Measuring the average outcome and inequality effects of segregation in the presence of social spillovers

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Question:

- What would the world be if social groups were configured differently?

- Is single sex schooling efficient?

- Many interesting policies are reallocations:

  - schooling policies: single sex schooling, ability tracking

  - armed forces academies: grouping recruits together in units, hoping to build cohesive units (Carell, Fullerton, West, 2008)

Motivation:

- The unsettled state of the literature reflects difficulty in identifying social interactions (cf., Manski, 1993)

- Literature focuses on traditional estimands. For example, the average partial effect of a unit increase in the fraction of students in a classroom who are minority.

  - Does not correspond to a feasible policy

  - Only indirectly helpful for predicting the effect of a given reallocation

- Little connection between empirical work and theory on segregation.

This Paper studies

- Causal effects of reallocating students across classrooms.
  - students are binary-typed (girl/boy) with unobs heterog.
  - Classrooms/teachers differ in unobserved ways.

- Nonparametric identification and estimation results for
  - average spillover effect
  - effects of small changes in segregation on average outcomes and inequality ($\star$)
  - maximum attainable average outcome gain available via reallocation ($\star$)

Application: Project STAR data

Data on 5781 kindergarten students in 325 classrooms ($\approx$ 18 per class). Outcome is Stanford Achievement Test score, normalized to have mean zero and unit variance.

Average outcome higher for girls than for boys:

$$\overline{Y}_{\text{girls}} = 0.08, \quad \overline{Y}_{\text{boys}} = -0.08$$

Proportion of girls in classrooms: average $\overline{P}_{\text{girl}} = 0.49$, standard deviation $S_{P_{\text{girl}}} = 0.06 \quad (0.28 < P_{\text{girl}} < 0.80)$

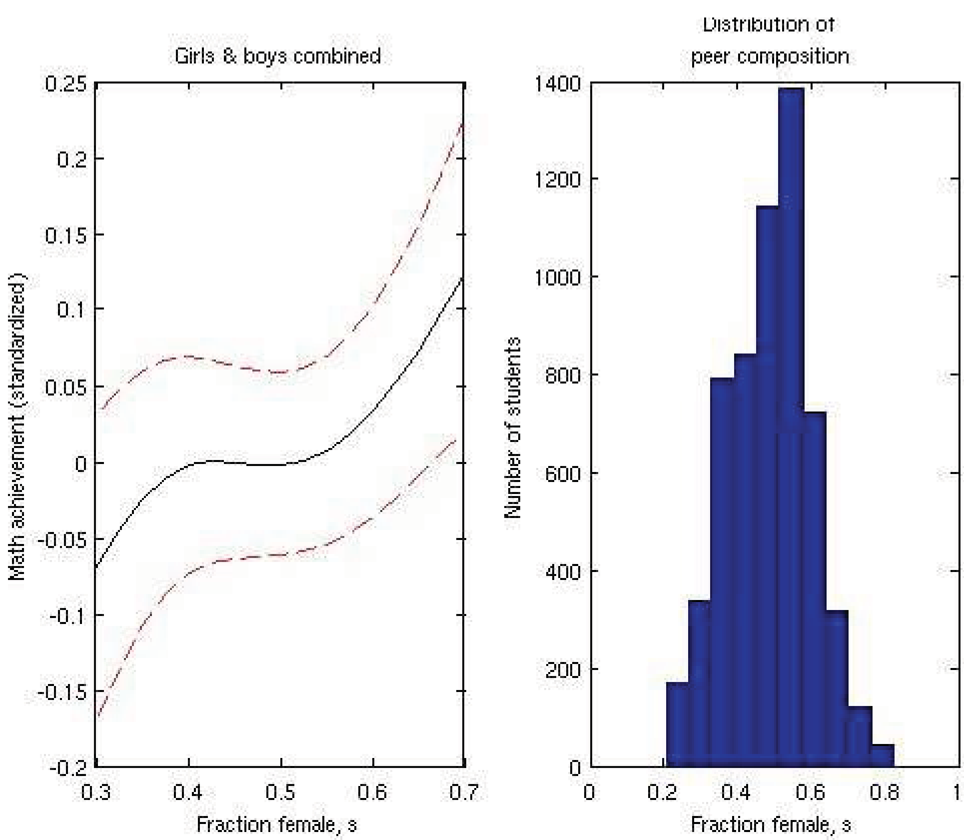
What would average outcomes and inequality between boys and girls have been like, had we configured allocated the 5781 kindergartners differently to the 325 classrooms?

- Create 163 all boys classrooms, 162 all girl classrooms, all classrooms 17-18 students.

- All classrooms 8-9 girls, 8-9 boys, with 17-18 students total.

- 1/3 classrooms all girls, 2/3 of the classrooms with 25% girls and 75% boys, all classrooms 17-18 students.

- Given current allocation, in all classes with more boys then girls, move a boy to a class with more girls than boys, and the other way around (raise segregation by small amount).


