markets and the rest of the economy are defined as $X_t = \left(V_t^{for}, U_t, \pi_t, \sigma_t, R_t, \mathbb{S}_t, \mathbb{V}_t^{NI}, d_t, \iota_t, Q_t\right)$. V_t^{for} is foreign volatility, that captures foreign risk factors; U_t is the unemployment rate, π_t the inflation rate and σ_t the real exchange rate, all summarizing the business cycle for the small open economy. To capture macro-financial policy we introduce the foreign reserves to GDP ratio, R_t, the supply of 30-day bonds, Q_t, the aggregate expectations of next period's deposit rate, d_t, and the nominal interest rate, i_t. Finally, $\mathbb{S}_t$ is aggregate risk and $\mathbb{V}_t^{NI}$ is the variance of the distribution of liquidity risk in the no-insurance case.

In line with the macroeconomic literature, we will empirically model the joint dynamics of these states by employing a log-linear Gaussian state space model at monthly frequency:

$$\tilde{X}_t = A\tilde{X}_{t-1} + B\epsilon_t$$

where $\tilde{X}_t$ signifies log-deviations of X from its long-run trend $\bar{X}$.

The set of measurements we employ for model estimation are denoted by $\tilde{Y}_t = \left(\hat{V}_t^{for}, \Delta\hat{U}_t, \hat{\pi}_t, \Delta\hat{\sigma}_t, \Delta\hat{R}_t, \hat{\mathbb{S}}_t^{NI}, \hat{\mathbb{V}}_t^{NI}, \hat{\iota}_t, \hat{Q}_t\right)$. Besides the mean and variance of $s_{i,t}^{NI}$, which we estimated using the resampling algorithm in section 6, and promissory note aggregate supply $\hat{Q}_t$[27], the macroeconomic data we use are the 30-day expectations of stock market volatility in the US (VIX) as a measure for $\hat{V}_t^{for}$, the unemployment rate for all persons above 15 years old for $\hat{U}_t$, the

[27]We compute $(\tilde{\mathbb{S}}_t^{NI}, \tilde{\mathbb{V}}_t^{NI})$ by computing the mean and variance of s_i^{NI}. Since the estimates of individual risk may fall outside $(0,1)$, we apply the monotone transformation $s_i^{NI} = \frac{\exp(s_{raw,i}^{NI})}{1+\exp(s_{raw,i}^{NI})}$.

Consumer Price Index Inflation for $\hat{\pi}_t$, the real broad effective exchange rate expressed in terms of foreign currency for $\hat{\sigma}_t$, the ratio of foreign reserves to (lagged) GDP for $\hat{R}_t$, and the benchmark monetary policy rate for $\hat{\iota}_t$.[28]

We next illustrate how we link the set of states ($\tilde{X}_t$) to the set of measurements ($\tilde{Y}_t$). Consistent with the macroeconomic model, we adopt a log-linearized version of equations (10) and (11) as follows:

$$
\begin{aligned}
\text{(12)} \qquad \tilde{\mathbb{S}}_t &= \frac{\bar{\mathbb{S}}^{NI}}{\bar{\mathbb{S}}}\left(1+\bar{p}\left(1-\left(\frac{1+\bar{d}}{1+\bar{\iota}}\right)(1-2\bar{\mathbb{S}}^{NI})\right)\right)\tilde{\mathbb{S}}_t^{NI} \\
&\quad +\frac{\bar{p}}{\bar{\mathbb{S}}}\left(-\bar{\mathbb{S}}^{NI}\left(\frac{\bar{d}-\bar{\iota}}{1+\bar{\iota}}\right)+\frac{1+\bar{d}}{1+\bar{\iota}}\left(\bar{\mathbb{V}}^{NI}+\left(\bar{\mathbb{S}}^{NI}\right)^2\right)\right)\tilde{p}_t \\
&\quad +\frac{\bar{p}}{\bar{\mathbb{S}}}\left(\frac{1+\bar{d}}{1+\bar{\iota}}\right)\left(-\bar{\mathbb{S}}^{NI}+\left(\bar{\mathbb{V}}^{NI}+\left(\bar{\mathbb{S}}^{NI}\right)^2\right)\right)\tilde{d}_t \\
&\quad -\left(1-(1+\bar{p})\frac{\bar{\mathbb{S}}^{NI}}{\bar{\mathbb{S}}}\right)\tilde{\iota}_t \\
&\quad +\frac{\bar{p}}{\bar{\mathbb{S}}}\left(\frac{1+\bar{d}}{1+\bar{\iota}}\right)\bar{\mathbb{V}}^{NI}\tilde{\mathbb{V}}_t^{NI} \\
\text{(13)} \qquad \tilde{p}_t &= (1-\bar{p})\boldsymbol{\alpha}'\Theta_{\mathfrak{J}}\tilde{X}_t
\end{aligned}
$$

Therefore, the link of unobservables to observables can be parameterized by a matrix H, which is the identity matrix apart from the sixth row that corresponds to $\tilde{S}_t$:[29]

$$\tilde{X}_t = H(\boldsymbol{\alpha})\tilde{Y}_t$$

Since H is invertible, the reduced form model that is taken to the data is a first order vector autoregression with cross equation restrictions implied by a

[28]We describe the macro dataset and variable transformations in Appendix A4. We compute deviations from trend for all variables by removing quadratic time trends, as well as computing first differences to remove stochastic trends in the case of the Reserve to GDP ratio, the unemployment rate and the real exchange rate in the baseline specification. We also report in Appendix A10 results for other specifications regarding reserves e.g. using the level instead of first differences for the reserves to GDP ratio. The results are similar.

[29]To make estimation feasible, we set $\tilde{d}_t = \tilde{\iota}_t$ as $\tilde{d}_t$ and $\tilde{\iota}_t$ are strongly correlated leading to near-collinearity. In Appendix A7, we plot the interest rate and two measures of expectations of one month ahead interest rate which confirm this strong co-movement. Nevertheless, the long run averages are different, and reflect the (constant) spread between the rate at which banks borrow from the central bank and the benchmark policy rate rate, $\bar{d} = \bar{\iota} + 0.25$.

combination of structural and measurement relationships in (B, H):

$$\begin{aligned} \tilde{Y}_t &= H^{-1}AH^{-1}\tilde{Y}_{t-1} + H^{-1}B\epsilon_t \\ &= C\tilde{Y}_{t-1} + D\epsilon_t = C\tilde{Y}_{t-1} + u_t \end{aligned} \tag{14}$$

where $\epsilon_t \sim N(0, \Sigma_\epsilon)$ with Σ_ϵ a diagonal matrix.[30]

We furthermore define $\epsilon_t = (\epsilon_{V_t^{for}}, \epsilon_{dem,t}, \epsilon_{sup,t}, \epsilon_{\sigma,t}, \epsilon_{R,t}, \epsilon_{R_t^{news}}, \epsilon_{s,t}, \epsilon_{\iota,t}, \epsilon_{Q,t})'$. The structural shocks we seek to identify are a foreign volatility shock $\epsilon_{V_t^{for}}$, a demand and supply shock, $\epsilon_{dem,t}$ and $\epsilon_{sup,t}$ respectively, a real exchange rate shock, $\epsilon_{\sigma,t}$, a non-anticipated and an anticipated (one period ahead) shock to the foreign reserve to GDP ratio, $(\epsilon_{R,t}, \epsilon_{R_t^{news}})$, a shock to aggregate beliefs about risk $\epsilon_{\mathbb{S},t}$, a shock to the nominal interest rate $\epsilon_{\iota,t}$, and a shock to bond supply, $\epsilon_{Q,t}$.

7.2.1. *Identification.* The parameters of interest are the elements of B, $\boldsymbol{\alpha}$ and Σ_ϵ, which are identified via solving

$$H^{-1}(\boldsymbol{\alpha})B\Sigma_\epsilon B'H^{-1}(\boldsymbol{\alpha}) = \Sigma_u \tag{15}$$

where Σ_u is the covariance matrix of the VAR residuals. This results in a system of nonlinear equations in $\boldsymbol{\alpha}$, the unrestricted elements of B and the diagonal covariance matrix of the shocks, Σ_ϵ.[31]

Identifying $\boldsymbol{\alpha}$ will enable the identification of distribution of liquidity risk at all times, while identifying B will allow us to trace the effect of aggregate shocks to the states in X_t, including the aggregate component of liquidity risk. This will be important later on when we investigate the effects of reserve accumulation and monetary policy shocks.

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[30]We have tested for autoregression lags up to 8th order. The Schwarz criterion selects one lag, which we adopt.

[31]As is standard in the macroeconometric literature, the covariance matrix of the VAR residuals is matched with $H^{-1}(\boldsymbol{\alpha})B\Sigma_\epsilon B'H^{-1}(\boldsymbol{\alpha})$ to obtain estimates of the underlying components of $\boldsymbol{\alpha}$, B and Σ_ϵ. It can be shown that the restrictions employed imply satisfying both the order and rank condition for identification, that is, there are 39 independent identifying conditions for the 39 unknown parameters, and the Jacobian matrix has full rank. In the supplemental material, we provide code for analytically checking the rank of the Jacobian for chosen restrictions.

In what follows, we discuss our identifying restrictions. The foreign volatility is ordered first as it is exogenous to the system. The demand shock is identified by ordering it first within the home variables, that is, unemployment responds contemporaneously to foreign and local demand conditions, and only with a lag to domestic financial conditions. Similarly, inflation reacts to foreign and domestic demand conditions, as well as to a supply shock. Since foreign reserves interventions might well be due to attempts to respond to developments in foreign risks and the exchange rate, foreign reserves are allowed to contemporaneously respond to both $\epsilon_{v_t^{for}}$ and the real exchange rate shock $\epsilon_{\sigma,t}$, as well as on the supply shock $\epsilon_{sup,t}$. The latter can also be justified if current inflation developments affect the level of the intervention, if the latter is only partially sterilized. Furthermore, we allow for some anticipation to changes in foreign reserves as announced policies usually specify gradual changes in the stock of reserves. Given the uncertainty that exists around the actual implementation i.e. timing and volume, we deem that allowing for one period is a reasonable approximation. The news shock is identified by imposing a zero restriction on the contemporaneous response of reserves and a zero restriction on the contemporaneous response of Q_t, as Q_t is a monetary policy operation that aims to absorb existing liquidity in the system.

A shock to foreign reserve accumulation is therefore an innovation to the process that happens contemporaneously, was not pre-announced, and cannot be explained by current developments such as unexpected movements in either foreign volatility, the real exchange rate or a supply-driven shock to inflation.

The real exchange rate is allowed to contemporaneously react to foreign and domestic conditions, as well as to actual foreign reserve accumulation shocks and news about these interventions.[32]

Aggregate beliefs about liquidity risk and $\tilde{\mathbb{V}}_t^{NI}$ are allowed to contemporaneously respond to all shocks but the macroeconomic shocks, $\epsilon_{dem,t}, \epsilon_{sup,t}, \epsilon_{\sigma,t}$,

[32]This is also in line with the results of the event study in (Chamon, Hofman, Magud, and Werner, 2019), which utilizes high frequency data on the bilateral Peso-US dollar nominal exchange rate and finds that announcements have clearer and more significant effects than the actual interventions. Our measure is of course based on the real broad effective exchange rate, but the US receives one of the highest weights, in the range of 0.15 and 0.2.

and $\epsilon_{Q,t}$. The rationale behind these restrictions is that banks' beliefs about liquidity risk depend on the macroeconomic conditions as signalled by interest rate changes and reserve accumulation rather that current shocks to unemployment, inflation or the real exchange rate.

Finally, the nominal interest rate is allowed to contemporaneously respond to shocks to macroeconomic conditions, whether domestic or foreign, which is consistent with a Taylor rule interpretation of monetary policy. Below, we summarize the set of restrictions placed on the impact matrix B.

$$\begin{pmatrix} & \epsilon_{v_t^{for}} & \epsilon_{dem,t} & \epsilon_{sup,t} & \epsilon_{\sigma,t} & \epsilon_{R,t} & \epsilon_{news,t} & \epsilon_{s,t} & \epsilon_{\iota,t} & \epsilon_{q,t} \\ V_t^{for} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_t & \star & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \pi_t & \star & \star & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_t & \star & \star & \star & 1 & \star & \star & 0 & 0 & 0 \\ R_t & \star & 0 & \star & \star & 1 & 0 & 0 & 0 & 0 \\ S_t & \star & 0 & 0 & 0 & \star & \star & 1 & \star & 0 \\ Var(s_t^{NI}) & \star & 0 & 0 & 0 & \star & \star & \star & \star & 0 \\ \iota_t & \star & \star & \star & \star & 0 & 0 & 0 & 1 & 0 \\ Q_t & 0 & 0 & 0 & \star & \star & 0 & \star & \star & 1 \end{pmatrix}$$

7.3. **Probability index.** Regarding the identification of the impact of aggregate states on the probability of liquidity insurance, $\boldsymbol{a}$, we restrict attention to the subset of states in X_t that are a priori relevant for the probability of liquidity insurance.[33] Both the macroeconomic literature as well as policy makers have stressed the importance of the credibility of the lender of last resort. Calvo (2006) highlighted the importance of foreign reserve accumulation in emerging market economies as a complementary tool to injections of domestic liquidity in the face of a bank run, as in the presence of currency substitution the additional liquidity can cause excessive volatility to the exchange rate and inflation. This can undermine the effectiveness of the LOLR policy itself. Obstfeld, Shambaugh, and Taylor (2010) argue that a large scale bailout might increase concerns of public insolvency, rendering reserve sales

[33] We have investigated the possibility of identifying the whole vector $\boldsymbol{a}$ but it is not possible unless we further restrict the impact matrix B.

necessary to support the exchange rate while they also find that reserve accumulation correlates strongly with financial stability variables. Bocola and Lorenzoni (2017)) make a similar point that accumulating reserves enhances the fiscal credibility of the lender of last resort. Reserves should therefore be one of the candidate factors, in conjunction with the real exchange rate, as the latter can also determine the effectiveness of the lender of last resort policy and of reserve accumulation itself, as a large depreciation may require larger reserve accumulation to maintain the credibility of the LOLR. In addition, as reserves are expressed as a ratio to lagged GDP, we partially control for fluctuations that are related to the business cycle.

The nominal interest rate itself can have an effect on the probability of having a succesfull match in the interbank market, yet the aggregate effect will depend on its relative impact on banks' liquidity positions, affecting therefore market tightness. Allen, Carletti, and Gale (2009) argue that setting the interest rate through open market operations might be enough to facilitate liquidity risk sharing in the interbank market. Freixas, Martin, and Skeie (2011) make a similar argument for a dynamic interest rate policy that enables efficient liquidity reallocation in the face of distributional liquidity shocks. Controlling for the interest rate is thus considered sufficient to capture these effects in a reduced form way.

