Choice by sequential procedures

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Introduction

- The traditional choice-theoretic approach takes behavior as rational if choice behavior can be explained as the outcome of maximizing a preference relation
- However, over the last decades mounting evidence has been accumulated documenting systematic and predictable violations of this notion of rationality
  - There are framing effects, menu effects, importance of reference points, cyclic choice patterns, choice overload effects, temporal inconsistencies, etc.

Introduction

- Here, we study an alternative model of choice: choice by sequential procedures
  - It encompasses the standard model of choice as a special case.
  - It is able to accommodate behavior often observed in empirical/experimental studies that the standard model of choice regards as irrational.
  - It is testable: not all choice patterns can be explained as choice by sequential procedures.

Introduction

- Choice by sequential procedures:
  - The DM applies a number of criteria (incomplete binary relations) in a fixed order of priority, gradually narrowing down the set of alternatives, until one is identified as the choice
    - Same set of criteria, applied in the same fixed order to every choice problem
  - Examples: individual and collective choice
    - Buying a house: first location, then layout, and then price
    - Social choice: first efficiency, then fairness
    - Hiring a new professor: first area of research, then letters, then job market paper, then seminar and interviews
    - Multiple selves, orderly applied

Concrete Examples

- Let $X = \{x, y, z\}$ and
  - $c(x, y, z) = x$
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Questions:

- Can we distinguish those choice functions that are SR, from those that are not?
- Can we find some property that characterizes SR, and that at the same time it is informative about the behavioral principles governing SR?
- Can we use such a property to establish the relation between SR and other models of choice?

Literature:

- Kalai, Rubinstein and Spiegler (2002, Econometrica)
- Masatlioglu and Ok (2005, JET)
- Rubinstein and Salant (2006, Theoretical Economics)
- Xu and Zhou (2007, JET)
- Bernheim and Rangel (2007, 2008, AER, QJE)
- Salant and Rubinstein (2008, Review of Economic Studies)
- Masatlioglu and Nakajima (2009, WP)
- Eliaz and Spiegler (2009, WP)
- Cherepanov, Feddersen and Sandroni (2009, WP)
- Green and Hojman (2009, WP)

- **Manzini and Mariotti (2007, AER)**

Notation: choice

- X finite set of alternatives
- $\mathcal{P}(X)$ collection of all non-empty subsets of X
- $c : \mathcal{P}(X) \to X$ with $c(A) \in A$
- $\mathcal{C}$ collection of all possible choice functions c given X

Notation: rationales

- A rationale: an acyclic binary relation $P \subseteq X \times X$
- Maximal elements in $A \subseteq X$ according to P:

$$M(A, P) = \{x \in A : (y, x) \in P \text{ for no } y \in A\}$$

- Given an ordered collection of rationales $\{P_1, \ldots, P_K\}$:

$$M_1^K(A) = M(M(\ldots M(M(A, P_1), P_2), \ldots, P_{K-1}), P_K)$$

Sequential rationalizability: definition

- Sequential Rationalizability (SR): A choice function c is sequentially rationalizable whenever there exists a non-empty ordered list $\{P_1, \ldots, P_K\}$ of rationales on X such that

$$c(A) = M_1^K(A) \text{ for all } A \subseteq X$$

Characterization

Characterization: definitions

- A binary selector f is a single-valued function that, for every choice problem A with at least two alternatives, gives a binary problem in A
- We say that the binary selector f is consistent if it satisfies the Strong Axiom.

Characterization: property

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- The classic IIA states that if an element x is chosen from a set A, it should also be chosen from any subset of A in which x is present.
- Independence of Irrelevant Alternatives (IIA): For any consistent binary selector f and any $A \subseteq X, c(A) = c(A \setminus \{x^*\})$ with $x^* = f(A) \setminus c(f(A))$.
- Independence of **One** Irrelevant Alternative (IOIA): There is a consistent binary selector f such that, for any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$

Characterization: result

- Theorem: c is sequentially rationalizable $\Leftrightarrow$ c satisfies IOIA

Characterization: remarks

- Assessing whether a particular c is SR reduces to check whether there is a linear order over the binary sets such that, for every choice problem A and for the first binary problem $B \subseteq A$, the choice from A does not depend on the dominated alternative in B

Characterization: remarks

- No Binary Cycles: For all $x_1, \ldots, x_{r+1} \in X$, $c(x_j, x_{j+1}) = x_j$, $j = 1, \ldots, r$, implies that $c(x_1, x_{r+1}) = x_1$.

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- Lemma: c satisfies IIA if and only if c satisfies IOIA and No Binary Cycles.