Girls & boys combined
Math achievement (standardized)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0.3
0.4
0.5
0.6
0.7
Fraction female, s
Distribution of
peer composition
Number of students
1400
1200
1000
800
600
400
200
0
0
0.2
0.4
0.6
0.8
1
Fraction female, s

Other Application: Forming Squadrons (Carell, Fullerton, West, 2008)

Incoming freshman at US Air Force Academy are assigned to squadrons, with 120 students per squadron.

They have limited contact with other students in their cohort who are in different squadrons.

Squadrons can be formed on the basis of sex, incoming test scores.

Question of interest: how does the formation of squadrons affect outcomes: test scores, cohesion, dropout rates?

Talk Outline

1. Population framework
    - potential outcomes set-up
    - average allocation response function

2. Spillover estimands
    - average spillover effect
    - local segregation outcome effect

3. Estimation and Large Sample Inference

4. Application to Project STAR data

- Population of students: $i \in \mathcal{I} = \{1, \ldots, I_P\}$
  - Observed types: $T_i \in \{0, 1\}$ (girls, boys, population frequency of high types: p_H)
  - Heterogenous in unobserved 'ability': A_i
  - Outcome: Y_i (math achievement, on stanford achievement test)
- Population of classrooms: $c \in \mathcal{C} = \{1, \ldots, C_P\}$
  - Unobserved classroom/teacher characteristics: U_c
  - Each classroom has N students ($N \times C_P = I_P$)
- Assignment indicator: $G_i = c$ if i is assigned to classroom c

- (with some abuse of notation): $U_i = U_{G_i}$, etc.

- Indexing by c will signal that the reference population is that of locations, i that of individuals

- Peers are classmates: $p(i) = \left\{ j \in \mathcal{J} : G_j = G_i, i \neq j \right\}$

- Peers' types are given by $\underline{T}_{p(i)} = (\underline{T}_{p(i),1}, \ldots, \underline{T}_{p(i),N-1})'$ with $\underline{A}_{p(i)}$ similarly defined

- $\underline{T}_i = (T_i, \underline{T}'_{p(i)})'$ with $\underline{A}_i$ defined analogously

- An individual's overall classroom quality is given by $Q_i = \left(\underline{T}'_{p(i)}, \underline{A}'_{p(i)}, U'_i \right)'$

There are mappings from allocations to individuals level and average outcomes:

$$Y_i(\mathbf{g}), \qquad \overline{Y}(\mathbf{g}), \qquad \overline{Y}_{\text{girls}}(\mathbf{g}) - \overline{Y}_{\text{boys}}(\mathbf{g})$$

$g \in \mathcal{G}$, where $\mathcal{G}$ is set of feasible allocations (5781 kids, 325 classrooms, 49% girls)

Estimands are averages over subsets of allocations, $\mathcal{G}_0 \subset \mathcal{G}$:

$$\frac{1}{\#\mathcal{G}_0} \sum_{\mathbf{g} \in \mathcal{G}_0} \overline{Y}(\mathbf{g}) \qquad \frac{1}{\#\mathcal{G}_0} \sum_{\mathbf{g} \in \mathcal{G}_0} \left(\overline{Y}_{\text{girls}}(\mathbf{g}) - \overline{Y}_{\text{boys}}(\mathbf{g}) \right)$$

- $\mathcal{G}_0$ is all alloc with equal number of boys/girls in each class,

- $\mathcal{G}_0$ is set of all allocations with single sex classrooms.

No Cross Neighborhood Spillovers

Let $\mathbf{g}$ and $\widetilde{\mathbf{g}}$ denote two feasible allocations with associated neighborhood qualities for individual i of q_i and $\widetilde{q}_i$. If $q_i = \widetilde{q}_i$ then

$$Y_i(\mathbf{g}) = Y_i(\widetilde{\mathbf{g}})$$

- Rules out general equilibrium effects

- Allows us to write

$$Y_i(\mathbf{g}) = Y_i\left(\underline{T}_{p(i)}, \underline{A}_{p(i)}, U_i\right) = Y_i\left(Q_i\right)$$

- Related to 'no interference' type assumptions in causal inference literature

Within-Type Peer Exchangeability

Let $\underline{\widetilde{A}}_{p(i)} = (\underline{\widetilde{A}}^{\text{girl}}_{p(i)}, \underline{\widetilde{A}}^{\text{boy}}_{p(i)})'$ where $\underline{\widetilde{A}}^{\text{girl}}_{p(i)}$ and $\underline{\widetilde{A}}^{\text{boy}}_{p(i)}$ are permutations of $\underline{A}^{\text{girl}}_{p(i)}$ and $\underline{A}^{\text{boy}}_{p(i)}$, and let $\underline{\widetilde{T}}_{p(i)}$ be a conformable reordering of $\underline{T}_{p(i)}$, for all such within-type permutations

$$Y_i(\underline{\widetilde{T}}_{p(i)}, \underline{\widetilde{A}}_{p(i)}, U_i) = Y_i(\underline{T}_{p(i)}, \underline{A}_{p(i)}, U_i)$$