Given our choice of observables, the index function that corresponds to p_t is as follows:

$$\boldsymbol{\alpha}'\Theta_{\mathfrak{J}}ln(X_t) \equiv \alpha_R ln(R_t) + \alpha_\iota ln(\iota_t) + +\alpha_e ln(\sigma_t) \tag{16}$$

Finally, as in the rest of the model, for estimation we employ a log-linearized version of the probability of insurance (13), and thus all variables are expressed in deviations from long run trends. Once $\boldsymbol{\alpha}$ is estimated, we use (11) to back out the probability.

7.4. **Details on Estimation.** The model is estimated using conventional Bayesian methods. Inference is based on 15000 posterior draws using the Gibbs sampler, discarding 2000 draws as burn in. We use the standard prior of independent Normal-Wishart, with $C \sim N(0, I_{ny})$ and $\Sigma_u^{-1} \sim Wishart(I_{n_y}, n_y)$, the degrees of freedom being equal to the number of observables (n_y).[34]Given the posterior draws of $(vec(C)^T, vec(B)^T, \alpha)^T$, we are able to recover both the distribution of liquidity over time, as well as the identified effects of shocks to the aggregate component of this distribution.

Since the auction based measures of $\tilde{\mathbb{S}}_t^{NI}$ and $\tilde{\mathbb{V}}_t^{NI}$ are necessarily noisy due to the finite simulation size, as well as due to other idiosyncratic factors in banks' valuations that our auction model may not capture, we need to account for measurement error in our estimation. To account for the first type of noise, we estimate the variance of these estimates using the bootstrap. Under classical assumptions for the measurement error, ν, we can use the bootstrap variance to calibrate Σ_ν and correspondingly modify Σ_U in (15). In Appendix B4 we lay out the details and plot the bootstrap distribution. To account for the second type of error, we report results based on inflating the measurement error variance (Σ_v) by 10% and 20%.

7.5. **Distribution Estimates.** In Figure 2, we present the estimated distribution of liquidity risk across time by summarizing it through a low-medium-high classification.[35] The figure plots the mass of banks that corresponds to the last two categories, with low risk being the complement. We can see that in normal times, at most 3% of the institutions faced an elevated level of liquidity risk, while in some periods there is essentially no financial institution with liquidity risks of concern. At the same time, we can see that in certain months the percentage of banks that are in the high risk region reaches $6-7\%$. This is especially true during the US Financial crisis of 2007-2008, with the Lehman Brothers episode in September 2008, as well as December 2011, where domestic money markets faced some increasing liquidity pressure leading to elevated funding costs (see IMF (2012)). What this also implies is that the skewness of this distribution increases during turbulent times, as expected.

[34]See Koop and Korobilis (2010) for details on the implementation and the posterior, which has the same Normal - Wishart form as the prior. For every posterior draw of Σ_u, we obtain a draw for $(\alpha, B, \Sigma_\epsilon)$ using GMM based on the moments: $vech(H^{-1}B\Sigma_\epsilon B' H^{-1} - \Sigma_u) = 0$.
[35]We classify the estimated $s_{i,t}$ into 'low', 'medium' and 'high' risk, using 0.3 and 0.6 as thresholds. Results are not sensitive to the choice of threshold, as most of the institutions feature risks much less than the first threshold, and very few of them are at high risk, above the upper threshold.

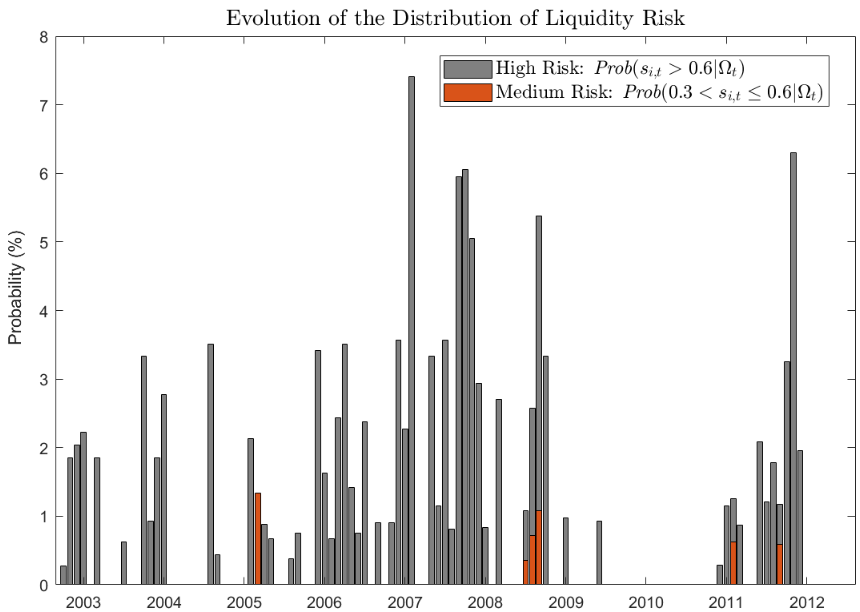


FIGURE 2

While interesting in itself for descriptive purposes, one can use the estimated distribution and the underlying methodology for further applications. In the rest of the paper we provide two such applications: we will firstly employ our estimates to improve our understanding of the effects of monetary policy and foreign reserve management on the aggregate beliefs about liquidity risk. Secondly, we will show how one could employ the methodology in settings similar to ours and how the corresponding estimates of s_i could be used as a calibration target for quantitative models with banking sectors subject to funding risk (see e.g. Corbae and D'Erasmo (2021); Bianchi and Bigio (2022)).

8. Does policy convey information about aggregate risk?

Prior to presenting the empirical results from the structural VAR, it is useful to be reminded of what our auction based measure of liquidity risk consists of, as well as what is captured by our measures of policy shocks. Both will be important for providing an intuitive interpretation to the SVAR results in the rest of our analysis.

Before the auction, each bank learns its individual liquidity risk (s_i), whose distribution F is conditional on Ω, the publicly available information about the true state of the economy, Ω^*. Given the information set Ω, banks form beliefs about the distribution of risk, and consequently about aggregate risk which we have defined as the average probability of becoming illiquid in the near future. Similarly, define S^* as the state of aggregate liquidity risk under complete information about the true state of the economy.

We distinguish between Ω and Ω^* as it is reasonable to assume that $\Omega \subseteq \Omega^*$, since banks may not have complete information about the true state of the economy. Moreover, it is also plausible that the Central Bank has additional private information about Ω^*, not contained in Ω. This could be a result of information choice, availability, or simply more targeted analysis of the likely path of the economy given the available information. If banks observed Ω^*, they would be able to accurately predict S^* and thus no additional information would affect financial market perceptions about aggregate risk. However, if banks cannot directly observe the true state of aggregate liquidity risk, policy actions taken by the Central Bank can enlarge their information set as they might reveal a response to superior information about this risk.

A concrete exposition of this observation is as follows. Suppose that the signal received by each bank has an additive form:

$$s_{i,t} \quad = \quad \mathbb{S}_t + \xi_{i,t}$$

where $\xi_{i,t}$ is the idiosyncratic component and $\mathbb{S}_t$ the aggregate component. The latter is an expectation of $S^\star$ given all available information, which we split into information known to banks excluding any CB actions (Ω_t^{banks}), and the CB actions themselves, which are functions of Ω_t^{CB}:

$$\mathbb{S}_t = \mathbb{E}\left(\mathbb{S}_t^\star \mid \Omega_t\right) = \mathbb{E}\left(\mathbb{S}_t^\star \mid \Omega_t^{banks}, \Omega_t^{CB}\right)$$

$\mathbb{S}_t$ will be insensitive to information generated by the CB actions if this was already contained in Ω^{banks}. Otherwise, an action by the Central Bank that reveals more information ($\Omega_t^{banks} \subset \Omega_t^{CB}$), will cause banks to contemporaneously update their beliefs as Ω_t^{CB} is not redundant. In the aggregate model, we consider two types of actions, changing foreign reserves and the monetary policy rate, while we also consider news about future foreign reserve accumulation.

To see how policy may move expectations, consider the extreme case in which banks do not have information about $\mathbb{S}^\star$ while the Central Bank has perfect information. Hence $\Omega_t^{banks} = \varnothing$ and $\Omega_t^{CB} = \Omega^\star$.

Focusing on the case of reserve accumulation, consider a policy rule that describes how the Central Bank sets its policy as a function of its current information about liquidity risk and other shocks, summarized by $\hat{\eta}_{R,t} \sim N(0, \sigma^2_{\hat{\eta},R})$:

$$R_t = R_{t-1} + \phi_s \mathbb{S}_t^\star + \hat{\eta}_{R,t}$$

At the same time, aggregate liquidity risk may endogenously respond to policy, as follows:

$$\mathbb{S}_t^\star = \delta_R \Delta R_t + \hat{\eta}_{S^\star,t}$$

with $\hat{\eta}_{S*,t} \sim N(0, \sigma^2_{\hat{\eta},S*})$. It can then be shown that observing a policy action will induce an update to banks' aggregate beliefs as follows:

$$\mathbb{E}\left(\mathbb{S}_t^\star \mid \Delta R_t\right) = \frac{\phi_s \sigma^2_{\hat{\eta},S^\star} + \delta_R \sigma^2_{\hat{\eta},R}}{\phi_s^2 \sigma^2_{\hat{\eta},S^\star} + \sigma^2_{\hat{\eta},R}} \Delta R_t$$

Changes in policy can therefore have two effects on aggregate beliefs about risk: a direct effect on the possibility of a severe liquidity shock as well as a signalling effect. The sign of the total effect depends on which effect dominates, that is, how strongly policy makers react to risk ($\phi_s > 0$) as well as how much the state of risk is in turn affected by the policy at hand, δ_R, which is expected to be negative.

Moreover, notice that in the case in which the CB does not respond to any other shock i.e. $\sigma^2_{\hat{\eta},R} = 0$, or shocks to aggregate risk are severe i.e. $\sigma^2_{\hat{\eta},S^\star} \to \infty$, private banks can perfectly infer the true state of aggregate risk from changes in reserves as $\mathbb{E}\left(\mathbb{S}_t^\star \mid \Delta R_t\right) = \frac{1}{\phi_s} \Delta R_t = \mathbb{S}_t^\star$. The only effect in this case is just informational.

The opposite is true when foreign reserves respond mostly to other macro shocks, or shocks to aggregate risk are very small. The same kind of analysis can be used for changes in the interest rate, where the relevant policy rule is the Taylor rule. We postpone the relevant discussion to later on.

Turning to the SVAR, we have allowed both reserve changes and the interest rate to contemporaneously react to external and domestic macro shocks that are independently identified in the system. In terms of the above example, these could correspond to $\hat{\eta}_{R,t}$ (and $\hat{\eta}_{\iota,t}$ in the case of monetary policy). Hence, the unsystematic component of these policies that is still not explained by reactions to $(\hat{\eta}_{R,t}, \hat{\eta}_{i,t})$ must contain reactions to information that cannot be inferred from the rest of the publicly observed variables. This is exactly the type of superior information that the Central Bank may have about the economy, as compared to private market participants. If then these discretionary policy changes have an impact on our measure of aggregate beliefs about liquidity risk, part of it could then be attributed to the purely informational effect. The remaining effect would necessarily be explained by an impact on the actual probability of a severe liquidity shock in the future.

While identifying (ϕ_s, δ_R) or $(\phi_\iota, \delta_\iota)$ is beyond the scope of this paper, we will use the above insights to interpret the sign of the responses to shocks in policy in the SVAR.[36] In Figure 3, we plot the impulse response function of aggregate risk as expected by the banks with respect to the identified shock to reserve accumulation. The impact effect is significantly positive; an increase in the growth rate of foreign reserves by 1% is associated with a median 3.8% increase in expected aggregate risk.[37]

In light of the earlier theoretical insights, we interpret our result as evidence for an *informational* channel of foreign reserve policy. As we already discussed, a positive shock to foreign reserve accumulation by the Central Bank can impact aggregate beliefs about risk in two opposite ways. First, an increase in the growth rate of reserves implies a permanent positive shock to the level of reserves, which may act as a deterrent to possible self fulfilling speculative

[36]Notice also that the microfounded model implies that the covariance of innovations to the reserve accumulation and the interest rate in (14) should not be zero but it is restricted to be so to achieve identification of the rest of the parameters.

[37]In all exercises, the lighter shade corresponds to the 90% pointwise credible sets and the darkest shade to the 68% credible sets.

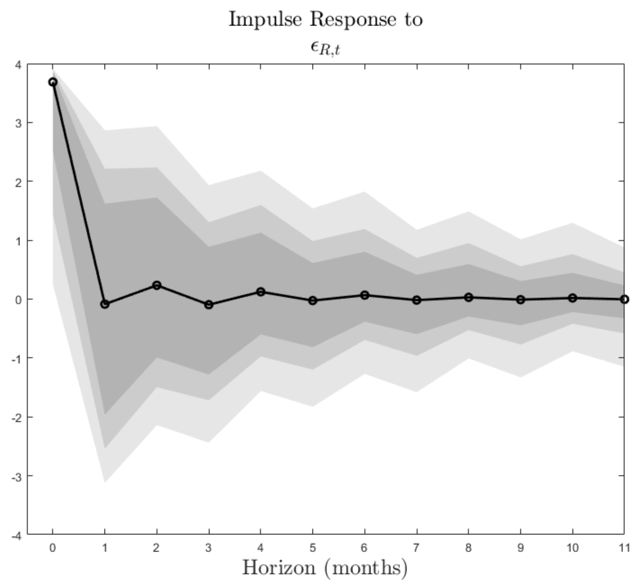


FIGURE 3. The effect of reserve accumulation on aggregate risk as expected by the banking sector.

attacks, leading to a fall in the probability of an externally induced liquidity shock in the next period. On the other hand, reserve accumulation may signal to private financial institutions that the Central Bank has information of an increase in the average probability of a liquidity shock in the next period. A net positive effect implies that the signalling channel dominates.[38]

Figure 4 presents more informal evidence on the efforts of policy makers to accumulate adequate reserves. The figure plots the ratio of the actual level of reserves to a measure of the efficient level for several emerging market economies. Reserve accumulation efforts during and after the crisis are consistent with striving to achieve a ratio above or close to one.

In Figure 5 we plot the corresponding ratio for Chile against the actual changes in reserves from 2000 to 2018. As can be observed, when the ratio falls below

[38]Reserve accumulation is indeed a signal for future liquidity issues as it is considered a precautionary measure against such crises, see e.g. Obstfeld, Shambaugh, and Taylor (2010); Bocola and Lorenzoni (2017); Céspedes (2019). In fact, Obstfeld, Shambaugh, and Taylor (2010) emphasize that the need for ample reserves is higher in financially open economies. In episodes of internal and external liquidity drains and concerns about public insolvency due to large-scale bailouts, there might be no other means for managing the exchange rate.