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- Lemma: c satisfies IIA if and only if c satisfies IOIA and No Binary Cycles.
    - IOIA can be understood as the interplay of a fully consistent component, the binary selector f, and a potentially irrational component, choices from binary problems.

Characterization: applications

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Our characterizing property IOIA can be used to study the relation of sequential rationalizability with other models:

- Rationalizability by Game Trees (Xu and Zhou, JET 2007)
- Agenda Rationalizability (voting models; choice by elimination)
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- Status Quo Bias Rationalizability (Masatlioglu and Ok, JET 2005)
  - Theorem: $\mathcal{C}^{SQB} \subset \mathcal{C}^{AR} \subset \mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$

Razionalizability by game trees

- The choices of the DM are the equilibrium outcome of an extensive game with perfect information
- Consider the class of extensive games with perfect information (G, P) such that:
  - The tree has alternatives of X as terminal nodes, each alternative appearing once and only once
  - Every node of the tree represents the decision of some agent i, with an associated linear order P_i
- $G|A$ is the reduced tree of G that retains all the branches of G leading to terminal nodes in A
- Rationalizability by Game Trees: A choice function c is rationalizable by game trees whenever there is a game tree G such that $c(A) = SPNE(G|A; P)$ for all $A \subseteq X$

Rationalizability by game trees

- The relation between RGT and SR is not clear a priori:
  - The structure of rationales is richer in RGT (tree against linearity)
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- Theorem:

$$\mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$$

Agenda rationalizability

- Alternatives linearly ordered (agenda): $1 < 2 < \cdots < n$
- Binary choice (a tournament) between 1 and 2. The winner faces 3, etc
- The final choice is the surviving alternative of this process: $e(<, T, A)$
- Related literature:
  - Individual choice: models of choice by ordered elimination: Rubinstein and Salant (TE, 2006), Salant and Rubinstein (REStud, 2008) or Masatlioglu and Nakajima (WP, 2007)
  - Collective choice: Voting by successive elimination as in Dutta et al (JET, 2002)
- Agenda Rationalizability: A choice function c is agenda rationalizable whenever there exists a linear order $<$ over the set of alternatives (an agenda) and a tournament T such that for every $A \in \mathcal{P}(X)$, $c(A) = e(<, T, A)$

Agenda rationalizability

- Theorem

$$\mathcal{C}^{AR} \subset \mathcal{C}^{SR}$$

Agenda rationalizability

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- Indeed,

$$\mathcal{C}^{AR} \subset \mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$$

Status quo bias rationalizability

- Individuals often evaluate an alternative more highly when it is regarded as the status quo
- Intense empirical and theoretical attention to this phenomenon
- We adapt the axiomatization of Masatlioglu and Ok (2005, JET), to our setting:
  - There is a status quo $\overline{x} \in X$
  - When the status quo is not present, the agent maximizes a multi attribute utility function over the set of alternatives
  - If the status quo is present, the agent maximizes the utility function over the set of alternatives that dominate the status quo in every single dimension, if there is any
  - Otherwise the agent sticks to the status quo

Status quo bias rationalizability

A choice function c is status-quo biased if there exists an element $\overline{x} \in X$, a positive integer q, an injective function $u : X \rightarrow \mathbb{R}^q$ and a strictly increasing map $h : u(X) \rightarrow \mathbb{R}$ such that:

1. For all $A \subseteq X$ with $\overline{x} \notin A$:

$$c(A) = \text{ argmax }_{y \in A} \quad h(u(x))$$

2. For all $A \subseteq X$ with $\overline{x} \in A$:
   - If $\hat{A} = A \cap \{x \in X : u(x) > u(\overline{x})\} = \emptyset$:

$$c(A) = \overline{x}$$

   - If $\hat{A} \neq \emptyset$:

$$c(A) = \text{ argmax }_{y \in \hat{A}} \quad h(u(y))$$

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- Indeed,

$$\mathcal{C}^{SQB} \subset \mathcal{C}^{AR} \subset \mathcal{C}^{RGT} \subset \mathcal{C}^{SR}$$

Final remarks

- We study choice by sequential procedures
- We offer a behavioral characterization of sequential choice
- Our characterizing property IOIA can be used to establish the relation between SR and other models. In particular we have shown that SR subsumes a number of prominent models like:
  - Rationalizability by Game Trees (Xu and Zhou, JET 2007)
  - Agenda Rationalizability (voting models; choice by elimination)
  - Status Quo Bias Rationalizability (Masatlioglu and Ok, JET 2005)
- Future research: nature and manipulability of f