- Among those of the same sex, peers are equally influential

Inclusive Definition of Type

T_i is independent of A_i

- Achieved via normalization: $A_i = F_{A^*|T}(A_i^* \big| T_i)$, with A_i^* unnormalized ability

- An individual's 'ability' is their rank amongst those of their own sex

- The allocation response function (function of share of types, ability of high types, ability of low types, and characteristics of classroom):

$$Y_i\left(S_{-i}, \underline{A}^{\text{girl}}_{p(i)}), \underline{A}^{\text{boy}}_{p(i)}, U_i\right)$$

defines an individual-specific mapping from peers' types, abilities and amenities into outcomes

- A 'treatment' is an assignment to a specific configuration of these variables

- Unobservability of peer ability and amenities implies that observationally identical assignments may be associated with distinct treatments (and hence potential outcomes)

- We deal with treatment heterogeneity by defining an intermediate object:
  - an average over all the potential outcomes associated with a given level of S_{-i}
  - the average is over a distribution of peer abilities and locational amenities (conditional on group structure)

$$F_{\underline{A},U|\underline{T}}\left(\underline{a}_c, u_c | \, \underline{t}_c\right)$$

  - we **define** this average to be unaffected by 'sorting' and/or 'matching'

Matching

- Matching occurs if students are assigned to a classroom on the basis of U_c and the utility derived from their choice varies with (T_i, A_i)
  - girls seek out high quality teachers
  - girls and boys have different preferences over (outcome affecting) amenities
- There is no matching if $(\underline{T}_i, \underline{A}_i) \perp U_i$
- This gives the density factorization

$$f_{\underline{A},U|\underline{T}}\left(\underline{a}_c, u_c \middle| \underline{t}_c\right) = f_{\underline{A}|\underline{T}}\left(\underline{a}_c \middle| \underline{t}_c\right) f_U\left(u_c\right)$$

Sorting

- Sorting occurs if individuals have preferences over their peers' ability/type and the strength of that preference varies with own ability

- There is no sorting if

$$\underline{A}_{p(i)} \perp A_i \Big| \underline{T}_i$$

- This (and inclusive types) gives the density factorization

$$f_{\underline{A},|\underline{T}}\left(\underline{a}_c | \, \underline{t}_c\right) = \prod_{j=1}^{N} f_{A|T}\left(a_{cj} \Big| \, t_{cj}\right) = \prod_{j=1}^{N} f_A\left(a_{cj}\right)$$

Average Allocation Response Function

In the absence of matching and sorting we have

$$f_{\underline{A},U|\underline{T}}\left(\underline{a}_c, u_c \middle| \underline{t}_c\right) = \left\{\prod_{j=1}^{N} f_A\left(a_{cj}\right)\right\} f_U\left(u_c\right)$$

Averaging over observationally identical treatments gives

$$Y_i^e(s_{-i}) = \int Y_i(s_{-i}, \tau_{K_{\text{girl}}}(\underline{a}_{p(i)}^{\text{girl}}), \tau_{K_{\text{boy}}}(\underline{a}_{p(i)}^{\text{boy}}), u_i)$$

$$\times \left\{\prod_{j \in p(i)} f_A\left(a_{p(i),j}\right) da_{p(i),j}\right\} f_U\left(u_i\right) du_i$$

$Y_i^e(s_{-i})$ gives $i's$ expected outcome when assigned to a group of peers with composition $S_{-i} = s_{-i}$

Average Allocation Response Function

- Averaging $Y_i^e(s_{-i})$ conditional on type yields

$$\mathbb{E}[Y_i^e(s_{-i})|\,T_i = 1] \;=\; m^*_{\text{girl}}(s_{-i})$$
$$\mathbb{E}[Y_i^e(s_{-i})|\,T_i = 0] \;=\; m^*_{\text{boy}}(s_{-i})$$

- We work with the one-to-one transformations

$$m_{\text{girl}}(s) = m^*_{\text{girl}}\left(\frac{sN}{N-1}\right) \qquad m_{\text{boy}}(s) = m^*_{\text{boy}}\left(\frac{sN-1}{N-1}\right)$$

- $m_{\text{girl}}(s)$ and $m_{\text{boy}}(s)$ are
  - averages over a prod of marginals, not a joint distribution
  - required because heterog is not just from the individual

- Group average outcomes are given by

$$m(s) = sm_{\mathsf{girl}}(s) + (1-s)m_{\mathsf{boy}}(s)$$

- Statistical analog of the deterministic production technology featured in, for example Benabou (1993, 1996)

- The marginal effect of a change in group composition is

$$\frac{\partial m}{\partial s}(s) = p(s) + e(s)$$

where

$$\begin{aligned} p(s) &= m_{\mathsf{girl}}(s) - m_{\mathsf{boy}}(s) && \textbf{compositional effect} \\ e(s) &= s\frac{\partial m_{\mathsf{girl}}}{\partial s}(s) + (1-s)\frac{\partial m_{\mathsf{boy}}}{\partial s}(s) && \textbf{spillover effect} \end{aligned}$$

Identification

Identification of each of our estimands follows if $m_{\text{boy}}(s)$, $m_{\text{girl}}(s)$ and their derivatives are identified.