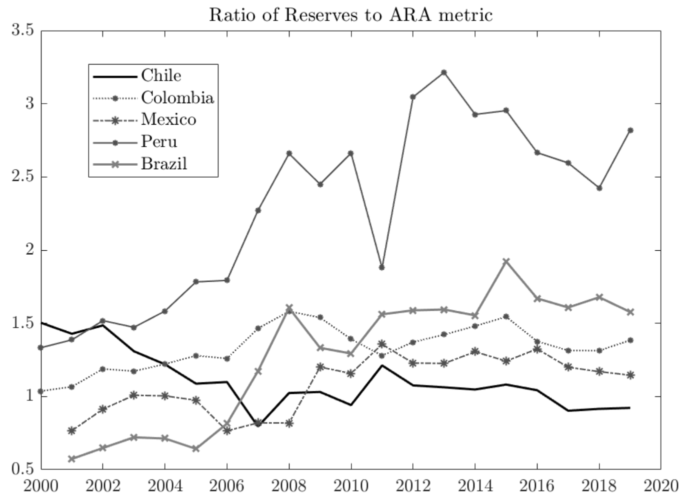


FIGURE 4. The ARA metric developed by the International Monetary Fund (IMF) estimates the efficient level of reserves that balances the benefits of holding reserves (e.g. reduced likelihood of Balance of Payment crises, financial stability, policy autonomy) against the corresponding opportunity costs (lower returns) . A ratio of reserves to ARA between 1.5 and 1 is considered adequate. Source: IMF, Assessing Reserve Adequacy.

one, as in e.g. 2006-2007 and in 2010, there is a subsequent increase of reserve accumulation. We interpret this a sign of commitment by the Central Bank to maintain financial stability, amongst other goals.

Furthermore, in Figure 6 we can see that there is no conclusive evidence of an effect of news about increments to reserves in the next period. One interpretation of this finding is that actual increases in reserves are more credible than announcements of future interventions as the former incur an actual cost.

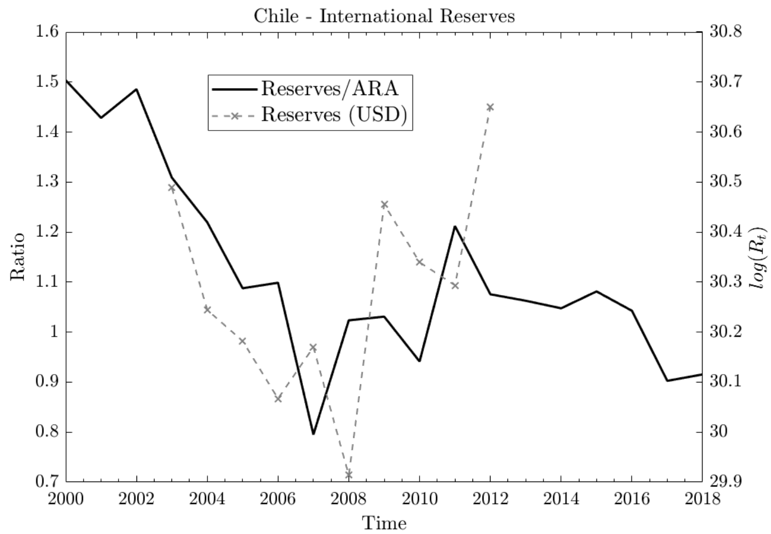


FIGURE 5. Reserve Accumulation

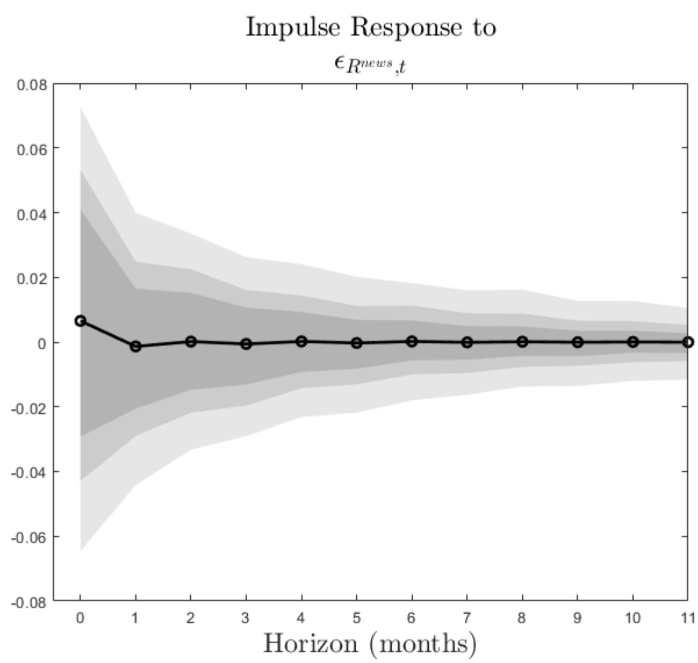


FIGURE 6

Indeed, there are cases where announcements of future interventions were only partially carried through in Chile. For example, while BCCh announced in April 2008 a series of interventions that were meant to last until the end of the year, they were prematurely ended with the Lehmann episode (Chamon, Hofman, Magud, and Werner, 2019). This can rationalize the absence of evidence on a significant effect of news on aggregate risk. These results are also consistent with the strand of literature that argues that the costliness of reserve accumulation is a feature that enhances credibility, and therefore the effectiveness of this policy, either in a direct way such as in Fanelli and Straub (2021)[39] or in an indirect way, by reducing uncertainty about future policy (Vitale, 2003).

An alternative interpretation for the positive co-movement between the exogenous movements to reserves and aggregate risk could be that the Central Bank and private banks are simply responding to a shock which leads the former to accumulate more foreign reserves and the latter to bid more aggressively, due to the higher opportunity cost of investing in the auction. However, given that we are separately identifying both a foreign risk shock, a real exchange rate shock as well as domestic demand and supply shocks, in addition to an independent shock to aggregate beliefs about liquidity risk, it is very unlikely that our measure of a shock to reserve accumulation is contaminated with an alternative, non-identified shock that has some explanatory power for aggregate risk as well.

Regarding the effects of monetary policy, Figure 7 presents posterior evidence that a rise in the nominal rate increases aggregate beliefs about liquidity risk.

[39]Interestingly, the theoretical results by Fanelli and Straub (2021) show that credibility is a necessary condition for positive optimal foreign exchange interventions, as long as there is some restriction on capital flows. A key ingredient of the theory is that interventions are costly in terms of carry trade profits made by foreign investors trying to exploit interest rate spreads, and the utility gain does not outweigh these costs if the commitment horizon shrinks to zero.

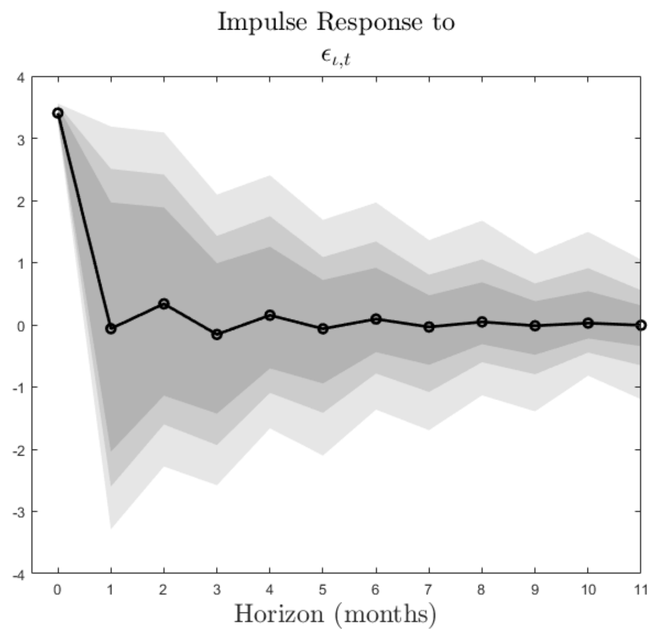


FIGURE 7

Our preferred explanation of this fact can also be found in the existence of asymmetric information between the private sector and the policy maker. Given our definition of $s_{i,t}$, and hence $\tilde{\mathbb{S}}_t$, liquidity risk is associated with events that have a direct impact on the liquidity position of banks, controlling for the fact that they might actively seek for additional liquidity in the interbank market in the event of a severe shock. Hence, while an increase in the interest rate absorbs liquidity from the market, liquidity risk will not necessarily be affected by this policy action.

Increases in the interest rate can actually decrease the risk of a severe liquidity shock as they put downward pressure on capital flight, which could be an illiquidity triggering event in an emerging market economy such as Chile. Moreover, an increase in the interest rate can signal information by the CB on higher liquidity risk due to capital flight in the near future. If this latter signalling effect dominates, $\tilde{\mathbb{S}}_t$ will eventually rise on impact. Since the model's horizon is only monthly, we also find it highly unlikely that increases in the interest rate would increase liquidity risk due to bank debtors' inability to meet repayments. Our result therefore adds to the results obtained in the literature on signalling effects of monetary policy on other macroeconomic aggregates, such as output growth (Nakamura and Steinsson, 2018) or inflation expectations (Melosi, 2016).

One might also wonder whether these results on foreign reserve accumulation and monetary policy are driven by the fact that we have allowed the probability of obtaining liquidity insurance to depend on both the reserve to GDP ratio and the nominal interest rate.

In Appendix A6 we present the estimated posterior distribution of the impact coefficients and the corresponding counterfactual where we subtract the contribution of the insurance probability. It is clear that this contribution is minimal, despite the fact that the estimated loading of the index on both policy variables is relatively high.

8.1. **Counterfactual with Competitive Market.** As we already mentioned earlier on, explicitly taking into account market power is essential for describing the strategic behavior of banks in the auction.

We next compare the estimated $\tilde{\mathbb{S}}_t$ to a counterfactual measure where the strategic component of bidding behavior is ignored, as well as how this changes the empirical results regarding the effects of policy shocks. Recall that in the absence of market power, no bank can affect the equilibrium price and quantity and thus equations (5) and (6) can be written as $s_i^{NI,0} = \frac{b_i}{1+b_i}$ and $s_i^{FI,0} = \frac{b_i}{1+\mathbb{E}d}$ respectively. We can therefore infer the counterfactual measure of aggregate risk by performing the same estimation as before, using that $s_{i,t}^0 = \frac{s_{i,t}^{NI,0}}{1+p_t\left(\frac{d_t-\iota_t}{1+\iota_t}-s_{i,t}^{NI,0}\frac{1+d_t}{1+\iota_t}\right)}$.

In Figure 8, we plot the recovered time series for aggregate risk against the counterfactual (in log deviation from the mean units, divided by their standard deviation to facilitate comparison). The shaded areas in Figure 8 indicate periods for which the Central Bank announced and implemented official foreign exchange interventions.

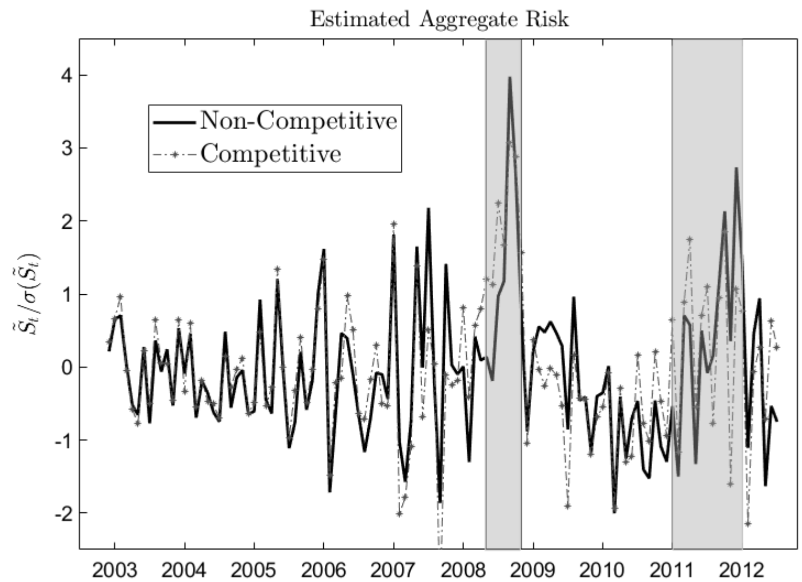
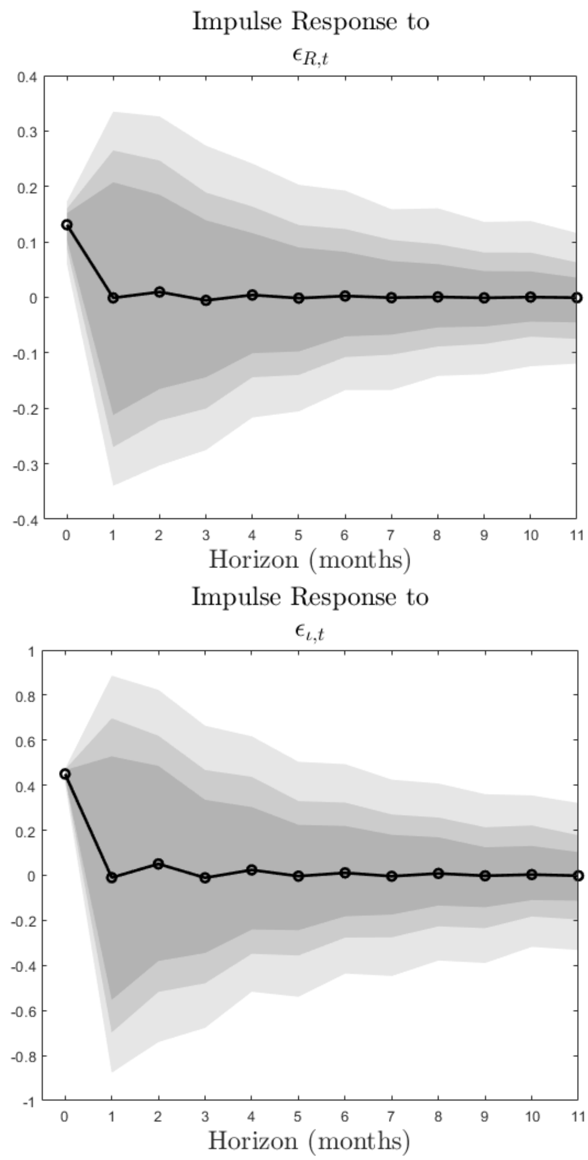


FIGURE 8. Aggregated Risk with/without Market Power

The first series of interventions were announced in April 2008, but as we already mentioned, they ended in September 2008. The second series of interventions were implemented during the year 2011, where the Central Bank scheduled daily purchases of $50 billion worth of foreign currency, strengthening the liquidity position to almost 16% of GDP. Eyeballing Figure 8, we can see that some of the increases in our proposed measure do coincide with these periods, partially justifying the more formal evidence using the identified impulse responses. While we also observe increases in the counterfactual measure during the periods of official interventions, the change is smaller in 2008-2009 and noisier in 2011. Although not visible from the figure due to standardizing the series, our estimate is also much more volatile than the counterfactual. Thus, accounting for market power reveals that aggregate beliefs about liquidity risk are much more responsive to new information.

