Private Information in Dynamic Macro Models

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Overview of two papers

1. Dynamic Higher Order Expectations
2. Speculative Dynamics in the Term Structure of Interest Rates

Dynamic Higher Order Expectations

A class of (linear) models:

- Private information, strategic interaction and dynamic choices
  - Every agent has his own "window to the world" but no agent is better informed than others on average
  - Individual pay offs depend on (average) action taken by others
  - Agents optimize intertemporally
- A framework to think about disagreement and uncertainty about the plans and actions of other agents
- The principal modeling difficulty: The infinite regress of "forecasting the forecasts of others" (Townsend 1983)

Dynamic Higher Order Expectations

Find an finite dimensional representation that is arbitrarily close to true model

Strategy:

1. Impose structure on higher order expectations through common knowledge of rational expectations
   - By it self does not solve the “infinite regress problem” but makes thinking about higher order expectations tractable
2. Show that variance of expectations non-increasing with order of expectation
3. Show that impact of expectations decreasing with order of expectation

Common knowledge of rational expectations and higher order expectations

Rational expectations allow us to solve for model consistent (first order) expectations

Treat average expectations as stochastic processes:

- Second order expectations should be rational, i.e. model consistent, expectations of first order expectations
- Third order expectation should be rational, i.e. model consistent, expectations of average second order expectations
- ...and so on.

The variance of higher order expectations

$$\underbrace{\theta_t^{(k)}}_{\text{"truth"}} \equiv \underbrace{\theta_t^{(k+1)}}_{\text{expectation}} + \underbrace{e_t^{(k+1)}}_{\text{expectation error}}$$

Errors are orthogonal to expectations so variances of right and left hand sides are simply given by

$$var\left(\theta_t^{(k)}\right) = var\left(\theta_t^{(k+1)}\right) + var\left(e_t^{(k+1)}\right)$$

Common knowledge of rational expectations thus implies that

$$var\left(\theta_t^{(k)}\right) \geq var\left(\theta_t^{(k+1)}\right)$$

The Impact of Higher Order Expectations

Full information solution is given by

$$Y_t = G\theta_t$$

Private information solution is of the form

$$Y_t = \begin{bmatrix} g_0 & g_1 & \cdots & g_\infty \end{bmatrix} \begin{bmatrix} \theta_t \\ \theta_t^{(1)} \\ \vdots \\ \theta_t^{(\infty)} \end{bmatrix}$$

and $\lim_{k\to\infty} g_k = 0$ since common knowledge of rationality implies that

$$\sum_{k=0}^{\infty} g_k = G$$

An application to the term structure of interest rates

Speculative dynamics and the term structure of interest rates

- Traders have private information about future short rates
- Long maturity bonds are traded frequently
- New term structure dynamics driven partly by speculative behavior in the sense of Harrison and Kreps (1978)
- Estimate model to quantify importance of speculative dynamics in US bond data

Decomposing forward rates

The forward rate f_t^n can be decomposed into the the average first order projection and higher order projection errors

$$
\begin{aligned}
f_t^n \quad = \quad & \underbrace{\int \mathcal{P}_{t,j} r_{t+n}}_{hold\ \ to\ \ maturity} \\
& \underbrace{-\int \mathcal{P}_{t,j} \left(r_{t+n} - \prod_{s=1}^{n-1} \int \mathcal{P}_{t+s,j} r_{t+n} \right)}_{"speculative\ \ dynamics"} \\
& + \left(\eta_t^n - \eta_t^{n+1} \right)
\end{aligned}
$$

"Speculative dynamics" are due to possibility of reselling a bond before it matures and orthogonal to (real time) public information.

Historical speculation

Speculative term in implied forward rate is orthogonal to public information

$$\int \mathcal{P}_{t,j} \left(r_{t+n} - \prod_{s=1}^{n-1} \int \mathcal{P}_{t+s,j} r_{t+n} \right) \tag{1}$$

Can we as econometricians still quantify its importance?

- The term (1) is only orthogonal to public information up to period t
- Use full sample and the Kalman simulation smoother to construct posterior estimate of $p\left(X^T \mid y^T\right)$
- Use estimate of $p\left(X^T \mid y^T\right)$ to construct a posterior estimate of (1)

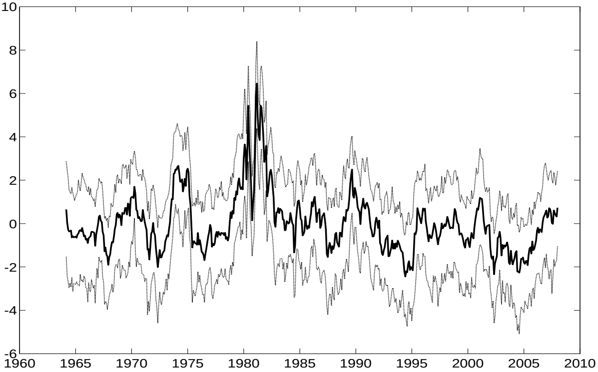

Figure: Estimated distribution of "speculative term" (percentage points) in implied 12 month forward rate. Median (solid) and 95% probability interval (dashed).

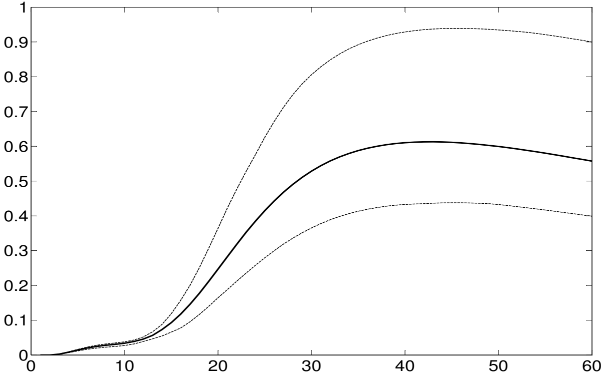

Figure: Fraction of variance (y-axis) of implied forward rates explained by speculative term across maturities (x-axis). Median (solid) and 95% probability interval (dashed).

Summing up

Develop methods to solve dynamic models with private information that are

- general enough to solve models that are not too different from standard macro models
- fast enough to use in empirical work

Private information may have quantitatively important implications for how asset prices are determined

- Speculative dynamics driven by rational agents systematically predicting average expectation errors
- Potentially quantitatively important even in a market where terminal value of asset is known (i.e. zero coupon bonds)