Sufficient (but not necessary) is

- the status quo assignment satifies the no matching/sorting conditions

- S_c (share of girls in classrooms) varies continuously

Under these conditions we have

$$\begin{aligned} \mathbb{E}\left[Y_i \middle| T_i = \text{girl}, S_i = s\right] &= m_{\text{girl}}(s) \\ \mathbb{E}\left[Y_i \middle| T_i = \text{boy}, S_i = s\right] &= m_{\text{boy}}(s) \end{aligned}$$

The no matching/sorting requirement can be weakened by using individual and location-level characteristics.

1st Estimand: Average Spillover Effect

The average spillover benefit of a unit increase in S_i is

$$\beta^{\text{ase}} = \mathbb{E}[e(S_i)]$$

- β^{ase} is average effect of an infeasible policy
- nonparametric version of Ciccone and Peri's (2006) 'constant composition' externality measure
- for statistical inference trimming is useful to ensure finiteness of variance bound and to eliminate boundary bias problems in estimating $e(s)$.

2nd Estimand: Local Segregation Outcome Effect

- Reallocations are reassignments of individuals across groups
- We assume that the planner does not observe individual ability A_i or location amenities U_c.
- A reallocation is thus a feasible group composition distribution, $F_S^{\mathsf{r}}(s)$
- Feasibility requires that $F_S^{\mathsf{r}}(s)$ satisfy the constraint

$$\int_0^1 s f_S^{\mathsf{r}}(s)\mathrm{d}s = p_{\mathsf{girl}}$$

We begin by considering the effect of a small increase in segregation on the average outcome

Our reallocation density is

$$f_S^{\mathsf{r}}(s;\lambda) = \frac{s}{1+\lambda} f_S^{\mathsf{sq}}\left(\frac{s+\lambda p_{\mathsf{girl}}}{1+\lambda}\right)$$

This is equivalent to implementing the rule

$$S_c^{\mathsf{r}} = S_c + \lambda(S_c - p_{\mathsf{girl}})$$

In classroom c, segregation is increased by $\lambda \times 100\%$

- After reallocation the average outcome is

$$\mathbb{E}\left[m(S_i^{\mathsf{r}})\right] = \mathbb{E}\left[m(S_i + \lambda(S_i - p_{\mathsf{girl}}))\right]$$

- Differentiating w.r.t λ and evaluating at $\lambda = 0$ gives the effect of a small increase in segregation

$$\begin{aligned}\beta^{\mathsf{lsoe}} &= \mathbb{E}\left[\frac{\partial m}{\partial s}(S_i)(S_i - p_{\mathsf{girl}})\right] \\ &= \mathbb{C}\left(\frac{\partial m}{\partial s}(S_i), S_i\right)\end{aligned}$$

Covariance of derivative $\frac{\partial m}{\partial s}(S_i)$ and S_i

Estimates of effects of local (to *status quo*) reallocation may be more credible (less reliant on extrapolation) than estimates of substantial reallocations.

Connections to Theory

- It is helpful to decompose $\beta^{\text{lsoe}} = \alpha^{\text{lppe}} + \alpha^{\text{lepe}}$ into a *local private peer effect* and a *local external peer effect*:
  - $\alpha^{\text{lppe}} = \mathbb{C}(p(S_i), S_i)$ is positive if movers benefit on net from reallocation
  - $\alpha^{\text{lepe}} = \mathbb{C}(e(S_i), S_i)$ is positive if stayers benefit on net from reallocation
- If $\alpha^{\text{lppe}} > 0$ suggests presence of incentives for additional sorting
- If $\alpha^{\text{lepe}} \neq 0$ suggest additional sorting has unpriced consequences

Example: Suppose all classes ten students, one class with 60% girls, one class with 60% boys: local reallocation moves one boy from the 50% girl class to the 60% boy class, and one girl the other way.

Effects (all peer effects)

Movers:
1. boy moves from 60% girl class to 60% boy class.
2. girl moves from 60% boy class to 60% girl class.

Stayers
3. 6 girls see their class change from 60% girls to 70% girls.
4. 3 boys see their class change from 60% girls to 70% girls.
5. 6 boys see their class change from 40% girls to 30% girls.
6. 3 girls see their class change from 40% girls to 30% girls.