To have a quantitative sense of whether employing the counterfactual measure would lead to distorted inference regarding the effects of foreign reserve interventions on aggregate risk, we repeat the impulse response exercise using the counterfactual measures for s_i. Below we plot the counterfactual impulse responses identified using the same restrictions. Employing a measure that conflates risk with market power leads to largely underestimating the effects. More particularly, the median reserve accumulation effect is estimated to be 28.5 times smaller, and the monetary policy effect 8.1 times smaller.



9. Other Applications: Calibrating Macro-Banking models

The estimated distribution of risk could potentially be used as a calibration target for structural macroeconomic models with a banking sector. In this section we briefly describe the mapping between this class of models and the distribution of liquidity risk.[40] In Section 4 we defined the probability of becoming illiquid as the probability of getting a shock so large such that excess reserves become negative. This obliges the bank to liquidate assets or to obtain additional funds by resorting to the interbank market or the lender of last resort. This identified probability, which is specific to each bank, is more precisely defined as

$$s_{i,t} = \mathbb{P}_t(\text{Excess reserves} + \eta_{i,t+1} < 0) \equiv G_t(\eta_i^\star)$$

where $\eta_{i,t+1}$ is the liquidity shock and η_i^* is the threshold below which excess reserves become negative. Probability $s_{i,t}$ is therefore equal to the mass of the distribution of liquidity shocks in the sector (G) which is below the individual specific threshold. As excess reserves depend by construction on the specific model of the bank, the threshold will depend on several objects that relate to the structure of the balance sheet, macroeconomic and macroprudential policy e.g. reserve requirements, asset returns etcetera.

Since we do not really take a stance on the structure of $\eta_i^\star$, $s_{i,t}$ is essentially unrestricted[41], which means that it is potentially consistent with many models, with different bank structures and policy.[42]

Therefore, depending on the amount of heterogeneity that the quantitative model allows, $\{s_{i,t}\}_{i=1,2..N}^{t=1,2..T}$ can be used as a calibration target to discipline the

[40] It goes without saying that applying this methodology to data from other countries will require a potentially different specification of the SVAR model or some adaptation of the auction model in order to match the specific country's setting.

[41] To be precise, we assume that s_i is constant within the month, and thus we also have to assume that all the bank characteristics stay constant within the month as well.

[42] An example can be found in Bianchi and Bigio (2022), where the threshold is defined by

$$\eta_i^\star = -\frac{\frac{Liquid\ Assets}{Deposits} - \kappa_2}{\frac{R^d_{t+1}}{\iota_t} - \kappa_2}$$

where κ_2 is the minimum reserve requirement given our notation. Additionally, in Appendix B2, we define excess reserves in the context of the postulated general model of the bank.

relevant quantities. For example, in a model with heterogeneous banks and thus heterogeneity in thresholds $\eta_i^\star$, $s_{i,t}$ can be used to calibrate the sources of this heterogeneity e.g. portfolio weights, as well as parameters governing $G_t(\cdot)$. As long as liquid assets do not include the Central Bank bond (as we define liquidity needs before the bond is sold), a heterogeneous and time varying $s_{i,t}$ would imply a heterogeneous and time varying $G_t(\eta_{i,t}^\star)$.

10. Conclusion

We have developed a structural model of a uniform multi unit auction where banks place interest rate and quantity bids to sell liquidity to the Central Bank. The main novelty of the model lies in the explicit introduction of macroeconomic information in characterizing banks' optimal bidding behavior. Macroeconomic policies can affect banks' expected losses in the case in which they become illiquid through different channels such as the cost of obtaining emergency liquidity as well as the credibility of the State in acting as a lender of last resort. We exploited the model's predictions about bidding behavior to recover the time varying cross sectional distribution of liquidity risk across banks conditional on public information in the case of Chile at a monthly frequency.

Going one step further, we provided new empirical evidence on the effects of macroeconomic policy on the mean of the distribution of liquidity risk as expected by the financial sector. In particular, we undertook a structural study of the effects of unpredictable shocks and news shocks about foreign exchange reserve accumulation, a popular tool amongst emerging economies to promote financial stability, as well as the effects of unpredictable shocks to the policy interest rate. The identified effects of unanticipated shocks to reserve accumulation and the policy rate are consistent with the presence of asymmetric information between financial market participants and the central bank regarding the distribution of liquidity risk. While banks are well aware of the purely idiosyncratic component - their exposure to liquidity risk that comes from their own financial state - expectations about the aggregate state of liquidity risk are conditional on public information. Central Bank actions therefore have two, possibly conflicting effects on aggregate risk as expected by the banks. The first is the direct mitigating effect, while the second is the signalling effect. Our results are informative about which effect actually dominates. We find that actual reserve accumulation interventions have strong signalling effects while news about future interventions are not effective. We find similar effects in the case of monetary policy.

These results have direct implications for policy design. While in the recent years Central Banks have placed a lot of emphasis on communication strategies regarding present and future policy, our findings indicate that when actions are costly, actual policy may have significantly stronger signalling effects than mere announcements.

Finally the paper shows that empirical work that aims to identify liquidity risk as well as policy effects from financial market data should take into account imperfect competition, as strategic behavior in these markets can mask the identification of the true effects.

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Appendix A

A1. Evidence on Supply Uncertainty

While positive differences between $\bar{Q}_t$ and Q_t are expected due to demand falling below supply, positive differences signify that supply is not fixed; it can increase to accommodate additional demand. Dashed lines indicate the US sub-prime crisis and the Lehman Brothers collapse respectively.

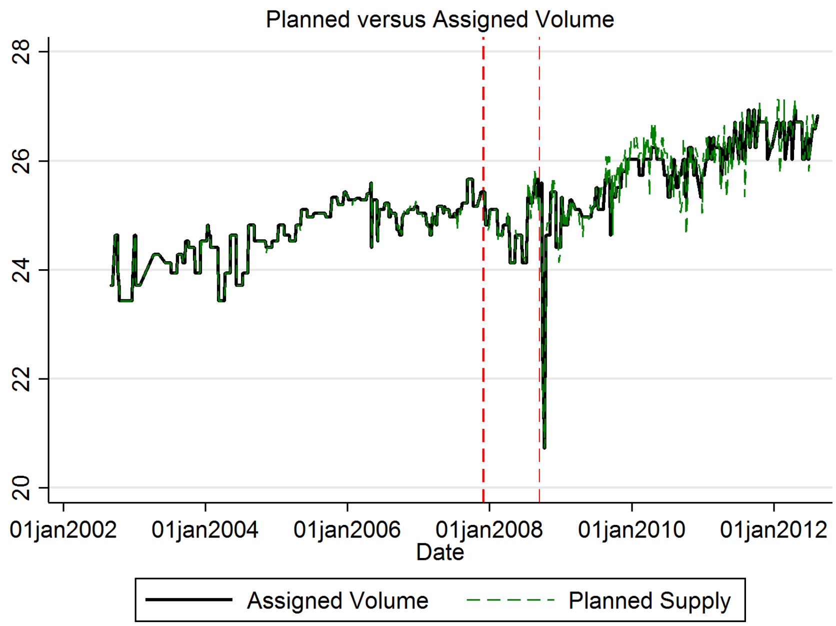


Figure 9

A2. Details on the optimization problem of bank i

Below we calculate the expected cost of being hit by the illiquidity shock when $p = 0$, TC_i.

$$TC_i = s_i \mathbb{P}(B^c > b_i \mid s_i) q_i \mathbb{E}(Q) + s_i \mathbb{P}(B^c = b_i \mid s_i) \mathbb{E}(q_i^c \mid b_i = B^c, s_i) \mathbb{E}(Q)$$

where the first component of the sum is the cost when firm i is not the cutoff bank, TC_i^{NC} and the second component is the cost when it is, TC_i^C. The derivative of TC_i with respect to b_i is then:

$$\begin{aligned}\frac{\partial TC_i}{\partial b_i} &= s_i \frac{\partial \mathbb{P}(B^c > b_i \mid s_i)}{\partial b_i} q_i \mathbb{E}(Q) + \\ &+ s_i \Big[\frac{\partial \mathbb{P}(B^c = b_i \mid s_i)}{\partial b_i} \mathbb{E}(q_i^c \mid b_i = B^c, s_i)\mathbb{E}(Q) \\ &+ \mathbb{P}(B^c = b_i \mid s_i) \frac{\partial \mathbb{E}(q_i^c \mid b_i = B^c, s_i)}{\partial b_i} \mathbb{E}(Q) \Big]\end{aligned}$$

where the first component of the sum is the derivative of the cost when firm i is not the cutoff bank $(TC_i^{NC})'$, and the second component is the corresponding derivative when it is, $(TC_i^C)'$.

Then $c_i = \frac{(TC_i)'}{(TC_i^C)'}$ is the ratio of the expected marginal cost of being hit by the liquidity shock to the expected marginal cost of being hit by the liquidity shock when being the cutoff bidder. When we decompose conditional expectations as $\mathbb{E}(.|s_i) = \mathcal{E}_{nc}(.) + \mathcal{E}_c(.) + \mathcal{E}_{nb}(.)$, we can rewrite it as $c_i := q_i \frac{\partial \mathbb{P}(B^c > b_i)}{\partial b_i} \left(\frac{\partial \mathcal{E}_c(q_i^c)}{\partial b_i} \right)^{-1} + 1$. Note that the expected cost of being hit by the illiquidity shock when $p = 1$, can be written as:

$$TC_{FI,i} = TC_i \left[1 + \frac{\mathbb{E}(d)}{1 + \iota} \right]$$

and thus c_i also satisfies $c_i = \frac{(TC_{FI,i})'}{(TC_{FI,i}^C)'}$.

A3. Computing Equilibrium Prices: Further Details

Given the demand for bonds of $N - 1$ bidders, and the supply by the Central Bank, we construct the excess supply faced by bank i, in order to derive all possible equilibrium prices in this market, and the corresponding equilibrium quantities assigned.

Let (b_i, q_i) be bank i's bid vector. We rank the interest rate bids of all but bank i from the lowest to the highest: $b_1 \leqslant b_2 \leqslant ... \leqslant b_{N-1}$, and denote their corresponding shares asked by $q_1, q_2, ..., q_{N-1}$.

Define $Q_j \equiv \sum_{l=1}^{j} q_l$ with $j \in [1, N - 1]$. Note that although $q_j < 1 \forall j$, it can well be that $Q_j > 1$, i.e. the Nth bank is facing no excess supply for bonds above some level of interest rate.

We distinguish between two cases. The first, is when there exists some bank $k \in [1, N-1]$ at which the residual supply of bonds becomes zero and thus bank i is not guaranteed a positive allocation of bonds. The second, is when the CB's supply of bonds is high enough such that there is positive residual supply at any level of interest rate, and thus bank i is guaranteed to receive at least part of its demanded quantity in equilibrium.

No Excess Supply after some level of interest rate. Take r_k to be the interest rate above which there is no excess supply of bonds for bank i, i.e: $Q_{k-1} < 1 \leqslant Q_k$, where $k \in [1, ..., N-1]$. In this case, the equilibrium interest rate received by all winning banks is:

$$B^c(b_i, q_i) = \begin{cases} b_k & \text{if } b_i > b_k \\ b_i & \text{if } \begin{cases} \text{either } b_{m-1} < b_i = b_m \text{ and } Q_{m-1} < 1 \leqslant Q_m + q_i \\ \text{or } b_{m-1} < b_i < b_m \text{ and } Q_{m-1} < 1 \leqslant Q_{m-1} + q_i \\ \text{or } b_i = b_1 = ... = b_m \text{ and } Q_m + q_i \geqslant 1 \end{cases} \\ b_{m+h} & \text{if } b_{m-1} \leqslant b_i < b_m \text{ and } Q_{m-1+h} + q_i < 1 \leqslant Q_{m+i} + q_i \end{cases}$$

with $1 \leqslant m \leqslant k$ and $h = 0, 1, ..., k-m$.[43]

In the first case bank i does not receive any shares in equilibrium, whereas it is assigned the asked share, q_i, in the third case. When the equilibrium interest rate is b_i, the bank is assigned the residual demand (or a proportion of it in case of ties), which is given by:

$$q^c(b_i, q_i) = \begin{cases} 1 - Q_{m-1} & \text{if } b_{m-1} < b_i < b_m \text{ and } Q_{m-1} < 1 \leqslant Q_{m-1} + q_i \\ \frac{q_i}{q_i + \sum\limits_{l: b_l = b} q_l}(1 - Q_{m-1}) & \text{if } b_{m-1} < b_i = b_m \text{ and } Q_{m-1} < 1 \leqslant Q_m + q_i \\ \frac{q_i}{q_i + Q_m} & \text{if } b_i = b_1 = ... = b_m \text{ and } Q_m + q_i \geqslant 1 \end{cases}$$

In Figure 10, we provide a graphical example of the resulting equilibrium interest rate and quantity assigned to bank i, in the absence of ties. Suppose that bank i bids (b, q). Since its interest rate bid is higher than b_k, the highest interest rate at which there is positive residual supply of bonds, bank i does

[43] Define $b_0 \equiv 0$ and $Q_0 \equiv 0$ for completeness of notation.

not receive any bonds in equilibrium. On the contrary, for any interest rate bid smaller than b_k, the bank will be awarded some bonds. If the quantity bid by bank i is smaller than the residual supply it faces at the interest rate it bid for, then it will be awarded the total quantity asked, and it will receive a higher interest rate than its own bid. An example of this is the bid (b', q') in Figure 10 where the bank is awarded (b_{k-1}, q'). However, when the bank asks for a quantity that is higher than the residual supply it faces at the interest rate it bid for, it will be the cutoff bidder. This is exactly the case for bid (b'', q''), where the bank is awarded the residual supply at the previous step, $1 - Q_1$, and the equilibrium interest rate is b''.

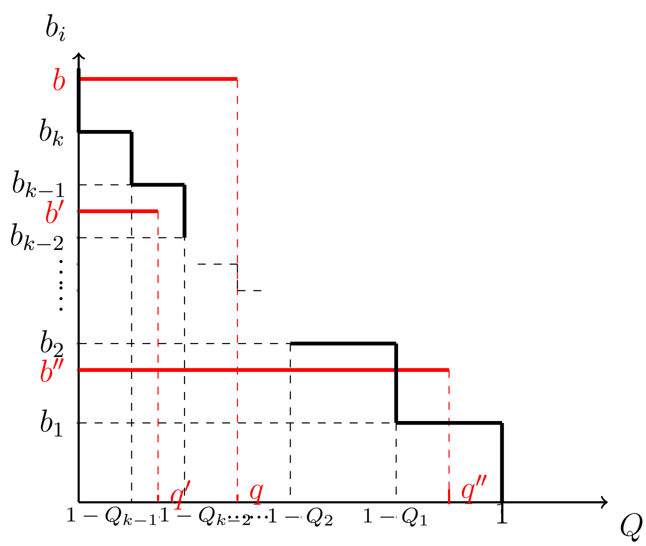


FIGURE 10

In figure 11 below, we consider the case where bank i places an interest rate bid b that is equal to the interest rate bid by another bank. Given the residual supply of bonds, b is going to be the equilibrium cutoff rate when bank i has asked for quantity share q. In this case, the two banks share the residual demand at the previous step. More precisely, bank i receives $\frac{q}{q+q_2}(1 - Q_{k-1})$.