Connections to Theory

β^{Isoe} equals the weighted average

$$2\mathbb{E}\left[\omega(S_i)\left\{\frac{\partial m_{\text{girl}}}{\partial s}(S_i) - \frac{\partial m_{\text{boy}}}{\partial s}(S_i)\right\}\right]$$

$$+\mathbb{E}\left[\omega(S_i)\left\{S\frac{\partial^2 m_{\text{girl}}}{\partial s^2}(S_i) + (1 - S_i)\,\frac{\partial^2 m_{\text{boy}}}{\partial s^2}(S_i)\right\}\right]$$

where the weights $\mathbb{E}\left[\omega(S_i)\right] = 1$.

• This is a local average of, respectively, own and peer type *complementarity* and *curvature*

• Important to allow for heterogeneity in second derivatives of regression function.

3rd Estimand: Local Segregation Inequality Effect

The local segregation inequality effect of the reallocation is the effect effect on the gap by sex in average outcomes, again of a small move towards more segregation:

$$\beta^{\text{lsie}} = \mathbb{E}\left[\frac{1}{p_{\text{girl}}}\left\{m_{\text{girl}}(S_i) + S_i \frac{\partial m_{\text{girl}}}{\partial s}(S_i)\right\}\Big(S_i - p_{\text{girl}}\Big)\right]$$

$$-\mathbb{E}\left[\frac{1}{1-p_{\text{girl}}}\left\{-m_{\text{boy}}(S_i) + (1-S_i)\frac{\partial}{\partial s} m_{\text{boy}}(S_i)\right\}\Big(S_c - p_{\text{girl}}\Big)\right].$$

Why Nonparametric Estimation?

Parametric models restrict answers to the questions:

- 'Linear-in-means' (e.g., Manski, 1993)

$$
\begin{aligned}
m_{\text{girl}}(s) &= \alpha_{\text{girl}} + \gamma s \\
m_{\text{boy}}(s) &= \alpha_{\text{boy}} + \gamma s
\end{aligned}
$$

$$\Longrightarrow \quad \beta^{\text{ase}} = \gamma \quad \beta^{\text{lsoe}} = 0$$

- Type-specific linear-in-means (e.g., Angrist and Lang, 2004)

$$
\begin{aligned}
m_{\text{girl}}(s) &= \alpha_{\text{girl}} + \gamma_{\text{girl}} \cdot s \\
m_{\text{boy}}(s) &= \alpha_{\text{boy}} + \gamma_{\text{boy}} \cdot s
\end{aligned}
$$

$$\Rightarrow \quad \beta^{\text{ase}} = p_{\text{girl}}\gamma_{\text{girl}} + (1 - p_{\text{girl}})\gamma_{\text{boy}} \quad \beta^{\text{lsoe}} = 2(\gamma_{\text{girl}} - \gamma_{\text{boy}})\mathbb{V}(S_i)$$

Estimation

- Each of our estimators is a two-step semiparametric M-estimator (e.g., Newey and McFadden, 1994)

- We estimate $m_{\text{girl}}(s)$, $m_{\text{boy}}(s)$ and their derivs by kernel methods

- We then estimate targets by plugging in our 1st step est's.

- Derive influence function by performing a pathwise derivative calculation (e.g., Newey, 1994)

- We use fixed trimming to deal with boundary issues (which redefines the estimands)

First Step Estimation

- Let $\mathcal{K}(u)$ denote a kernel function, $K_b(s - S_i) = \frac{1}{b}\mathcal{K}(\frac{s-S_i}{b})$, then

$$\widehat{m}_{\text{girl}}(s) = \frac{\widehat{g}_{1,\text{girl}}(s)}{\widehat{g}_{2,\text{girl}}(s)}, \quad \widehat{m}_{\text{boy}}(s) = \frac{\widehat{g}_{1,\text{boy}}(s)}{\widehat{g}_{2,\text{boy}}(s)}$$

$$\widehat{g}_{1,\text{girl}}(s) = \frac{1}{I_1}\sum_{i=1}^{I_1} K_b(s - S_i)Y_i \quad \widehat{g}_{2,\text{girl}}(s) = \frac{1}{I_1}\sum_{i=1}^{I_1} K_b(s - S_i)$$

- and $\widehat{g}_{1,\text{boy}}(s)$ and $\widehat{g}_{2,\text{boy}}(s)$ defined similarly

- $\frac{\partial}{\partial s}m_{\text{girl}}(s)$ and $\frac{\partial}{\partial s}m_{\text{boy}}(s)$ are estimated by differentiating $\widehat{m}_{\text{girl}}(s)$ and $\widehat{m}_{\text{boy}}(s)$

Second Step Estimation of β^{ase}

- With trimming function $d(s)$, we use the analog estimator

$$\widehat{\beta}^{\text{ase}} = \frac{1}{I}\sum_{i=1}^{I} d(S_i)\left\{S_i \frac{\partial}{\partial s}\widehat{m}_{\text{girl}}(S_i)\right.$$

$$\left. + (1 - S_i)\frac{\partial}{\partial s}\widehat{m}_{\text{boy}}(S_i)\right\}$$