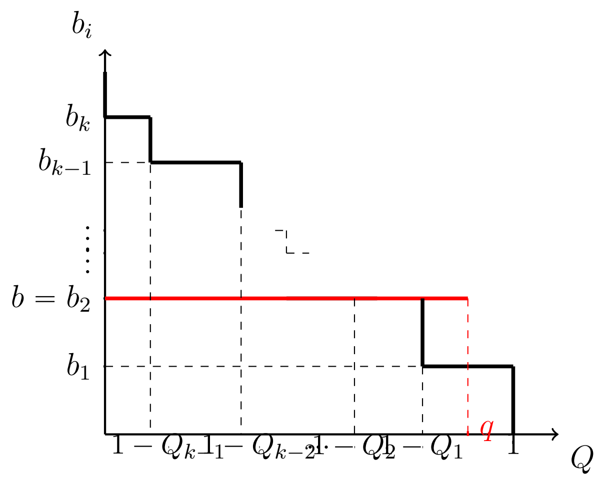


FIGURE 11. Weakly Positive Residual Supply

Excess Residual Supply. In this case the CB's supply of bonds is too high so that there is positive residual supply for bank i for any level of interest rate i.e.: $Q_{N-1} < 1$.

The equilibrium cutoff interest rate is:

$$B^c(b_i, q_i) = \begin{cases} b_{N-1}, & \text{if } b_i < b_{N-1} \text{ and } 1 - Q_{N-1} > q_i \\ b_i & \text{if } \begin{cases} \text{either } b_i > b_{N-1} \\ \text{or } b_{m-1} < b_i = b_m \text{ and } Q_{m-1} < 1 \leqslant Q_m + q_i \\ \text{or } b_{m-1} < b_i < b_m \text{ and } Q_{m-1} < 1 \leqslant Q_{m-1} + q_i \\ \text{or } b_i = b_1 = ... = b_m \text{ and } Q_m + q_i \geqslant 1 \end{cases} \\ b_{m+h} & \text{if } b_{m-1} \leqslant b_i < b_m \text{ and } Q_{m-1+h} + q_i < 1 \leqslant Q_{m+h} + q_i \end{cases}$$

with $m \leqslant N - 1$ and $h = 0, 1, ..., N - 1 - m$.

In the first and third cases the bank receives exactly q_i, whereas in the second case it is assigned its residual supply (or a proportion of it in case of ties), which is the same as in before. In this case the bank i is always guaranteed a strictly positive quantity of bonds in equilibrium.

Figure 12 below illustrates examples of the possible scenarios when there exists positive residual supply for bank i at any level of interest rate. When bidding

(b', q') where $b_{k-3} < b < b_{k-2}$ and $q' \in (1 - Q_{k-1}, 1 - Q_{k-2})$, the bank receives their full quantity asked at the cutoff interest rate b_{k-1}. Instead, the bank's interest rate is the cutoff in both the cases in which they bid b and b''.

In the first case the bank receives $1 - Q_{k-1}$ since $b > b_{k-1}$ and $q > 1 - Q_{k-1}$; in the second they receive $1 - Q_1$ since $b'' \in (b_1, b_2)$ and $q'' > 1 - Q_1$.

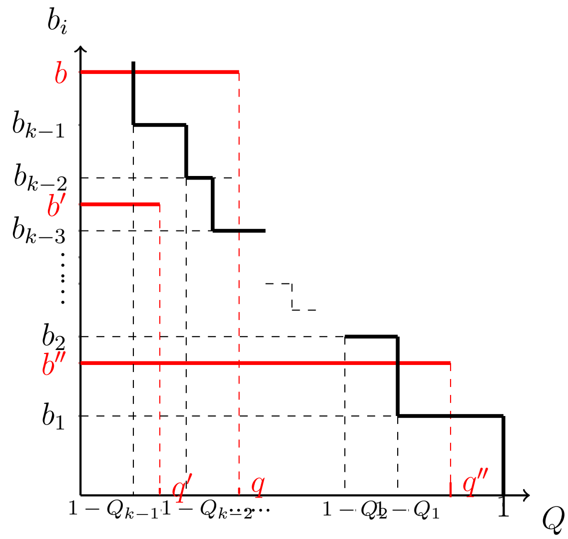


FIGURE 12. Strictly Positive Residual Supply

A4. Dataset Description and Transformations

Auction Dataset. The dataset contains all bidding information for the open market operations of the BCCh from September 2002 to August 2012. The information includes the total volume of bonds allotted by the Central Bank in each auction, the cuttoff interest rate, the bidders' identities and the rates and quantities of bonds asked by each bidder. For the structural estimation, we pool auctions that occured after the 20th day of month t and the first 15 days of the month $t+1$, in order to measure liquidity risk of month $t+1$.

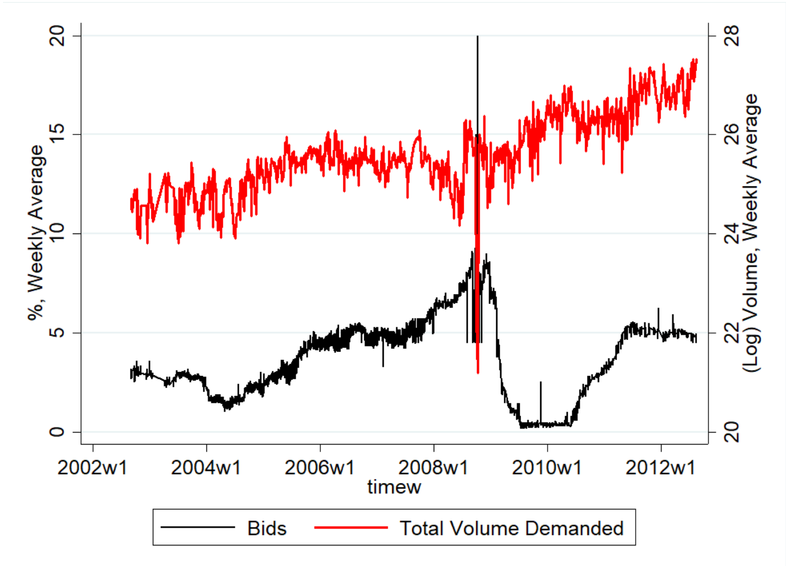


Figure 13

Macro Data and Calibration.

TABLE 1. Sources and Transformations

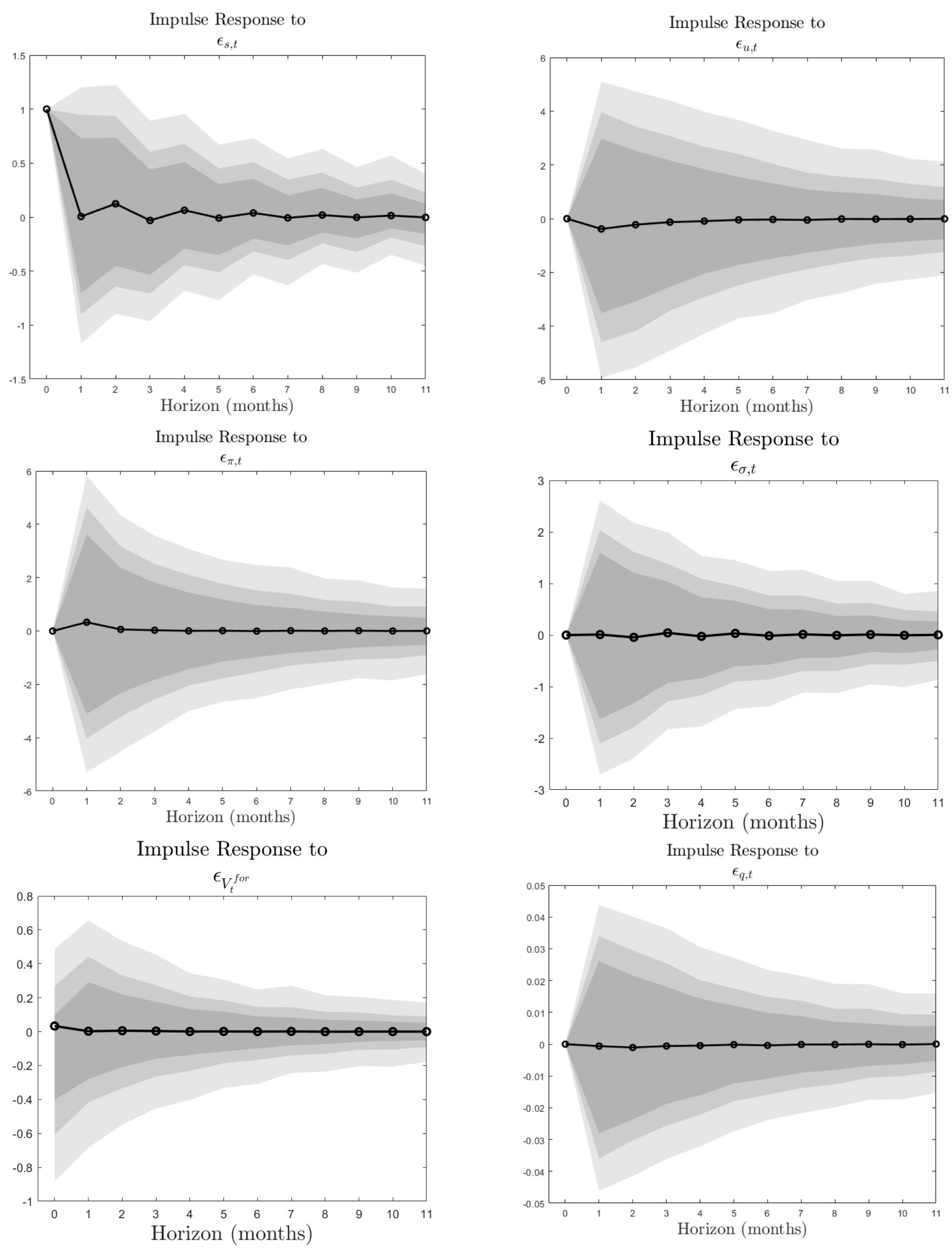
Variable	Data	Source	Final Transformation
VIX	CBOE Volatility Index: VIX, Index, Monthly	FRED	$ln(VIX)$
U_t	Unemployment Rate: Aged 15 and Over: All Persons for Chile, Percent, Monthly, S.A	FRED (OECD)	ΔU_t
e_t	Real Broad Effective Exchange Rate for Chile, Index 2010=100, Monthly	FRED (OECD)	$\Delta ln(e_t)$
$e_t^{P/US}$	Nominal Exchange Rate Peso per $, Monthly	FRED (OECD)	–
P_t	Consumer Price Index, Chile (2005=100), Monthly	FRED (OECD)	–
P_t^{USA}	Consumer Price Index, USA (2005=100), Monthly	FRED (OECD)	–
π_t	CPI Inflation	–	$\Delta ln(P_t)$
GDP_t	Gross Domestic Product, Current Prices, Quarterly	FRED	$ln(GDP_t) - ln(P_t/100)\star\star\star$
R_t	International [Currency] Reserves at the BCCh, USD, Monthly	Thomson Reuters (BCCh)	$\Delta(ln(R_t) + ln(e_t^{P/USA}) - ln(P_t/100))\ \star\star$
ι_t	Monetary Policy Rate , Monthly	BCCh	$ln(\iota_t)$
d_t	MPR expectations within the next two months⋆, Monthly	Expectations Survey - BCCh	–
d_t	3m, 6m, 12 m SWAP rates ⋆	BCCh	$\frac{1}{3}(0.7\text{SWAP90}_t - 0.5\text{SWAP180}_t + \text{SWAP360}_t)$
$\mathbb{S}_t^{NI}$	Aggregate Risk (NI), Monthly	Own Calculations	$ln(\sum_{i=1..N}\hat{s}_{i,t}^{NI})$
$\mathbb{V}_t^{NI}$	Risk Variance (NI), Monthly	Own Calculations	$ln(\hat{Var}(\hat{s}_{i,t}^{NI}))$
Reserve to GDP	International Reserves for Mexico, Chile and Brazil, USD,Yearly	Thomson Reuters	–
ratios [descriptive]	Nominal Gross Domestic Product for Mexico, Chile and Brazil, USD, Yearly	FRED	–
Calibrated Parameter	Information	Source	–
0.31	Metadata	Pizzo (2020)	–

⋆ There are several measures of interest rate expectations. A market based measure can be constructed based on swaps, yet 30 Day swaps (SWAP30) do not explicitly exist as instruments. As in Becerra, Claeys, and Martínez (2016), we compute the implied rate using a combination of 3 month, 6 month and 1 year swap rates. Nevertheless, 3 month swap rates are not available before May 2015, so we impute expectations for the one month ahead interest rate using the "within two month" measure of interest rate expectations survey of the BCCh.

⋆⋆ Foreign Reserves and Foreign Currency reserves are virtually identical before the 2007-2008 crisis and almost identical after the crisis. We have used both of them and they give very similar results.

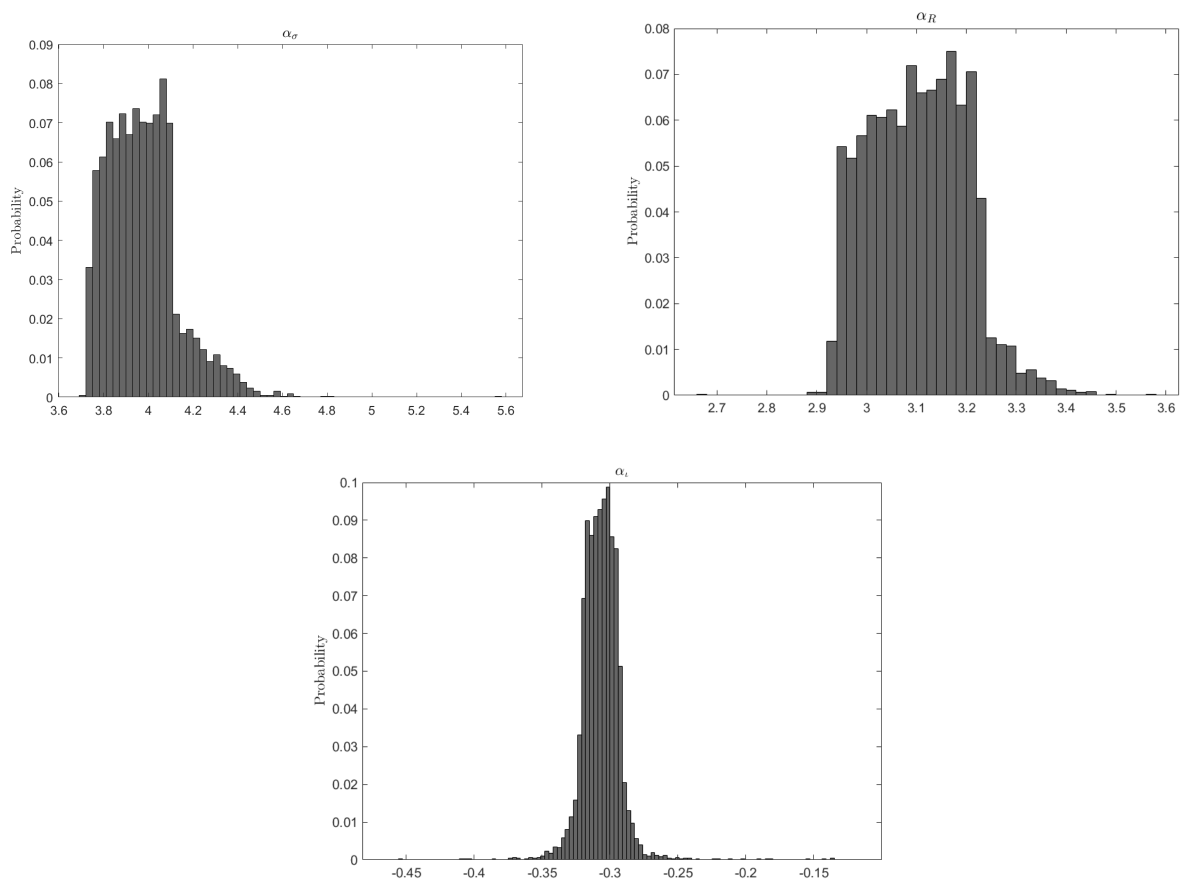
⋆ ⋆ ⋆Due to lack of monthly estimates of GDP, we impute missing data by linearly interpolating quarterly data.