- With influence function $\widetilde{\phi}_c = \sum_{i\in\{i:G_i=c\}} \phi\left(Z_i\right)$

$$\phi(Z_i) = \frac{d(S_i)}{N}\left\{e(S_i) - \beta^{\text{ase}} - \frac{\frac{\partial}{\partial s} f_S\left(S_i\right)}{f_S\left(S_i\right)}(Y_i - m(S_i))\right.$$

$$\left. - \left(\left[\frac{T_i Y_i}{S_i} - \frac{\left(1 - T_i\right) Y_i}{1 - S_i}\right] - \left[m_{\text{girl}}(S_i) - m_{\text{boy}}(S_i)\right]\right)\right\}$$

Second Step Estimation of β^{lsoe}

$$\widehat{\beta}^{\text{lsoe}} = \frac{1}{I}\sum_{i=1}^{I} d(S_i)\left\{\widehat{m}_{\text{girl}}(S_i) - \widehat{m}_{\text{boy}}(S_i)\right.$$

$$\left. S_i\frac{\partial\widehat{m}_{\text{girl}}}{\partial s}(S_i) + (1-S_i)\,\frac{\partial\widehat{m}_{\text{boy}}}{\partial s}(S_i)\right\}(S_i - \widehat{p}_{H,\kappa})$$

With influence function $\widetilde{\phi}_c = \sum_{i\in\{i:G_i=c\}}\phi\left(Z_i\right)$ with

$$\phi(Z_i) = \frac{d(S_i)}{N}\left\{\frac{\partial m}{\partial s}(S_i)(S_i - p_{H,\kappa}) - \beta^{\text{lsoe}}\right.$$

$$-\frac{\frac{\partial}{\partial s}f_S\left(S_i\right)}{f_S\left(S_i\right)}(Y_i - m(S_i))(S_i - p_{H,\kappa}) - (Y_i - m(S_i))$$

$$\text{-}\mathbb{E}\left[\frac{\partial m}{\partial s}(S_i)\middle|\, d_\kappa(S_i) = 1\right](T_i - p_{H,\kappa})\left.\right\}$$

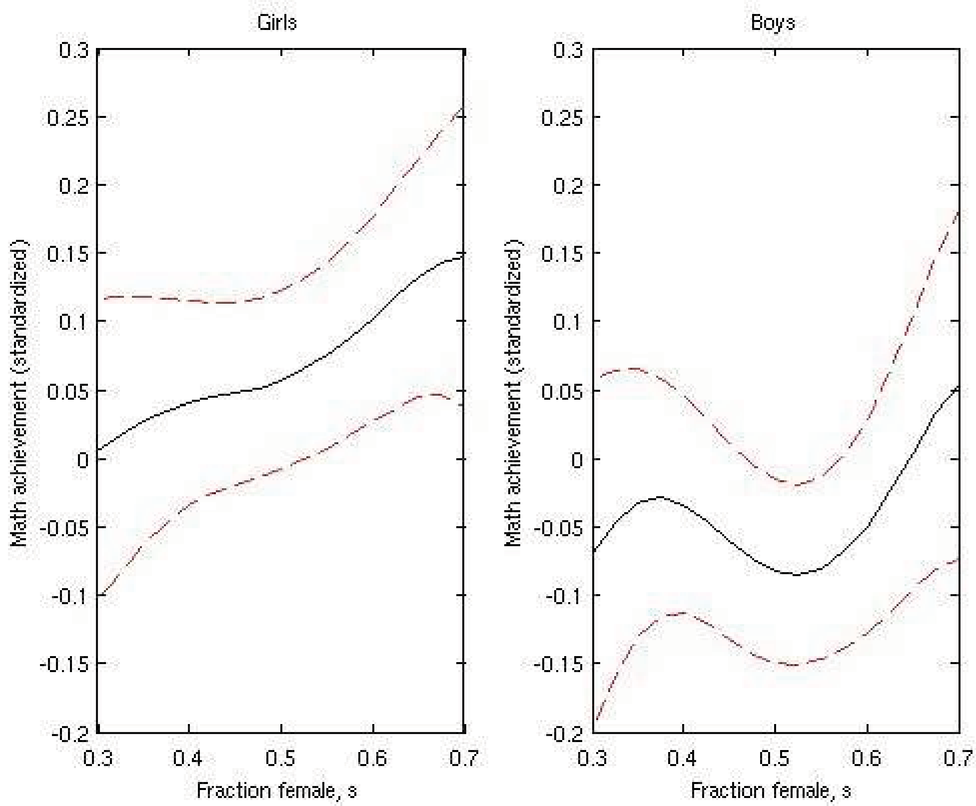
Empirical Application

- Stanford Achievement Test (math) for 5,871 kindergarten students (325 classrooms, 79 schools), normalized to have mean zero and standard deviation one.