A5. Additional Impulse Responses



A6. Empirical Results on the Probability of Insurance

(a) Estimated Index Coefficients for p_t



(b) Evidence of minimal contribution of p_t to impact effect

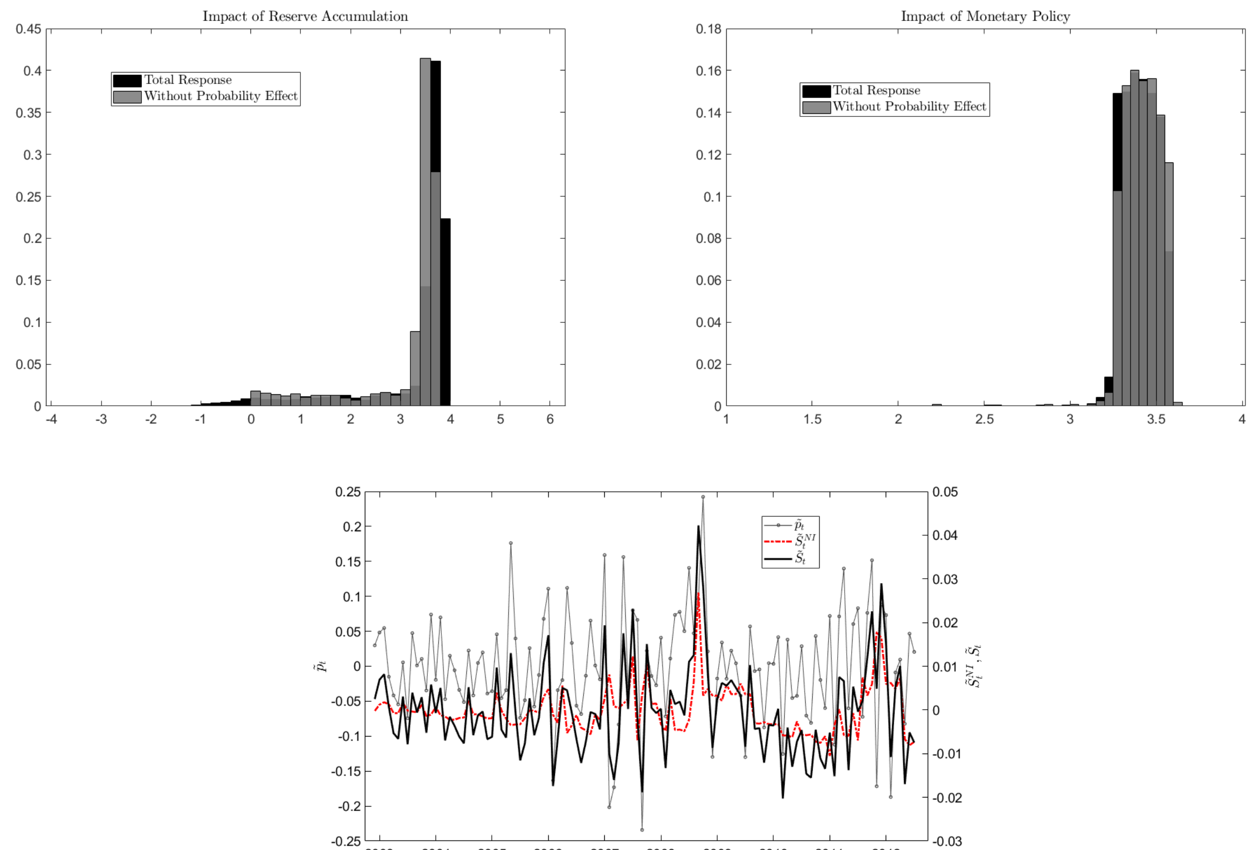
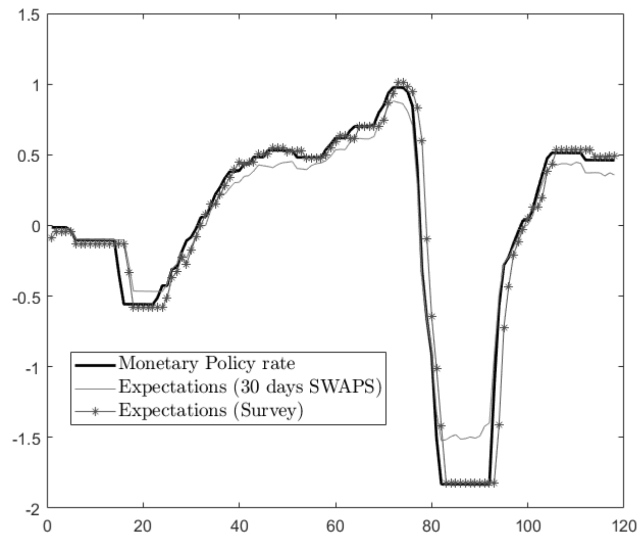


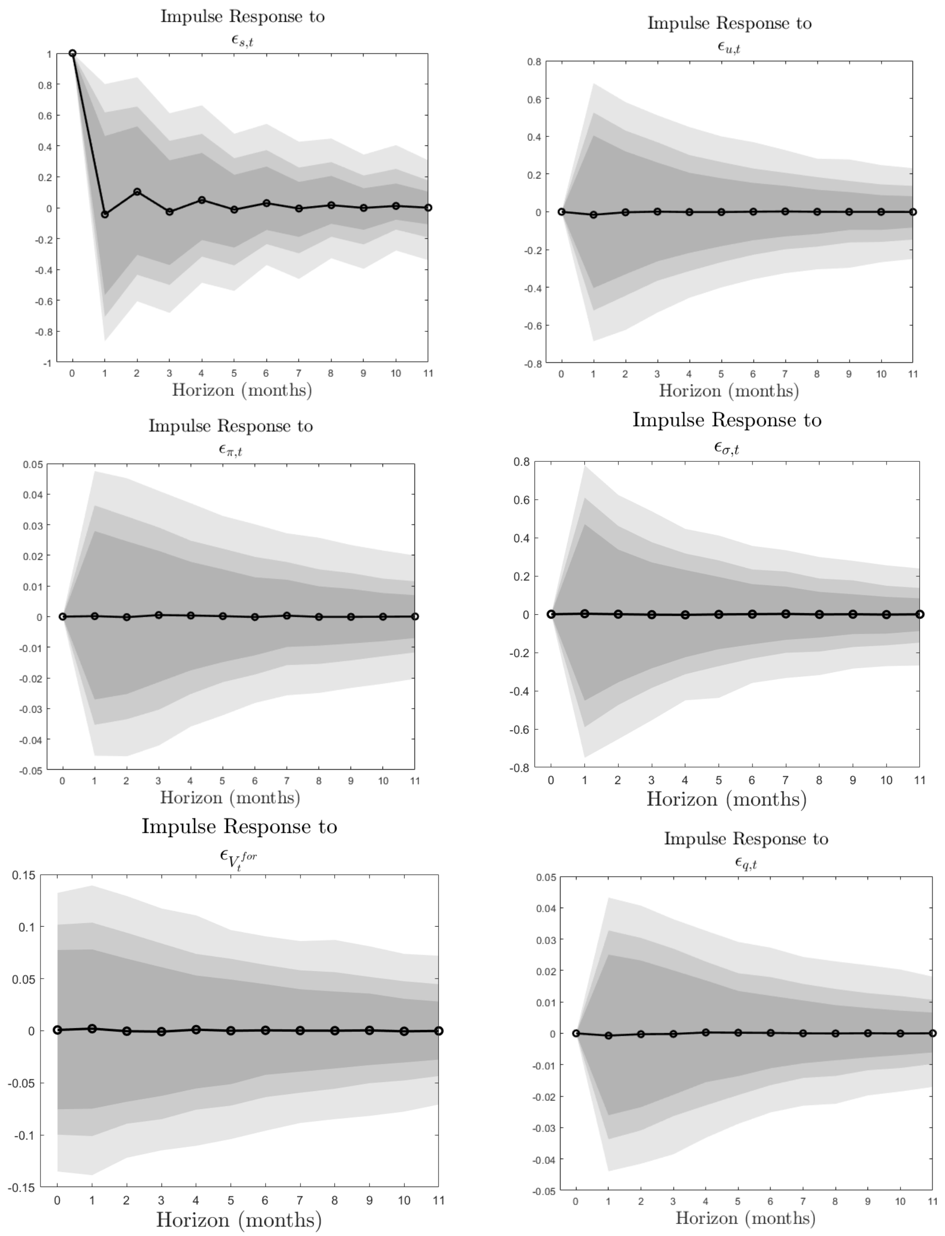
FIGURE 14. The panel displays the estimated impact effects with and without the contribution of the probability of insurance, and the corresponding estimates of $\tilde{p}_t$ and $\tilde{S}_t$. Fluctuations in $\tilde{\tilde{p}}_t$ are not the major determinant of $\tilde{\tilde{S}}_t$, while the latter is strongly, but imperfectly correlated with $\tilde{S}_t^{Ni}$, as expected.

A7. Comparing Expectations Measures

Below we plot the current monthly interest rate, the constructed 30-day measure based on SWAPS post May 2015 and the within two month interest rate expectations BCCh survey pre May 2015, and the average of within two month and current month interest rate expectations of the BCCh survey. As evident they are all strongly correlated.

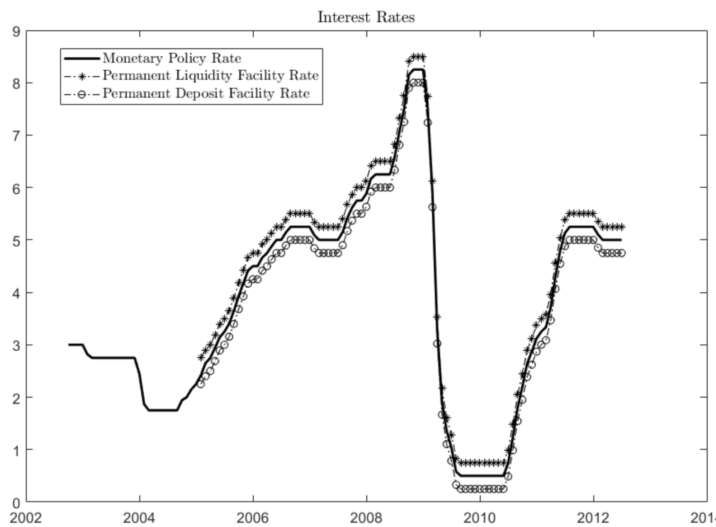


A8. Additional Impulse Responses - Competitive Outcome



A9. Liquidity Facilities at the Central Bank of Chile

The central Bank of Chile implements its monetary policy using different instruments, such as the intraday liquidity facility, the liquidity inflow and withdrawal facilities, and the adjustment and structural open market operations (BCCh, 2012). In order to achieve an interbank rate (IBR) close to the monetary policy rate (MPR), the Bank sets a daily floating band of 25 basis points above and below the MPR, while it offers standard liquidity facilities at the corresponding ceiling rate and standing deposit facilities at the floor rate.



If there is tendency for the IBR to hit either limits, the Bank uses adjustment operations at the standing MPR, in order to provide or absorb liquidity. In particular, adjustmemt operations have to do either with cash inflow, such as repurchase of credit bonds (Repos), sale of foreign currency swaps and the FLAP[44] or cash outflow, through purchase of foreign currency swaps and liquidity deposits. Longer term changes in the supply of liquidity are implemented through the purchase and sale of instruments such as promissory notes

[44]The FLAP represents the term liquidity facility introduced by the BCCh between 7/2009 and 6/2010, which provided collateralized liquidity up to 6 months in order to align asset prices with the path of monetary policy at that time (Céspedes, García-Cicco, and Saravia, 2014).

with short (1 month) and longer maturities (1 year), as well as other types bonds issued by the Central Bank.

A10. Credible Sets for Impact effects on $\mathbb{S}_t$

Credible Sets for Impact effects on $\mathbb{S}_t$ for alternative transformations for reserves and measurement error inflation factor

	$\Delta ln\left(\frac{R_t}{GDP_{t-1}}\right)$, Σ_v		$\Delta ln\left(\frac{R_t}{GDP_{t-1}}\right)$, $\Sigma_v \times 1.1$		$\Delta ln\left(\frac{R_t}{GDP_{t-1}}\right)$,$\Sigma_v \times$ 1.2		$ln(R)$	
Shock	$q_{5\%}$	$q_{95\%}$	$q_{5\%}$	$q_{95\%}$	$q_{5\%}$	$q_{95\%}$	$q_{5\%}$	$q_{95\%}$
ϵ_{VIX}	−0.884	0.486	−0.825	0.561	−0.649	0.496	−0.876	0.449
ϵ_R	0.473	3.892	0.3446	3.884	0.3321	3.86	0.2516	3.908
$\epsilon_{R_{news}}$	−0.065	0.073	−0.098	0.116	−0.125	0.139	−0.062	0.073
ϵ_ι	3.259	3.558	3.219	3.545	2.873	3.564	3.257	3.556

Appendix B

B1. The auction in a dynamic model of a bank

Consistent with the literature, see e.g. Bianchi and Bigio (2022), we consider a two step budgeting problem where in the first step the bank decides how much equity it should hold across time subject to capital requirements. Conditional on the choice of equity, there is a portfolio choice step where the bank has to decide how to split its equity between assets and deposits.

Let d_t denote deposits, a_t total assets (including reserves $r_{0,t}$, net of loans and liabilities other than deposits) and c_t the remuneration of the owner (dividend) in terms of consumption units. The bank chooses its balance sheet structure as well as dividends to maximize its total lifetime utility, with discount factor $\beta \in (0,1)$. The bank's constrained maximization problem therefore reads:

$$\begin{aligned} V(d_t, a_t) &= \max_{d_{t+1}, a_{t+1}, c_t} (1-\beta)u(c_t) + \beta \mathbb{E}_t V(d_{t+1}, a_{t+1}) \\ s.t. & \\ P_{a,t+1}a_{t+1} - P_{d,t+1}d_{t+1} &= a_t - d_t - c_t \\ \frac{d_{t+1}}{a_{t+1} - d_{t+1}} &\leqslant \kappa_1 \\ \frac{r_{0,t+1}}{d_{t+1}} &\geqslant \kappa_2 \end{aligned}$$

where $P_{a,t+1}$ is the price paid for acquiring assets a_{t+1} and $P_{d,t+1}$ is the price received for depositing d_{t+1}. In turn, a capital requirement constraint places an upper bound on how leveraged the bank can be (κ_1), while a reserve requirement imposes a lower bound on the reserve to deposit ratio (κ_2).

We further reformulate the dynamic problem of the bank using beginning of period equity (that is, before dividend payout): $e_t := a_t - d_t$ and $e_{t+1} := a_{t+1} - d_{t+1}$, as well as equity price $P_{e,t+1} = P_{a,t+1}\frac{a_{t+1}}{e_{t+1}} - P_{d,t+1}\frac{d_{t+1}}{e_{t+1}} := P_{a,t+1}\psi_{a,t+1} - P_{d,t+1}\psi_{d,t+1}$, where ψ_j is the portfolio weight of asset or liability j, and $R_{e,t+1} =$

$\frac{1}{P_{e,t+1}}$ such that:

$$\begin{aligned}
V(e_t) &= \max_{e_{t+1},c_t,\psi_{a,t+1},\psi_{d,t+1},\psi_{0,t+1}} (1-\beta)u(c) + \beta\mathbb{E}_t V(e_{t+1}) \\
s.t. & \\
e_{t+1} &= R_{e,t+1}(\psi_{a,t+1},\psi_{d,t+1})(e_t - c_t) \\
\psi_{d,t+1} &\leqslant \kappa_1 \\
\psi_{a,t+1} - \psi_{d,t+1} &= 1 \\
\frac{\psi_{0,t+1}}{\psi_{d,t+1}} &\geqslant \kappa_2
\end{aligned}$$

We have thus reduced the state variables from (a_t, d_t) to just e_t .

Using the guess solution $c_t = \delta_t e_t$ and indirect value function $V_t = u(\zeta_t e_t)$, the problem becomes:

$$\begin{aligned}
u(\zeta_t e_t) &= \max_{\delta_t,\psi_{a,t+1},\psi_{d,t+1},\psi_{0,t+1}} (1-\beta)u(\delta_t e_t) + \beta\mathbb{E}_t u(\zeta_{t+1} e_{t+1}) \\
s.t. & \\
e_{t+1} &= R_{e,t+1}(\psi_{a,t+1},\psi_{d,t+1})(1-\delta_t)e_t \\
\psi_{d,t+1} &\leqslant \kappa_1 \\
\psi_{a,t+1} - \psi_{d,t+1} &= 1 \\
\frac{\psi_{0,t+1}}{\psi_{d,t+1}} &\geqslant \kappa_2
\end{aligned}$$

The first order condition with respect to δ_t yields,

$$u(\delta_t e_t) = \frac{\delta_t}{1-\delta_t}\mathbb{E}u(\zeta_{t+1}e_{t+1})\frac{\beta}{1-\beta}$$

Plugging this back to the objective function, matching coefficients and imposing that $u(.)$ is homogeneous of degree λ, we get that $\zeta_t = \delta_t^{1-\frac{1}{\lambda}}(1-\beta)^{\frac{1}{\lambda}}$. Using the budget constraint, we arrive at the Euler equation for equity

$$1 = \beta\mathbb{E}_t u\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}} R_{e,t+1}$$

e.g. in the case of CRRA utility, $\lambda = 1-\gamma$ and thus

$$1 = \beta\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{e,t+1}$$

The dynamic allocation of dividends and thus equity is independent of capital and reserve requirements.

We turn next to the problem of portfolio allocation. Decompose a_{t+1} into J different assets, and use that

$$R_{e,t+1} = \frac{1}{\sum_{j=1..J} \psi_{j,t+1} P_{j,t+1} - \psi_{d,t+1} P_{d,t+1}} \approx \sum_{j=0..J} \psi_j R_{j,t+1} - \psi_{d,t+1} R_{d,t+1}$$

Substituting in the optimality condition for equity, and using that $e_{t+1} = R_{e,t+1} \frac{1-\delta_t}{\delta_t} c_t$, the optimization problem becomes as follows:

$$\max_{\{\psi_{j,t+1}\}_{j=1..J}, \psi_{d,t+1}, \psi_{0,t+1}} \mathbb{E}_t u \left(\left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\lambda}} R_{e,t+1}^{\frac{1}{\lambda}} \right)$$

$$\begin{aligned} & s.t. \\ \psi_{d,t+1} &\leqslant \kappa_1 \\ \frac{\psi_{0,t+1}}{\psi_{d,t+1}} &\geqslant \kappa_2 \\ R_{e,t+1} &= \sum_{j=1..J} \psi_{j,t+1} R_{j,t+1} - \psi_{d,t+1} R_{d,t+1} + \psi_{0,t+1} R_{0,t+1} \\ 1 &= \sum_{j=1..J} \psi_{j,t+1} - \psi_{d,t+1} + \psi_{0,t+1} \end{aligned}$$

where we have factored out predetermined variables from the objective.

Perfect Competition. The corresponding first order condition for any asset in a perfectly competitive market yields

$$\begin{aligned} \mathbb{E}_t u' \left(\left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\lambda}} R_{e,t+1}^{\frac{1}{\lambda}} \right) \left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\lambda}} R_{e,t+1}^{\frac{1}{\lambda}-1} \frac{\partial R_{e,t+1}}{\partial \psi_{j,t+1}} &= 0 \\ \mathbb{E}_t u \left(\left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\lambda}} \right) (R_{j,t+1} - R_{0,t+1}) &= 0 \end{aligned}$$

and therefore

$$\mathbb{E}_t u \left(\left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\lambda}} \right) (R_{j,t+1} - R_{0,t+1}) = 0$$

The asset portfolio allocation is not affected by capital and reserve requirements.
Finally, for deposits, the first order conditions are:

$$\begin{aligned} \mathbb{E}_t u\left(\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}}\right)(R_{d,t+1} - R_{0,t+1}) &= \mu_{1,t} + \mu_{2,t} \\ \mu_{1,t}[\kappa_1 - \psi_{d,t+1}] &= 0 \\ \mu_{2,t}\left[\frac{\psi_{0,t+1}}{\kappa_2} - \psi_{d,t+1}\right] &= 0 \end{aligned}$$

When capital or reserve requirements bind, they affect the choice of deposits vis a vis equity and not how equity is distributed over assets $j = 1..J$.

The case of the Central Bank Bond. Applying the results previously derived, the corresponding first order condition for bonds (which we let be the asset J with weight ψ_J) yields

$$\begin{aligned} \mathbb{E}_t u'\left(\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}} R_{e,t+1}^{\frac{1}{\lambda}}\right)\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}} R_{e,t+1}^{\frac{1}{\lambda}-1}\frac{\partial R_{e,t+1}}{\partial \psi_{J,t+1}} &= 0 \\ \mathbb{E}_t u\left(\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}}\right)(R_{J,t+1} - R_{0,t+1}) &= 0 \end{aligned}$$

and therefore

$$\mathbb{E}_t u\left(\left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\lambda}}\right)(R_{J,t+1} - R_{0,t+1}) = 0$$

For $\lambda = 1$, the asset pricing problem becomes simpler as the optimal choice for $\psi_{J,t+1}$ requires that expected returns are equalized. Thus,

$$\mathbb{E}_t R_{J,t+1} = R_{0,t+1} \tag{17}$$

Equivalence to maximizing present value. The asset of interest in this case is that of the Central Bank bond. Abstracting momentarily from the oligopolistic structure, the bank is contemplating how much to invest in the bond, taking into account the fact that it can always keep its current cash as reserves at the Central Bank. We take the corresponding deposit facility rate as a reasonable benchmark for the opportunity cost of investing in the Central Bank bond; we thus set $R_{0,t+1} = 1 + \iota_{t+1}$.[45]

We next distinguish between the different events that can affect the payoff from investing in the bond. If the bank does not receive a liquidity shock that is highly negative such that reserves need to be replenished, then the bank gets to keep the bond up to maturity, earning the interest b_J. If on the contrary the shock is large enough such that additional liquidity is required, then the bank has different options. It can borrow reserves from the interbank market, with probability $\hat{p}_{2,t}$, while with probability $(1 - \hat{p}_{2,t})$ it will have to apply for emergency liquidity assistance from the Central Bank. The Central bank itself will be able to provide support support with probability $\hat{p}_{3,t}$. Thus, the bank will be able to obtain funds and insure the shock with probability $\hat{p}_{2,t} + (1 - \hat{p}_{2,t})\hat{p}_{3,t}$.[46]

We can redefine the probabilities of successfully obtaining additional liquidity as $p_{2,t} := \hat{p}_{2,t}$ and $p_{3,t} := (1 - \hat{p}_{2,t})\hat{p}_{3,t}$ respectively. They can then be best interpreted as *reduced form* probabilities, as they reflect both market tightness as well as the ability of the Central Bank to successfully intervene. Both of these factors depend on aggregate conditions.

Moreover, despite the fact that we have already incorporated the possibility of liquidating the bond and borrowing from the interbank market or the lender of last resort, there is also the possibility that the bank cannot accommodate for the liquidity shock to its full extent. This probability is equal to $(1 - \hat{p}_{2,t})(1 - \hat{p}_{3,t})$ or simply, $1 - p_{2,t} - p_{3,t}$. In this case, we assume that the bank liquidates the bond in the secondary market, yielding a net return equal to ι^{sec}_{t+1}.

[45]Once could also argue that an alternative use of the extra liquidity would be to lend to other banks in the interbank market. We subsume this possibility as just investing in another asset in $j = 1..J$.

[46]Note that the sequence of events is without loss of generality. It can be shown that liquidating the bond first only if the bank cannot borrow from either the interbank market or the LOR yields the same probability.

The expected profit from investing in the Central Bank bond can therefore be written as follows:

$$\begin{aligned}
& \mathbb{E}_t \Pi_{J,t+1} \\
= \quad & q_J \left[(1-s_i)\left(\frac{1+b_{J,t}}{1+\iota_{t+1}} - 1 \right) + s_i(1-p_{2,t}-p_{3,t}) \left(\frac{\mathbb{E}_t \iota^{sec}_{t+1}}{1+\iota_{t+1}} - 1 \right) \right] \\
& + q_J \left[s_i p_{2,t} \left(\frac{b_{J,t} - \mathbb{E}_t \iota^{ib}_{t+1}}{1+\iota_{t+1}} - 1 \right) + s_i p_{3,t} \left(\frac{b_{J,t} - \mathbb{E}_t d_{t+1}}{1+\iota_{t+1}} - 1 \right) \right]
\end{aligned}$$

Since ι_{t+1} is predetermined, next period's payoff per unit of bond is

$$\begin{aligned}
& \mathbb{E}_t \Pi_{J,t+1}(1+\iota_{t+1}) \\
= \quad & (1-s_i)\,(1+b_{J,t}) + s_i(1-p_{2,t}-p_{3,t})\mathbb{E}_t \iota^{sec}_{t+1} + s_i p_{2,t} \left(b_{J,t} - \mathbb{E}_t \iota^{ib}_{t+1} \right) \\
& + s_i p_{3,t} \left(b_{J,t} - \mathbb{E}_t d_{t+1} \right) - (1+\iota_{t+1}) \\
:= \quad & \mathbb{E}_t R_{J,t+1} - R_{0,t+1}
\end{aligned}$$

We can therefore see that at the optimum, the expected return of the bond should be equal to the benchmark return R_0, just as in the general problem in (17).

We next explain how the bond price is determined in this case, and how this maps to the identifying restrictions we derive in the main body of the paper. Equalizing expected returns, yields that:

$$(18) \quad b_J = \frac{\iota_{t+1} + s_i \left(1 - (1-p_{2,t}-p_{3,t})\iota^{sec}_{t+1} + p_{2,t}\mathbb{E}_t d_{t+1} + p_{3,t}\mathbb{E}_t \iota^{ib}_{t+1} \right)}{1 - s_i + s_i(p_{2,t}+p_{3,t})}$$

Furthermore, banks will wish to borrow in the interbank market only if the cost of doing so is lower than paying the discount rate, while banks that wish to lend will only do so if the yield is higher than the opportunity cost e.g. the policy rate, or the rate at which reserves are remunerated by the Central Bank. Thus, the interbank market rate is expected to fluctuate between $\mathbb{E}i_{t+1}$ and $\mathbb{E}d_{t+1}$, and how close the interbank market rate will be to these bounds will depend on the relative market power of buyers and sellers, ϕ_t (see e.g. Bianchi and Bigio (2017)). We thus set $\mathbb{E}_t \iota^{ib}_{t+1} = \phi_t \mathbb{E} i_{t+1} + (1-\phi_t)\mathbb{E}d_{t+1} = \mathbb{E}d_{t+1} - \phi_t(\mathbb{E}d_{t+1} - \mathbb{E}i_{t+1})$.[47]

[47] We do not explicitly consider non-pecuniary costs of borrowing from the Central Bank, such as the cost of stigma, as we do not find strong evidence for the existence of such cost in the data. Please see Section B5 for details.

The bond price can be therefore rewritten as follows, for $p_t = p_{2,t} + p_{3,t}$:

$$(19) \quad b_J = \frac{\iota_{t+1} + s_i\left(1 + p_t\mathbb{E}_t d_{t+1} - p_{3,t}\phi_t\mathbb{E}_t\left(d_{t+1} - \iota_{t+1}\right) - (1-p_{,t})\iota^{sec}_{t+1}\right)}{1 - s_i + s_i p_t}$$

Notice that in the secondary market for bonds, the maximum return the bank can hope for is zero as bond buyers will only accept a price that is weakly lower than the nominal price of the bond. Thus, ι^{sec}_{t+1} is expected to be weakly negative. The solution to 4 in the perfectly competitive case is thus identical to the upper bound to (19) as follows:

$$(20) \quad b_J = \frac{\iota_{t+1} + s_i\left(1 + p_t\mathbb{E}_t d_{t+1}\right)}{1 - s_i + s_i p_t}$$

There are two reasons for which this is reasonable. First, since $d_{t+1} > \iota_{t+1}$, the second term $p_{3,t}\phi_t\mathbb{E}_t\left(d_{t+1} - \iota_{t+1}\right)$ is negative , and more importantly, it is of lower order than the other terms. Moreover, i^{sec} is very likely to be zero as if a bank tries to liquidate a bond that is maturing soon, it will have to forgo any returns.