- Collected in conjunction with Tennessee class size reduction experiment Project STAR

- Girls are high types, boys low types:

$$\frac{1}{N_H}\sum_{i:T_i=H} Y_i = 0.08, \qquad \frac{1}{N_L}\sum_{i:T_i=L} Y_i = -0.08$$


Girls
Boys
Math achievement (standardized)
Fraction female, s
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0.3
0.4
0.5
0.6
0.7

	Nonpar	Linear	Type-specific-lin
β^{ase} (ave spillover effect)	0.347 (0.134)	0.443 (0.200)	0.473 (0.201)
β^{lsoe} (local segreg outc eff)	−0.000 (0.021)	0	0.001 (0.007)
β^{lsie} (local segreg ineq eff)	0.055 (0.027)	0.063 (0.026)	0.066 (0.027)

Both boys and girls benefit from higher proportion of girls.

Additional segregation by sex benefits girls, and hurts boys, by approximately the same amount.

Additional segregation by sex would increase the achievement gap between girls and boys.

Social Planner's Problem

- Knowledge of the maximum and minimum average outcome available via reallocation is useful for
  - bounding positive/negative effects of reallocating policies
  - measuring the efficiency of the status quo

Social Planner's Problem

- The planner's problem is to choose $F_S(\cdot) \in \Gamma_S$ to maximize

$$\int m(s) f_S(s) \, \mathrm{d}s$$

subject to the feasibility constraint

$$\int s f_S(s) \, \mathrm{d}s = p_{\text{girl}}.$$

- Non-convex functional (i.e., infinite dimensional) optimization problem

- ...but it can be transformed into a finite-dimensional concave programming problem