Therefore, p_t in our model can be interpreted more generally as the probability of insurance: the trading probability for buying liquidity in the interbank market and the probability of a succesfull lender of last resort, both as functions of aggregate conditions.

The non-competitive case. The above derivation makes the implicit assumption that once $\psi_{j,t+1}$ is chosen, the expected gross payoff is determined by the market return $R_{j,t+1}$ as there is perfect competition. In the case of an auction, such as in our setup, the payoff is determined by a simultaneous choice of bid (q_J, b_J), and the equilibrium outcome can be different than (q_J, b_J) as the bank may not be allocated the total amount requested and the equilibrium price may be affected by its bid. Thus, for any choice of the fraction of equity invested in bonds, $\psi_{J,t+1}$, the bank has to formulate an expectation on the payoff that takes into account the possibility that bank i may or may not affect the equilibrium outcome:

$$\begin{aligned} \mathbb{E}_t\Pi^i_{J,t+1} &= \mathbb{E}_t(q_J R_{J,t+1} | \text{Cutoff}; (b_J, q_J))\mathbb{P}(\text{Cutoff}; (b_J, q_J)) + \\ &\quad \mathbb{E}_t(q_J R_{J,t+1} | \text{No Cutoff}; (b_J, q_J))\mathbb{P}(\text{No Cutoff}; (b_J, q_J)) - q_J R_{0,t+1} \end{aligned}$$

The modified portfolio choice problem should therefore be as follows:

$$
\begin{aligned}
& \max_{\{\psi_{j,t+1}\}_{j=1..J},\psi_{d,t+1},\psi_{0,t+1},b_{J,t+1}} \mathbb{E}_t u\left(R_{e,t+1}\right) \\
& s.t. \\
\psi_{d,t+1} &< \kappa_1 \\
\frac{\psi_{0,t+1}}{\psi_{d,t+1}} &\geqslant \kappa_2 \\
R_{e,t+1} &= \sum_{j=1..J-1} \psi_{j,t+1}(R_{j,t+1} - R_{0,t+1}) \\
& +\psi_{J,t+1}(R_{J,t+1}(b_J, q_j) - R_{0,t+1}) + \psi_{d,t+1}(R_{d,t+1} - R_{0,t+1}) \\
1 &= \sum_{j=1..J} \psi_{j,t+1} + \psi_{d,t+1} + \psi_{0,t+1} \\
\psi_{J,t+1} &\equiv \frac{q_J}{e_{t+1}}
\end{aligned}
$$

Given homogeneity of degree one of the utility function, for asset J the problem reduces to

$$\max_{b_J,q_J} \quad \mathbb{E}_t q_J \left[R_{J,t+1}(b_J, q_J) - R_{0,t+1}\right]$$

which is *identical* to the problem solved by the bank in equation (1). One can therefore see that the problem of the bank we describe is simply a sub-problem of the more general portfolio choice problem solved by the bank. Furthermore, the constraint we impose, $q_i Q \leqslant m_i$ is simply acknowledging that there is an upper bound on how much the bank can invest in the auction, given by m_i. That is, conditional on some choice of equity share $\psi_{j,t+1}$ for all other assets and liabilities, the bank can invest at most

$$
\begin{aligned}
m_i &:= \left(1 - \sum_{j=1..J-1} \psi_{j,t+1} + (1-\kappa_2)\psi_{d,t+1}\right) e_{t+1} \\
&\geqslant \left(1 - \sum_{j=1..J-1} \psi_{j,t+1} + \psi_{d,t+1} - \psi_{0,t+1}\right) e_{t+1} = \psi_{J,t+1} e_{t+1}
\end{aligned}
$$

The amount invested in the auction $\psi_{J,t+1}e_{t+1}$ must be equal to $q_i Q$, which is bounded above by m_i.

B2. Mapping to Liquidity Risk

According to our definition of m_i, it is the *upper bound* on how much cash the bank could allocate on investing in the auction, which depends on the other portfolio decisions, within the context of the model in Appendix B1. More particularly,

$$\begin{aligned}
\frac{m_i}{e_{t+1}} &:= 1 - \sum_{j=1..J-1} \psi_{j,t+1} + (1-\kappa_2)\psi_{d,t+1} \\
&\geqslant 1 - \sum_{j=1..J-1} \psi_{j,t+1} + \psi_{d,t+1} - \psi_{0,t+1} \\
&= 1 - \sum_{j=1..J-1} \psi_{j,t+1} + \psi_{d,t+1}(1-\kappa_2) - \underbrace{(\psi_{0,t+1} - \kappa_2 \psi_{d,t+1})}_{\text{Excess Reserves}}
\end{aligned}$$

Given a liquidity shock (η_{t+1}) e.g. a large withdrawal, the level of reserves will fall in order to settle the deposit withdrawal, while the level of reserves required to stay in the bank will fall as well since the level of deposits is lower. Assuming that the bank may partially liquidate some of the other assets (apart from the bond in question), generating revenue equal to $\chi\left(\{\psi_{j,t+1}\}_{0<j<J}\right)$, the level of reserves (per unit of equity) becomes as follows:

$$\chi\left(\{\psi_{j,t+1}\}_{0<j<J}\right) + \psi_{0,t+1} - \eta_{t+1}\psi_{d,t+1}$$

Thus, excess reserves become

$$\chi\left(\{\psi_{j,t+1}\}_{0<j<J}\right) + \psi_{0,t+1} - \eta_{t+1}\psi_{d,t+1} - \kappa_2\psi_{d,t+1}(1-\eta_{t+1})$$

where we have assumed for simplicity that the interest rate on deposits and reserves at the Central Bank is the same. Similar to Bianchi and Bigio (2022), we define the event of illiquidity as the threshold below which reserve suplus is zero. The probability of becoming illiquid is thus the probability that these excess reserves become negative, so that there is a need for liquidating the bond and/or seek for additional liquidity from the interbank market and the lender of last resort. This yields that

$$s_i := \mathbb{P}_t\left(\eta_{t+1} > \frac{\chi\left(\{\psi_{j,t+1}\}_{0<j<J}\right) + \psi_{0,t+1} - \kappa_2\psi_{d,t+1}}{\psi_{d,t+1}(1-\kappa_2)}\right)$$

As an example, consider the case in which reserve requirements become strict e.g. $\kappa_2 = 1$. Then $s_i \to 0$ as deposit risk is the only relevant risk to reserves. Essentially, since all deposits are kept as reserves at the Central Bank, this risk goes to zero.

B3. Evidence on the (non)variability of types' shares

When pooling auctions from different point in time (t), we have to make the assumption that *conditional on auction-specific observables* bidder values are drawn from the same distribution and bidders play the same equilibrium in all auctions in the pooled panel. Following Hortaçsu and McAdams (2010, 2018), since the number of bidders is observable, we explicitly take into account the fact that the auctions we pool have a different number of participants by using the following kernel weights in the resampling algorithm:

$$w_m = \frac{\mathcal{K}(\frac{N_\tau - N}{h})}{\sum_{\tau=1..T} \mathcal{K}(\frac{N_\tau - N}{h})}$$

Auctions (equilibria) whose number of participants are close to the number of participants in the auction that bank i participates receive a higher weight and are likely to be drawn more frequently. Since we also assume that there are two types of players, Banks and non-Banks, we resample from bids of these groups separately in order to take into account this heterogeneity in the distributions of bond valuations. Given that the number of banks and non banks that participate in these auctions also varies in the auctions we pool, one should in principle also use kernel weights for these numbers too. Nevertheless, since we account for the change in the total number of participants, there would be no need to do this if the share of banks and non banks participating stays constant within the month we pool the auctions. That is, since $N_B + N_{NB} = (sh_B + (1 - sh_B))N$, a constant sh_B would imply that controlling for covariate N is sufficient. We have computed the share of banks in all auctions, and then compute the intra-monthly variation in this share. As we mentioned, in theory this should be strictly zero. Below we plot both the distributions of the standard deviation and the interquartile range:

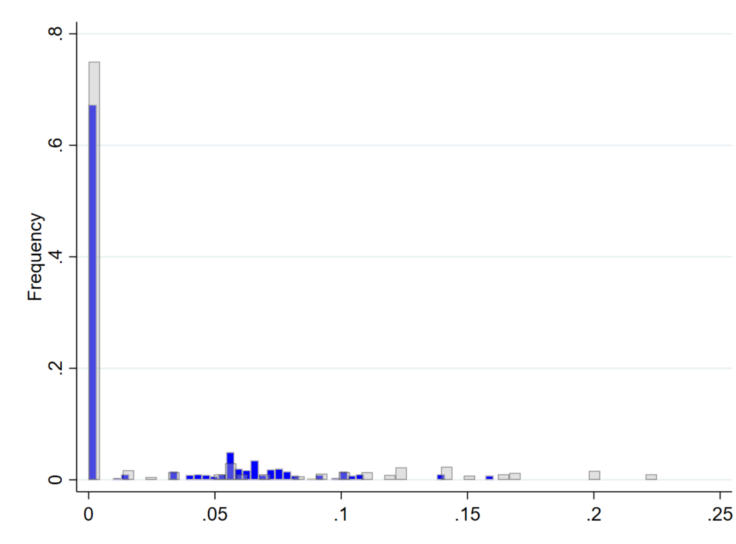


Figure 15. Within pool standard deviation (blue) and interquartile range (grey)

In $70 - 75\%$ of the months, dispersion is *exactly* zero, while when non-zero, median standard deviation is approximately 0.065 and median IQR is 0.1. 90% of *positive* standard deviations are below 0.1 and 90% of *positive* IQR statistics are below 0.2. Thus, even when shares vary, they do not vary in a substantial way.

B4. Measurement Error

We estimate the variance of the estimates of the mean and dispersion, $Var(\mathbb{S}_t^{NI})$ and $Var(\mathbb{V}_t)$ respectively using the bootstrap. Due to computational constraints, we do not re-sample for all 118 months, but we do this once every ten months. Below are the resulting estimates.

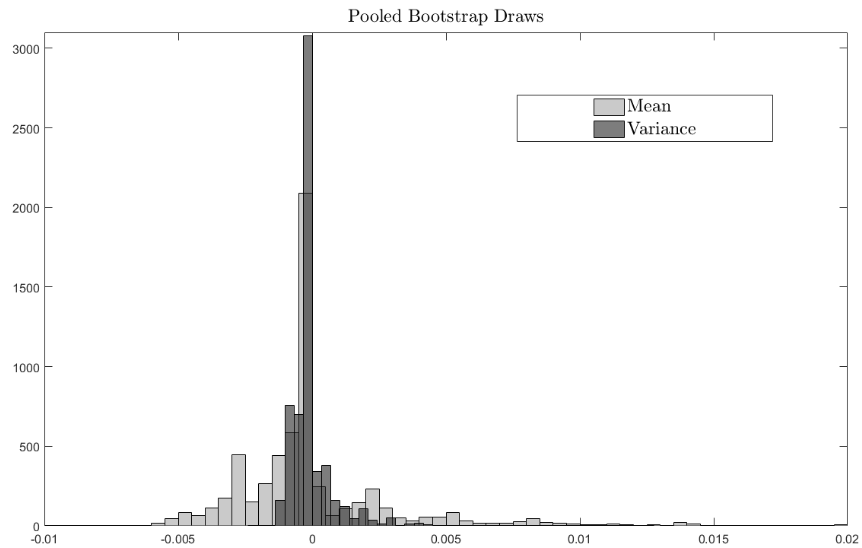


FIGURE 16. Distribution of Bootstrap draws for estimates of aggregate risk and dispersion in the no insurance case (deviation from the mean)

Under classical assumptions about the measurement error $\mathbb{E}(\nu_t|U_t) = 0$, i.e. it is uncorrelated with the shocks, the covariance matrix of the reduced form residuals of the VAR can be decomposed as follows:

$$\Sigma_{\hat{U}} = \Sigma_U + \Sigma_\nu + \hat{B}'\Sigma_\nu \left(\Sigma_{\hat{Y}} - \Sigma_\nu\right)^{-1} \Sigma_\nu \hat{B} + \hat{B}'\Sigma_\nu \hat{B}$$

The variance of the estimates of mean and dispersion across the bootstrap samples are used to calibrate the non-zero diagonal components of Σ_ν. Notice that when $\Sigma_\nu = 0$, $\Sigma_{\hat{U}}$ collapses to Σ_U.

B5. EVIDENCE ON STIGMA

While we have shown that our empirical approach takes as given that the discount rate is the maximum rate that a bank could face when in need of liquidity, part of the literature on the lender of last resort has suggested that banks may choose not to borrow from the Central Bank due to fear of stigma, and borrow from the interbank market at a *higher* rate. The difference between the latter and the discount are though of as measures of this non-pecuniary cost. Below we plot the distribution amongst daily observations of this difference, both in our sample period and in the period extended up to 2021. Only 5% – 7% of the observations are actually negative, suggesting that stigma is possible but not very commonly observed in our sample. The fact that we are using data at the monthly frequency makes this even less important as a potential valuation factor for the bonds we consider.

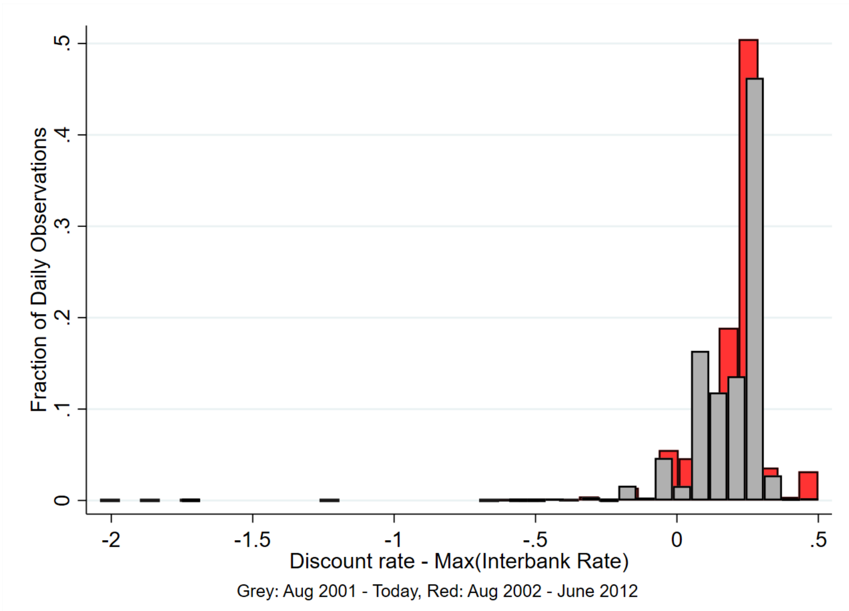


FIGURE 17