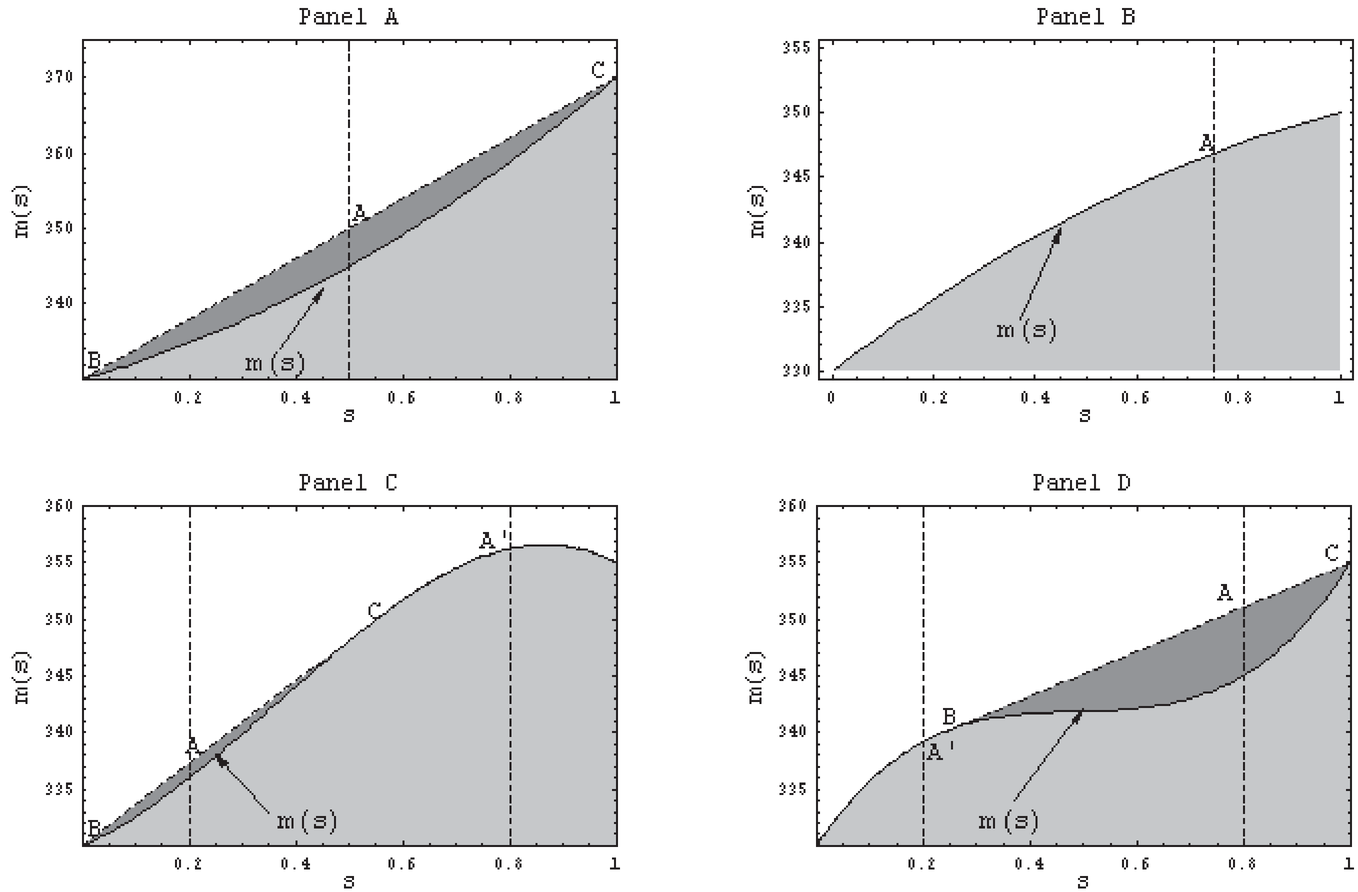


Figure 1: Optimal allocations for different $m(s)$ and p_H

NOTES: Each panel plots a different expected allocation response function, $m(s)$ (solid dark line). The concave envelopes of these expected allocation response functions, $M(s)$, are the given by the dashed lines at or above $m(s)$. The vertical dashed lines indicate the population frequency of high types, p_H. For figures with two such lines the second line (i.e., the right-most line) gives the location of a second population frequency, p'_H. The point labeled A marks the location of $(p_H, M(p_H))$. The points labeled B and C mark the locations of, respectively, $(s_L, m(s_L))$ and $(s_U, m(s_U))$ (when $s_L \neq s_U$). The point labeled A', if present, marks the location of $(p'_H, M(p'_H))$.

Solution to the planner's problem

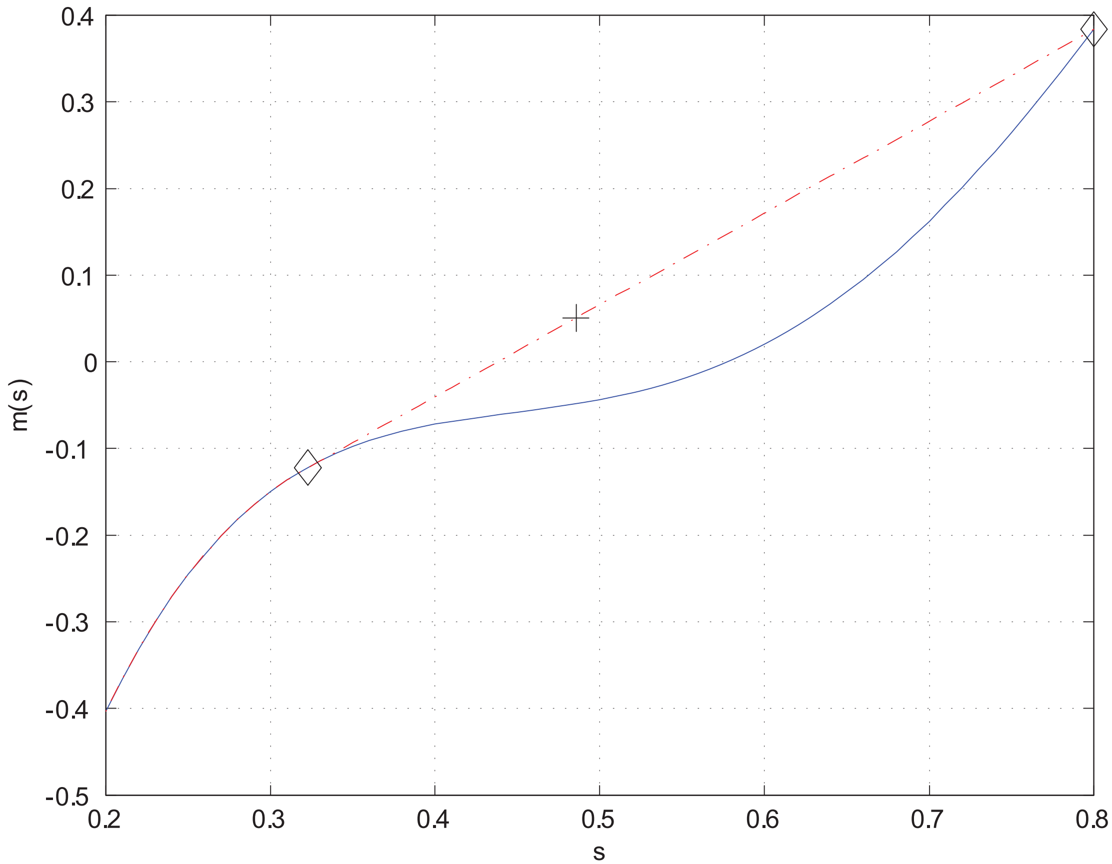


Figure: $m(s)$, its concave envelope $\overline{m}(s)$ and the average outcome maximizing allocation.

Summary

- This paper develops several reallocation-specific estimands
- The estimands connect to theoretical literature on locational sorting
- Proposes nonparametric estimators and characterizes their large sample properties
- Characterizes the solution to the social planner's problem